TEMPERATURE COEFFICIENT OF FREQUENCY

The AT-Cut

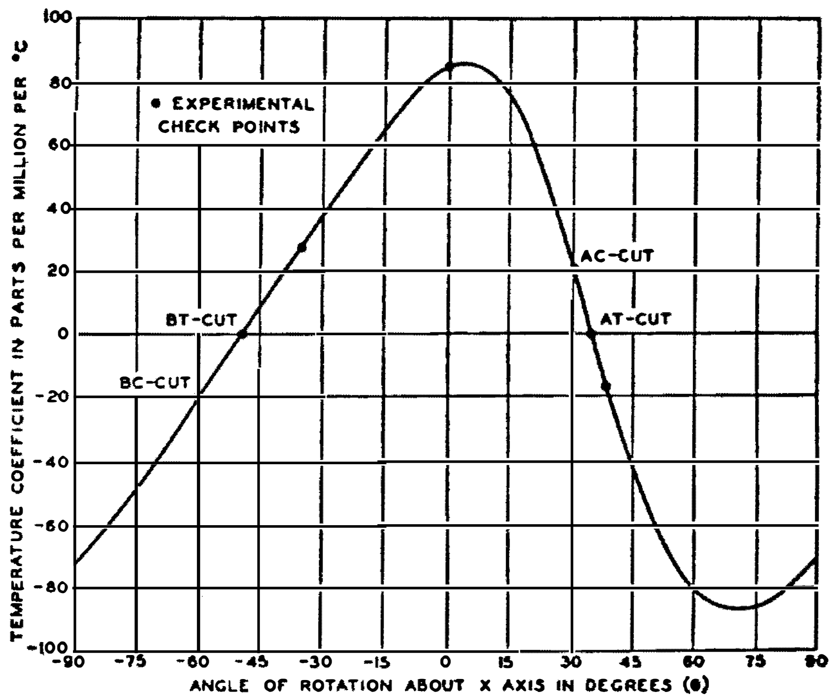
There are two common methods of specifying the frequency tolerance of a quartz resonator plate:

1. A percentage of the nominal frequency plus or minus deviation from the nominal over a specified temperature range. This type of specification is often used for units to be operated over a wide temperature range, such as -55° to +90°C. It has the effect of lumping together in one figure the frequency deviation resulting from temperature change, the inaccuracy in adjusting the resonator to nominal frequency, and short term aging.

2. A percentage of the nominal frequency at room temperature, or at the operating temperature of a unit to be operated in an oven ("controlled temperature operation"), and a percentage of the actual frequency at this temperature point over a temperature range. This type of specification can be used either for units to be operated over a wide temperature range or for units to be operated under temperature control, but is most often used for units intended for temperature control. The control temperature may be specified as a single temperature ± a small tolerance, or as any temperature in a specified range ± a small deviation from the operating temperature of the individual unit.

The major factor determining the frequency deviation with temperature change of a resonator unit is the orientation of the resonator plate with reference to the natural crystallographic axes.

FIGURE 35



Temperature coefficients of oriented Y cut crystals

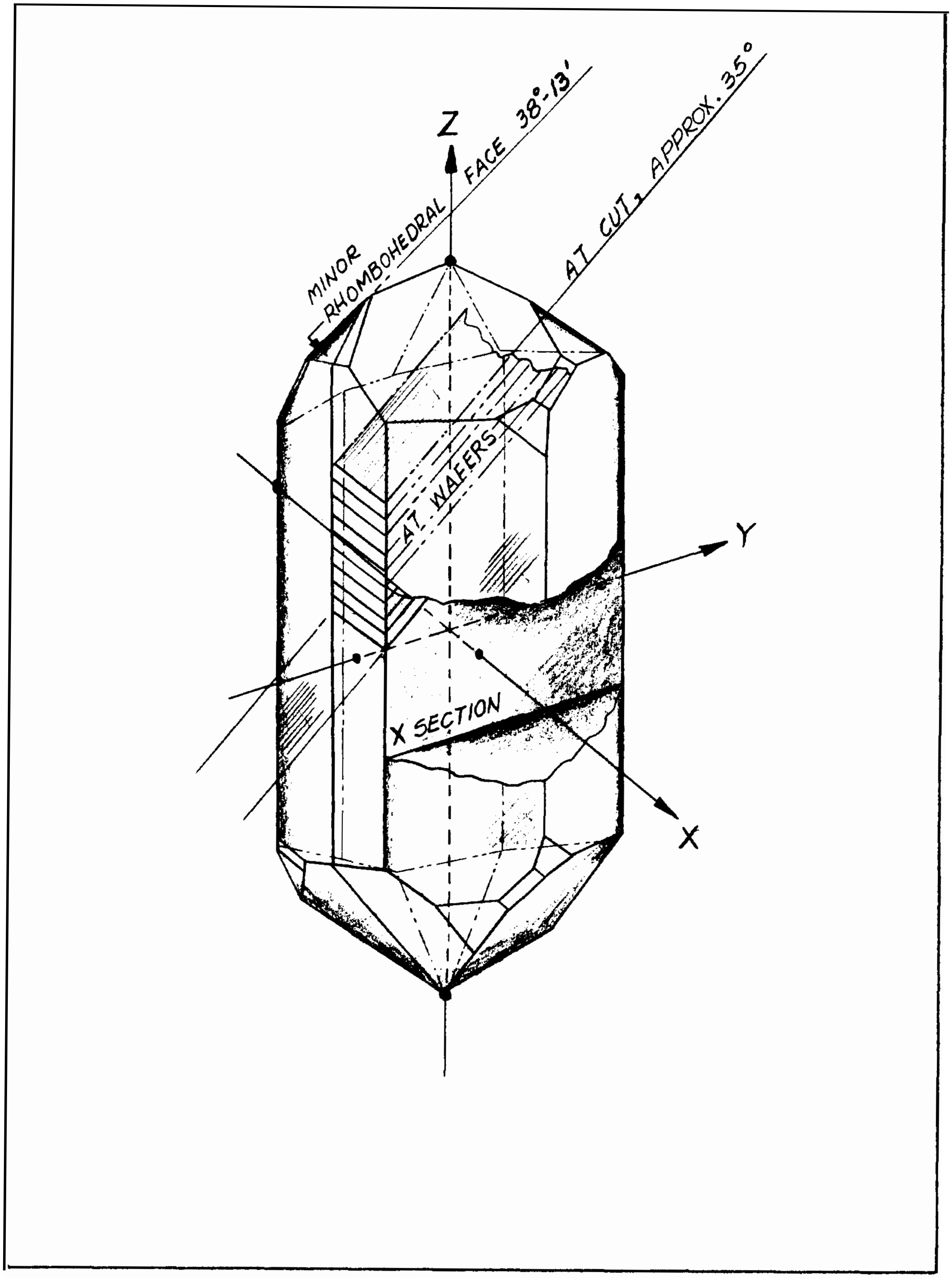
From Raymond A. Heising, Quartz Crystals for Electrical Circuits. Copyright 1946, D. Van Nostrand Co., Inc., Princeton, New Jersey, Chapter VI, which see for a more thorough treatment of the entire subject.

Definitions

The zero-temperature-coefficient angle is the angle which results in a frequency-temperature curve with zero slope at only one temperature. For the AT cut this temperature is usually estimated to be between 20° and 30°C.* In Figures 38-40 this is the curve designated as 0'. (It is to be noted that this does not mean 35°0', but stands for a value which must be determined by the category of the resonator, as in Table 4.)

* Quartz can be grown, with the additions of impurities, to have inflection temperatures above this range. See D. L. Hammond, A. R. Chi, and J. M. Stanley, Engineering Report E-1162, Effects of Impurities on the Resonator and Lattice Properties of Quartz, (Signal Corps Engineering Laboratories, Fort Monmouth, N. J., 3 November, 1955). Practical application of this fact has not been developed. Also see Clevite Corporation's Patent No. 2-871-192, January 27, 1959.

FIGURE 36



ORI

QUARTZ STONE, SHOWING X AND Z AXES, MINOR RHOMBOHEDRAL FACE, ONE SURFACE OF X SECTION EXPOSED, AND AT WAFERS IN X SECTION.

The minimum-total-deviation angle or optimum angle is the angle which, for some specific temperature range, will result in a minimum frequency excursion. In Figure 38, if the temperature range is 0° to +52°C (read assumed real temperatures in brackets in the figure), maximum total deviations, analyzed by finding maximum in each quadrant.

zero-temperature-coefficient angle,	+0'	0 / −1.7*	+2.3* / 0	4.0 ppm
zero-temperature-coefficient angle,	$+\frac{1}{2}$'	+.3 / −.5*	+.9* / −.3	1.4 ppm
zero-temperature coefficient angle,	+1'	−.9* / −.6	−.2 / −.9*	1.8 ppm

Therefore, the optimum angle for this temperature range is approximately $\frac{1}{2}$' higher than the zero-temperature coefficient angle.

Further examination of Figure 38 (or of Figure 39 or 40 on a smaller scale) will show that if the specified temperature range is symmetrical about the reference temperature, and is from 80°C below the reference temperature to 80°C above the reference temperature, minimum frequency deviation will be achieved with an angle approximately 6' above the zero angle. If the reference temperature were exactly +25°C (not +26° as suggested in the figures), the familiar specification -55°C to +105°C would have an optimum angle nearly 6' higher then the zero angle. Moreover, study of the figures will show that since the optimum angle in this case is determined by the deviation at the two turning points where the curve changes direction and by either extreme of the temperature range, the same will hold if the temperature range on the high side is lowered to +90°C.

The design-center angle is midway between the lowest angle which will just meet a specific temperature-frequency-deviation specification and the highest angle which will just meet the same specification. That is, whereas, the optimum or minimum deviation angle is defined in terms of temperature range alone, the design-center angle is defined in terms of temperature range and specified maximum frequency deviation. As the specified maximum frequency deviation is made smaller the design-center angle approaches the optimum angle.

THE DESIGN-CENTER ANGLE IS THE MOST USEFUL FOR THE PRACTICAL DESIGN OF RESONATOR UNITS.

For narrow ranges of oven-controlled units, such as 75°C ±5°C, the difference between the design-center angle and the optimum angle is not significant.

The name and definition of the design-center angle also assume a certain type of manufacturing, as will appear at the end of the following example.

If the temperature range is -55° to +90°C and the maximum frequency deviation is ±0.005% or 50 ppm, we make the following notations from Figure 39, which, for convenience, we consider as divided into quadrants:

Upper left: at about -34°C an angle 14' higher than the zero angle has a deviation of +48.5 ppm.

Lower right: at about +88°C this same +14' angle results in a deviation of -48.5 ppm.

Therefore, the highest angle which will meet the specification is only slightly more than 14' higher than the zero angle.

Lower left: the zero angle fails to meet specification, resulting in a deviation of -56 ppm at -55°C. By interpolation it appears that an angle about 1' higher will have a deviation slightly less than -50 ppm at -55°C.

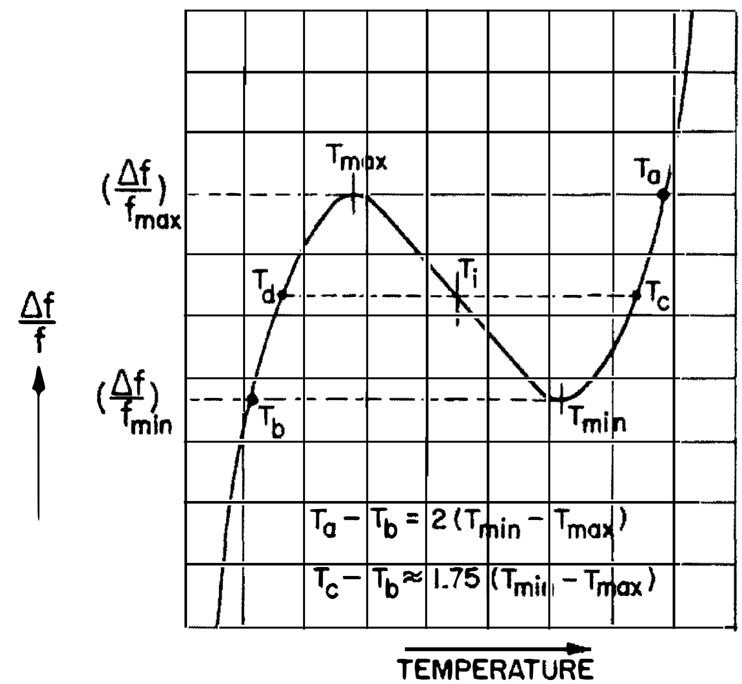
Upper right: Quick inspection shows that an angle considerably less than +1' will meet the specification. Close examination is unnecessary since it has already been established that an angle nearer the zero angle than +1' cannot be used.

Therefore, the angular range is from 1' above the zero angle to +14' above the zero angle, the design center is +7.5' and the tolerance is ±6.5'.

A closer examination of the angle line designated as 0' will show why the definition implies a certain type of manufacturing. Although as presented above the zero angle fails to meet the specification, it results in a total deviation of less than 90 ppm. Therefore, if a manufacturer were to handle plates cut to the zero angle separately and adjust them to nominal frequency at the midpoint of their frequency excursion, they would meet the specification. It is assumed, then, by the definition of design-center angle that all plates intended to meet a certain specification are adjusted to nominal frequency at one temperature.

Turning points (T_{max} and T_{min}) are the temperature points at which the slope of the frequency vs. temperature curve reverses direction (the slope is zero).

FIGURE 37



Based on R. Bechamnn
Proc. I.R.E., 1956, p.1603

T_c and T_d have been added.

At the low temperature turning point the slope changes from positive to negative. This is the temperature (T_{max}) of maximum positive frequency deviation. At the high temperature turning point (T_{min}) it becomes positive again. This is the temperature of maximum negative frequency deviation. Both turning points occur if the orientation angle is higher than the zero-temperature coefficient angle. As the angle is reduced toward the value of the zero-temperature-coefficient angle, the frequency difference between the two turning-point temperatures is reduced, the slopes on either side of the turning points are reduced, and the temperature difference between the turning points is reduced. At the unique zero-temperature-coefficient-angle the high and low temperature turning points coincide and there is a unique temperature at which the slope of the curve is zero.

The inflection temperature, Ti, is the temperature for which the derivative of the temperature coefficient of frequency becomes zero.

Ta is the temperature above T_{min} at which the frequency is the same as the frequency at T_{max}.

Tb is the temperature below T_{max} at which the frequency is the same as the frequency at T_{min}.

Tc is the temperature above T_{min} at which the frequency is equal to $\frac{F_{max} + F_{min}}{2}$.

Td is the temperature below T_{max} at which the frequency is equal to $\frac{F_{max} + F_{min}}{2}$.

Variables Determining the Temperature Coefficient of an "AT" Cut

The major variable is the ZZ' angle.

The angle which will produce the same frequency-temperature characteristic is different for the fundamental and for the third harmonic, and slightly different for the fifth and higher harmonics.

The shape of the quartz blank (contour and diameter to thickness ratio) effects the temperature coefficient.

The drive level effects the temperature coefficient.

Some measure of quantitative evaluation of all of the above variables is available. There are other variables, such as the mount, the thickness and size of plated electrodes, and the raw material, of which no significant quantitative evaluation can yet be made.*

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* Good evaluations have been made of certain types of cultured quartz which show a distinct difference from natural quartz. See D. L. Hammond, A. R. Chi, and J. M. Stanley, Effects of Impurities on the Resonator and Lattice Properties of Quartz, Engineering Report E-1162, (Signal Corps Engineering Laboratories, Fort Monmouth, N.J., 3 November, 1955); Rudolph Bechmann, "Frequency-Temperature-Angle Characteristics of AT-Type Resonators made of Natural and Synthetic Quartz," Proceedings of the I.R.E., vol. 44 (November 1956), pp. 1600-1607.

The effect of the smaller variables, and the exact quantitative values of the proved and larger variables are obscured by difficulties of measurement. Determining temperature coefficients requires very precise measurements of the ZZ' angle, of frequency, and precise measurement and control of the drive level. The most difficult measurement is, however, of temperature.

It is customary to test quartz crystal units by placing them in a metal slug, the temperature of which is continuously varied until the desired temperature range is covered, usually proceeding from cold to hot. The amount of difference between the temperature of the slug and the temperature of the quartz plate depends upon a number of factors: the speed and linearity of the temperature change, the drive level of the oscillator which is driving the crystal under test, the mass of the quartz plate, and the thermal conductivity of the mount, holder, and the atmosphere in the holder. In normal tests, such as those specified for military units, the temperature of the slug usually leads, by an increasing amount as the hot end is approached. Differences of five degrees C between the slug and the quartz plate should be considered relatively small.

Generalized Family of Curves

At the present time it is not possible to define any differences in the shape of the characteristic curves (frequency deviation vs. temperature) as functions of the shape, frequency, or harmonic order of the oscillator plate, or of other variables. Consequently, within the limits of presently available data, we can employ one set of master curves for all situations, making no change in the shape of the curves, but using different sets of reference values for the interpretation of the curves.

Figures 38-40 have been calculated from equations by R. Bechmann.*

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* "Frequency-Temperature-Angle Characteristics of AT- and BT-Type Quartz Oscillators in an Extended Temperature Range," Proceedings of the I.R.E., vol. 48 (August, 1960), p. 1494. The equations and the values of the constants were based upon extensive data. Before using them for Figures 38-40, additional and extensive data was examined in an effort to detect any clear systematic departure. It was concluded that if curves were created entirely by empirical means from the data available, they would not differ significantly. For purposes of calculation Dr. Bechmann's material was used in the following form:

$$\frac{\Delta f}{f} = a(T-Ti) + b(T-Ti)^2 + c(T-Ti)^3 \text{ ppm}$$

$a = 0 - 0.08583\ (\theta-\theta i) \times 10^{-6}$

$b = 0.39 \times 10^{-9} - 0.07833\ (\theta-\theta i) \times 10^{-9}$

$c = 109.5 \times 10^{-12} - 0.033\ (\theta-\theta i) \times 10^{-12}$

θ is the ZZ' angle.

θi is the reference angle, which may be considered to be identical with the zero-temperature coefficient angle.

$\theta-\theta i$ is the value shown on the individual curves in Figures 38-40. } in minutes of arc

T is the actual temperature.

Ti is the inflection temperature.

T-Ti is the value shown on the abscissa of Figures 38-40. The values in brackets, however, assume that Ti is +26°C.

To calculate the temperatures of the turning points:

$$\Delta T = \frac{1}{3c}\left(-b \pm \sqrt{b^2 - 3ac}\right)^3$$

where ΔT is T_{max} or T_{min} - Ti.

The zero-frequency-deviation line represents the frequency for each particular angle which is equal to $\frac{f_{max} + f_{min}}{2}$ regardless of what temperature that frequency may occur at. That is, the zero frequency deviation line is always midway between the frequencies at the low temperature and the high temperature turning points. This constitutes a departure from a method often used, according to which all characteristic curves are brought through the zero frequency line at one temperature point. In effect, it bases the analysis upon frequency measurements at the turning points instead of upon frequency measurement at the inflection temperature. The importance of this approach will appear later when the use of the curves is discussed.

The zero temperature line (marked 0 on the upper scale of the abscissa and 26°C in parentheses on the lower scale) represents the inflection temperature of plates cut at the reference or zero angle. It will be noted that these calculated curves show the temperature at which $f = \frac{f_{max} + f_{min}}{2}$ increasing as the angle is increased.

The individual curves in Figures 38-40 are designated in minutes + or - from the reference angle. This angle must be established for particular categories (see Table 4).

Accuracy of Figures 38-40 and Table 4

It is estimated that the accuracy of the generalized curves and the Table is sufficient for such specifications as ±0.005% from -55° to +105°C and that it approaches the maximum accuracy achievable within the limits of data on secondary variables. The value given for the reference temperature, +26°C, represents an average of what may be genuine variations over a 10° range. The values given in Table 4 for the reference angles are no better than $\pm\frac{1}{2}'$. *

* The following is a typical correlation between measurements made in two laboratories and between those measurements and Figure 38 and Table 4. All values in parentheses were derived from frequency measurements, Table 4, and Figure 38. Other values are direct measurements, except that Ti is normalized for 35°20', the Table 4 value for the reference angle for the third harmonic.

	Laboratory A			Laboratory B		
	Unit 1	Unit 2	Unit 3	Unit 1	Unit 2	Unit 3
Angle	35°24'	35°24'	35°24'	(35°24')	(35°24.5')	(35°23.5')
T_{max}	-5°	-6.5°	-4°	-5° (-6.5°)	-9° (-8.5°)	-6° (-5.5°)
T_{min}	61°	64°	61.5°	63° (58°)	63° (60.5°)	61° (56.8°)
Ti	30°	26°	26°	28.5° (26°)	26.5° (26°)	28° (26°)

It is to be assumed that Table 4, and possibly the curves, will be less valid for a number of less usual situations, such as very small electrodes, very thin or very thick electrodes, etc.

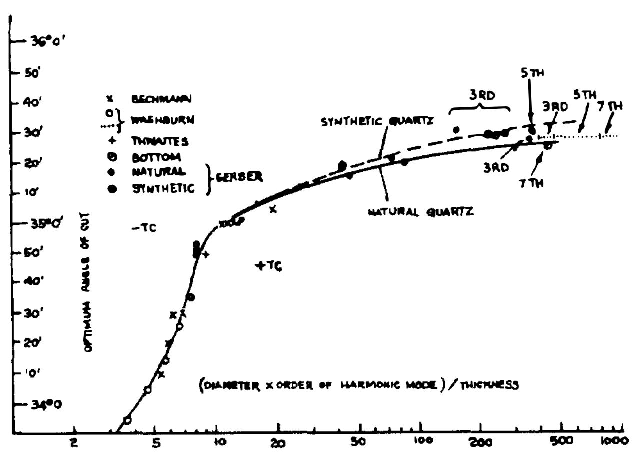
Scale and Scope of Figures 38-40

Figures 38-40 are from the same calculations, but on different scales. Figure 38 uses an expanded scale over a narrow range and is useful as a guide for relatively close tolerance designs. Figure 39 uses a less expanded scale and covers a sufficient range for most types of specification. Figure 40 uses the smallest scale and covers a very wide range of angles and temperatures.

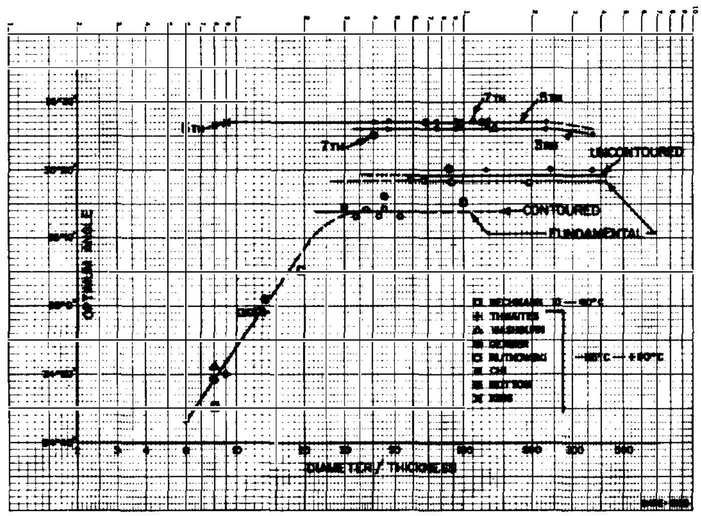
Table 4

Table 4 covers three important and large categories, the fundamental, third harmonic, and fifth and higher harmonics, of uncontoured plates. It also covers one special case, that of plano-convex plates with diameter to thickness ratios greater than 27, and designed according to Figures 31 and 32. The first column gives the reference angle for each category. This is sufficient information with which to enter the curves in one of the three figures (38, 39, or 40). The approximate optimum angle is also given, as well as approximate values for the design-center angles for the temperature range -55° to +90°C according to two tolerances, ±0.005% and ±0.0025%. For the ±0.005% tolerance the angular tolerance is between ±6.5' and ±7', and for the 0.0025% frequency tolerance it is approximately ±2'. These are maximum tolerances with no allowances made for inaccuracies of production measurement, adjustment to nominal frequency, etc.

It will be noted that the table, by assigning only one set of values to these large categories assumes that so long as the diameter to thickness ratio is relatively large, the effect of the d/t ratio is not very large. In October, 1955, Dr. E. A. Gerber published in the Proceedings of the I. R. E., the following curve,



showing optimum orientation angle as a function of the diameter to thickness ratio. Further data was collected, and revisions of the curve were made, the last being by Mr. Chester Rutkowski on page 310 of the Proceedings of the 12th Annual Symposium on Frequency Control (May, 1958). The differences between the early form of this curve

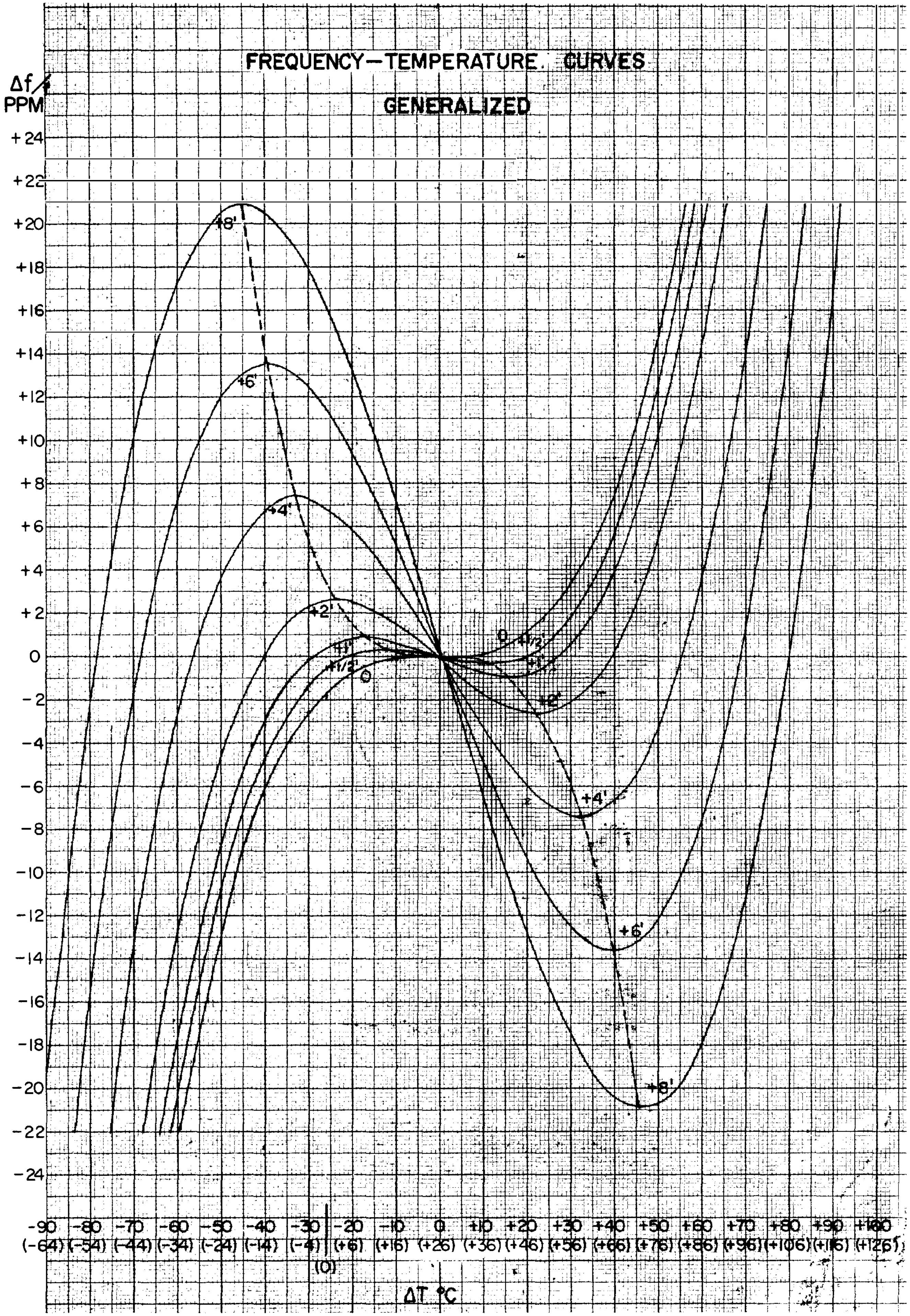


and the later is the result of progress in the understanding and evaluation of two principles: (1) that diameter to thickness ratio is not a significant factor (that is, is within the current error factor) if the ratio is above a minimum value, and (2) that it is always desirable, and frequently possible, to design units so that the diameter to thickness ratio is not critical, using contour when the diameter has to be small relative to the thickness.

TABLE 4

SUGGESTED REFERENCE ANGLES FOR GENERALIZED CURVES AND OTHER ANGLES FOR SPECIAL CASES

	Reference Angle	Approximate Optimum Angle for -55° to +105°C	Approximate Design-Center Angle for -55° to +105°C and Maximum Freq. Deviation of ±0.005%	Approximate Design-Center Angle for -55° to +105°C and Maximum Freq. Deviation of ±0.0025%
Plano-convex designed near optimum Figures 31-32	35°8.5'	35°14.5'	35°16'	35°15.5'
Uncontoured fundamentals, diameter to thickness ratio greater than 67	35°12.5'	35°18.5'	35°20'	35°19.5'
Uncontoured third harmonics, large diameter to thickness ratio	35°20'	35°26'	35°27.5'	35°27'
Uncontoured fifth harmonics, and higher harmonics, large diameter to thickness ratio	35°21'	35°27'	35°28.5'	35°28'


FREQUENCY—TEMPERATURE CURVES
GENERALIZED
Δf/f PPM
+24
+22
+20
+18
+16
+14
+12
+10
+8
+6
+4
+2
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
-22
-24
+8'
+6'
+4'
+2'
+1'
+1/2'
0
-90 -80 -70 -60 -50 -40 -30 -20 -10 0 +10 +20 +30 +40 +50 +60 +70 +80 +90 +100
(-64) (-54) (-44) (-34) (-24) (-14) (-4) (+6) (+16) (+26) (+36) (+46) (+56) (+66) (+76) (+86) (+96) (+106) (+116) (+126)
(0)
ΔT °C

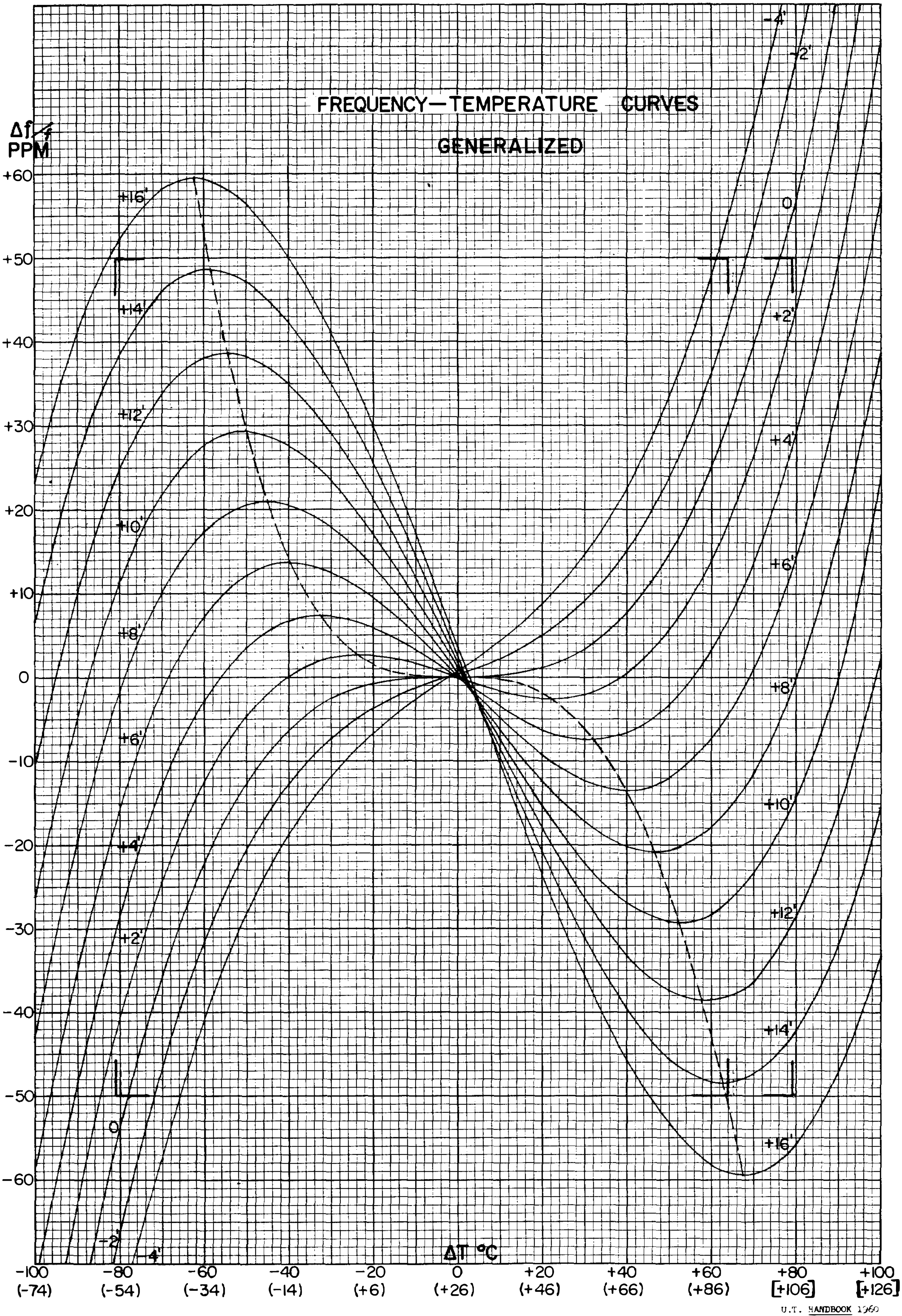
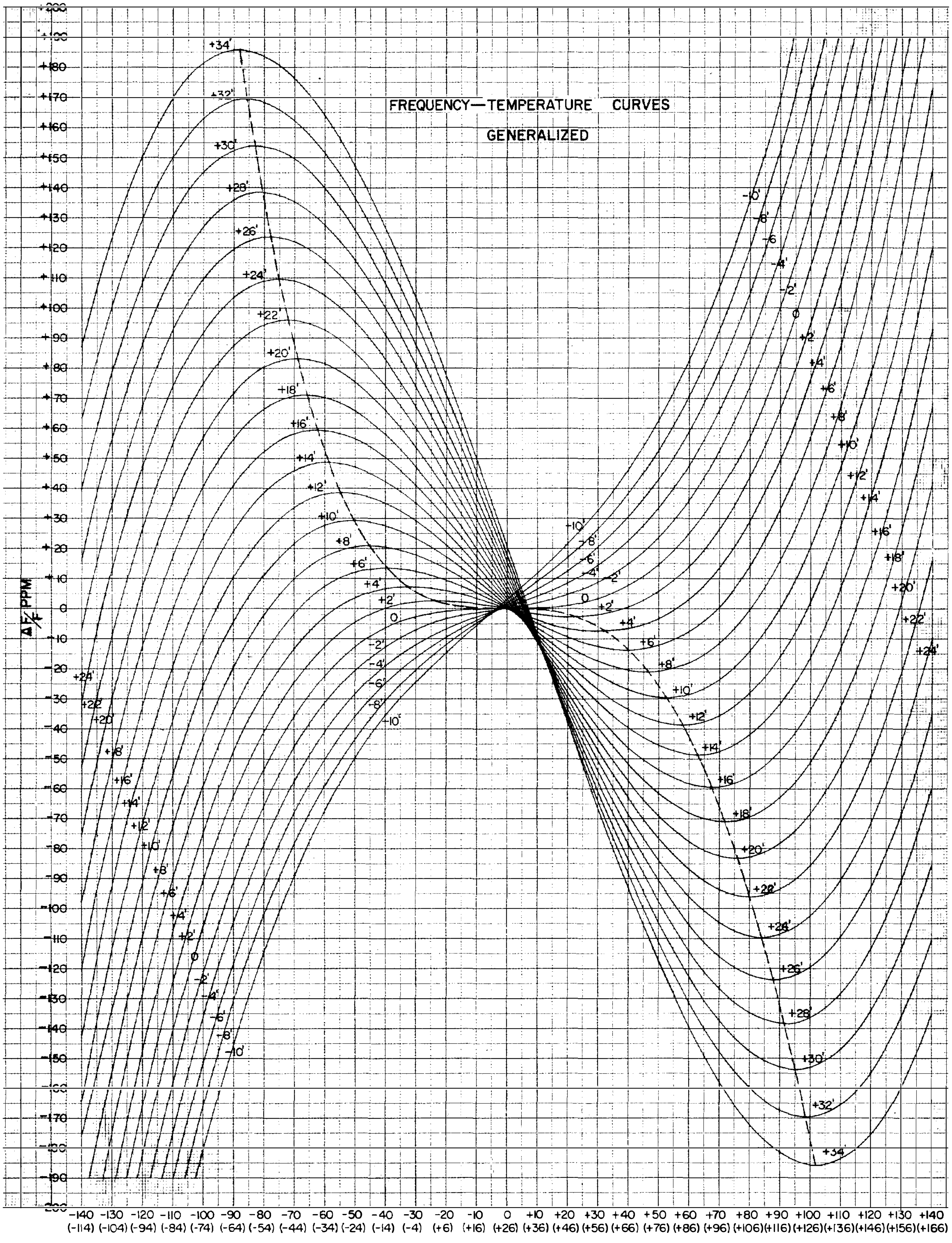

FREQUENCY—TEMPERATURE CURVES
GENERALIZED
Δf/f PPM
+60
+50
+40
+30
+20
+10
0
-10
-20
-30
-40
-50
-60
+16'
+14'
+12'
+10'
+8'
+6'
+4'
+2'
0
-2'
-4'
ΔT °C
-100 (-74)
-80 (-54)
-60 (-34)
-40 (-14)
-20 (+6)
0 (+26)
+20 (+46)
+40 (+66)
+60 (+86)
+80 [+106]
+100 [+126]

FIGURE 40



Use of the Generalized Curves with Table 4

For comparatively wide specifications of common types the first column of Table 4 should be adequate. To illustrate its use, the steps by which the values in the ±.0025% column were determined to be 7' greater than the reference angle values in Column 1 are given:-

In Figure 39 it is observed that in the upper left-hand corner and lower right-hand corner a line half-way between the lines marked +8' and +10' will have a maximum deviation of 25 ppm before the extreme temperatures of the range are reached. This establishes the highest possible angle as +9'. In the lower left-hand corner and upper right-hand corner it is noted that a +5' line would have a maximum deviation of 25 ppm at -55°C and +105°C. This establishes the lowest angle as +5'. The center is 7' higher than the reference angle, and the tolerance is ±2'. Therefore, 7'±2' is added to each value in Column 1.

If the temperature range specified is not symmetrical about the reference temperature, it is necessary to select the lower of two high angles and the higher of two low angles, which are the angles at the end of the temperature range farther from +26°C unless the frequency deviation specification is also unbalanced.

The tolerances so derived are maximum tolerances, and make no allowance for production errors. It is usual practice to reduce a ±6.5' tolerance to ±5' or even to ±3' to allow for inaccuracies in the X-ray sort, for inaccuracies in adjustment to frequency at the reference temperature, etc.

It may be discovered during production that the reference values, or the design-center, should be adjusted somewhat. The certainty of the reference values for the angles is no better than $\pm\frac{1}{2}$' and difference in X-ray calibration may add further cause for readjustment. Moreover, the uncertainty about the reference temperature is ±5°C.

In proportion as the specification is tighter it will be more necessary to make careful adjustment of the reference temperature and reference angle, but unless a secondary variable is known to be the cause, or the case is different from those covered by Table 4, departures of much more than 1' of angle should direct suspicion upon the accuracy of the measurements.

It is, however, to be noted that errors in X-ray sort and in adjustment to frequency at the reference temperature, as well as other errors, such as small changes of angle during lapping processes subsequent to the sort, are not necessarily cumulative, but may be mutually corrective. Therefore some adjustment in terms of the particular error factors of a production plant may be profitable.

Use of Generalized Curves by Starting with Frequency Measurements

The following method of using the generalized curves will yield much more precise results. It can be applied to a wide variety of cases not covered by Table 4. It is independent, when used for analysis, of errors in the measurement of the inflection temperature, and also independent of systematic errors in measuring the angles by X-ray. Table 4 is used, if at all, only as a very general guide. Resonator units within or close to the desired range are selected. It is not necessary that their angles be known with absolute accuracy, but only that in the particular factory or laboratory the angles can be repeated, or changed upward or downward by small increments. That is, if the true angle is 35°15' and the measured angle is 35°17', it makes no difference for this approach providing that all units measured as 35°17' are actually 35°15' units and all units measured as 35°20' are actually 35°18' units. The turning point frequencies are next determined with maximum possible accuracy. The turning point temperatures are not essential. These frequency measurements give the total deviation between turning points. Because of the manner in which the generalized curves are constructed, this total deviation value will be twice the deviation at either the low temperature or the high temperature turning point. A value for a line corresponding to this deviation is found on the curves, and noted in the + or - figure used on the curves. The + or - value for the design-center angle is next determined from the curves in the same way as if the Table were being used, except that values are kept in terms of the + or - difference from the zero angle. This value is now compared with the value obtained with the measured resonator units, and the cutting and sorting angle is adjusted upward or downward accordingly. In short, the cutting and sorting angle is adjusted up or down by means of the comparison of the frequency deviation between turning points with the desired deviation between turning points.

For example, a manufacturer is producing plates to be operated on the fundamental, which are carefully sorted to what he thinks is 35°20'. The total deviations between turning points of these plates is closely centered around a value of 27 ppm. The -13.5 ppm and +13.5 ppm turning points on the curves are for the +6' angle line.

The manufacturer's new specification is for ±25 ppm from -55° to +105°C, which, according to the curves, calls for a design angle of +7'. All that is necessary is to change the cut or sort angle to "35°21'."

This method also may require that some adjustment be made of the reference temperature.

Compressional Stress and Temperature Coefficient

A compressional stress applied to the edges of a plate vibrating in thickness-shear effects the frequency. Therefore the effective temperature coefficient of frequency of the resonator unit can, in part, be a function of the thermal coefficient of expansion of the mounting supports. This may be regarded as a probable cause of some of the apparently erratic data on the temperature coefficient of frequency. The principle has also been applied in laboratory experiments to create a desirable change in the normal temperature coefficient, by so arranging bi-metal strips with reference to the edges of the quartz blank that the frequency change created by the stress counteracts the extreme frequency changes and flattens the characteristic curves. The effect of the stress upon frequency is maximum in a negative direction when the stress is at about 90° to the X-axis, and maximum in a positive direction when the stress is along the X-axis.*

Drive Level and Temperature Coefficient

The same quartz resonator unit will have a different temperature-frequency characteristic if operated at significantly different drive levels, even if measured at stabilized temperatures. The data presented in this Handbook was for the most part taken with drive levels set at or below the drive levels specified for such military units as CR-18.**

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* See E. A. Gerber, "Reduction of Frequency-Temperature Shift of Piezoelectric Crystals by Application of Temperature-Dependent Pressure," Proceedings of the I.R.E., (February, 1960) vol.48, pp. 244-245; and A. D. Ballato and R. Bechmann, "Effect of Initial Stress in Vibrating Quartz Plates," Proceedings of the I.R.E., (February, 1960), vol.48, pp. 261-262.

** For problems of drive level and frequency see Marvin Bernstein, Frequency Repeatability of Crystal Impedance Meter TS-330/TSM, U. S. Army Research and Development Laboratories, Technical Report 2118 (15 May 1960).

Temperature Coefficients of Frequency for Temperature Controlled Units

The generalized curves show a locus line for the high-temperature turning points. This line locates within the limits of its accuracy, the angular values for control temperatures. Normal practice is for the control temperature to be selected well above the maximum ambient which is expected so that an oven alone, without provision for cooling, can be used. Military specifications commonly call for control at either +75° or +85°C; and there are newer requirements for control at +125°C. Civilian requirements for fixed installations permit the use of lower temperatures, such as +60°C, and even lower. The lower-temperature control points are used whenever possible because the frequency deviation on each side of the turning point is proportional to the temperature of the turning point.

Figure 41 shows the approximate magnitude.

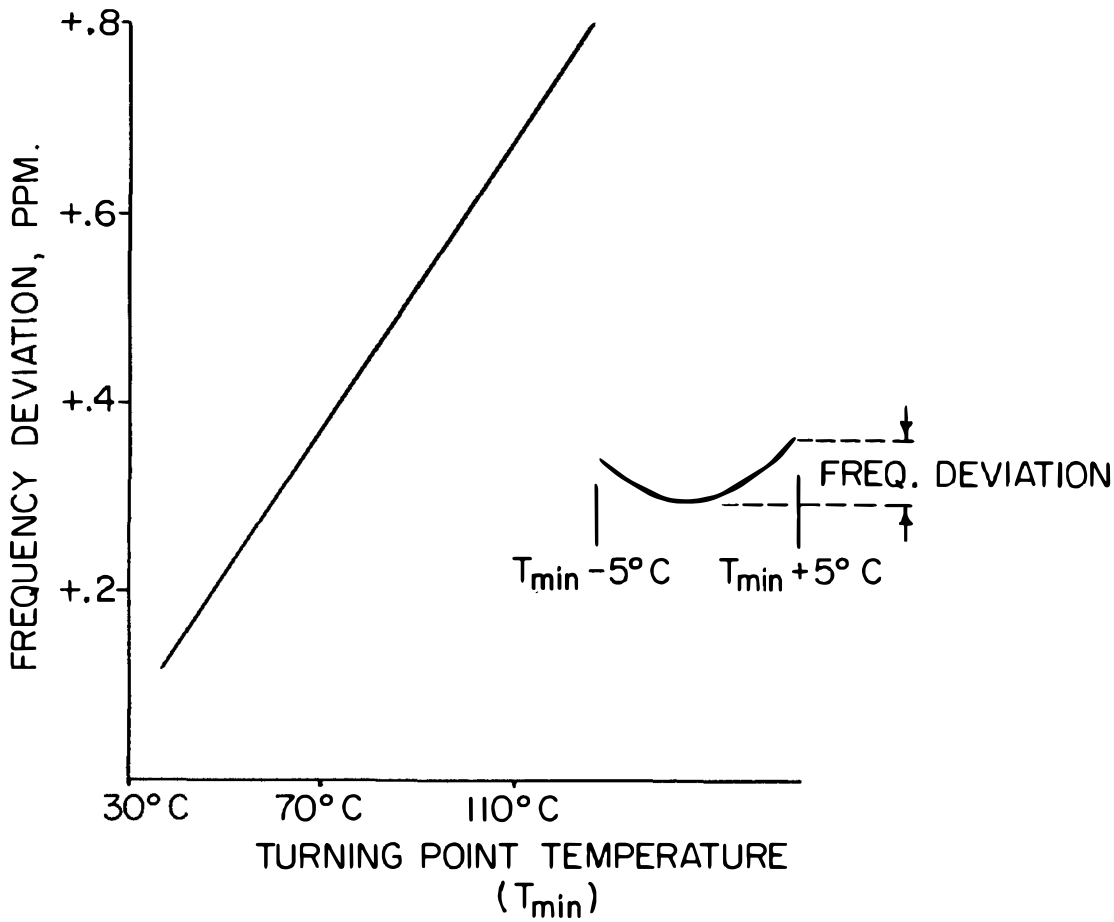
Frequency Deviation at the Inflection Temperature

Measurement of the slope of the characteristic curves through the inflection temperature can be valuable tool in the checking of other measurements. In the generalized curves, the rate of change of slope at the inflection temperature is 0.0858 ppm/°C/minute of angle. That is, for the zero-temperature coefficient angle the slope is zero, for an angle 1' greater the slope is 0.0858, for an angle 10' greater, it is 0.858, etc. For practical purposes the change in slope is linear, and 0.0858 ppm/°C/minute of angle, or some adjusted value, can be regarded as a constant. Actual experimental, direct measurements, result in values ranging from 0.078 to 0.1 for this constant, with the probability favoring a value between 0.08 and 0.086.

Other factors than the inaccuracy of the X-ray measurement of angle are possibly of significance, but empirical data on turning point temperatures and frequencies usually plots more smoothly when plotted against the slope at the inflection temperature (Tf) than when plotted against the X-ray reading of angles.

FIGURE 41

APPROXIMATE FREQUENCY DEVIATION,
FROM FREQUENCY AT TURNING POINT,
AT POINTS +5° and -5°C FROM TURNING POINT,
AS A FUNCTION OF TURNING POINT TEMPERATURE



This figure is based upon equations, using empirically derived constants, by R. Bechmann and A. R. Chi. There is a slight difference between the frequency deviation T_{min} +5° and at T_{min} -5°, which is not indicated on the graph. For direct experimental data in the neighborhood of +85°C see the reports of Scientific Radio Products on Contract DA36-039-sc-73007. Also see Bell Telephone Laboratories, Seventh Interim, on Contract DA36-039-sc-73078 (1958), Figure 6, for an indication of the deviation near +40°C. The accompanying discussion indicates that aging may be less when units are operated near the inflection temperature. In addition there is unpublished data at Union Thermoelectric and three curves in the James Knights leaflet (1960) on their 1 mc glass mounted precision crystals. Empirical data is characterized by scatter, but tends to indicate that any correction of the graph shown above would be to increase the slope of the line.

Temperature Coefficient of Frequency of Contoured Units

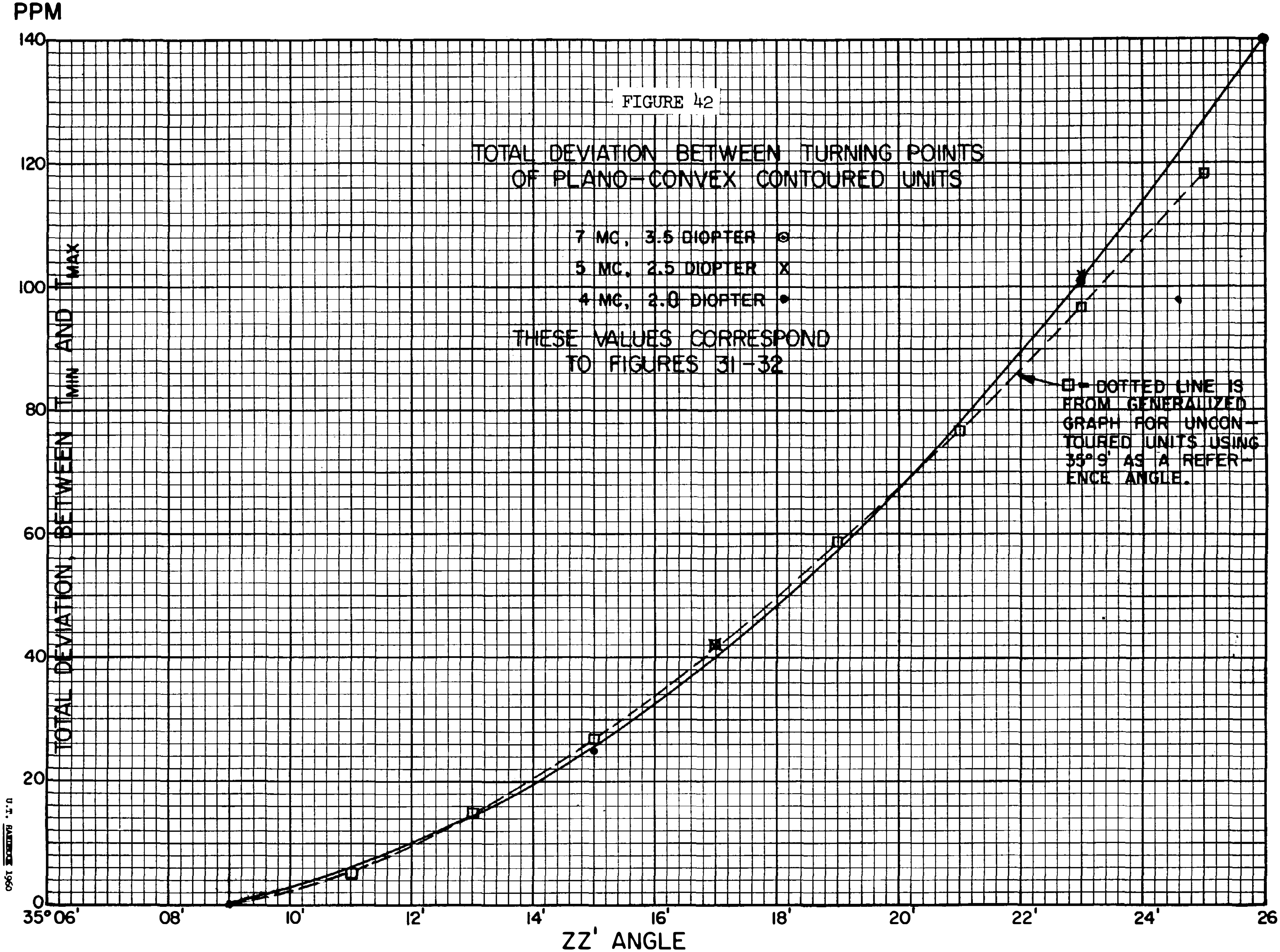
The several different kinds of contour, each of which may be used in different ways with varying effectiveness, have an influence on the temperature coefficient of frequency which does not yield itself to comprehensive generalization. The data on certain contour systems, however, indicates that practical use may be made of the generalized curves when designing contoured units.

Figure 42 shows a curve drawn to data collected with plano-convex blanks conforming to the design shown in Figures 31-32. In order to show the close conformity to the Generalized Curves, 35°9' is made the zero reference for the Generalized Curves, and the deviation between turning points is read from that graph and plotted as a dotted line. The differences are not necessarily significant. [Table 4 gives 35°8.5' for the zero angle. The range of uncertainty is 35°8' to 35°9'.]

Figure 43 shows for plano-convex units both the value of the optimum angle for the temperature range -55°C to +90°C, and the value of the zero temperature angle as functions of relative curvature (thickness/radius of curvature). Since the difference between these two angles remains approximately 6', which is also true of the generalized curves, the generalized curves may be used with the values for the zero angle given here.

It will be noted that in both of these figures which show the optimum angle as a function of relative curvature, the lines are not extended to 0 relative curvature. The case of 0 relative curvature belongs to the category of uncontoured blanks whose diameter/thickness ratio is too small for satisfactory performance in other respects, a category which is not being considered here.

Figure 44 shows optimum or minimum-deviation angle as a function of relative curvature for bi-convex units, and Figures 45 and 46 present the material for both shapes in a different form, optimum angle as a function of frequency for various curvatures.



PPM
FIGURE 42
TOTAL DEVIATION BETWEEN TURNING POINTS OF PLANO-CONVEX CONTOURED UNITS
7 MC, 3.5 DIOPTER ⊙
5 MC, 2.5 DIOPTER X
4 MC, 2.0 DIOPTER •
THESE VALUES CORRESPOND TO FIGURES 31-32
□ • DOTTED LINE IS FROM GENERALIZED GRAPH FOR UNCONTOURED UNITS USING 35°9' AS A REFERENCE ANGLE.
TOTAL DEVIATION, BETWEEN T_{MIN} AND T_{MAX}
0
20
40
60
80
100
120
140
35° 06'
08'
10'
12'
14'
16'
18'
20'
22'
24'
26
ZZ' ANGLE

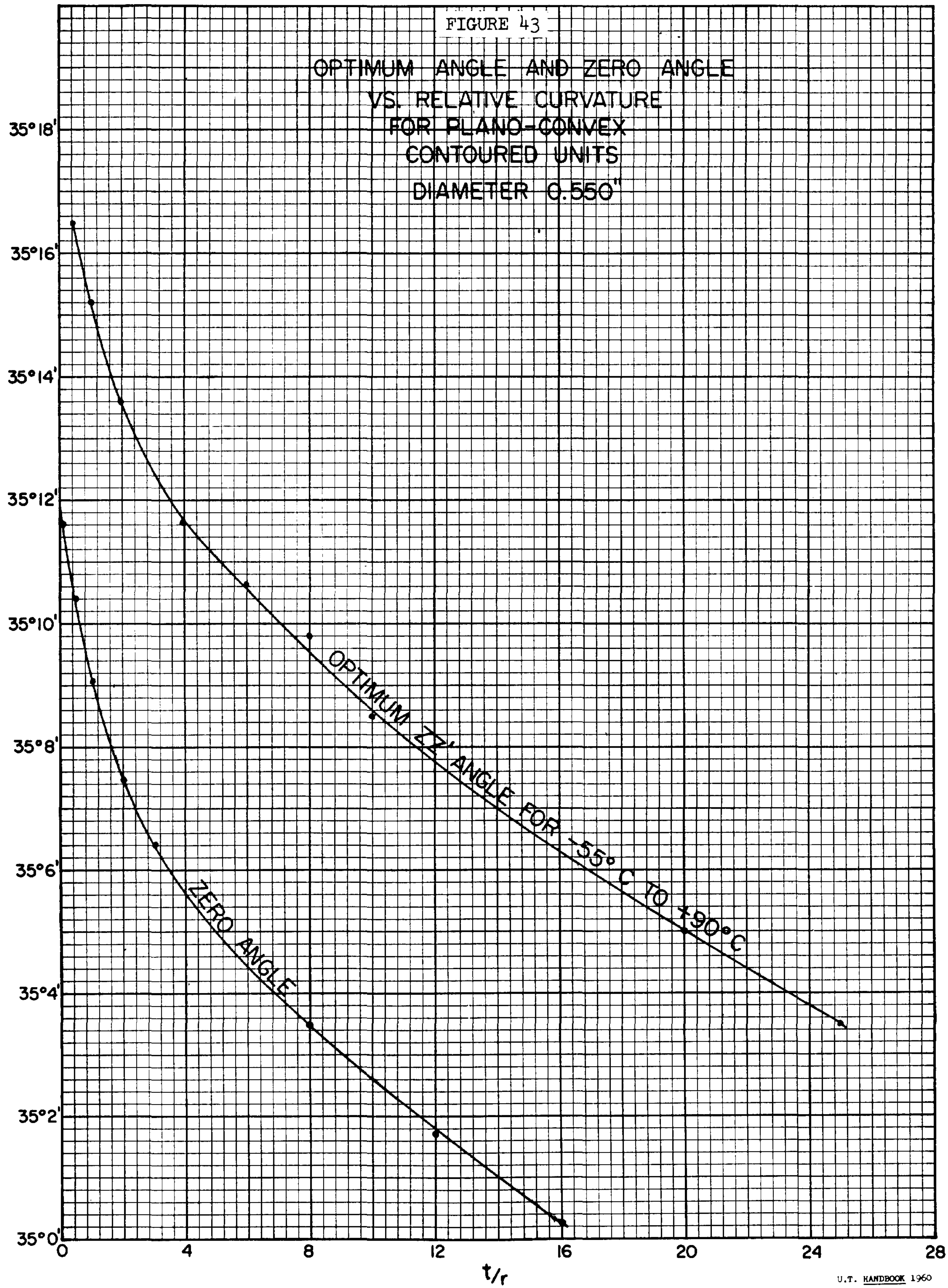


FIGURE 43
OPTIMUM ANGLE AND ZERO ANGLE
VS. RELATIVE CURVATURE
FOR PLANO-CONVEX
CONTOURED UNITS
DIAMETER 0.550"
OPTIMUM ZZ' ANGLE FOR -55° C TO +90° C
ZERO ANGLE
35°18'
35°16'
35°14'
35°12'
35°10'
35°8'
35°6'
35°4'
35°2'
35°0'
0
4
8
12
16
20
24
28
t/r

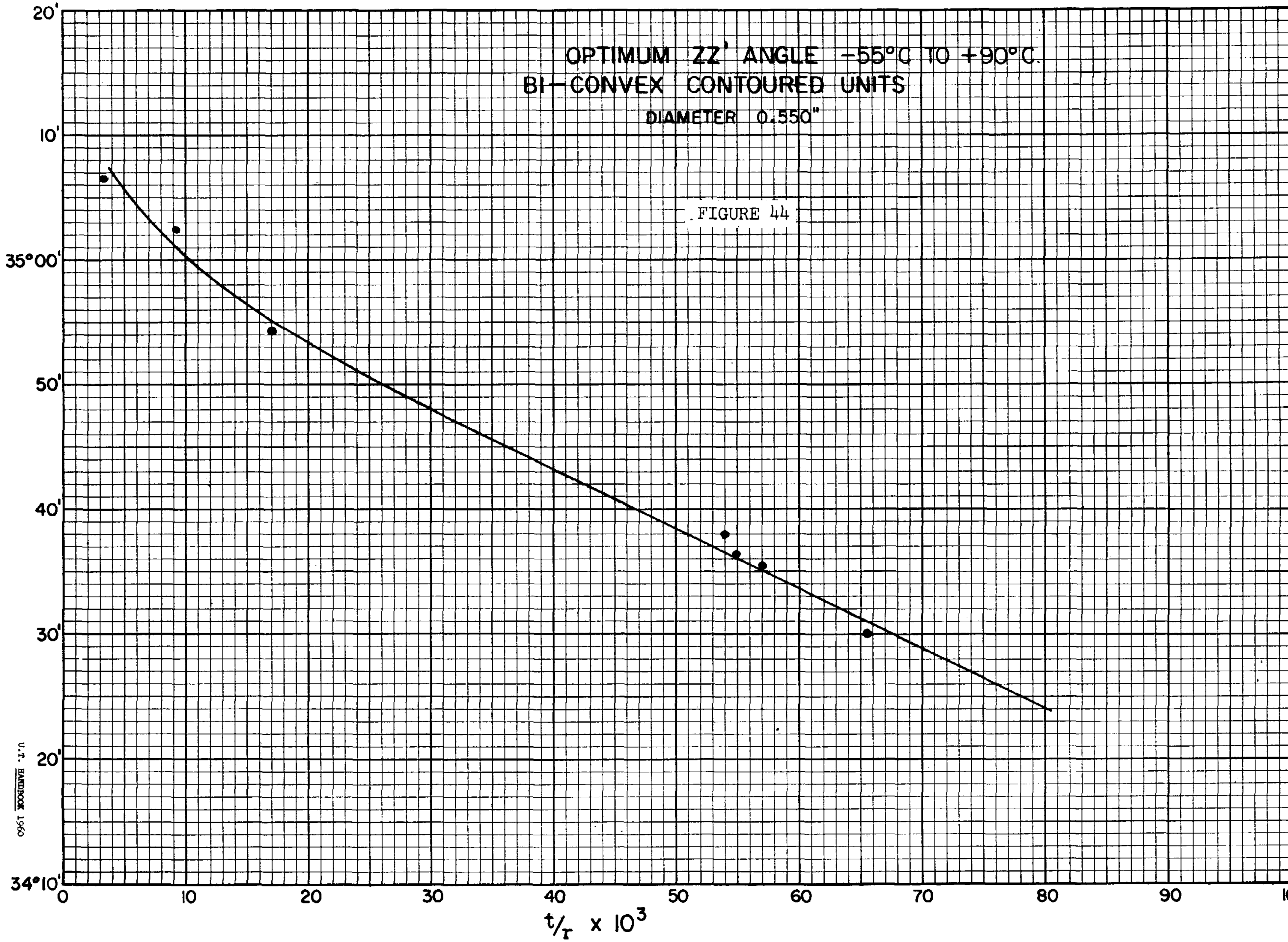

OPTIMUM ZZ' ANGLE -55°C TO +90°C
BI-CONVEX CONTOURED UNITS
DIAMETER 0.550"
20'
10'
35°00'
50'
40'
30'
20'
34°10'
0
10
20
30
40
50
60
70
80
90
100
t/r x 10³

FIGURE 44

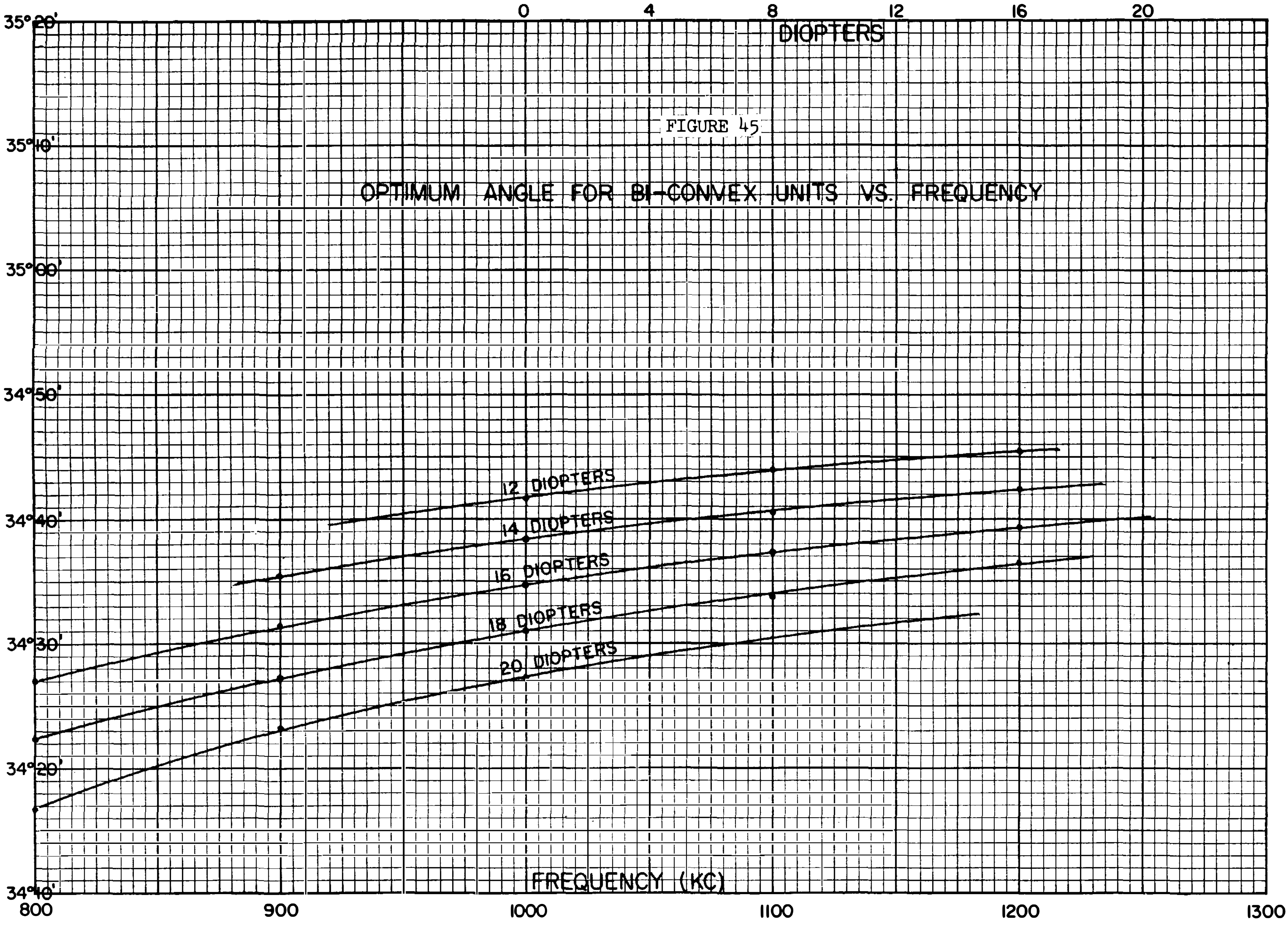
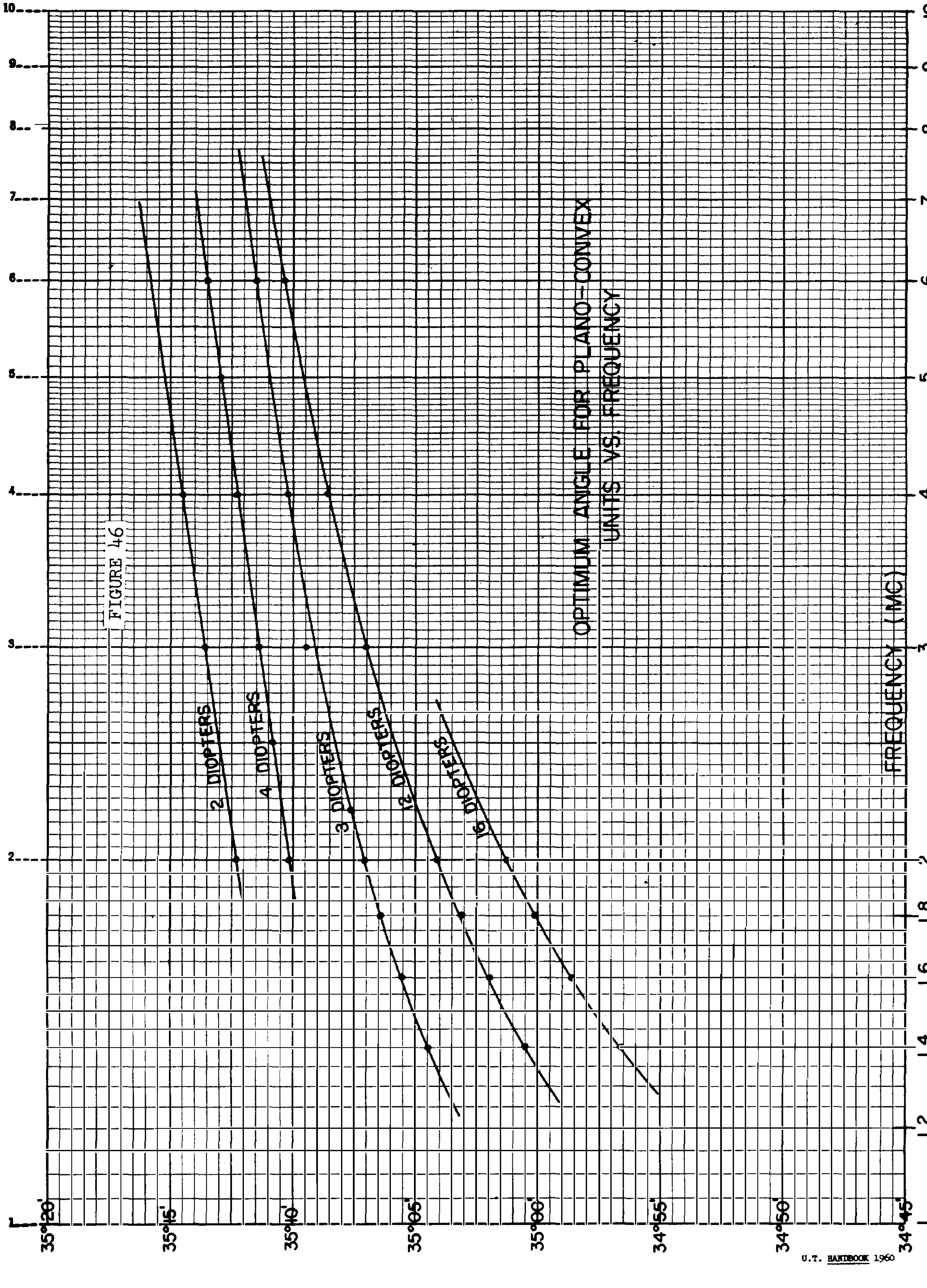


FIGURE 45

OPTIMUM ANGLE FOR BI-CONVEX UNITS VS. FREQUENCY


OPTIMUM ANGLE FOR PLANO-CONVEX
UNITS VS. FREQUENCY
FIGURE 46
FREQUENCY (MC)
2 DIOPTERS
4 DIOPTERS
3 DIOPTERS
12 DIOPTERS
16 DIOPTERS
35°20'
35°15'
35°10'
35°05'
35°00'
34°55'
34°50'
34°45'
1
1.2
1.4
1.6
1.8
2
3
4
5
6
7
8
9
10