Chapter 2

Acoustic Waves in Elastic Solids

Many different types of acoustic wave can propagate in solid materials. Here we are particularly concerned with surface waves, though several other kinds of acoustic wave are also relevant to surface wave devices. This chapter gives a brief account of the analysis and properties of the waves. Propagation in piezoelectric materials is emphasised, because such materials are used in most surface-wave devices.

2.1. ELASTICITY IN ANISOTROPIC MATERIALS

We first describe the elastic behaviour of anisotropic materials, summarising the development given in more detail elsewhere [26–31]. It is convenient to consider the non-piezoelectric case first, and then consider piezoelectric materials later.

2.1.1. Non-piezoelectric materials

Elasticity is concerned with the internal forces within a solid and the related displacement of the solid from its equilibrium, or force-free, configuration. It is assumed here that the solid is homogeneous. The forces will be expressed in terms of the *stress*, T, while the displacements are expressed in terms of the *strain*, S.

We first consider the strain. Suppose that, in the equilibrium state, a particle in the material is located at the point $\mathbf{x} = (x_1, x_2, x_3)$. When the material is not in its equilibrium state, this particle is displaced by an amount $\mathbf{u} = (u_1, u_2, u_3)$, where the components u_1, u_2 and u_3 are in general functions of the coordinates x_1, x_2, x_3. Thus a particle with equilibrium position $\mathbf{x}$ has been displaced to a new position $\mathbf{x} + \mathbf{u}$. For the present, the displacement $\mathbf{u}$ is taken to be independent of time, t. Now clearly there will be no internal forces if $\mathbf{u}$ is independent of $\mathbf{x}$, since this simply denotes a displacement of the material as a whole. There will also be no forces if the material is rotated. To avoid these cases the strain at each point is defined by

$$S_{ij}(x_1, x_2, x_3) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \quad i, j = 1, 2, 3. \tag{2.1}$$

With this definition, any displacements or rotations of the material as a whole cause no strain, and the strain is related to the internal forces. The strain is a second-rank tensor and is clearly symmetrical:

$$S_{ij} = S_{ji} \tag{2.2}$$

so that only six of the nine components are independent.

The internal forces are described by a stress tensor T_{ij}. To define this, consider the plane $x_1 = x_1'$ within the material, where x_1' is a constant. If the material is strained, the material on one side of the plane exerts a force on the material on the other side. The force may be in any direction, and may vary with the coordinates (x_2, x_3) in the plane. The stress is defined such that the force per unit area has an x_i-component equal to $T_{i1}(x_1', x_2, x_3)$, with $i = 1, 2, 3$. This is the force exerted on the material at $x_1 < x_1'$. The force exerted on the material at $x_1 > x_1'$ is the negative of this. The definition applies for any value of x_1', so we can write the stress as $T_{i1}(x_1, x_2, x_3)$. Similarly, we may consider forces on planes perpendicular to the x_2 and x_3 axes, defining the stresses in the same way, to arrive at the second-rank stress tensor $T_{ij}(x_1, x_2, x_3)$. Although we have only considered planes perpendicular to the coordinate axes, it can be shown that the forces acting on any plane can be deduced from this tensor. It can also be shown that the stress tensor is symmetric, that is,

$$T_{ij} = T_{ji}. \tag{2.3}$$

In most materials the stresses can be taken to be proportional to the strains, provided the strains are sufficiently small. If this is true, the material is said to be *elastic*. The relationship is a generalisation of Hooke's law, which states that stress is proportional to strain for the one-dimensional case. Unless stated otherwise, it will be assumed throughout that the material is elastic, and hence each component of the stress is given by a linear combination of the strain components. The coefficients required are given by the *stiffness tensor*, c_{ijkl}, defined such that

$$T_{ij} = \sum_k \sum_l c_{ijkl} S_{kl}, \quad i, j, k, l = 1, 2, 3. \tag{2.4}$$

The stiffness is a fourth rank tensor, with 81 elements. However many of these elements are related. The symmetry of S_{ij} and T_{ij}, equations (2.2) and (2.3), implies that the stiffness is unaltered if i and j are interchanged, or if k and l are interchanged, that is,

$$c_{jikl} = c_{ijkl}, \tag{2.5}$$

$$c_{ijlk} = c_{ijkl}, \tag{2.6}$$

Thus only 36 of the 81 elements are independent. It can also be shown, from thermodynamic considerations, that the second pair of indices can be interchanged with the first pair:

$$c_{klij} = c_{ijkl}. \tag{2.7}$$

This reduces the number of independent elements to 21. These elements are of course physical properties of the material under consideration, so that the number of independent components may well be reduced further by the symmetry of the material. For example, a crystalline material with cubic symmetry has only three independent elements. It should be noted that the coordinate axes x_1, x_2, x_3 will not in general be parallel to the axes of the crystal lattice.

Equation of motion. If the stress and strain are functions of time as well as position, the motion is subject to Newton's laws in addition to the above equations, and these constraints can be combined in the form of an equation of motion. Consider an elementary cube within the material, centred at $\mathbf{x}' = (x_1', x_2', x_3')$. The edges are parallel to the x_1, x_2, and x_3 axes, and each edge is of length δ. The material surrounding the cube exerts forces on all six faces. For the faces at $x_1 = x_1' \pm \delta/2$, the components of force in the x_i direction are $\pm\, \delta^2 T_{i1}(x_1' \pm \delta/2, x_2', x_3')$. The forces on the faces normal to x_2 and x_3 are obtained in the same way, and we add these to obtain the total force on the cube. Noting that δ is small, the total force has an x_i component

$$\delta^3 \left[\sum_j \frac{\partial T_{ij}}{\partial x_j} \right]_{\mathbf{x}'}.$$

This must be equal to the acceleration $\partial^2 u_i(\mathbf{x}')/\partial t^2$, multiplied by the mass $\varrho\delta^3$, where ϱ is the density. This is valid for all points $\mathbf{x}'$, and hence

$$\varrho \frac{\partial^2 u_i}{\partial t^2} = \sum_j \frac{\partial T_{ij}}{\partial x_j}, \quad i, j = 1, 2, 3, \tag{2.8}$$

which is the equation of motion.

2.1.2. Piezoelectric Materials

Piezoelectricity is the phenomenon which, in many materials, couples elastic stresses and strains to electric fields and displacements. It occurs only in anisotropic materials whose internal structure lacks a centre of symmetry. It occurs in many crystal classes but is often weak, thus having little effect on the elastic behaviour. However, here we are concerned with devices that make crucial use of piezoelectricity, so it is necessary to take account of the effect in the analysis. Only insulating materials will be considered here.

In a homogeneous piezoelectric insulator, the stress components T_{ij} at each point are dependent on the electric field $\mathbf{E}$ (or, equivalently, the electric displacement $\mathbf{D}$) in addition to the strain components S_{ij}. Assuming all these quantities are small enough we can take the relationship to be linear, so that T_{ij} is given by the linear relation

$$T_{ij} = \sum_k \sum_l c^E_{ijkl} S_{kl} - \sum_k e_{kij} E_k. \tag{2.9}$$

Here, the superscript on c^E_{ijkl} identifies this as the stiffness tensor for constant electric field; that is, if $\mathbf{E}$ is held constant this tensor relates changes of T_{ij} to changes of S_{kl}. Similarly, the electric displacement $\mathbf{D}$ is usually determined by the field $\mathbf{E}$ and the permittivity tensor ε_{ij}, but in a piezoelectric material it is also related to the strain:

$$D_i = \sum_j \varepsilon^S_{ij} E_j + \sum_j \sum_k e_{ijk} S_{jk}, \tag{2.10}$$

where ε^S_{ij} is the permittivity tensor for constant strain. The forms of these equations are justified by thermodynamic arguments which are not considered here. The tensor

e_{ijk}, relating elastic to electric fields, is called the piezoelectric tensor. From equation (2.9) and the symmetry of T_{ij}, this tensor has the symmetry

$$e_{ijk} = e_{ikj}. \tag{2.11}$$

It is equally valid to relate **D** to the stress instead of the strain, and this can be done by eliminating S_{ij} from equations (2.9) and (2.10). The result is expressed in the form

$$D_i = \sum_j \varepsilon_{ij}^T E_j + \sum_j \sum_k d_{ijk} T_{jk}, \tag{2.12}$$

where the new tensors ε_{ij}^T and d_{ijk} are related in a rather complicated manner to the tensors in equations (2.9) and (2.10). The tensor ε_{ij}^T is the permittivity tensor for constant stress. We can also eliminate **E** between equations (2.9) and (2.10) to obtain an equation giving T_{ij} in terms of S_{kl} and **D**; the coefficients of S_{kl} then give a stiffness tensor for constant electric displacement.

The mechanical equation of motion, equation (2.8), is valid for a piezoelectric material. It is convenient to express this in terms of the displacements u_i and the electric potential Φ. Since elastic disturbances travel much more slowly than electromagnetic ones the electric field can be taken to be quasi-static, that is, it is given by the gradient of the potential, so that

$$E_i = -\partial\Phi/\partial x_i. \tag{2.13}$$

Using this relation in equation (2.9), and equation (2.1) for the strain, the equation of motion becomes

$$\varrho \frac{\partial^2 u_i}{\partial t^2} = \sum_j \sum_k \left[e_{kij} \frac{\partial^2 \Phi}{\partial x_j \partial x_k} + \sum_l c_{ijkl}^E \frac{\partial^2 u_k}{\partial x_j \partial x_l} \right]. \tag{2.14a}$$

In addition there are no free charges, since the material is assumed to be an insulator. Hence div $\mathbf{D} = 0$, and using equation (2.10) this gives

$$\sum_i \sum_j \left[\varepsilon_{ij}^S \frac{\partial^2 \Phi}{\partial x_i \partial x_j} - \sum_k e_{ijk} \frac{\partial^2 u_j}{\partial x_i \partial x_k} \right] = 0. \tag{2.14b}$$

Equations (2.14) give four equations relating the four quantities u_i and Φ, and hence the motion is determined if appropriate boundary conditions are specified.

When specifying the stiffness and piezoelectric tensors for a particular material, it is usual to adopt a special notation known as the matrix notation. This is convenient because it reduces the number of elements to be specified. The stiffness tensor c_{ijkl}^E has only 36 independent components because of its symmetry, equations (2.5) and (2.6), and is expressed in terms of a stiffness matrix c_{mn}^E. This is defined by

$$c_{mn}^E = c_{ijkl}^E, \quad m, n = 1, 2, \ldots, 6. \tag{2.15}$$

where m is related to i and j by

$$m = i \qquad \text{for } i = j$$

$$m = 9 - i - j \quad \text{for } i \neq j, \qquad i, j = 1, 2, 3.$$

A similar definition relates n to k and l. A simplified piezoelectric matrix is also used, defined by

$$e_{km} = e_{kij}, \quad k = 1, 2, 3, \quad m = 1, 2, \ldots, 6 \tag{2.16}$$

with m related to i and j as above.

2.2. WAVES IN ISOTROPIC MATERIALS

In this book we are concerned mainly with wave motion in anisotropic materials. The complexity of the equations of elasticity, described in the previous section, is such that the properties of the waves can usually be found only by numerical techniques. In contrast, solutions for isotropic materials are much easier to obtain, and since they have many features in common with the solutions for anisotropic materials it is helpful to consider the isotropic case first [30–39]. Numerical examples are given here for fused quartz which has acoustic properties somewhat similar to crystalline quartz, used in many surface wave devices.

In an isotropic material the stiffness tensor c_{ijkl} has only two independent components. From the symmetry it can be shown [30] that the stiffness can be written in the form

$$c_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}), \tag{2.17}$$

where $\delta_{ij} = 1$ for $i = j$ and $\delta_{ij} = 0$ for $i \neq j$. The constants λ and μ are known as Lamé constants and in practice are always positive; μ is also called the rigidity. Substituting into equation (2.4), the stress can be written in the form

$$T_{ij} = \lambda\delta_{ij}\Delta + 2\mu S_{ij} \tag{2.18}$$

where

$$\Delta = \sum_i S_{ii} = \sum_i \frac{\partial u_i}{\partial x_i}. \tag{2.19}$$

The equation of motion, equation (2.8), becomes, on substituting equation (2.18),

$$\varrho\frac{\partial^2 u_j}{\partial t^2} = (\lambda + \mu)\frac{\partial\Delta}{\partial x_j} + \mu\nabla^2 u_j \tag{2.20}$$

where

$$\nabla^2 = \sum_i \frac{\partial^2}{\partial x_i^2}.$$

2.2.1. Plane Waves

We first consider an infinite medium supporting plane waves, with frequency ω, in which the displacement $\mathbf{u}$ takes the form

$$\mathbf{u} = \mathbf{u}_0 \exp\left[j(\omega t - \mathbf{k}\cdot\mathbf{x})\right], \tag{2.21}$$

where $\mathbf{u}_0$ is a constant vector, independent of $\mathbf{x}$ and t. The actual displacement is the real part of equation (2.21), but the complex form can be used throughout the analysis because the equations are linear. The wave vector is $\mathbf{k} = (k_1, k_2, k_3)$, which gives the direction of propagation. The wavefronts are solutions of $\mathbf{k} \cdot \mathbf{x} =$ constant, and are perpendicular to $\mathbf{k}$. The phase velocity of the wave is $V = \omega/|\mathbf{k}|$. With this form for $\mathbf{u}$, we have $\partial \mathbf{u}/\partial x_j = -jk_j\mathbf{u}$, and on substituting into equation (2.20) we obtain

$$\omega^2 \varrho u_j = (\lambda + \mu)(\mathbf{k}\cdot\mathbf{u})k_j + \mu|\mathbf{k}|^2 u_j, \quad j = 1, 2, 3$$

where

$$|\mathbf{k}|^2 = k_1^2 + k_2^2 + k_3^2.$$

Substituting for u_j using equation (2.21), and writing the result in vector form, gives

$$\omega^2 \varrho \mathbf{u}_0 = (\lambda + \mu)(\mathbf{k}\cdot\mathbf{u}_0)\mathbf{k} + \mu|\mathbf{k}|^2 \mathbf{u}_0. \tag{2.22}$$

Here there are two terms parallel to $\mathbf{u}_0$ and one term parallel to $\mathbf{k}$, with the latter including the scalar product $\mathbf{k}\cdot\mathbf{u}_0$. There are therefore two cases to consider. Firstly, if $\mathbf{u}_0$ is perpendicular to $\mathbf{k}$ the scalar product $\mathbf{k}\cdot\mathbf{u}_0$ is zero, and the remaining terms in the equation are parallel. Secondly, if $\mathbf{u}_0$ is not perpendicular to $\mathbf{k}$ the product $\mathbf{k}\cdot\mathbf{u}_0$ is non-zero, so that for non-trivial solutions we must have $\mathbf{u}_0$ parallel to $\mathbf{k}$. These two cases give shear wave solutions and longitudinal wave solutions, respectively.

Taking $\mathbf{u}_0$ to be perpendicular to $\mathbf{k}$ gives *shear*, or transverse, waves. For these the wave vector is denoted by $\mathbf{k}_t$, and equation (2.22) gives

$$|\mathbf{k}_t|^2 = \omega^2 \varrho/\mu.$$

The phase velocity for shear waves is denoted by V_t, equal to $\omega/|\mathbf{k}_t|$, so that

$$V_t = \sqrt{\mu/\varrho}, \tag{2.23}$$

taking V_t to be positive. Since this is independent of the frequency ω, the wave is non-dispersive. The displacement $\mathbf{u}_0$ can have any direction in the plane of the wavefront, perpendicular to $\mathbf{k}_t$.

For *longitudinal* waves we consider solutions of equation (2.22) in which $\mathbf{u}_0$ is parallel, or anti-parallel, to $\mathbf{k}$. Thus $\mathbf{k}$ is given by

$$\mathbf{k} = \pm\, \mathbf{u}_0 \frac{|\mathbf{k}|}{|\mathbf{u}_0|}. \tag{2.24}$$

With this relation we find

$$(\mathbf{k}\cdot\mathbf{u}_0)\,\mathbf{k} = \mathbf{u}_0|\mathbf{k}|^2$$

irrespective of the sign in equation (2.24). In this case the wave vector is denoted $\mathbf{k}_l$ and substitution into equation (2.22) gives

$$|\mathbf{k}_l|^2 = \omega^2\varrho/(\lambda + 2\mu).$$

The velocity for this case is denoted by V_l and is thus given by

$$V_l = \sqrt{\frac{\lambda + 2\mu}{\varrho}} \tag{2.25}$$

and hence the wave is non-dispersive. Since λ and μ are always positive, the velocity of longitudinal waves is always greater than the velocity of shear waves.

The velocities are typically in the region of 3000 m/s for shear waves, and 6000 m/s for longitudinal waves. For example, in fused quartz [44], $V_t = 4100$ m/s and $V_l = 6050$ m/s. The displacements involved are usually very small. In fused quartz, a shear wave with a power density of 1 mW/mm^2 and a frequency of 10 MHz has maximum displacements of about 0.3 nm, and a similar figure applies for longitudinal waves. This displacement is some six orders of magnitude smaller than the wavelengths.

We now consider several configurations involving plane boundaries. The solutions are obtained by summing plane wave solutions of the above types. We consider solutions in which the displacements are proportional to $\exp(-j\beta x_1)$, with the x_1 axis parallel to the boundaries. In each case this gives one or more characteristic solutions, and these are often called modes. It should however be noted that these modes are not the only solutions that can exist.

In most cases we are concerned with waves propagating on a half-space of some material, which may have a layer of another material on the surface. The solutions of most interest are described as *surface acoustic waves* (SAW) or, more simply, as surface waves. These are solutions for which the displacements in the half-space decay rapidly in the direction normal to the surface, and the energy of the wave is transported parallel to the surface. The Rayleigh wave to be considered next is one type of surface acoustic wave.

2.2.2. Rayleigh Waves in a Half-Space

Consider an isotropic medium with infinite extent in the x_1 and x_2 directions but with a boundary at $x_3 = 0$, so that the medium occupies the space $x_3 < 0$. The space $x_3 > 0$ is a vacuum. The surface wave solution for this case is named after Lord Rayleigh, who first discovered it [37]. We assume propagation in the x_1 direction, so that the wavefronts are parallel to x_2, as depicted in Figure 2.1. The (x_1, x_3) plane, which contains the surface normal and the propagation direction, is known as the *sagittal plane*. The solution must satisfy the equation of motion,

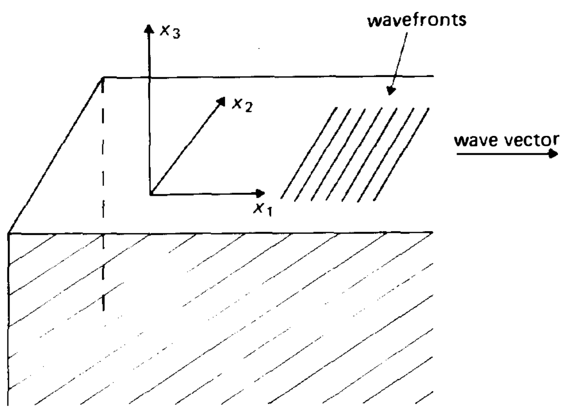

FIGURE 2.1. Axes for surface wave analysis.

equation (2.20), and the boundary conditions that there should be no forces on the free surface at $x_3 = 0$.

The solution is found by adding two components corresponding to plane shear and longitudinal waves, and these components are described as *partial* waves. The wave vectors, $\mathbf{k}_t$ and $\mathbf{k}_l$, have magnitudes given by

$$|\mathbf{k}_t|^2 = \omega^2/V_t^2; \qquad |\mathbf{k}_l|^2 = \omega^2/V_l^2.$$

Now since the Rayleigh wave has no variation in the x_2 direction these vectors must have no x_2-components, and are therefore in the sagittal plane. In the x_1-direction the displacements are assumed to vary as $\exp(-j\beta x_1)$, where β is the wavenumber of the Rayleigh wave. The x_1-components of $\mathbf{k}_t$ and $\mathbf{k}_l$ must therefore be equal to β. The x_3-components are denoted respectively by T and L, and we thus have

$$T^2 = \omega^2/V_t^2 - \beta^2, \tag{2.26}$$

$$L^2 = \omega^2/V_l^2 - \beta^2. \tag{2.27}$$

The displacement of the longitudinal wave, $\mathbf{u}_l$, must be parallel to the wave vector $\mathbf{k}_l = (\beta, 0, L)$. Thus, ommitting a factor $\exp(j\omega t)$, we can write

$$\mathbf{u}_l = A(1, 0, L/\beta) \exp[-j(\beta x_1 + Lx_3)], \tag{2.28}$$

where A is a constant. The displacement of the shear wave, $\mathbf{u}_t$, is perpendicular to the wave vector $\mathbf{k}_t = (\beta, 0, T)$.This does not determine the direction of $\mathbf{u}_t$, but we assume for the present that $\mathbf{u}_t$ is the sagittal plane (as is $\mathbf{u}_l$). Thus $\mathbf{u}_t$ is given by

$$\mathbf{u}_t = B(1, 0, -\beta/T) \exp[-j(\beta x_1 + Tx_3)], \tag{2.29}$$

where B is a constant. The total displacement $\mathbf{u}$ is the sum

$$\mathbf{u} = \mathbf{u}_t + \mathbf{u}_l. \tag{2.30}$$

Now, in the x_3-direction, the shear wave displacement $\mathbf{u}_t$ varies as $\exp(-jTx_3)$. For surface wave solution the displacement must decay for negative x_3, and hence the value of T must be positive imaginary. Thus β must be large enough to make the right hand side of equation (2.26) negative, and since the Rayleigh wave velocity V_R will be given by $V_R = \omega/\beta$ we must have

$$V_R < V_t. \tag{2.31}$$

Similarly, L must be positive imaginary and this implies $V_R < V_l$, but this is already implied by equation (2.31) because V_l is greater than V_t. With T and L imaginary, the partial wave solutions, $\mathbf{u}_l$ of equation (2.28) and $\mathbf{u}_t$ of equation (2.29), are no longer plane waves; however, they are still valid as solutions for an infinite medium, satisfying the equation of motion, equation (2.20). If the total displacement $\mathbf{u}$ of equation (2.30) is to be a valid solution for the half-space, it must also satisfy the boundary conditions that $T_{13} = T_{23} = T_{33} = 0$ at the surface $x_3 = 0$, where the stresses T_{i3} are given by equation (2.18). This gives two linear homogeneous equations

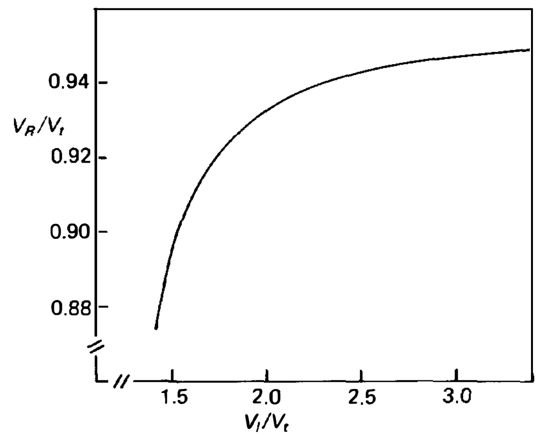

FIGURE 2.2. Normalised Rayleigh wave velocity for isotropic materials.

relating the constants A and B. For non-trivial solutions the determinant of the coefficients must be zero, and this gives

$$(T^2 - \beta^2)^2 + 4\beta^2 LT = 0. \tag{2.32}$$

Using equations (2.26) and (2.27) for T and L, and defining the phase velocity $V = \omega/\beta$, this gives

$$[2 - V^2/V_t^2]^2 = 4[1 - V^2/V_l^2]^{1/2}[1 - V^2/V_t^2]^{1/2}. \tag{2.33}$$

For Rayleigh waves we require a solution giving V^2 a real positive value, less than V_t^2. The equation has only one such solution, which is denoted V_R^2, and we take V_R to be positive. The ratio V_R/V_t is determined by the ratio V_l/V_t of the plane wave velocities and is shown in Figure 2.2, which thus gives the Rayleigh velocity for any isotropic material. V_R is usually quite close to V_t, and is independent of frequency.

The above analysis also gives the displacements. Omitting a factor $\exp[j(\omega t - \beta x_1)]$ and an arbitrary multiplier, the displacements are

$$u_1 = \gamma \exp(a\beta x_3) - \exp(b\beta x_3),$$

$$u_3 = j[\gamma a \exp(a\beta x_3) - b^{-1} \exp(b\beta x_3)], \tag{2.34}$$

where a, b and γ are real positive quantities given by $a = -jL/\beta$, $b = -jT/\beta$ and $\gamma = (2 - V_R^2/V_t^2)/(2ab)$, and $\beta = \omega/V_R$. These displacements are shown in Figure 2.3, as functions of the depth normalised to the Rayleigh wavelength $\lambda_R = 2\pi V_R/\omega$. Since u_3 is in phase quadrature with u_1, the motion of each particle is an ellipse. Because of the change of sign of u_1 at a depth of about 0.2 wavelengths, the ellipse is described in different directions above and below this point; at the surface the motion is retrograde, while lower down it is prograde. The distortion of the material at one instant is shown in Figure 2.4, with the displacements exaggerated. The dots in this figure represent the equilibrium positions of particles within the

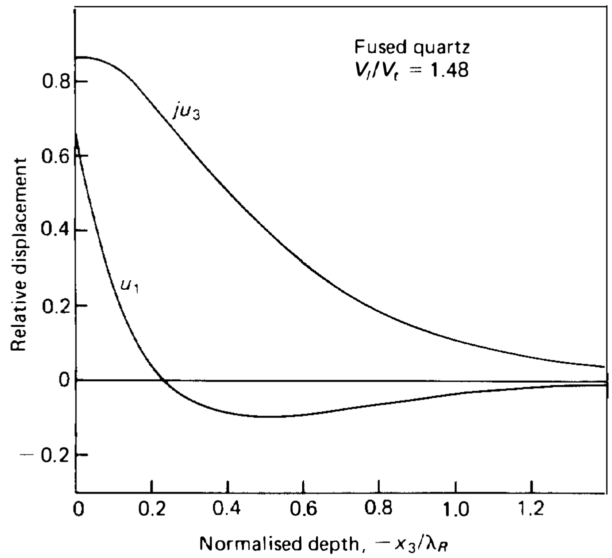

FIGURE 2.3. Rayleigh wave displacements for isotropic material.

material, while the lines show the displacements when a Rayleigh wave is present. Note that there is little motion at depths greater than one wavelength.

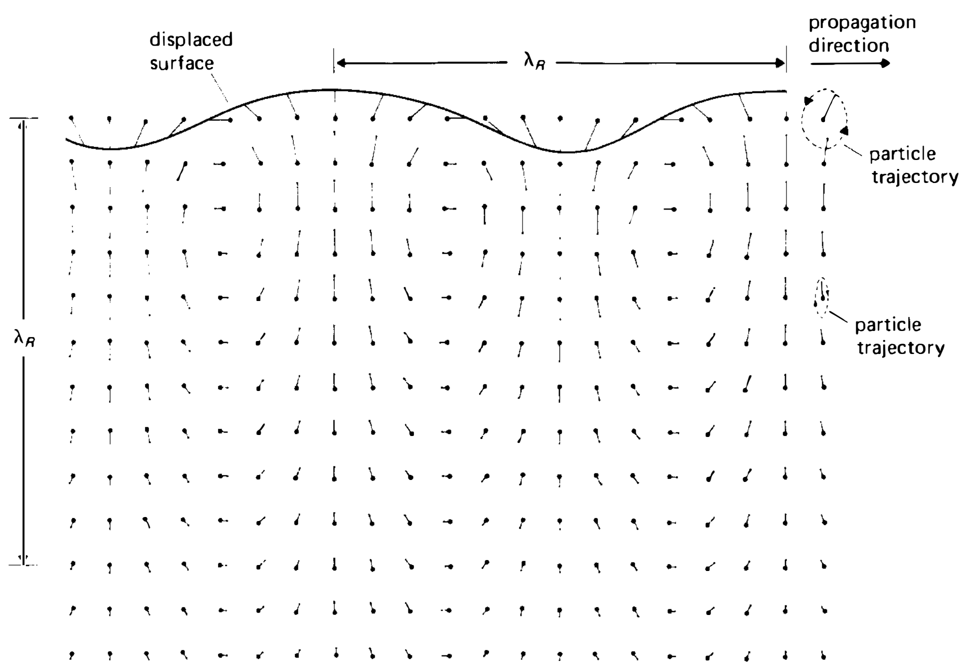

FIGURE 2.4. Instantaneous displacements for Rayleigh wave propagation in isotropic material.

2.2.3. Shear-horizontal Waves in a Half-space

It was assumed above that for surface wave solutions the displacements would be confined to the sagittal plane (x_1, x_3). We now consider whether a valid solution can be obtained with a perpendicular component, in the x_2-direction. As before, the wave vector of any partial wave must be confined to the sagittal plane, and hence a displacement normal to this plane can only be produced by shear waves. The wave vector must therefore be $\mathbf{k}_t = (\beta, 0, T)$, as before, and the displacement must be

$$\mathbf{u} = A(0, 1, 0) \exp[-j(\beta x_1 + Tx_3)],$$

where A is a constant. For this wave the stresses T_{13} and T_{33} are zero, while $T_{23} = -j\mu T \cdot u_2$. The boundary conditions, $T_{i3} = 0$ at $x_3 = 0$, cannot therefore be satisfied if T is finite, and hence there is no solution representing a wave bound to the surface. However, if $T = 0$ the stress components T_{i3} are zero everywhere, and this satisfies the boundary conditions. This solution is simply a plane shear wave propagating parallel to the surface, with its amplitude independent of x_3 within the material. It is called a shear-horizontal, or SH, wave, since its displacements are parallel to the surface. The phase velocity is equal to V_t.

2.2.4. Waves in a Layered Half-space

Now consider a half space of material with a layer of another material, of thickness d, on top, as shown in Figure 2.5. Structures of this type are common in surface wave devices, where the layer may for example be a metal film. Usually, one is concerned with minimising the perturbing effect of the film, for example to minimise the dispersion. In seismology the structure is of considerable interest because the layering of rocks influences the propagation of surface waves, and consequently the solutions have been studied in detail [32, 38–44].

We first consider waves with their displacements confined to the sagittal plane. If the layer thickness is small, the solution will be similar to the Rayleigh wave for a half-space, described in Section 2.2.2 above, so the solutions here are described as

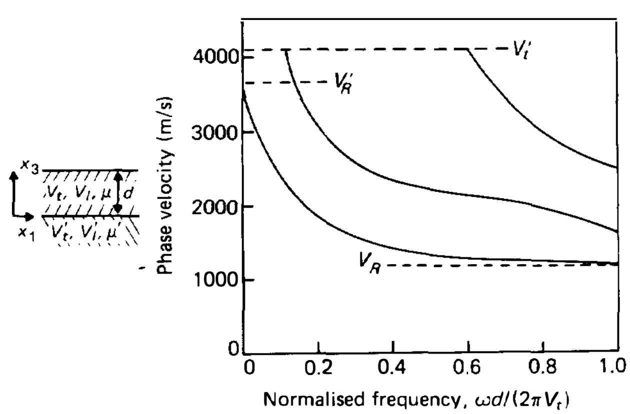

FIGURE 2.5. Velocities for layered Rayleigh waves, gold layer on fused quartz substrate. After Farnell and Adler [44], with permission.

"layered Rayleigh waves". The solutions may be found by summing partial waves, as before, though the calculation is more complex. The layer material is taken to have plane wave velocities V_t and V_l. Assuming the displacements are proportional to $\exp(-j\beta x_1)$, a partial shear wave in the layer gives displacements with the form of equation (2.29), with T given by equation (2.26). However, there are two solutions for T. To account for this, we define T as the positive solution of equation (2.26), so that the partial shear waves have wavenumbers $\mathbf{k}_t = (\beta, 0, \pm T)$. The displacements of these two waves then give

$$\mathbf{u}_t = A(1, 0, -\beta/T)\exp(-jTx_3) + B(1, 0, \beta/T)\exp(jTx_3), \qquad (2.35)$$

where A and B are constants, and the variations with x_1 and t have been omitted. Similarly, there are two longitudinal partial waves in the layer, with displacements

$$\mathbf{u}_l = C(1, 0, L/\beta)\exp(-jLx_3) + D(1, 0, -L/\beta)\exp(jLx_3), \qquad (2.36)$$

where L is the positive solution of equation (2.27), and C and D are constants. The total displacement in the layer is $\mathbf{u} = \mathbf{u}_t + \mathbf{u}_l$.

We define V_t' and V_l' as the velocities of plane shear and longitudinal waves in the half-space material. The displacement $\mathbf{u}'$ in the half-space is obtained as before; comparing with equations (2.28) and (2.29) we can write

$$\mathbf{u}' = E(1, 0, -\beta/T')\exp(-jT'x_3) + F(1, 0, L'/\beta)\exp(-jL'x_3), \qquad (2.37)$$

where E and F are constants. T' and L' are the x_3-components of the wavenumbers, given by equations (2.26) and (2.27) but with V_t' and V_l' replacing V_t and V_l. For a surface wave solution, with displacements decaying in the half-space, T' and L' must be positive imaginary, and hence the phase velocity must be less than V_t'.

The boundary conditions require that the stresses T_{i3} should be zero on the free upper surface, while on the lower surface the stresses T_{i3} and the displacements should be continuous. This gives six homogeneous equations relating the constants A, B, . . . , F, and the determinant of coefficients is set to zero to give the dispersion relation. This in turn gives the allowed values for β, and hence the velocities.

Figure 2.5 shows the calculated phase velocities, for a gold layer on a fused quartz substrate, after Farnell and Adler [44]. The horizontal axis here gives normalised frequency, but may also be read as the thickness of the layer divided by the wavelength of plane shear waves in the layer material. The result is typical of cases in which the layer material has acoustic velocities much less than those of the half-space material. The fundamental mode, that is, the solution with lowest velocity, is of primary importance. At low frequencies the layer thickness is much less than the wavelength, so the velocity approaches V_R', the Rayleigh velocity for the half-space material. At high frequencies the structure can support a Rayleigh wave with its energy concentrated near the upper surface, so the velocity approaches the Rayleigh velocity for the layer material, denoted V_R. The velocity thus varies from V_R' to V_R. Clearly, if we wish to minimise the dispersion, the materials should be chosen such that V_R is not substantially less than V_R'. For this reason the metal film used for the electrodes in surface wave devices is usually aluminium, which has a Rayleigh velocity similar to those of the common substrate materials quartz and lithium niobate. In addition to the fundamental, there is a series of higher modes, which are named after Sezawa [40].

If the layer material has acoustic velocities greater than those of the substrate, the velocity of the fundamental is V_R' at zero frequency, rising to a value V_t', at which point there is a cut-off. There is therefore little dispersion in this case.

In addition to layered Rayleigh waves, the layered system can also support surface waves with the displacements normal to the sagittal plane. These are known as Love waves [43]. In this case the partial waves are shear waves, with wave vectors $\mathbf{k}_t = (\beta, 0, \pm T)$ in the layer and $\mathbf{k}_t' = (\beta, 0, T')$ in the substrate. Thus the displacements in the layer can be written

$$\mathbf{u}_t = A(0, 1, 0) \exp(-jTx_3) + B(0, 1, 0) \exp(jTx_3) \tag{2.38}$$

and the displacement in the substrate is

$$\mathbf{u}_t' = C(0, 1, 0) \exp(-jT'x_3),$$

where A, B and C are constants. The boundary conditions are the same as for the Rayleigh wave case, and applying these gives the dispersion relation

$$\tan(Td) = j\mu' T'/(\mu T), \tag{2.39}$$

where μ and μ' are respectively the rigidities of the layer material and the half-space material. Solving for β gives in general a number of modes, and the velocities for gold on fused quartz are shown in Figure 2.6. Solutions are obtainable only for $V_t < V_t'$, and the solutions must have velocities less than V_t', so that T' is imaginary and the displacement decays in the half-space. At zero frequency the Love wave solution becomes identical to the SH plane wave solution for a half-space. Thus Love waves can be regarded as modified forms of the SH plane wave, where the presence of a layer with low acoustic velocity converts the plane wave into a surface wave and causes dispersion.

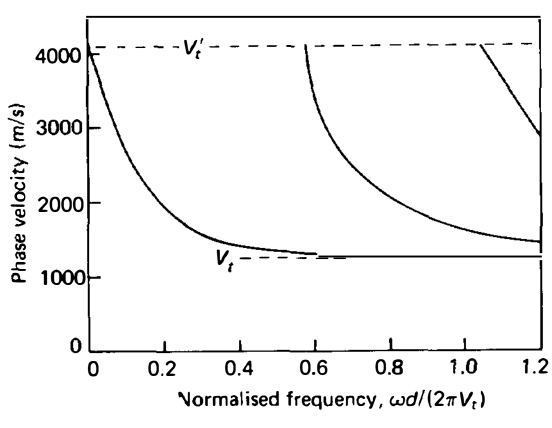

FIGURE 2.6. Velocities for Love waves, gold layer on fused quartz substrate. After Farnell and Adler [44], with permission.

2.2.5. Waves in a Parallel-sided Plate

We now consider a plate of material with boundaries at $x_3 = \pm d/2$ and with infinite extent in the x_1 and x_2 directions. As for the layered substrate there are two types of solution [32, 34]. For displacements confined to the sagittal plane (x_1, x_3) the solutions are called Lamb waves, while there are also SH-wave solutions with displacements perpendicular to the sagittal plane. Both types have some relevance to surface wave devices.

The parallel plate is rather similar to the layered half-space considered above, with the half-space omitted. Thus, for Lamb waves, the partial waves have the same form as those for layered Rayleigh waves, given by equations (2.35) and (2.36). The allowed values for β, and the relative values of the constants A, B, C and D, are obtained by applying the boundary conditions $T_{i3} = 0$ at the two surfaces, $x_3 = \pm d/2$. This gives two families of dispersive solutions, known as symmetric and antisymmetric modes. The dispersion relation is

$$\left[\frac{\tan (Ld/2)}{\tan (Td/2)}\right]^{\pm 1} = -\frac{(T^2 - \beta^2)^2}{4LT\beta^2}, \tag{2.40}$$

taking the upper sign for symmetric modes and the lower sign for antisymmetric modes

At high frequencies, Td and Ld are large, and if we also assume T and L to be imaginary the left side of equation (2.40) approaches unity. Comparison with equation (2.32) shows that the velocity approaches the Rayleigh velocity, V_R. Thus the two Lamb modes, one symmetric and one antisymmetric, each become equivalent to a Rayleigh wave on the upper surface plus a Rayleigh wave on the lower surface. This gives some insight into the behaviour of a wave generated on one surface of a parallel-sided plate. The wave is equivalent to the sum of the two Lamb waves, with equal amplitudes and phases; owing to the symmetry the Lamb wave displacements are additive near the upper surface but cancel near the lower surface, so that if d is large enough the wave is essentially a Rayleigh wave on the upper surface. However, for finite d the Lamb waves have different wave numbers, β_s and β_a, say, so that the disturbance at the upper surface has the form $[\exp(-j\beta_a x_1) + \exp(-j\beta_s x_1)]$, with an amplitude proportional to $\cos[(\beta_a - \beta_s)x_1/2]$. Thus, after travelling a distance

$$x_c = \frac{\pi}{|\beta_a - \beta_s|}$$

the amplitude at the top surface is zero. At this point, the two Lamb waves are in anti-phase, so that they reinforce at the lower surface; in effect, the Rayleigh wave has been transferred from one surface to the other. The distance x_c is therefore called the coupling distance. At a distance $2x_c$, the amplitude on the upper surface is again maximised, so that the Rayleigh wave has been transferred back again.

The coupling distance may be evaluated by solving equation (2.40) for β_a and β_s, giving the result shown in Figure 2.7, where x_c is normalised to the Rayleigh wavelength. The coupling length is many thousand wavelengths, even when the plate

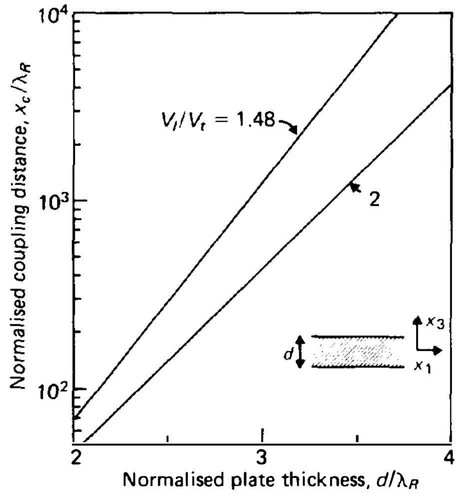

FIGURE 2.7. Coupling distance for transfer of Rayleigh waves between the two sides of a parallel-sided plate.

is only a few wavelengths thick. In surface-wave devices the wave is generated on a rectangular substrate, and it can be concluded that a substrate thickness of a few wavelengths should be sufficient to prevent the rear surface from having any significant effect. This conclusion remains valid if the substrate is mounted on a carrier using an adhesive (as is usually the case).

Setting $\beta = 0$ in equation (2.40) gives the resonant frequencies of the plate. These are the frequencies at which $2d$ is a multiple of the shear wavelength $2\pi V_t/\omega$, or of the longitudinal wavelength $2\pi V_l/\omega$. Resonances of this type are used in the parallel-plate crystal resonator. They are sometimes observed in surface wave devices, though usually most of them are damped out because of the mounting of the substrate.

The parallel-sided plate also supports shear horizontal (SH) modes, with the displacements in the x_2-direction. These solutions can be obtained by assuming partial waves of the form given by equation (2.38). This gives a series of modes, most of which are dispersive. However, at low frequencies there is only one solution, a non-dispersive mode with velocity V_t; this is simply a plane shear wave propagating in the x_1 direction. The same solution was found in Section 2.2.3 for propagation in a half-space, and it was shown that this wave gives no stresses on planes perpendicular to x_3.

The non-dispersive SH mode of the parallel-sided plate is used in a type of dispersive delay line known as the IMCON, in which the wave is reflected by arrays of grooves. Dispersive modes (Lamb waves and SH modes) have also been used in dispersive delay lines.

2.3. WAVES IN ANISOTROPIC MATERIALS

This section is concerned with acoustic waves in piezoelectric materials, which must of course be anisotropic. Because of the complexity of the equations for this case, the solutions can usually be found only by using numerical techniques. The account here is therefore mainly descriptive.

2.3.1. Plane Waves in an Infinite Medium

For a piezoelectric material, the equation of motion takes the form of equations (2.14), in terms of the displacements $\mathbf{u}$ and the potential Φ. We consider plane wave solutions with frequency ω and wave vector $\mathbf{k}$, with $\mathbf{u}$ and Φ having the forms

$$\mathbf{u} = \mathbf{u}_0 \exp[j(\omega t - \mathbf{k}\cdot\mathbf{x})],$$

$$\Phi = \Phi_0 \exp[j(\omega t - \mathbf{k}\cdot\mathbf{x})],$$

where $\mathbf{u}_0$ and Φ_0 are constants, independent of $\mathbf{x}$ and t, and $\mathbf{k}$ is real. For isotropic materials, Section 2.2.1, there are two solutions, the shear wave and the longitudinal wave, with $\mathbf{u}_0$ respectively perpendicular to and parallel to $\mathbf{k}$.

To find the solutions for anisotropic materials, the above functions $\mathbf{u}$ and Φ are substituted into equations (2.14), giving four equations in the four variables $\mathbf{u}_0$, Φ_0. Setting the determinant of coefficients to zero then gives four solutions. One of these solutions is essentially electrostatic in nature — it corresponds to the electrostatic solution for a non-piezoelectric material, for which $e_{ijk} = 0$ so that equation (2.14b) reduces to Laplace's equation. This solution is of little interest here. The other three solutions are non-dispersive acoustic waves. Usually, one solution has the displacement $\mathbf{u}_0$ almost parallel to $\mathbf{k}$, and is called the "quasi-longitudinal", or simply longitudinal, wave. The other two solutions (with different velocities) usually have $\mathbf{u}_0$ almost perpendicular to $\mathbf{k}$, are are called "quasi-shear", or shear, waves. For particular propagation directions the longitudinal wave has $\mathbf{u}_0$ parallel to $\mathbf{k}$, and is then called a "pure longitudinal" wave. Similarly, shear waves may have $\mathbf{u}_0$ perpendicular to $\mathbf{k}$, and are then called "pure shear" waves. Owing to anisotropy, each of the three waves has a phase velocity (equal to $\omega/|\mathbf{k}|$) dependent on the propagation direction. In addition, all three solutions may have associated electric potentials, though for particular propagation directions the potential may disappear.

2.3.2. Theory for a Piezoelectric Half-space

In Section 2.2.2 we saw that the Rayleigh wave solution for an isotropic half-space can be obtained by adding two partial waves, corresponding to plane shear and longitudinal waves, with the x_1-components of their wave vectors equal. For anisotropic materials the method [30, 45–50] is essentially the same, though a numerical procedure must be adopted to obtain the solutions.

Care is needed in specifying the orientation of the material. For most crystalline materials the internal structure is referenced to an orthogonal set of axes denoted by upper-case symbols X, Y, Z, with directions defined in relation to the crystal lattice [52]. The surface orientation and the wave propagation direction must be defined in relation to these axes. The convention usually adopted is to define the surface normal x_3, followed by the propagation direction x_1. For example "Y, Z lithium niobate" indicates that x_3 is parallel to the crystal Y-axis, and x_1 is parallel to the crystal Z-axis.

The orientation of x_3 is also referred to as the cut, so that for Y, Z lithium niobate the crystal is Y-cut. The material tensors, the stiffness, permittivity and piezoelectric tensor, are specified in relation to the internal axes X, Y and Z, so for the analysis they must be rotated into the frame defined by x_1, x_2, x_3. For cubic crystals, the orientation is usually defined directly in relation to the lattice by using Miller indices.

For a piezoelectric material it is necessary to use an electrical boundary condition at the surface, in addition to the stress-free condition which applies for isotropic materials. Two cases are usually considered. In the first case the space above the surface is a vacuum and conductors are excluded, so that there are no free charges. This is known as the *free-surface* case. In general there will be a potential in the vacuum above the surface. In the second case the surface is assumed to be covered with a thin metal layer with infinite conductivity, which shorts out the horizontal component of **E** at the surface but does not affect the mechanical boundary conditions. This is called the *metallised* case. These two cases generally give different velocities. The velocity difference is a measure of the coupling between the wave and electrical perturbations at the surface, and will be seen to be of crucial importance to the performance of surface wave transducers.

For the free-surface case the potential in the vacuum satisfies Laplace's equation $\nabla^2\Phi = 0$. If the wavenumber of the surface wave is β, the potential Φ in the vacuum can be written

$$\Phi = f(x_3) \exp [j(\omega t - \beta x_1)].$$

Using Laplace's equation shows that the function $f(x_3)$ has the form $\exp(\pm \beta x_3)$, and since Φ must vanish at $x_3 = +\infty$ the potential is given for $x_3 \geqslant 0$ by

$$\Phi = \Phi_0 \exp(-|\beta| x_3) \exp [j(\omega t - \beta x_1)], \tag{2.41}$$

where Φ_0 is a constant. Since there are no free charges D_3 must be continuous, so that in both the piezoelectric and the vacuum we have

$$D_3 = \varepsilon_0 |\beta| \Phi, \quad \text{at } x_3 = 0. \tag{2.42}$$

For the metallised case the potential at the surface is zero:

$$\Phi = 0, \quad \text{at } x_3 = 0. \tag{2.43}$$

In addition, for either case there are no forces on the surface, so

$$T_{13} = T_{23} = T_{33} = 0, \quad \text{at } x_3 = 0. \tag{2.44}$$

To find the surface wave solutions, we first consider partial waves in which the displacements and potential, denoted by $\mathbf{u}'$ and Φ', take the form

$$\mathbf{u}' = \mathbf{u}_0' \exp(j\gamma x_3) \exp [j(\omega t - \beta x_1)],$$

$$\Phi' = \Phi_0' \exp(j\gamma x_3) \exp [j(\omega t - \beta x_1)], \tag{2.45}$$

where β is the wave number of the surface wave, assumed to be real. These expressions are to satisfy the equations of motion, equations (2.14), for an infinite material. As in the isotropic case, if β is fixed there are a number of specific solutions for γ, the

x_3-component of the wave vector. We assume a particular real value of β and substitute equations (2.45) into equations (2.14). These can then be solved numerically, giving eight solutions for γ, and for each solution the relative values of $\mathbf{u}_0'$ and Φ_0' are obtained. The values of γ are generally complex, and we can only allow values whose imaginary parts are negative, so that $\mathbf{u}'$ and Φ' vanish at $x_3 = -\infty$. There are four such values of γ in general, and these are denoted $\gamma_1, \ldots, \gamma_4$. The partial waves are therefore

$$\begin{aligned} \mathbf{u}_m' &= \mathbf{u}_{0m}' \exp(j\gamma_m x_3) \exp[j(\omega t - \beta x_1)], \\ \Phi_m' &= \Phi_{0m}' \exp(j\gamma_m x_3) \exp[j(\omega t - \beta x_1)], \quad m = 1, 2, 3, 4, \end{aligned} \tag{2.46}$$

where $\mathbf{u}_{0m}'$ and Φ_{0m}' are the displacement and potential corresponding to γ_m.

In the half-space it is assumed that the solution is a linear sum of these partial waves, so that

$$\begin{aligned} \mathbf{u} &= \sum_{m=1}^{4} A_m \mathbf{u}_m', \\ \Phi &= \sum_{m=1}^{4} A_m \Phi_m'. \end{aligned} \tag{2.47}$$

The coefficients A_m are to be such that the solution satisfies the boundary conditions, given by equations (2.44) and either equation (2.42) (for the free-surface case) or equation (2.43) (for the metallised case). These conditions give a determinant which must be zero for a valid solution, and when zero gives the relative values of the constants A_m. However, the determinant will only be zero if the correct value for β has been chosen. Thus, to find the solution the entire procedure is iterated using different values for β until the boundary condition determinant vanishes. The velocity of the wave is then ω/β, and the displacements and potential are given by equation (2.47).

2.3.3. Surface-wave Solutions

The solutions obtained by the above procedure are of course significantly affected by the anisotropy of the material and by its orientation. The determination of surface wave characteristics in general is a considerable task because of the variety of crystal symmetries, and because for any one symmetry the orientation depends on three angular variables. However, a very extensive range of cases has been studied, as shown for example by Refs. [45–51]. It appears that one or more surface wave solutions can always be found, whatever the symmetry and orientation. In general the solution may involve all three components of the displacement, so that the motion is not confined to the sagittal plane. However, since there is no variation in the x_2-direction the electric field $\mathbf{E}$, given by the negative gradient of Φ, is always in the sagittal plane. For a metallised surface the parallel component, E_1, is always zero at the surface, though the normal components, E_3, is not necessarily zero. In some orientations the solution has no associated electric field, and can thus be described as non-piezoelectric. In this case the solution is not affected by metallisation of the surface.

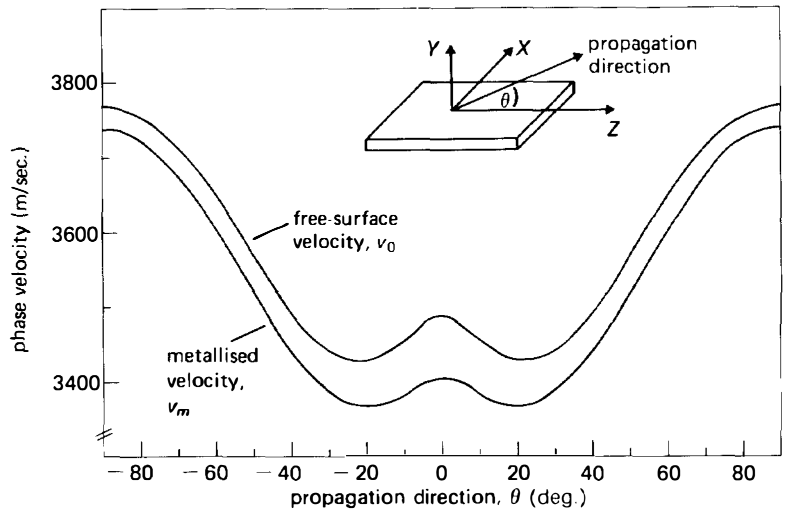

FIGURE 2.8. Rayleigh-wave velocities for Y-cut lithium niobate.

In general, the velocity of a surface wave must be less than the velocities of plane waves propagating in the x_1 direction in an infinite material. The reason for this is that, as for isotropic materials, the wave vectors of the partial waves must not have real x_3-components. Of the three plane waves (longitudinal, fast shear, slow shear) the slow shear wave has the lowest velocity, so the surface wave velocity must be less than this. In practice, the surface wave velocity is usually quite close to the slow shear velocity.

The solution encountered most frequently has its displacement **u** directed parallel, or almost parallel, to the sagittal plane, and has an associated electric field. This solution is known as a *piezoelectric Rayleigh wave*. It is similar to the Rayleigh wave for an isotropic material, with its behaviour modified somewhat by anisotropy and piezoelectricity. The penetration depth is typically about one wavelength. This type of solution is found in, for example, *Y*-cut lithium niobate. For this material the velocities are shown in Figure 2.8, as functions of the propagation direction. The free-surface velocity is denoted v_0 and the metallised velocity is denoted v_m. Note that v_m is less than v_0, which is always the case for a piezoelectrically coupled wave. For *Y*-cut lithium niobate the marked difference between the two velocities shows that the piezoelectric coupling is strong for this case. The coupling is strongest for propagation in the Z direction, and this orientation, described as Y, Z, is often used for surface-wave devices. The displacements and potential are shown in Figure 2.9, where the scales are appropriate for a power density of 1 mW/mm and a frequency of 100 MHz.

For some particular orientations the piezoelectric Rayleigh wave can have its displacement confined to the sagittal plane. This occurs if the sagittal plane is a plane of mirror symmetry for the crystal. The wave is then called a *pure* mode, and the propagation direction is called a pure mode direction. For a given surface orientation, the wave velocity is symmetrical with respect to a pure mode direction. For *Y*-cut

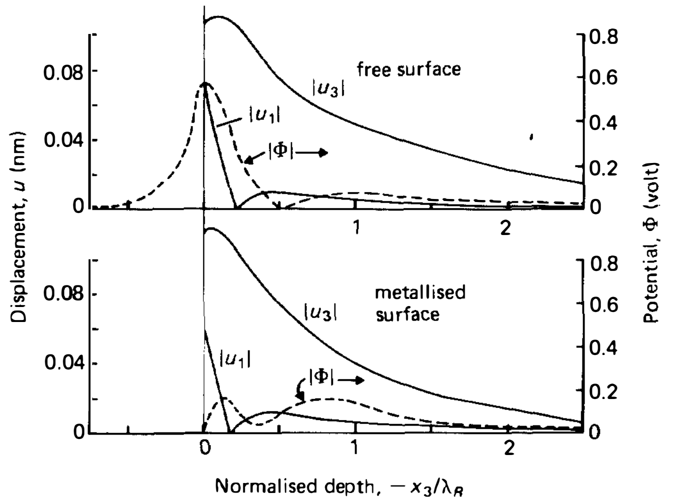

FIGURE 2.9. Displacements and potential for surface waves on Y, Z lithium niobate. After Farnell [49], with permission.

lithium niobate the X and Z directions are pure mode directions, and the symmetry can be seen in Figure 2.8. Some authors use the term "pure mode directions" for directions about which the velocity is symmetrical, and for these the displacements are not necessarily confined to the sagittal plane.

A quite different solution, the *Bleustein–Gulyaev wave* [53–56], occurs if the sagittal plane is normal to a two-fold axis of the crystal, or an axis of higher even order. This wave has its displacement normal to the sagittal plane, and has an associated electric field. It is closely related to the plane SH wave that can propagate in an isotropic half-space; in effect, piezoelectricity has caused the plane wave to become bound to the surface. However the wave is not very strongly bound, even in a strongly piezoelectric material; for example, in cadmium sulphide the penetration depth is typically 4 wavelengths for a metallised surface, changing to 44 wavelengths for a free surface. The velocity is very close to the velocity of slow shear waves. For the same orientation there is also a separate non-piezoelectric Rayleigh wave solution, with its displacements in the sagittal plane. For a given surface orientation the Bleustein–Gulyaev solution is found over a range of propagation directions, though not usually for all directions in the surface [56].

The Rayleigh wave and Bleustein–Gulyaev wave solutions are strongly related to the Rayleigh and SH wave solutions for an isotropic half-space, and these are in turn related to the solutions for a layered half-space. These relationships are summarised in Table 2.1, which also includes other solutions described below.

2.3.4. Other Solutions

In addition to the above surface wave solutions, the boundary conditions for a piezoelectric half-space can sometimes be satisfied by a plane shear wave propagating parallel to the surface, as in the isotropic case.

TABLE 2.1

Isotropic half-space		Anisotropic half-space
Layered	Non-layered	
Rayleigh waves (dispersive)	Rayleigh wave	Rayleigh wave Pseudo-surface wave Leaky surface wave
Love waves (dispersive)	SH plane wave	Bleustein–Gulyaev wave Plane wave

There can also be in some cases yet another type of surface wave solution, known as a *pseudo-surface wave*. Rather surprisingly, the pseudo-surface wave has a phase velocity higher than that of the slow shear plane wave, though it is less than that of the fast shear wave. Despite this, the pseudo-surface wave is a true surface wave, since its displacements decay exponentially with depth. This behaviour is possible because the displacement of the pseudo-surface wave is perpendicular to the displacement of the slow shear wave, so that the surface wave does not have a partial wave component corresponding to the slow shear wave. An example is the Y–Z plane of quartz [48, 57] which gives the velocities shown in Figure 2.10. For all propagation directions in this plane there is a Rayleigh wave solution, but in addition a pseudo-surface wave exists for propagation at an angle 153° away from the Z-axis. The Rayleigh wave and pseudo-surface wave are both piezoelectric. The plane shear wave velocities for an unbounded medium are also shown.

For propagation in a slightly different direction it is found that the boundary conditions cannot be satisfied without including a partial wave corresponding to the slow shear wave. This implies that a surface wave solution cannot exist, since the slow

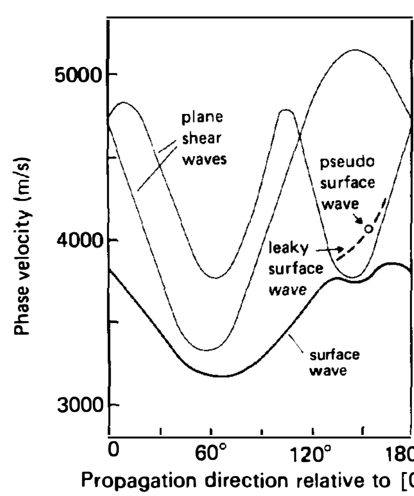

FIGURE 2.10. Velocities for surface waves and leaky waves on the YZ plane of quartz. After Farnell [48], with permission.

shear wave component will carry energy away from the surface. However, if the slow shear component only contributes a small fraction of the total displacement, the energy lost in this way will be small. In this case there is a solution with characteristics very similar to a true surface wave, except that it has a small amount of attenuation. This is known as a *leaky surface wave*. The solution may be found by the procedure outlined above (Section 2.3.2) for surface waves, allowing the wave number β to become complex. For propagation on the Y–Z plane of quartz, the velocity of the leaky surface wave is shown in Figure 2.10. For a range of propagation directions covering about 30°, the attenuation of the wave is less than 0.006 dB per wavelength. Experimentally, an attenuation as low as this is difficult to detect, and the wave appears to behave as a true surface wave.

Pseudo-surface waves and leaky waves are known to exist in many materials, including lithium niobate [59, 60], bismuth germanium oxide [60], and many non-piezoelectric anisotropic materials. In most cases the attenuation of the leaky wave is substantially larger than that of the quartz case described above.

2.3.5. Materials for Devices

For a surface-wave device the choice of an appropriate material is a vital part of the design procedure. Nearly always, the material and orientation are chosen such that only one surface wave mode can be excited. Bleustein–Gulyaev waves are usually excluded because their large penetration depths would require thicker substrates in order to avoid coupling to the rear surface.

It is usual to choose a material and orientation such that the one surface wave mode is a piezoelectric Rayleigh wave. Strong piezoelectric coupling, as shown by a relatively large difference between the free-surface and metallised velocities, is sometimes, but not always, desirable. A pure mode direction is usually chosen as the axis of the device. There are also many other factors such as temperature effects, diffraction and attenuation, and these will be considered later, in Chapter 6. The commonest materials used in devices are quartz and lithium niobate.

Perturbations Due to Thin Films. The effect of a layer on the surface of a half-space was considered for isotropic materials in Section 2.2.4, where it was shown that the layer causes Rayleigh waves to become dispersive, and a series of modes can exist. For anisotropic materials the behaviour is much more complex, though there are strong parallels with the isotropic case.

The case of most interest concerns a conducting metal layer on a piezoelectric substrate, since a metal layer is used for transducers and other structures. We consider a half-space with a uniform layer, as in Figure 2.5. The layer material can be taken to be isotropic, and it is assumed that the wave is of the piezoelectric Rayleigh type. A very thin layer has the effect of reducing the wave velocity because of the change of electrical boundary conditions, as discussed above. This effect is known as *electrical loading*. If the layer thickness is finite its elastic properties become relevant, causing an additional dispersive change of velocity, and this is known as *mass loading*. At high frequencies a series of higher modes can sometimes exist, as in the isotropic case (Figure 2.5), but these are not usually relevant in surface-wave devices.

In practical cases the velocity changes are small; electrical loading changes the velocity by at most a few percent, while mass loading must be controlled to avoid undue dispersion. For such cases a perturbation theory has been developed to enable the velocity change to be evaluated approximately, using a calculation much simpler than the exact analysis. An important result of this theory is a relation giving the velocity change due to electrical loading. If v_0 is the free-surface Rayleigh wave velocity and v_m the metallised velocity, the relation is [32, 49, 61]

$$\frac{v_m - v_0}{v_0} \approx -(\varepsilon_0 + \varepsilon_p^T)|\Phi(0)|^2\omega/(4P_s), \tag{2.48}$$

where

$$\varepsilon_p^T = [\varepsilon_{33}^T\varepsilon_{11}^T - (\varepsilon_{13}^T)^2]^{1/2}.$$

The permittivity components ε_{ij}^T are measured at constant stress, as in equation (2.12). Here $\Phi(0)$ is the potential at the surface $x_3 = 0$ for the free-surface case, and P_s is the power carried by this wave per unit length in the x_2 direction. The perturbation theory can also be applied to many other problems, including mass loading due to an isotropic film [32, 49].

Some special properties of the layered system are sometimes exploited in devices. For example, surface waves can be generated on a non-piezoelectric substrate by first depositing a piezoelectric film and then fabricating an interdigital transducer in the usual way. An example is a zinc oxide layer on silicon. Thin films are also used to guide surface waves, or to deliberately introduce dispersion. For these reasons, waves in layered anisotropic media have been studied quite extensively [44, 60].