APPENDIX I

QUARTZ AT-TYPE FILTER CRYSTALS FOR THE FREQUENCY RANGE 0.7 to 60 MC

The following is incorporated from the manuscript of a forthcoming article by Dr. R. Bechmann without editorial modifications, that is, retaining Dr. R. Bechmann's symbols, and the metric system of units. The only exception is that Figures 55 and 56, originally one figure, have been separated, and slight changes made in the text accordingly. A few notes by the editors are added.

"QUARTZ AT-TYPE FILTER CRYSTALS FOR THE FREQUENCY RANGE 0.7 to 60 MC"

by

R. Bechmann

U. S. Army Signal Research and Development Laboratory
Fort Monmouth, New Jersey

1. INTRODUCTION

For application of quartz crystals to filters, the knowledge of the motional parameters, i.e., the capacitance C_1 or inductance L_1, the resistance R_1 or Q, and the shunt capacitance C_o of the crystal is necessary. In addition, the crystal must be free from unwanted modes in a specific range in the vicinity of the wanted frequency.

In the following, design data for AT-type crystals in the frequency range 0.7 to 60 mc are given, leading to a suppression of unwanted modes. In general, unwanted modes must be suppressed at least to 40 db below the main mode.

The behavior of unwanted modes in thickness-shear vibrating quartz plates e.g., AT fundamental or overtone crystals is determined by:

(a) The ratio of plate diameter, ϕ_a, to plate thickness, t: $\frac{\phi_a}{t}$

(b) The ratio of electrode diameter, ϕ_e, to plate thickness, t: $\frac{\phi_e}{t}$

The electrodes are assumed to be circular and accurately centered.

2. QUARTZ CRYSTALS HAVING A DIAMETER-THICKNESS RATIO LARGER THAN 60

For quartz crystals for frequencies higher than 10 mc the diameter-thickness ratio is usually larger than 60. When the diameter-thickness ratio is larger than 60, unwanted modes depend in first approximation on the electrode diameter to thickness ratio and are practically independent of the plate diameter to thickness ratio.

A. Circular Quartz Crystals in the Range 10 to 30 mc AT-Type, Fundamental

It has been found that unwanted modes are suppressed to 50 db and more below the main mode when the electrode diameter to thickness ratio equals 18.

B. Triangular Quartz Crystals in the Range 10 to 30 mc AT-Type, Fundamental

Triangular shaped quartz crystals[1] are very suitable for filter crystals when the electrode diameter-thickness ratio is $\frac{\phi_e}{t} = 24$ or less.

The performance of triangular crystals is superior when one side of the triangle is parallel to the X-axis. The plate size of an equilateral triangle is defined by the circumscribed circle and the length of the triangle equals the radius of the circumscribed circle multiplied by the cos 30°, ($\frac{\sqrt{3}}{2} = 0.8660$).

C. Circular Quartz Crystals in the Range 30 to 60 mc AT-Type, Third Overtone

Circular plates are sufficiently free of unwanted modes for an electrode to thickness ratio $\frac{\phi_e}{t} = 10$. Triangular crystals behave similarly to circular crystals. The limitation for practical application is the electrode size which becomes very small for higher frequencies.

D. The Motional Capacitance

The motional capacitance C_1 follows from the equation

$$C_1 = \Gamma \frac{a}{t} \qquad (1)$$

where Γ * is the motional capacitance constant measured in 10^{-6} pF mm^{-1}, "a" the area of the electrode measured in mm^2, "t" is the thickness of the plate measured in mm and the motional capacitance C_1 measured in pF. The motional capacitance constant Γ depends slightly on the electrode diameter ϕ_e and the values for Γ are given in Table 6. The values of Γ for the fundamental mode are fairly accurate (±5%), however the accuracy of Γ for the third overtone is estimated to be ±10% and may be improved. Thickness "t" and frequency "f" are related by the frequency constant $N = f \cdot t$ when f is measured in mc: for the fundamental mode $N_1 = 1.660$ mc mm, for the third overtone $N_3 = 4.980$ mc mm.

* For the definition of Γ , see the "I.R.E. Standards on Piezoelectric Crystals: Determination of the Elastic, Piezoelectric and Dielectric Constants - The Electromechanical Coupling Factor," Proceedings of the I.R.E., vol. 46, 1958, pp. 765-778, (April, 1958).

Equation (1) can be written as

$$C_1 = \frac{K}{f} \qquad (2)$$

where

$$K = N\,\Gamma\left(\frac{\phi_e}{t}^2\right)\frac{\pi}{4}\ \text{pF}\cdot\text{mc}\ . \qquad (2a)$$

The values for K for circular quartz plates, fundamental mode (10 to 30 mc) using $\frac{\phi_e}{t} = 18$, are shown in Table 7; for triangular plates, fundamental mode, (10 to 30 mc) when $\frac{\phi_e}{t} = 24$, are shown in Table 8; and for circular plates, third overtone (30 to 60 mc) $\frac{\phi_e}{t} = 10$ are shown in Table 9.

3. QUARTZ CRYSTALS HAVING A DIAMETER-THICKNESS RATIO SMALLER THAN 60

The range for the diameter-thickness ratio smaller than 60 can be divided in the ranges:

(a) $\frac{\phi_a}{t} = 60$ TO 25, (b) $\frac{\phi_a}{t} < 25$.

In the range $\frac{\phi_a}{t} < 25$, the unwanted modes can be suppressed by adequate bevelling of the edges of the plate[2,3]. Bevelling affects the boundary conditions of a plate and removes coupling with other modes, mainly low frequency contour modes. The size of the electrodes becomes unessential. For plates with larger diameter-thickness ratios in the range $\frac{\phi_e}{t} = 25$ TO 60, bevelling becomes less effective with increasing $\frac{\phi_a}{t}$. The parallelism of the plate becomes more and more essential.

The electrode diameter to thickness ratio must be sufficiently small. No general geometric rules can be given for bevelling; bevelling is mainly based upon empirical data. Suppression of unwanted modes can be attained using different bevels.

Small changes of the radius and the extent of the bevel give rise to essential changes in the performance of the crystal. However, for the frequency range .07 to 10 mc, suitable dimensions for both ratios $\frac{\phi_a}{t}$ and $\frac{\phi_e}{t}$ can be found to suppress unwanted modes.

The motional capacitance constant in this range can be calculated from a value for the motional capacitance constant $\Gamma = 210 \ \ 10^{-6}$ pF mm ^{1} ±5%.

Figure 55, where the frequency in the range 0.7 to 60 mc is plotted against the diameter-thickness ratio $\frac{\phi_a}{t}$ and Figure 56 electrode diameter-thickness ratio $\frac{\phi_e}{t}$, give a survey which has been found to result in an optimum suppression of unwanted modes. The symbol ① △1 refers to fundamental mode, circular and triangular shaped crystals respectively; ③ refers to crystals excited in the third overtone. Experimental points for the diameter-thickness ratio $\frac{\phi_a}{t}$ are shown for both plate sizes corresponding to crystal units HC-6/U and HC-18/U which follow on practically straight lines. In the frequency range 1 to 5 mc, two cross-hatched areas are shown: one for the diameter-thickness ratio $\frac{\phi_a}{t}$ and the corresponding range for the electrode diameter-thickness ratio $\frac{\phi_e}{t}$. These areas represent dimensions which have been found preferable by several manufacturers. As we can see from Figure 55, for low frequencies in the range 0.7 to 2 mc, very small diameter-thickness ratios can be used provided the plates are adequately bevelled.

It may be mentioned that the strips forming the electrode connections may be arranged so that they form an angle other than 180° and particularly for crystals for the third overtone, this arrangement is advantageous to suppress unwanted modes.

This information was obtained from measurements by the author over a long period of time. In addition, the author acknowledges with grateful appreciation information obtained from Hermes Electronics Company, U.S. Army Signal Research and Development Laboratory, Fort Monmouth, Contracts Nos. DA36-039-sc-73234 and DA36-039-sc-78242; James Knights Company, McCoy Electronics Company, Piezo Crystal Company, Reeves-Hoffman Corp. and Union Thermoelectric Corporation.

REFERENCES

1. R. Bechmann, "High-Frequency Quartz Filter Crystals," Proceedings of the I.R.E., vol. 46, 1958, pp. 617-618 (March 1958).

2. R. Bechmann, "Single Response Thickness Shear Mode Resonators Using Circular Bevelled Plates," Journ. Sci. Instruments, vol. 29, 1952, pp. 73-76.

3. R. Bechmann, "Filter Crystals," Proceedings of the Twelfth Annual Symposium on Frequency Control, U. S. Army Signal Research and Development Laboratories, Fort Monmouth, N. J., (6-8 May 1958), pp. 437-474.

TABLE 6

Observed Values for the Motional Capacitance Constant Γ For Fundamental and Third Overtone as Function of the Electrode Diameter

ϕ_e mm	Γ 10^{-6} pF mm^{-1} Fundamental	Γ 10^{-6} pF mm^{-1} 3rd Overtone
0.5	220	24
1.0	220	24
1.5	215	22
2.0	210	20
2.5	205	
3.0	200	
3.5	195	

TABLE 7

DATA FOR CIRCULAR FILTER CRYSTALS, 10 to 30 mc, $\frac{\phi_e}{t} = 18$

f mc	ϕ_e mm	Γ 10^{-6} pF mm^{-1}	K 10^{-4} pF mc	C_1 10^{-4} pF
10	2.988	200	845	84.5
20	1.494	215	908	45.4
30	0.996	220	929	31.0

TABLE 8

DATA FOR TRIANGULAR FILTER CRYSTALS, 10 to 30 mc, $\frac{\phi_e}{t} = 24$

f mc	ϕ_e mm	Γ 10^{-6} pF mm^{-1}	K 10^{-4} pF mc	C_1 10^{-4} pF
10	3.984	190	142.7	142.7
20	1.992	210	157.7	78.9
30	1.328	217	163.0	54.3

TABLE 9

DATA FOR FILTER CRYSTALS, 30 to 60 mc, $\frac{\phi_e}{t} = 10$

f mc	ϕ_e mm	Γ 10^{-6} pF mm^{-1}	K 10^{-4} pF mc	C_1 10^{-4} pF
30	1.660	21.5	84.1	2.80
40	1.245	22.7	88.8	2.22
50	0.996	24	93.9	1.88
60	0.830	24	93.9	1.57

FIGURE 55

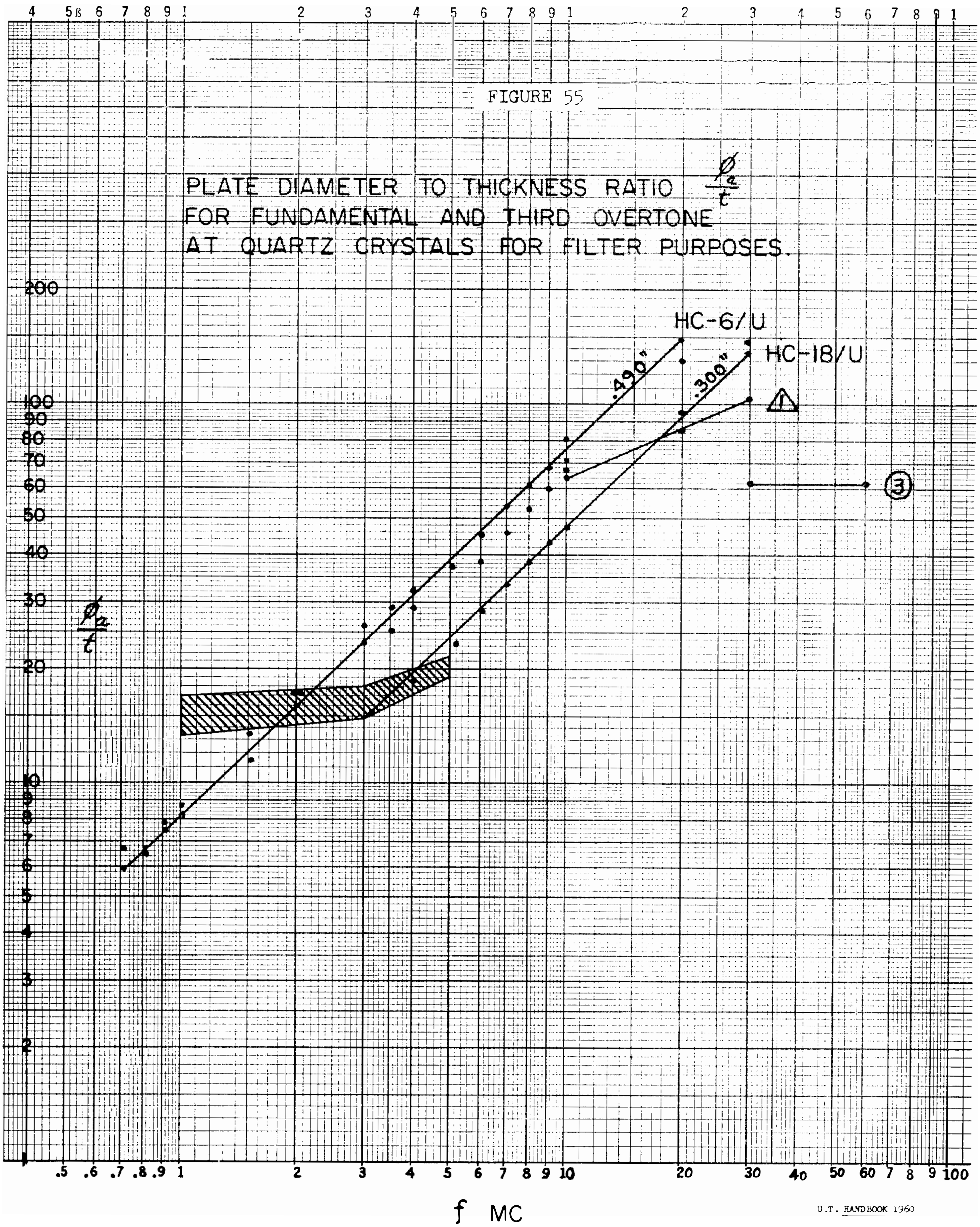


PLATE DIAMETER TO THICKNESS RATIO Ø/t
FOR FUNDAMENTAL AND THIRD OVERTONE
AT QUARTZ CRYSTALS FOR FILTER PURPOSES.
HC-6/U
HC-18/U
.490"
.300"
1
3
Ø/t
f MC

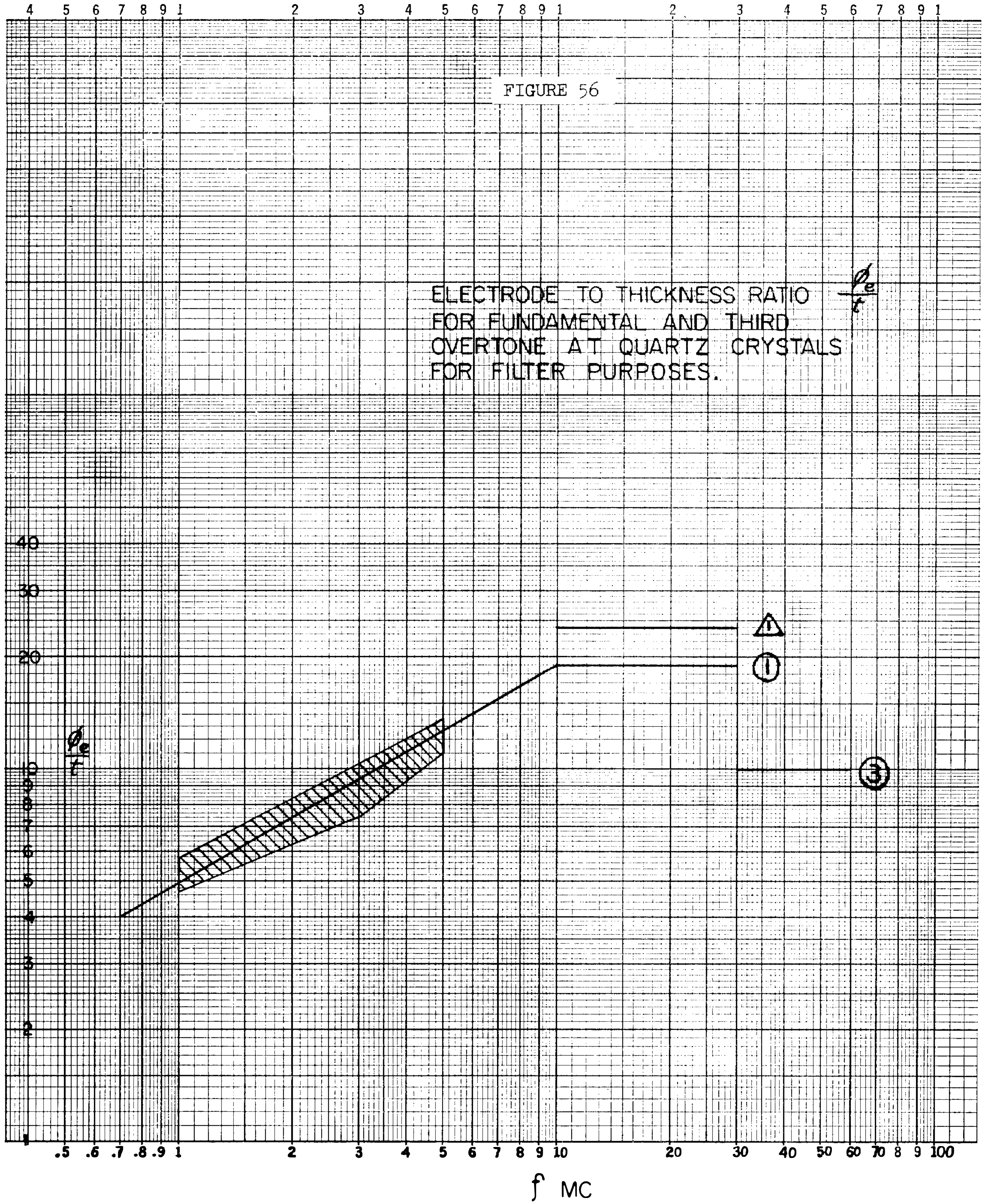


FIGURE 56
ELECTRODE TO THICKNESS RATIO $\frac{\phi_e}{t}$
FOR FUNDAMENTAL AND THIRD
OVERTONE AT QUARTZ CRYSTALS
FOR FILTER PURPOSES.
$\frac{\phi_e}{t}$
f MC

Editorial Notes on Motional Capacitance

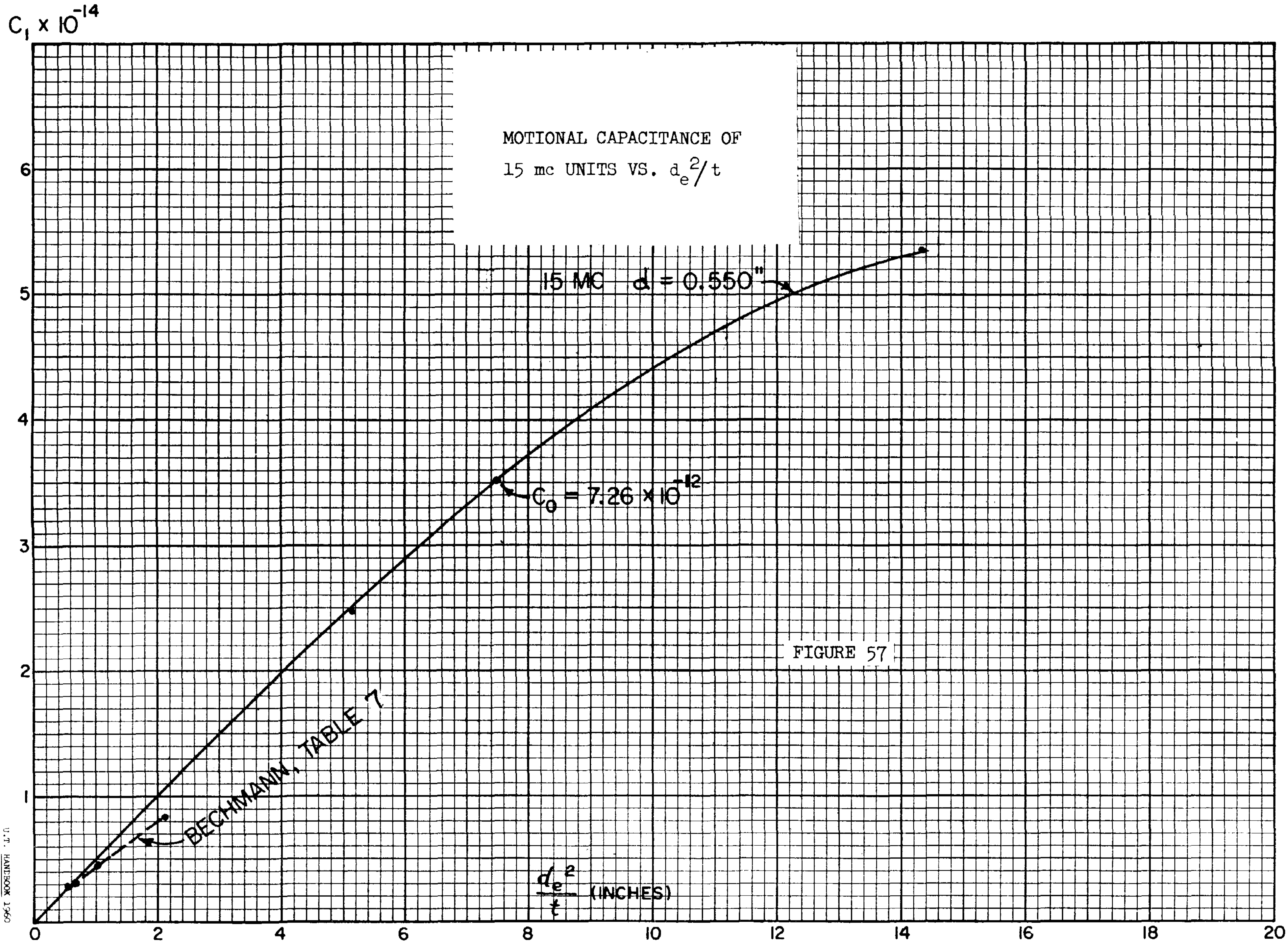
In order to convert the material on motional capacitance from area in mm^2 and thickness in mm, to the electrode diameter in $inches^2$ and the thickness in inches:

$$C_n = 0.002\,\Gamma\,\frac{d_e^{\,2}}{t}\,.$$

Figures 57 and 58, added by the editors are examples of measurements made by Union Thermoelectric on specific cases. Dr. Bechmann's values are plotted beside them. There appears to be no question about the general shape of the curve, C_n vs. d_e^2/t, that is, that "The motional capacitance constant Γ depends slightly on the electrode diameter." About the exact values, and the effect upon these values of the thickness and diameter of the plate there is considerable uncertainty. Therefore, examples only are given here, and no attempt is made to give a general formula.

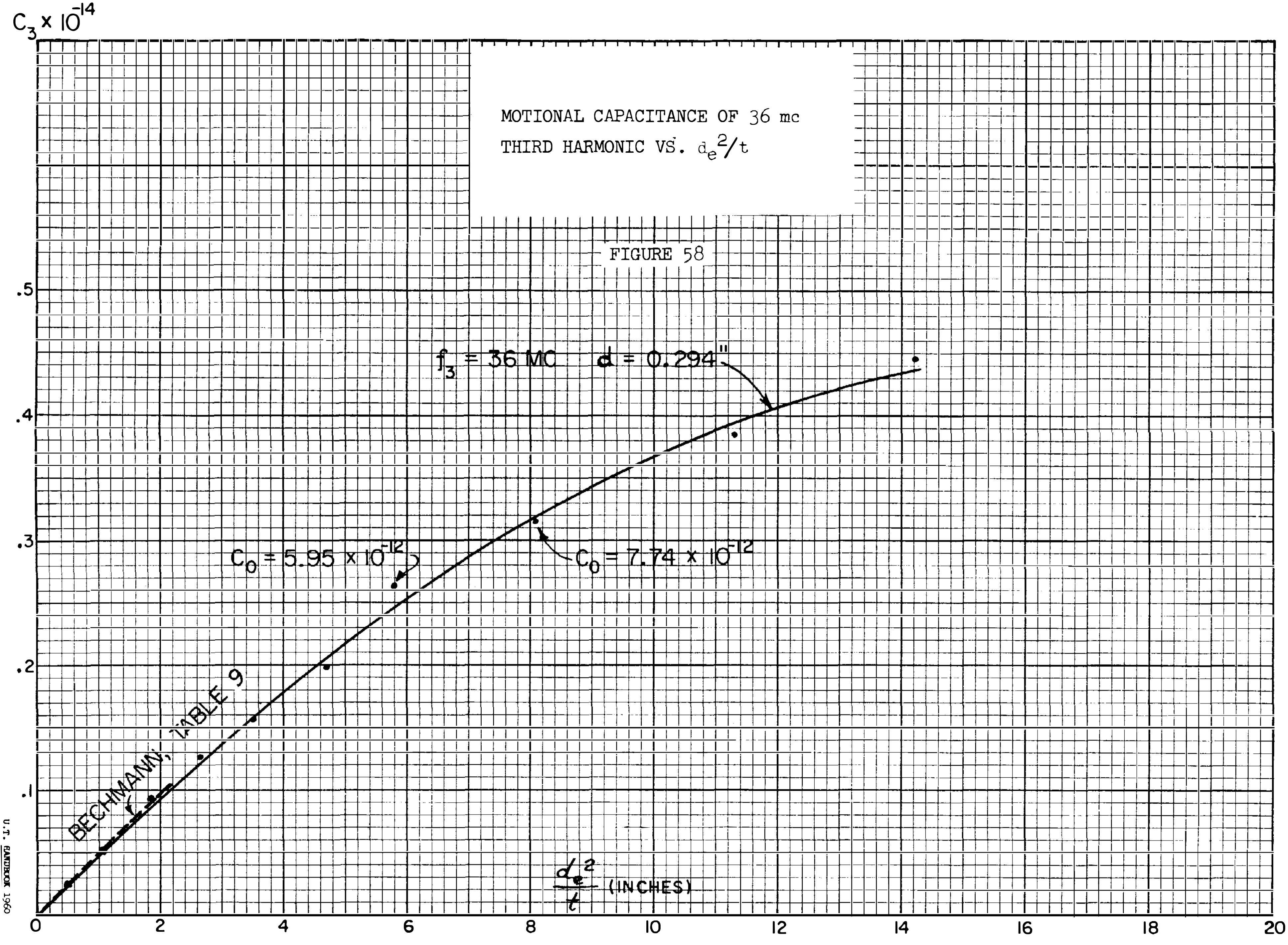
It will be noted that in no part of this Appendix do the values given for C_1 and C_3 correspond to an n^2 ratio. From available data it appears that when the same unit is measured on different orders C_1/C_3 may become as small as 9 and C_1/C_5 as small as 25 under certain conditions, among which is a very small electrode. For typical units with about 7 μμf shunt capacitance C_1/C_3 is likely to be in the neighborhood of 10 or 11 rather than 9, and C_5/C_1, to be greater than 30. Likewise C_3/C_5 is likely to be between 3 and 4 rather then 25/9 = 2.78.

Figures 59-64 show the motional capacitance and reactance, as measured, for several cases of the contoured plate.



MOTIONAL CAPACITANCE OF 15 mc UNITS VS. d_e^2/t

FIGURE 57



$C_3 \times 10^{-14}$
MOTIONAL CAPACITANCE OF 36 mc THIRD HARMONIC VS. d_e^2/t
FIGURE 58
f_3 = 36 MC d = 0.294"
$C_0 = 5.95 \times 10^{-12}$
$C_0 = 7.74 \times 10^{-12}$
BECHMANN, TABLE 9
$\frac{d_e^2}{t}$ (INCHES)
.1
.2
.3
.4
.5
0
2
4
6
8
10
12
14
16
18
20

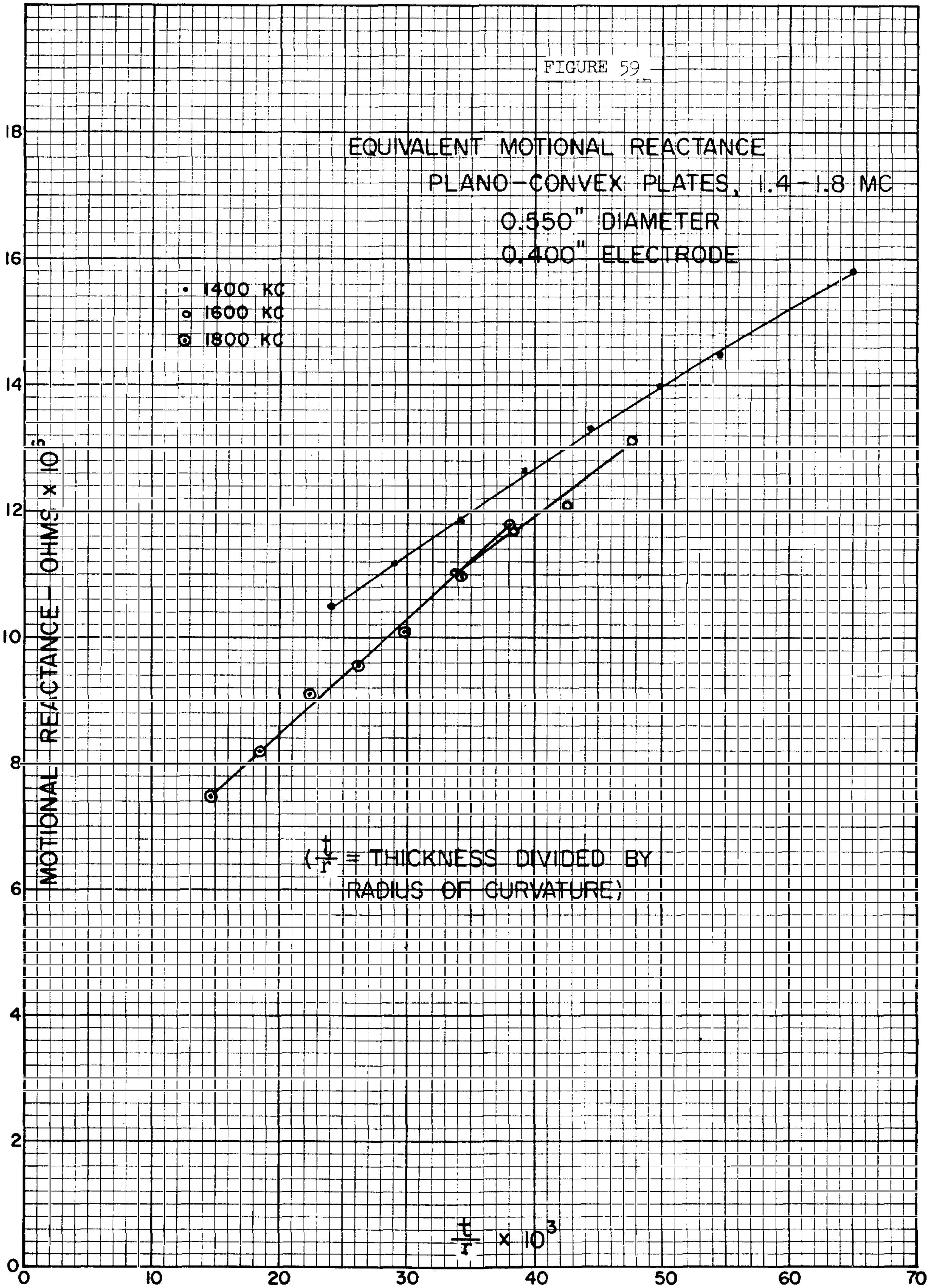


FIGURE 59
EQUIVALENT MOTIONAL REACTANCE
PLANO-CONVEX PLATES, 1.4-1.8 MC
0.550" DIAMETER
0.400" ELECTRODE
• 1400 KC
o 1600 KC
⊙ 1800 KC
($\frac{t}{r}$ = THICKNESS DIVIDED BY RADIUS OF CURVATURE)
MOTIONAL REACTANCE - OHMS x 10^5
$\frac{t}{r} \times 10^3$
0
2
4
6
8
10
12
14
16
18
0
10
20
30
40
50
60
70

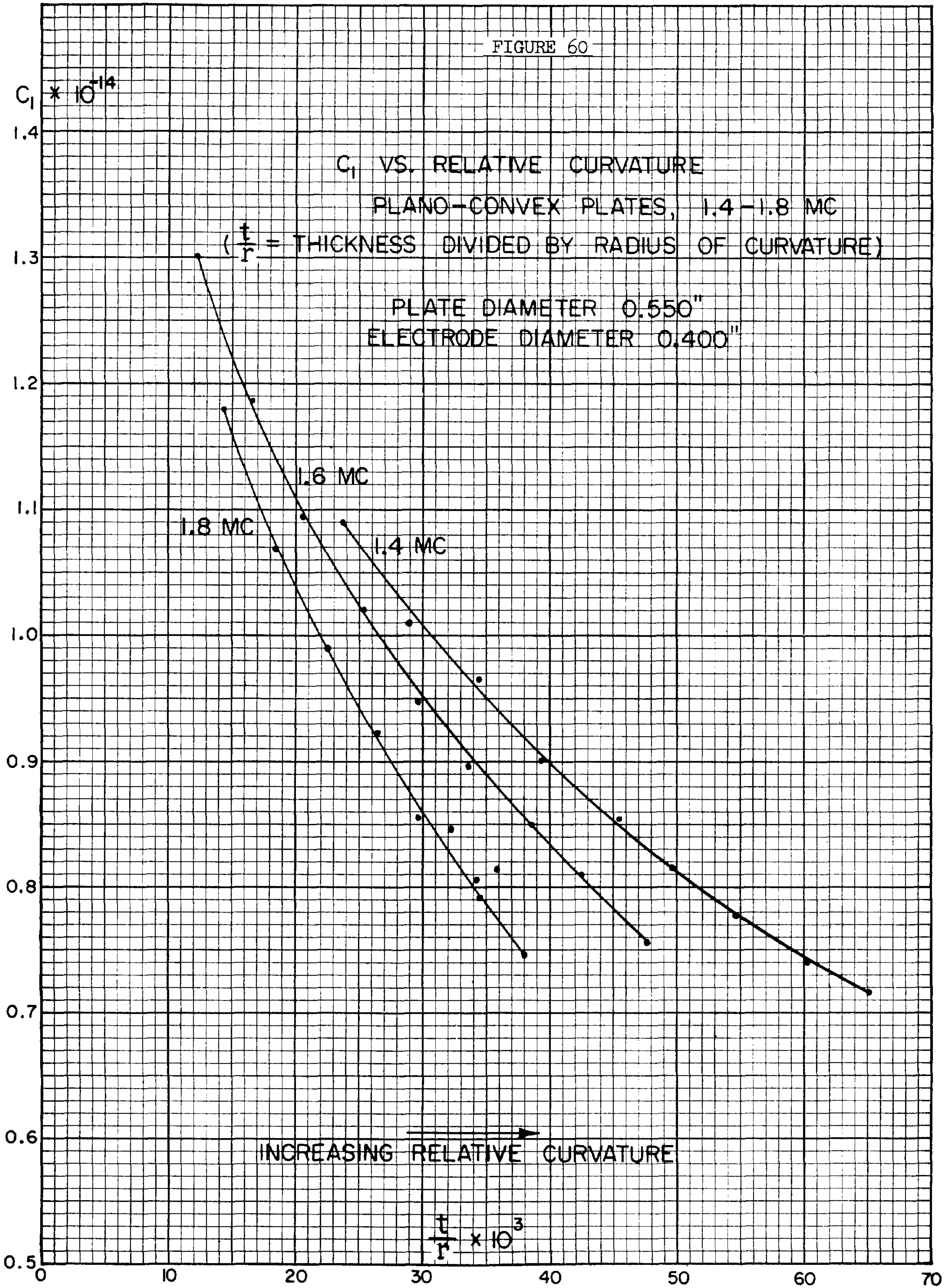
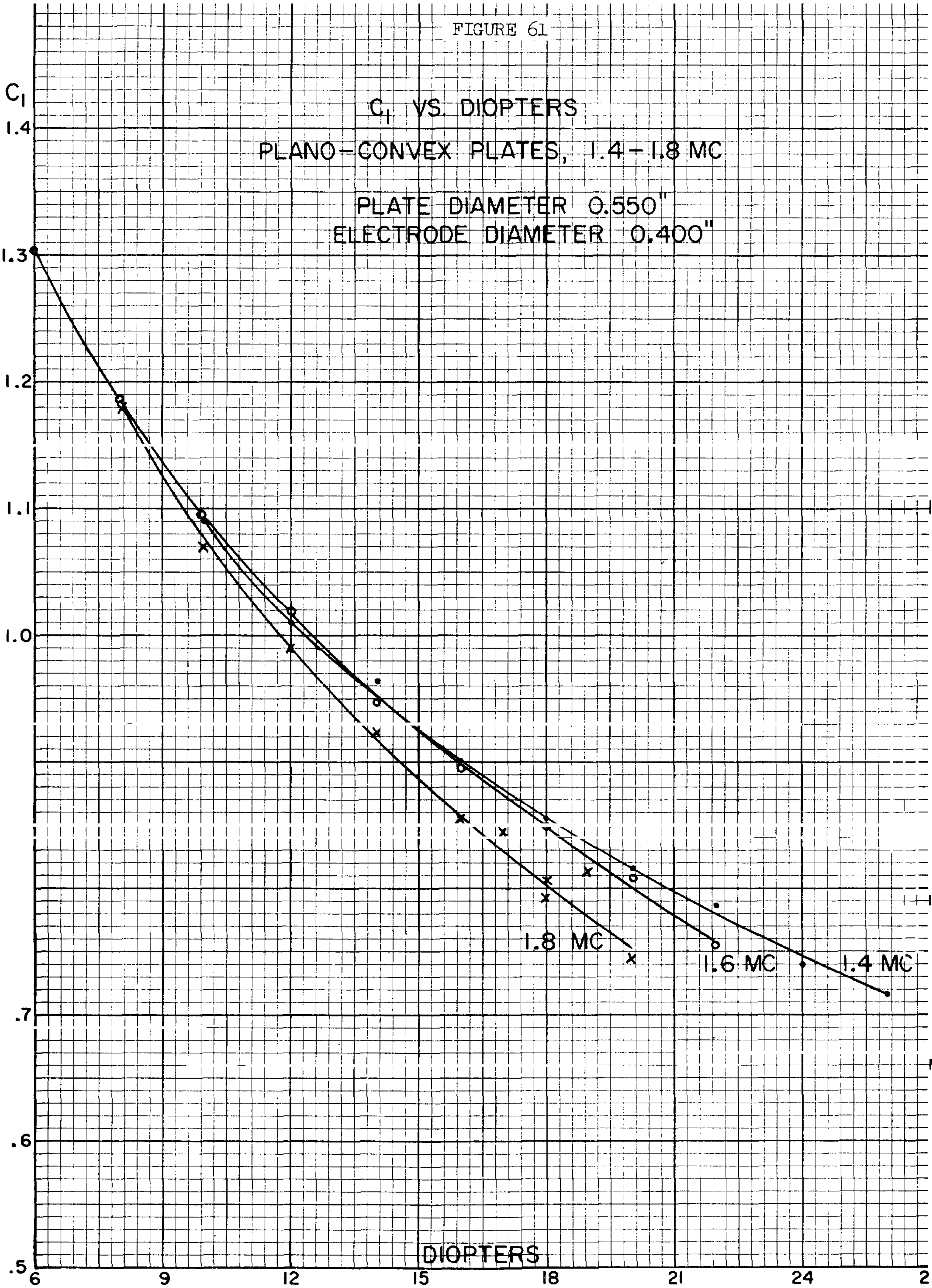


FIGURE 60
C_1 VS. RELATIVE CURVATURE
PLANO-CONVEX PLATES, 1.4 - 1.8 MC
($\frac{t}{r}$ = THICKNESS DIVIDED BY RADIUS OF CURVATURE)
PLATE DIAMETER 0.550"
ELECTRODE DIAMETER 0.400"
$C_1 \times 10^{-14}$
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
1.6 MC
1.8 MC
1.4 MC
INCREASING RELATIVE CURVATURE
$\frac{t}{r} \times 10^3$
0
10
20
30
40
50
60
70

FIGURE 61


C1
C1 VS. DIOPTERS
PLANO-CONVEX PLATES, 1.4 - 1.8 MC
PLATE DIAMETER 0.550"
ELECTRODE DIAMETER 0.400"
1.4
1.3
1.2
1.1
1.0
.7
.6
.5
1.8 MC
1.6 MC
1.4 MC
6
9
12
15
18
21
24
27
DIOPTERS

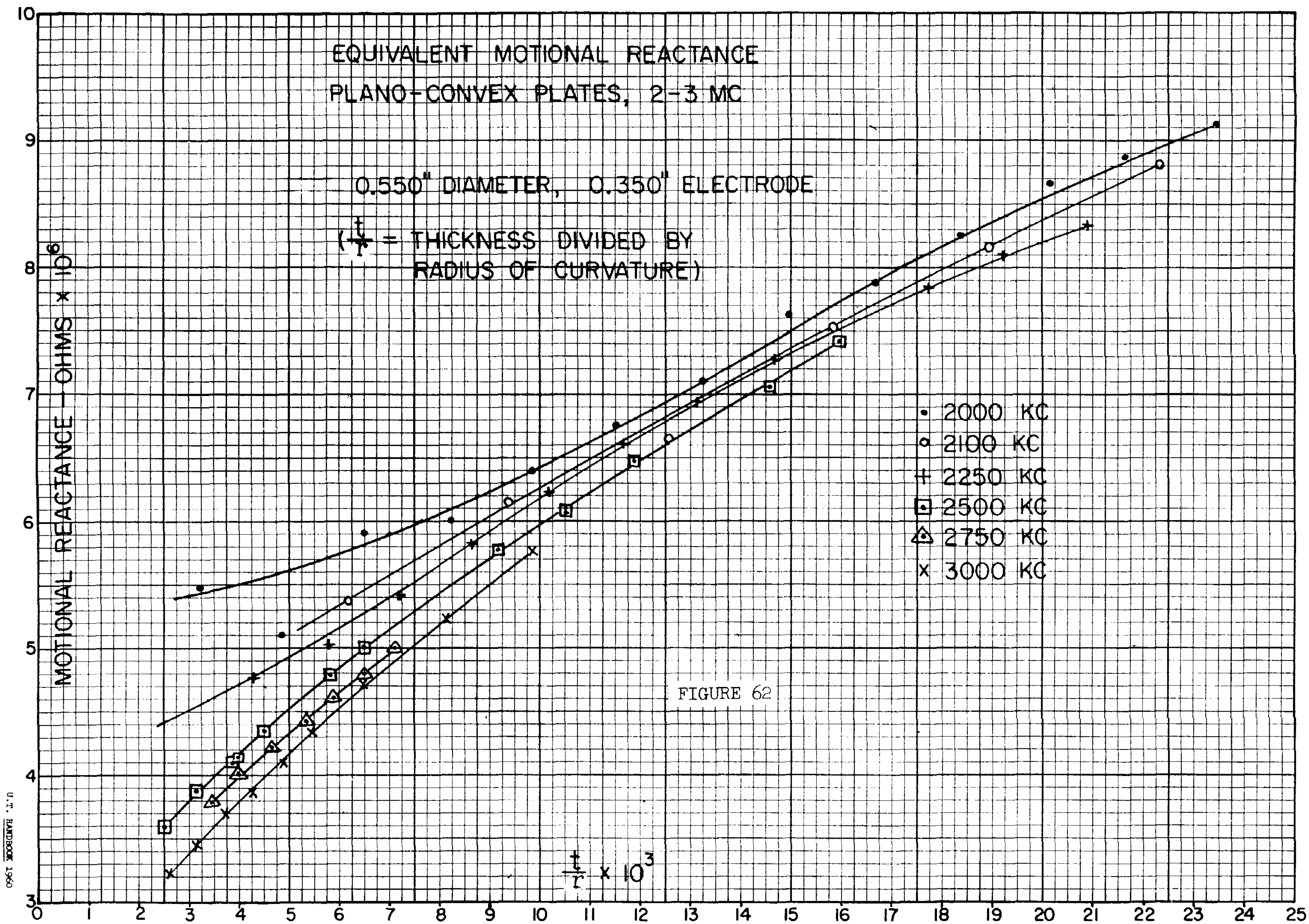

EQUIVALENT MOTIONAL REACTANCE
PLANO-CONVEX PLATES, 2-3 MC
0.550" DIAMETER, 0.350" ELECTRODE
(t/r = THICKNESS DIVIDED BY RADIUS OF CURVATURE)
MOTIONAL REACTANCE - OHMS × 10^6
t/r × 10^3
• 2000 KC
o 2100 KC
+ 2250 KC
⊡ 2500 KC
△ 2750 KC
× 3000 KC

FIGURE 62

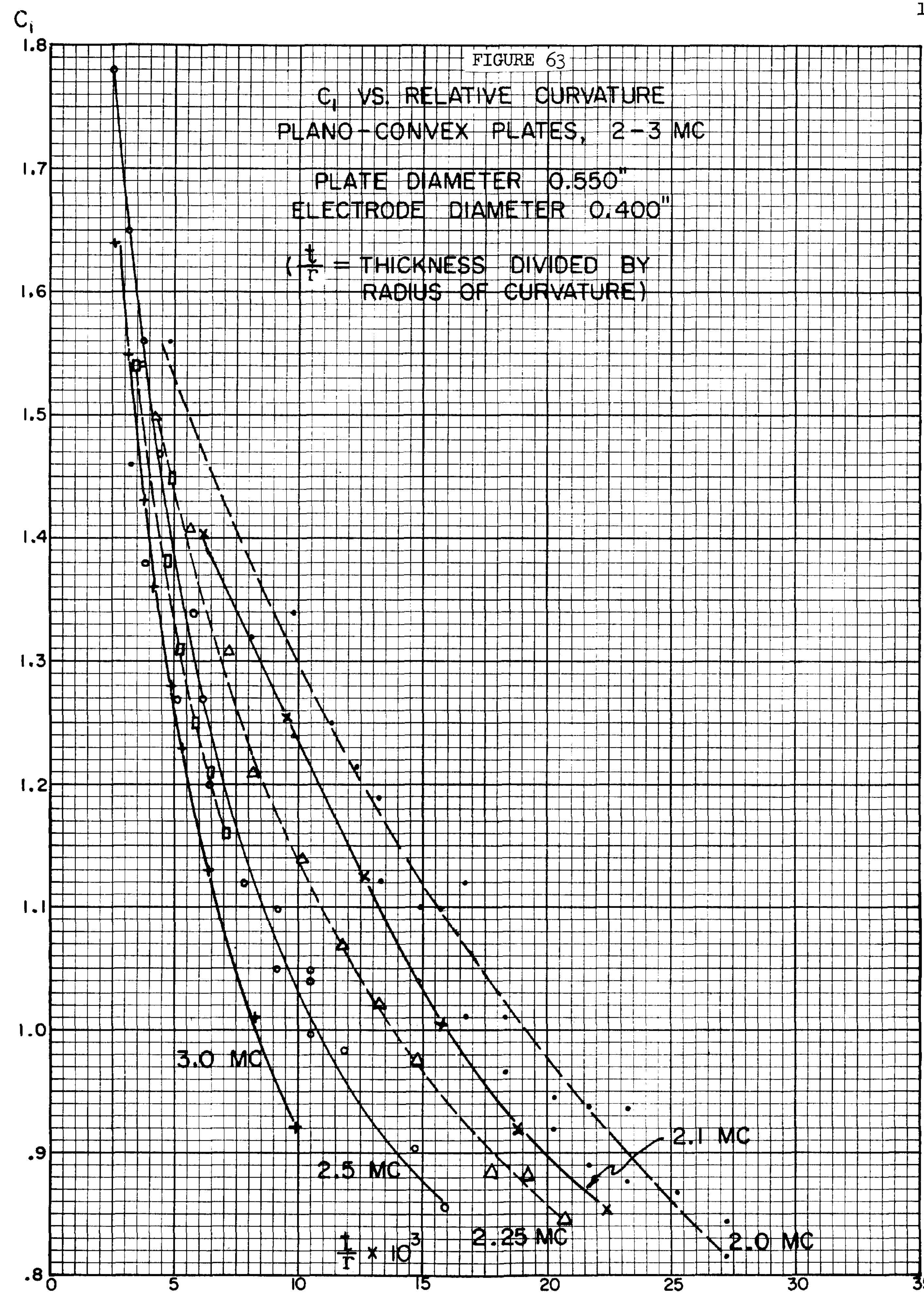


FIGURE 63
C_i VS. RELATIVE CURVATURE
PLANO-CONVEX PLATES, 2-3 MC
PLATE DIAMETER 0.550"
ELECTRODE DIAMETER 0.400"
($\frac{t}{r}$ = THICKNESS DIVIDED BY RADIUS OF CURVATURE)
C_i
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
.9
.8
0
5
10
15
20
25
30
35
3.0 MC
2.5 MC
2.25 MC
2.1 MC
2.0 MC
$\frac{t}{r} \times 10^3$

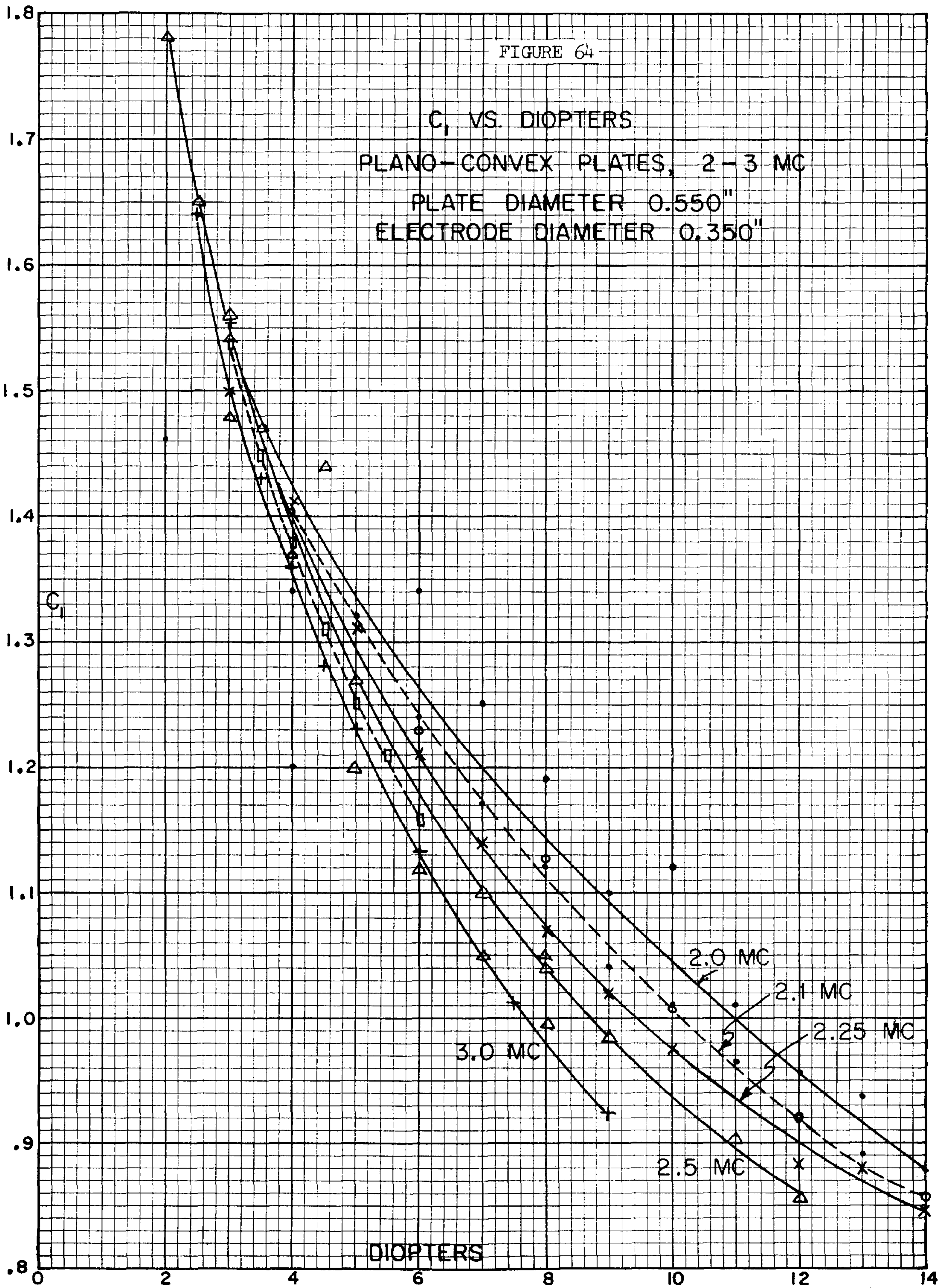


FIGURE 64
C_1 VS. DIOPTERS
PLANO-CONVEX PLATES, 2-3 MC
PLATE DIAMETER 0.550"
ELECTRODE DIAMETER 0.350"
C_1
DIOPTERS
2.0 MC
2.1 MC
2.25 MC
2.5 MC
3.0 MC
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
.9
.8
0
2
4
6
8
10
12
14