THE DIAMETER OF THE RESONATOR PLATE

Performance and Diameter

If a resonator unit is specified mainly in terms of electrical performance, design starts with the selection of a diameter which will result in the specified performance, and a holder is selected or designed which will accomodate plates of this diameter. If the specification is for optimum performance (such as Q and stability), as is sometimes the case with resonators intended for use in frequency standards or for calibration purposes, harmonic order and shape (contour) must be chosen together with diameter. For such a design problem the procedure followed by Bell Telephone Laboratories in the design of the 35-AA, 5 mc, 5th harmonic unit is an excellent guide.*

---

* The 35-AA, in ruggedized form is the military type CR-71/U. See reports on following government contracts with Bell Telephone Laboratories: AF30(602)-13; AF28(099)204; DA36-039-sc-73078. Reports on Contract DA36-039-sc-64586, "Fundamental Studies of the Properties of Natural and Synthetic Quartz Crystals," contain much relevant material. Also see: A. W. Warner, "High Frequency Crystal Units for Primary Frequency Standards," Proceedings of the IRE, vol.40 (September 1952) pp. 1030-1033; H. E. Bommel, W. P. Mason, and A. W. Warner, "Experimental Evidence for Dislocations in Crystalline Quartz," Physical Review, vol.99 (1955) pp. 1894-1896; "Dislocations, Relaxations and Anelasticity of Crystal Quartz," Physical Review, vol.102, No.1 (April 1956), pp. 64-71; A. W. Warner, "Ultra Precise Quartz Crystal Frequency Standards," IRE Transactions on Instrumentation, vol.1-7 (December 1958), pp. 185-188; "Crystal Unit Design for Use in a Ground Station Frequency Standard," Proceedings of the 10th Annual Symposium on Frequency Control, Signal Corps Engineering Laboratories, Fort Monmouth, N.J. (1956), pp. 190-196; W. L. Smith and A. W. Warner, "Fundamental Studies on an Improved Crystal-Controlled Standard," Proceedings of the 12th Annual Symposium on Frequency Control, U. S. Army Signal Research and Development Laboratories, Fort Monmouth, N.J. (1958), pp. 131-161; A. W. Warner and W. L. Smith, "Highly Stable Crystal Oscillators for Missile Applications," Proceedings of the 13th Annual Symposium on Frequency Control, U. S. Army Signal Research and Development Laboratories, Fort Monmouth, N.J. (1959), pp.191-206; A. W. Warner, "Design and Performance of Ultra Precise 2.5 MC Quartz Crystal Units," Bell System Technical Journal, vol.39 (September 1960), pp. 1193-1217.

Economy of Cost and Diameter

If economy is a consideration it is important to select a diameter which can be accomodated in a readily accessible holder, and one which will, if possible, not require that the plate be contoured, or, if this is impossible, a diameter which will permit the use of a relatively simple contour.

If the ratio of diameter to thickness is greater than approximately 67, resonators of good quality can be made without contouring. This means that in many cases relatively high frequency resonators can be made at any convenient diameter. For example, a 20 mc plate has a sufficiently high diameter to thickness ratio if the diameter is greater than 0.220", although for practical reasons, such as standardizing on as few diameters as possible, such plates are seldom made so small, and are often made 0.375", or 0.500", and even larger.

Changes in diameter, even when all diameter to thickness ratios are quite large, may however, have certain effects, the most likely being corresponding changes in the thickness-shear spectrum (inharmonic modes).

If the diameter to thickness ratio is less than about 67, it will be necessary to shape (contour) the resonator plate; but relatively simple contour systems can be used if the ratio is greater than 27. Ratios less than about 27 should be avoided whenever possible; but it is to be noted that because of the difficulty in procuring large stones, and of processing blanks with very large diameters, it is not always practical to avoid contour, or even to avoid difficult contours. Let us assume, for example, that it is very uneconomical to cut, process, mount, and seal units larger in diameter than 1.25". The downward limit of frequency for uncontoured plates is, then, approximately:

$$F(\text{mc}) = \frac{0.0654}{t(\text{inches})},$$

$$F\text{"min"} = \frac{67 \times 0.0654}{1.25} = 3.5 \text{ mc},$$

and for what we call "relatively simple" contours:

$$F\text{"min"} = \frac{27 \times 0.0654}{1.25} = 1.4 \text{ mc}.$$

Economy of Size and Diameter

If the resonator is rigidly specified not to exceed a certain size, and this size is small relative to the desirable diameter of the plate, design must begin with the holder, its internal dimensions, and the maximum size of a plate which can be mounted in the holder. Typical military holders will accomodate round plates with maximum diameters from 0.600" (or by special mounting in the HC-6/U, 0.640") down to a maximum of about 0.310" for the very small military glass holder, and including about 0.350" for the very small, HC-18/U metal holder. Differences in mounting structures, and the fact that inside dimensions are usually not specified make exact figures impossible, particularly with reference to miniature glass holders where there is likely to be a great variation in wall thickness.

Even greater miniaturization is desired, and projects are active which call for circular plates at least as small as 0.195" in diameter.

In addition to the military holders, various standard sized vacuum tube envelopes are used to enclose very high precision units, intended for civilian use.* When Bell Telephone Laboratories were developing the high precision 2.5 MC 5th harmonic unit, 30 mm was judged to be the largest practical size.

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* One high precision military unit, employing an AT cut, and housed in a vacuum tube (T-$5\frac{1}{2}$) envelope, is currently specified, the CR-71/U.

MOUNTS

Most mounts provide both electrical connection to and mechanical support for the plated quartz element. Mounts are usually not specified directly. The mount is a factor in determining the maximum size of plate which can be put in a holder of specified internal dimensions, the ruggedness and stability of the unit, and the resistance of the unit.

TABLE 1

REPRESENTATIVE MOUNTS FOR AT-CUT PLATES

Type of Mount	Use
Wire	Standard for ordinary specifications (Mil-C-3098B) down to 1 or 1.4 mc.
Tab-Clip	An adaptation of a mount furnished to Candaian Radio by Phillips of Eindhoven. Less rigid than the RCA mount, and maintains alignment better than the wire mount.
P. R. Hoffmann	Similar to Tab-clip, and available in an altered form for use in HC-18/U.
RCA, rigid	Developed primarily for the support of thick plates (near 1 mc), but adapted for use as a more vibration-resistant mount for thin, high frequency plates.
Floating	Developed for the support of thick plates, in such a manner as to leave the edges relatively unrestrained.
RCA Strap Type	Used for very thick plates. Plate is held rather firmly. May or may not be bonded.
Bumper-Clip	Easier to assemble than the RCA strap type, and has about the same characteristics.

Discussion

The wire mount is the most commonly used. Support is inadequate for thick blanks with small diameters (such as 1 mc blanks, about 0.065" thick and about 0.600" in diameter). At high frequencies skin effect results in excessive resistance losses. It is difficult to maintain alignment so that manual adjustment of the alignment is frequently necessary before plating to frequency (which is normally done after the plate has been mounted). Special jigs can be used to facilitate the assembly of the plate in the wire clips, but complete mechanization of the operation has not been accomplished.

When subjected to severe vibration (MIL-STD. 202A, method 204, Condition C: 10-2000 cps at 10 g's) electrical continuity in the bonding cement tends to be broken. It is also virtually impossible to use the wire mount with glass because of the alignment problem.

The various rigid mounts maintain alignment better and there is less resistance resulting from skin effect. The tab-clip mount, in particular, yields to mechanized assembly, and has quite low resistance at high frequencies. Too much of the plate is covered by the tabs for maximum avoidance of damping at the lower frequencies. When subjected to severe vibration (10-2000 cps at 10 g's) the bond may fail electrically or mechically, or the tab may break just above the spot weld. In this respect, and in resistance to vibration, the rigid RCA mount ("claw" mount) is better. It has also been used successfully by the U. S. Army Signal Research and Development Agency for high frequency units employed in satellites.

The RCA "strap" or "channel" mount, using a good bonding cement, has been found noticeably superior to the spring-clip and tab-clip mounts in withstanding severe vibration, such as 10-2000 cps at 10 g's.

The bumper-clip mount has characteristics similar to those of the RCA "strap" mount and is easier to assemble.

The floating mounts are designed for very thick plates with relatively small diameters (1 mc plates to be housed in the HC-6/U), in order to provide more support than is furnished by the wire mount and less damping of the edges than occurs with the rigid or wire mounts. In general, the behavior of these mounts is uncertain and marginal, especially when they are used for the purpose for which they were designed, that is, support of a heavy plate which is active at its extreme edges. They are also likely to result in units which shift frequency and resistance when subjected to shock and vibration; and those which lack a bonded electrical connection may allow the blank to rotate so as to break the electrical connection. The RCA "strap" mount can be used without bonding cement, as a floating mount.

No completely satisfactory mount for a small, thick plate, which is active at the edges has been devised.

With reference to the resistance of various mounts, it should be noted that in addition to the size of the electrical conductors, the configuration of the contact surfaces (spring clip, tab-clip, grooved pillar, etc. and the plated strip) together with the type of bonding cement (or solder), and the method of application of the bond, have a considerable effect upon the resistance of the unit.

The total resistance of mount, bond, and plating, as distinguished from the dynamic or motional resistance, can be measured by treating the completed unit as a capacitor, and measuring at a frequency off resonance, as in a Q meter.

$$Q = \frac{1}{\omega C_o R_m}\ ; \qquad R_m = \frac{1}{\omega C_o Q}\ ;$$

where C_o is the capacitance of the unit when the current applied is not at any resonant frequency of the quartz plate; and R_m is the total static, or non-motional, resistance.

At all frequencies the mount contributes to the static capacitance of the unit (C_o, measured at the terminal pins of the unit), and at very high frequencies, especially above 100 mc, the inductance of the mount can be significantly large.

In all of the accompanying drawings of mounts which hold the plate in a vertical position, the plating strips are in line with each other along the horizontal axis of the plate. That is, the connecting points are 180° apart on the circumference of the plate. Therefore, the holder must have space inside sufficient to accommodate both the plate and the mounting supports and clips. Considerable economy of space can be achieved by plating the connecting strips 90° apart so that the mounting supports and clips can occupy space between the lower edge of the plate and the base of the holder. Such an arrangement is useful when putting a 0.350" diameter plate in an HC-18/U holder, if the design of the plate does not require orientation of the mounting at points 180° apart.

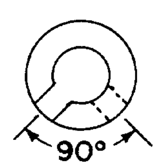
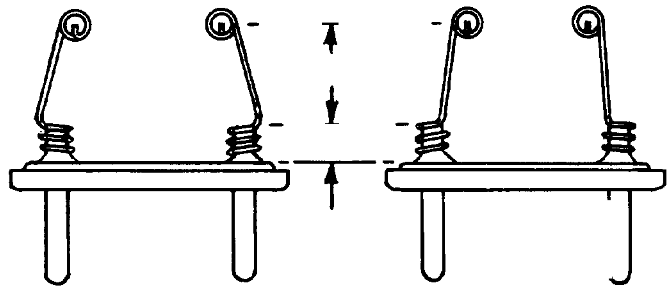
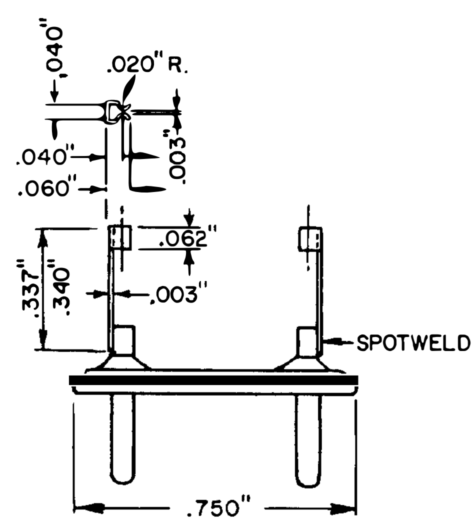


Figure 14



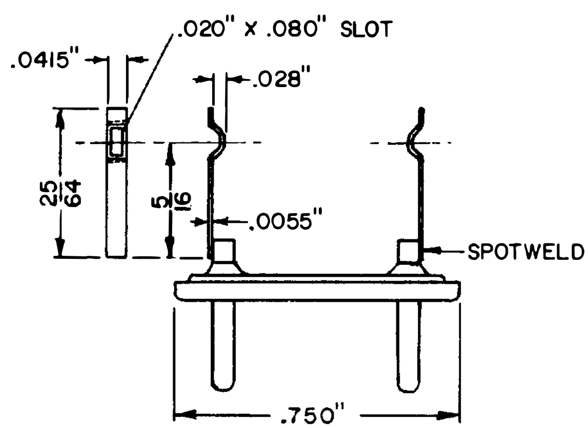
Music wire supports as commonly furnished consist of wire from 0.005" to 0.009" in diameter (0.006" and 0.009" being the most common sizes). The turns around the base pin occupy approximately 1/16" (a). The spring clip is 0.078" ID (0.048" OD available for the HC-18/U holder). The heights from top of base pin to center of spring clip (b) range from 1/16" to 5/16".

Figure 15



TAB CLIP

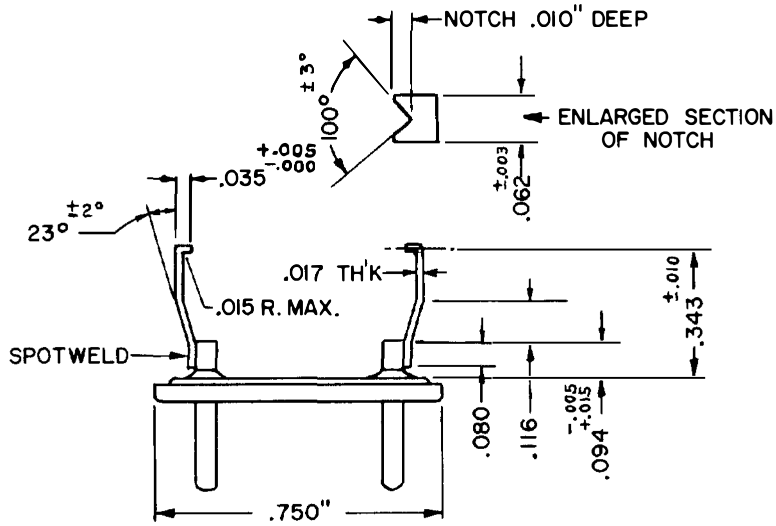
Figure 16



P. R. HOFFMAN
(PATENTED)

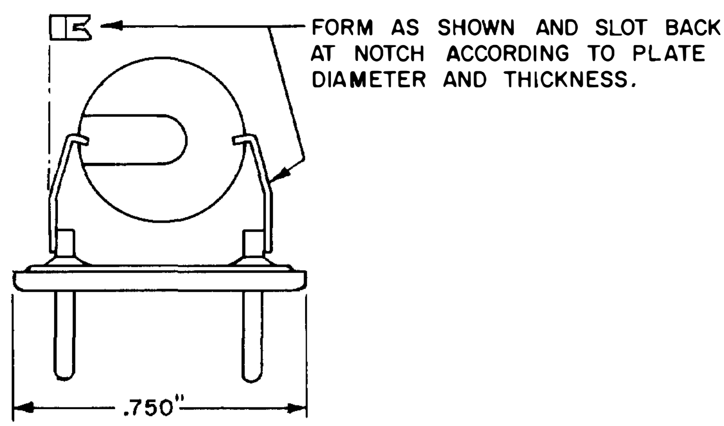
HC-6/U BASES MEET SPEC. MIL-H-10056/2A

Figure 17



R.C.A. "RIGID" OR "CLAW" TYPE MOUNT
DESIGNED FOR SMALL THICK PLATES
(1 MC IN HC-6/U)

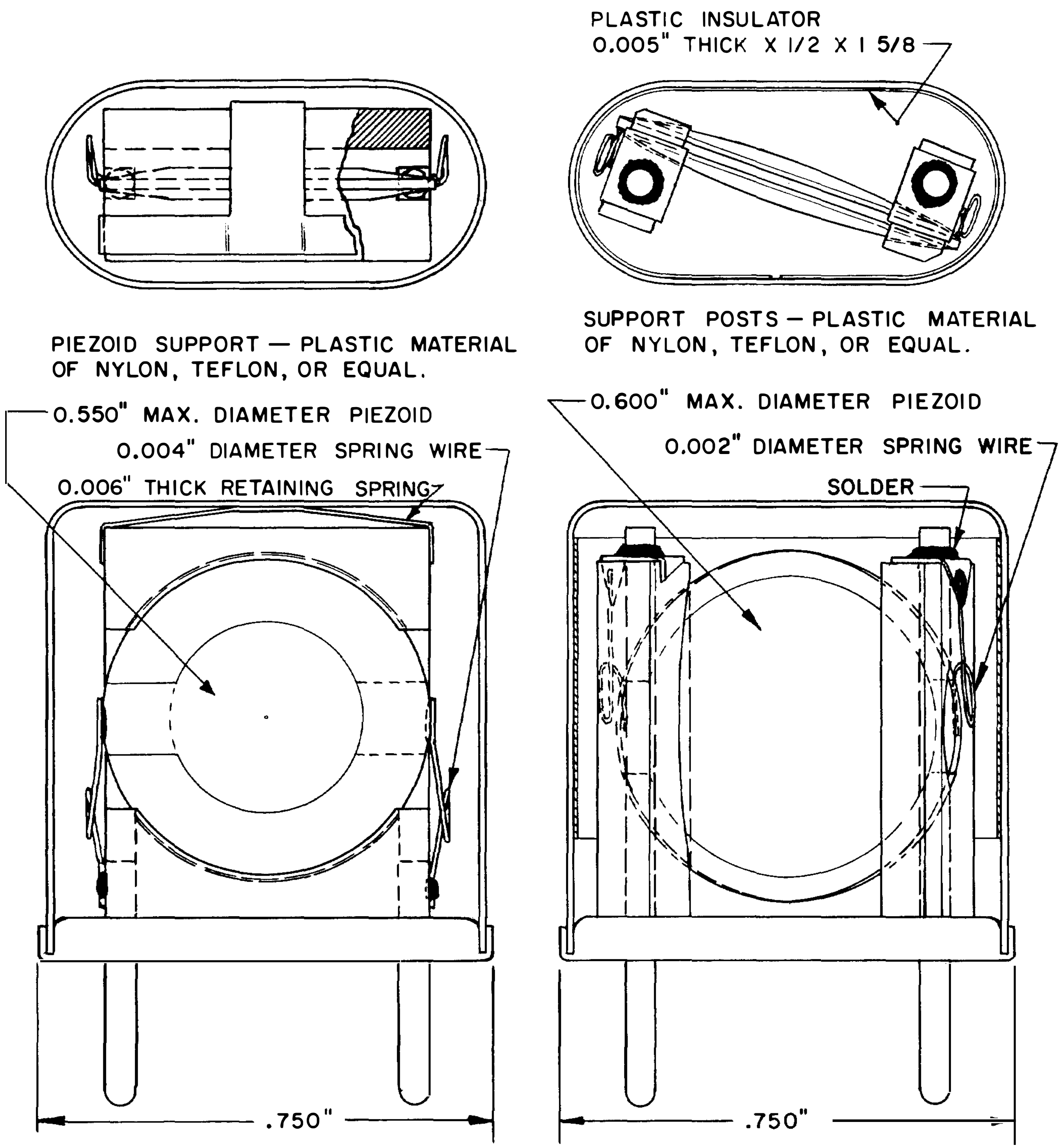
HC-6/U BASE MEETS SPEC. MIL-H-10056/2A



R.C.A. TYPE MOUNT
ADAPTED TO VHF PLATES

Figure 18

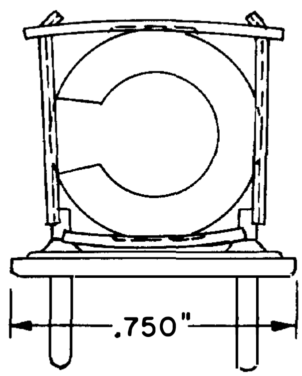
TWO TYPES OF FLOATING CRYSTAL MOUNTS IN HC-6/U BASE AND CAN



Reeves-Hoffman
Patent # 2,705,760

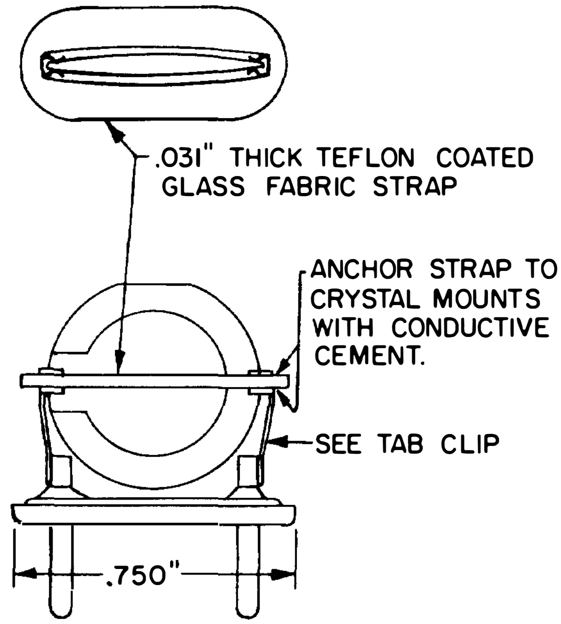
Patent applied for by
Piezo Crystal Company

Figure 19



R.C.A. STRAP TYPE

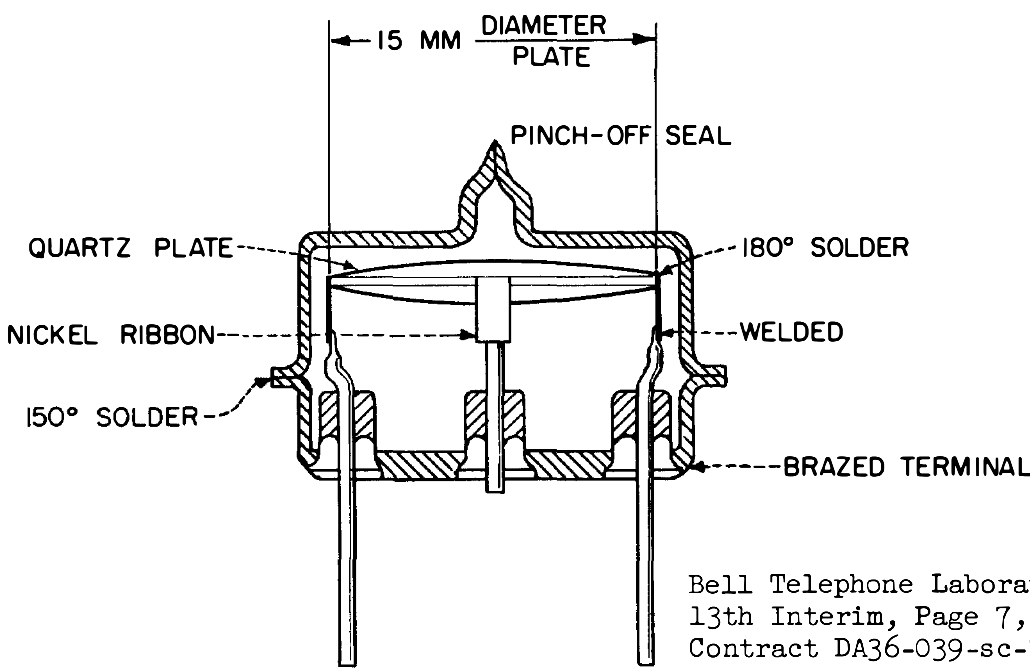
Figure 20



BUMPER CLIP

HC-6/U BASES MEET SPEC. MIL-H-10056/2A

Figure 21



Bell Telephone Laboratories,
13th Interim, Page 7, Figure 2.
Contract DA36-039-sc-73078

RUGGEDIZED MOUNT FOR HIGH PRECISION UNIT

CEMENTS FOR BONDING

Quartz resonator plates vibrating in thickness shear, and with plated electrodes, are usually bonded to the mounting supports with a conductive cement. The type of cement and the method of application and method of curing effect the resistance of the resonator unit, its aging characteristics, and its ability to withstand high temperatures both while the unit is being sealed, and after it is finished.

All known cements are marginal with respect to ability to withstand temperatures in excess of 200°C, and with respect to the possibility of applying them so that they do not contaminate the surface of the quartz plate. Satisfactory substitutes for cement are, however, not easily found.

A list of commonly used cements follows:

Bakelite Silver Mixture

6.5 grams Bakelite cement, BC-6035.
5 grams flake silver, MD-750 of Metals Disintegrating Company
Elizabeth, N. J.
3.0 cc 95% ethyl alcohol.

Do not use cement which is more than two months old. Stir cement thoroughly. Mix the silver thorougly with one half of the alcohol. Add cement and blend well. Then add remaining alcohol. If the mixture is too thick, or thickens during use, add small amounts of alcohol until a heavy, cream-like consistency results.

Dupont Silver Mixture No. 5605

Dupont silver mixture No. 5605, thinned with butyl acetate to a heavy cream-like consistency. This is a ready mixed paste with long shelf life. It is cured, at 150°C, from $1\frac{1}{2}$ to 5 hours.

Dupont - 5504A

Dupont - 5504A is a ready mixed conductive cement. It is cured at 150°C from 1 to 4 hours.

<u>Union Thermoelectric Mixture, Type 3269</u>

2.5 grams of Bondmaster M-640, Rubber and Asbestos Corp.
225 Belleview Avenue
Bloomfield, N. J.

7.5 grams of "Silflake #131," Handy and Harman,
Bridgeport, Conneticut

Mix thoroughly in mortar with pestle for about five minutes.

1.8 to 1.9 ml "Cellosolve Acetate" (ethylene gylcol monoethyl ether acetate), Carbide & Carbon Chemicals Company
230 N. Michigan Avenue
Chicago, Illinois

Add in a mixing dish by means of a pipette. Stir until mixture reaches a creamy consistency, satin appearance. Do not use if it appears granular or sandy. Very small amounts of solvent may be added if necessary to achieve the proper consistency.

Curing cycle of bond: $1\frac{1}{2}$ hours at 175°C.

The mixture is kept refrigerated. More than 30 days refrigerated storage is not recommended.

Scientific Radio Products have increased the curing temperatures for units to be enclosed in glass: 2 hours at 200°C plus 15 minutes in a vacuum at 375°C. Otherwise contamination results from the temperatures resulting from sealing the glass envelope.

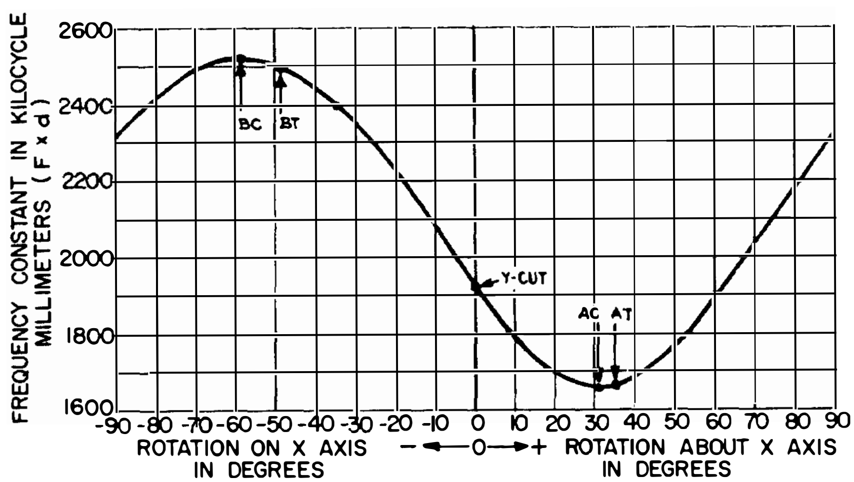
THICKNESS-FREQUENCY COEFFICIENT

The major frequency determining dimension of an AT resonator plate is its thickness, and the frequency can be expressed approximately by:

$$f = \frac{k}{t}; \qquad k = ft$$

f = fundamental frequency
t = thickness
k is a constant for certain conditions.

FIGURE 22



FREQUENCY CONSTANT OF ORIENTED
Y CUT CRYSTALS

From Raymond A. Heising, Quartz Crystals for Electrical Circuits. Copyright 1946, D. Van Nostrand Co., Inc., Princeton, New Jersey, Chapter VI, which see for a more thorough treatment of the entire subject.

Under the following conditions k = ft = 65.4 kilocycle inches, or in millimeters, k = 1660, may be used as an approximation:

ZZ' angle is between 34° and 36°.
Resonator plate is uncontoured; ie., major surfaces are parallel
Diameter to thickness ratio is greater than 100.

The factor, k, becomes larger as the angle is increased (or decreased below about 30°), as the diameter to thickness ratio is decreased, and as the curvature of a contoured resonator plate is increased.

For practical manufacturing purposes, it is usually not necessary to know the thickness precisely (precision measurements being made electrically), but it is frequently necessary to know the approximate thickness or the approximate change in thickness corresponding to a change in frequency. In Figure 23 the approximate thickness in inches corresponding to frequencies between 3 and 30 mc can be read on the right hand scale. If the approximate effect upon frequency of a change in thickness is desired, the Δf line is used, and the number of kilocycles change per one millionth of an inch change in thickness is read on the left-hand scale. In Figure 24 this line is graphically interpreted in a convenient form for use in comparing the effect upon frequency of changes in thickness at different thickness-frequency center points. From either graph one can learn, for example, that if a certain amount of stock removal by lapping, or increase in thickness by plating, causes a change in frequency of 110 kc at 5 mc, the same process will cause a change in frequency of about 1790 kc at 20 mc. In Figure 24 we can read this directly. From Δf line in Figure 23 we read that at 5 mc, 1 millionth of an inch causes a change in frequency of about 0.380 kc. Approximately, the change in thickness corresponding to a frequency change of 110 kc is $\frac{110}{.380} = 289 \times 10^{-6}$ inches. At 20 mc the change in frequency for one millionth of an inch is about 6.1 and 6.1 x 289 = 1760 kc, which is within the limits of accuracy of these graphs.

For contoured plates the equation which is a basis for these graphs is not an equally good approximation, and therefore available data is given directly in terms of thickness in Figures 25 and 26.*

The same data is also given in tabular form in <u>Appendix IV.</u>

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* It is possible to represent this data by smooth curves in which ft is plotted as a function of t/r. Such curves are, however, difficult to use for immediate practical pruposes. The t/r and ft values are given in the table in <u>Appendix IV.</u>

FIGURE 23

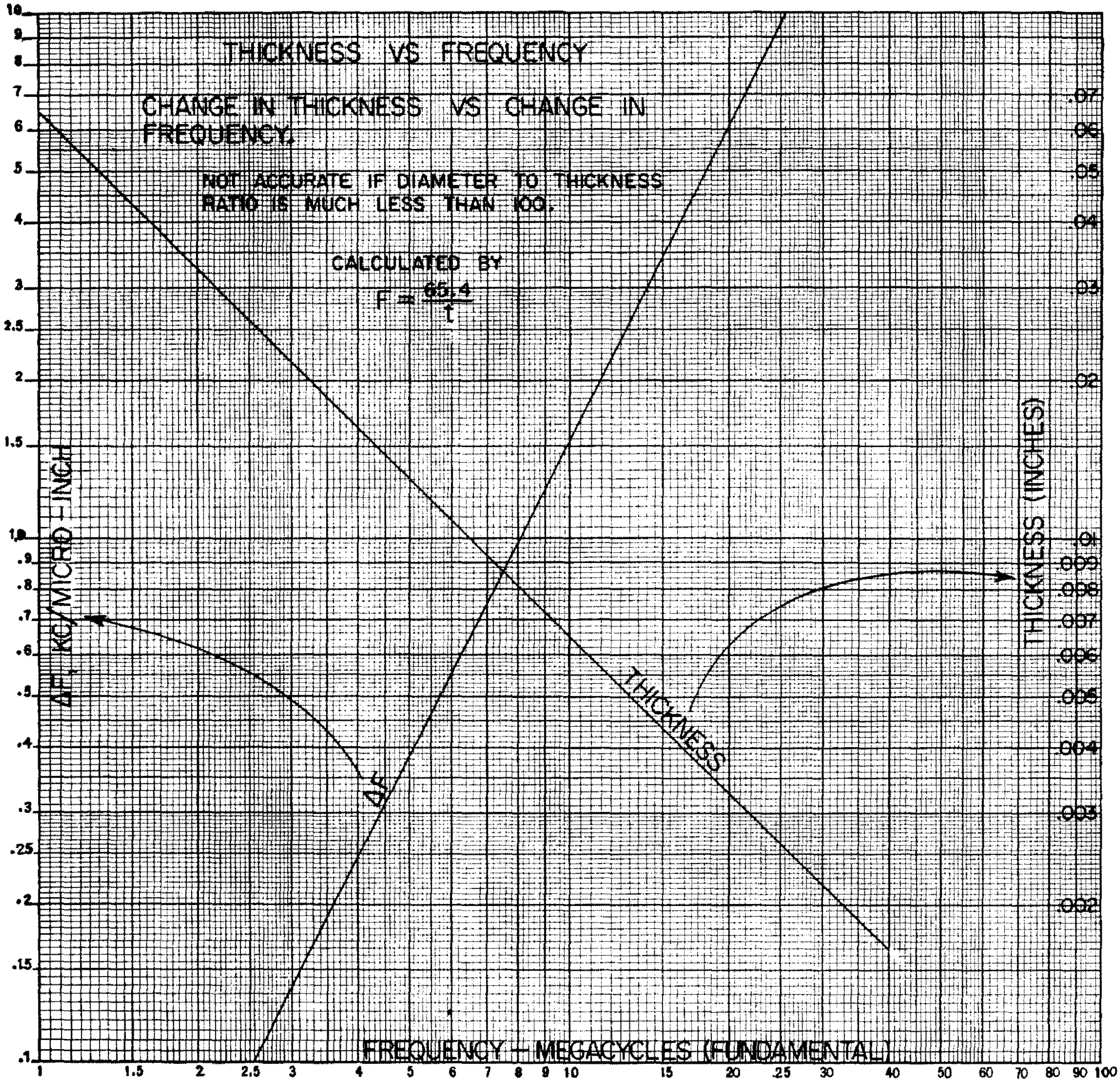
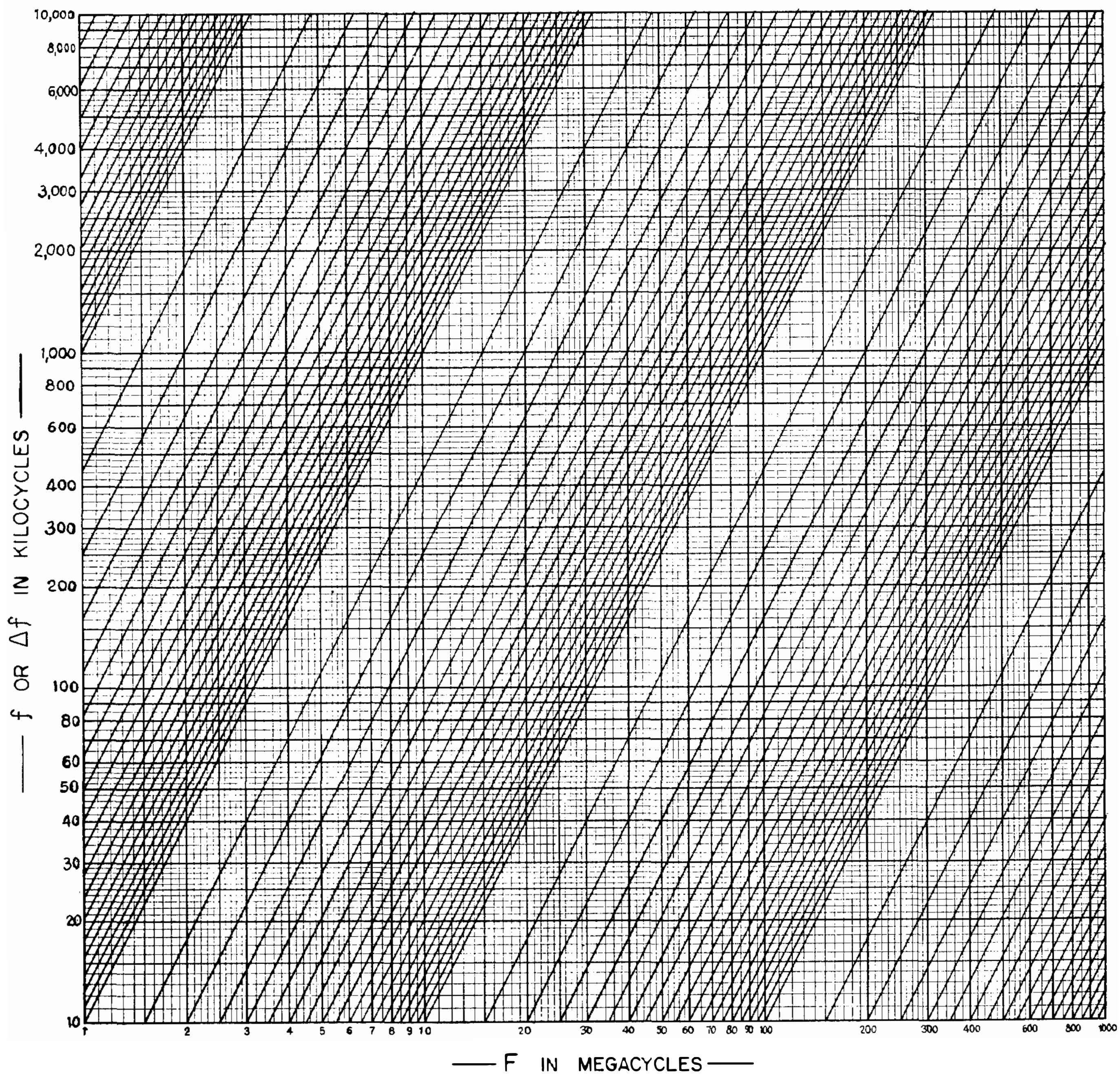

THICKNESS VS FREQUENCY
CHANGE IN THICKNESS VS CHANGE IN FREQUENCY.
NOT ACCURATE IF DIAMETER TO THICKNESS RATIO IS MUCH LESS THAN 100.
CALCULATED BY
F = 65.4 / t
ΔF, KC/MICRO – INCH
THICKNESS (INCHES)
THICKNESS
ΔF
FREQUENCY – MEGACYCLES (FUNDAMENTAL)
10 9 8 7 6 5 4 3 2.5 2 1.5 1.0 .9 .8 .7 .6 .5 .4 .3 .25 .2 .15 .1
.07 .06 .05 .04 .03 .02 .01 .009 .008 .007 .006 .005 .004 .003 .002
1 1.5 2 2.5 3 4 5 6 7 8 9 10 15 20 25 30 40 50 60 70 80 90 100

FIGURE 24

The graph is an arithmetical aid to the use of the Δf line of Figure 23. That is, it relates frequency change to thickness. If a given amount of stock removal at 10 mc produces an increase in frequency of 30 kc, at 20 mc the same amount of stock removal will produce an increase of 120 kc. That is, each slanting line represents a fixed amount of change in thickness. Values are not given for the slant lines, since the graph is for use in going from one process situation to another. Approximate values can be read from Figure 23. For example, the line which corresponds to Δf = 13 at 8 mc and to Δf = 20 at 10 mc:- at 8 mc we read from Figure 23 that Δf equals almost 1 kc per millionth of an inch, indicating that this line represents a little more than 0.000013" stock removal; and checking at 10 mc, Δf = 1.52 kc per millionth of an inch, and $\frac{20}{152}$ gives us again 0.000013". Normally, however, it will not be necessary to assign values to the lines in terms of thickness.

$$\Delta f_1 = \Delta f_2 \frac{F_1^2}{F_2^2}$$

f IN KILOCYCLES
F IN MEGACYCLES



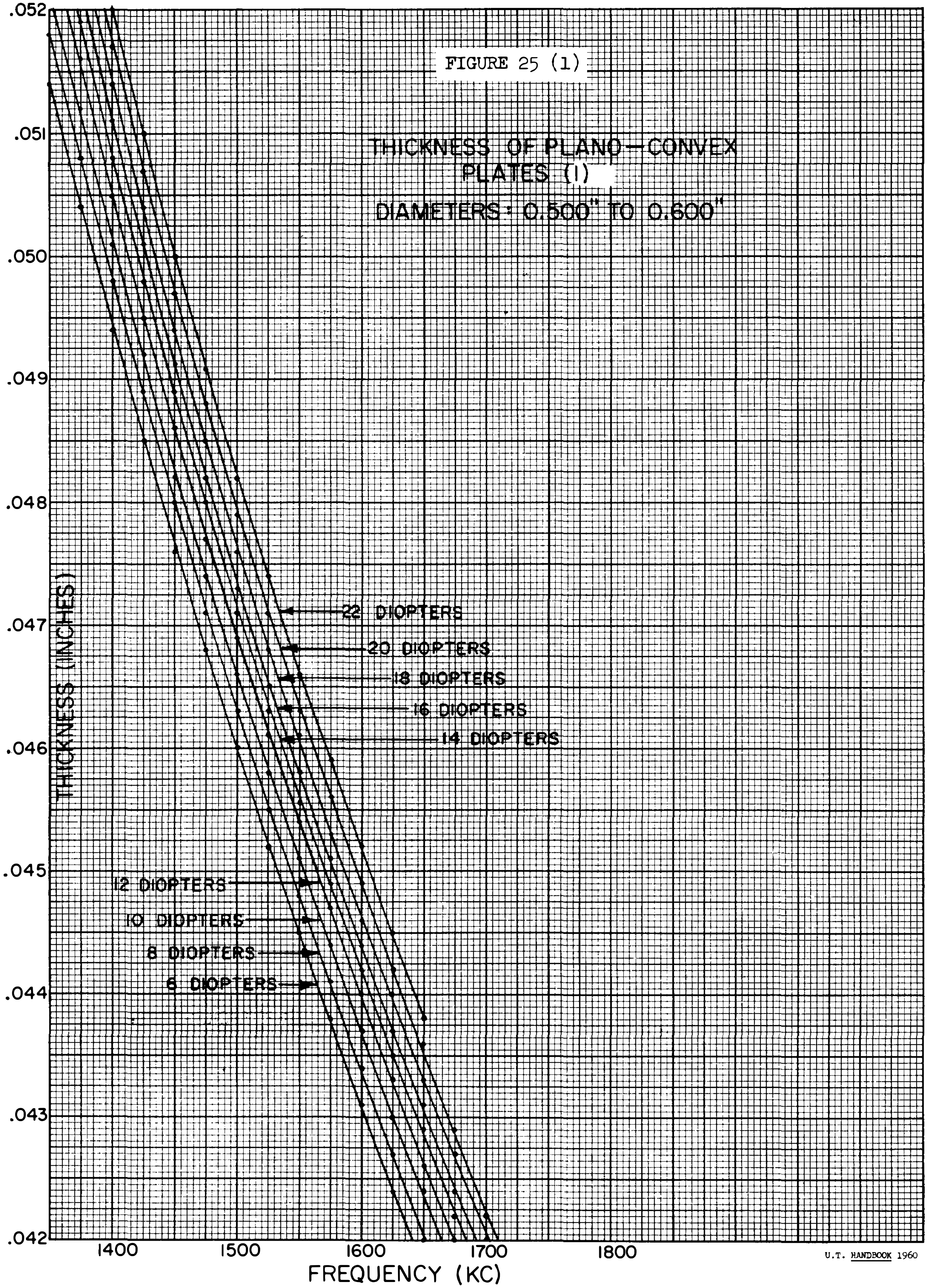
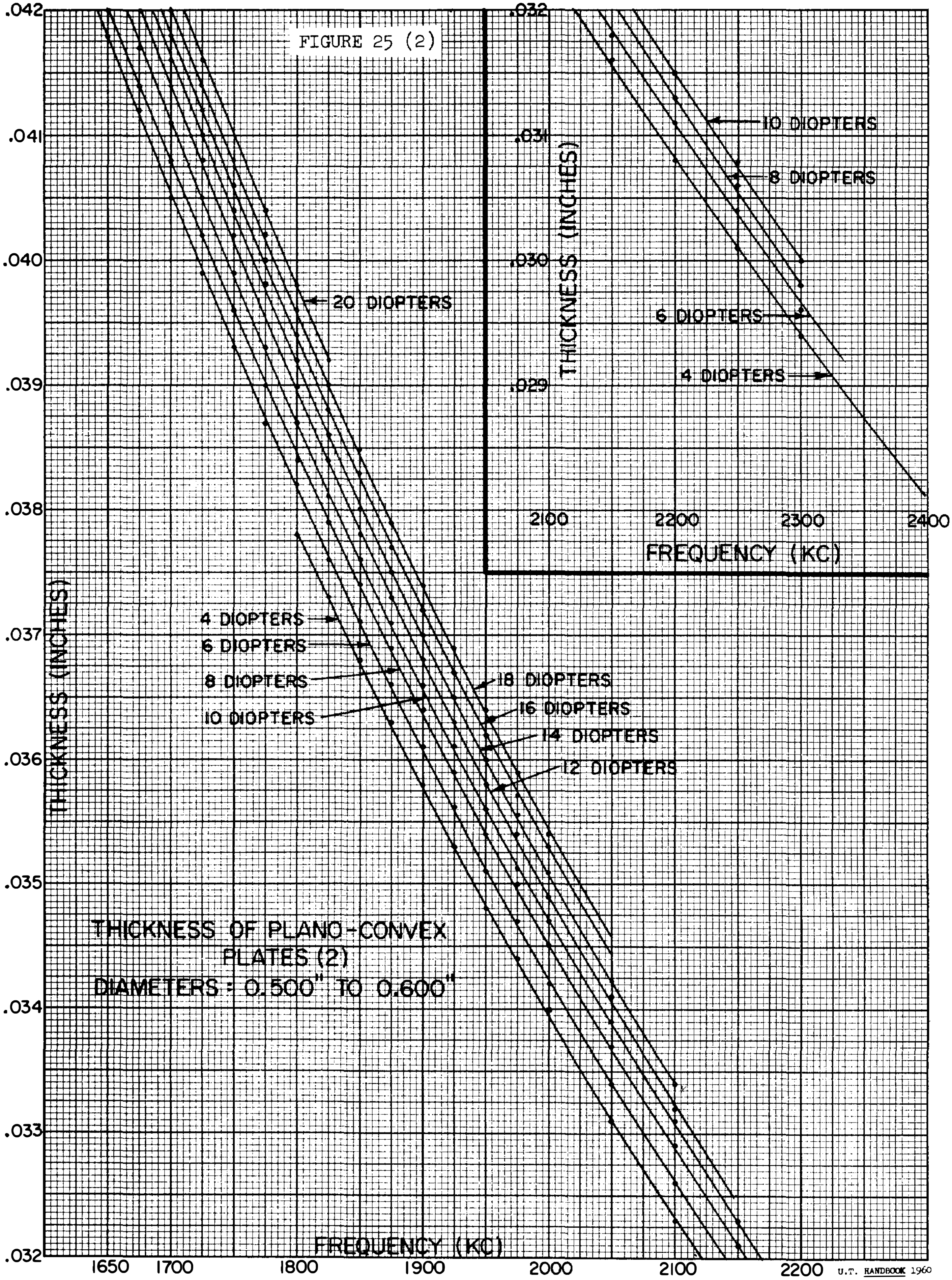


FIGURE 25 (1)
THICKNESS OF PLANO—CONVEX PLATES (I)
DIAMETERS: 0.500" TO 0.600"
22 DIOPTERS
20 DIOPTERS
18 DIOPTERS
16 DIOPTERS
14 DIOPTERS
12 DIOPTERS
10 DIOPTERS
8 DIOPTERS
6 DIOPTERS
THICKNESS (INCHES)
.052
.051
.050
.049
.048
.047
.046
.045
.044
.043
.042
1400
1500
1600
1700
1800
FREQUENCY (KC)



U.T. HANDBOOK 1960

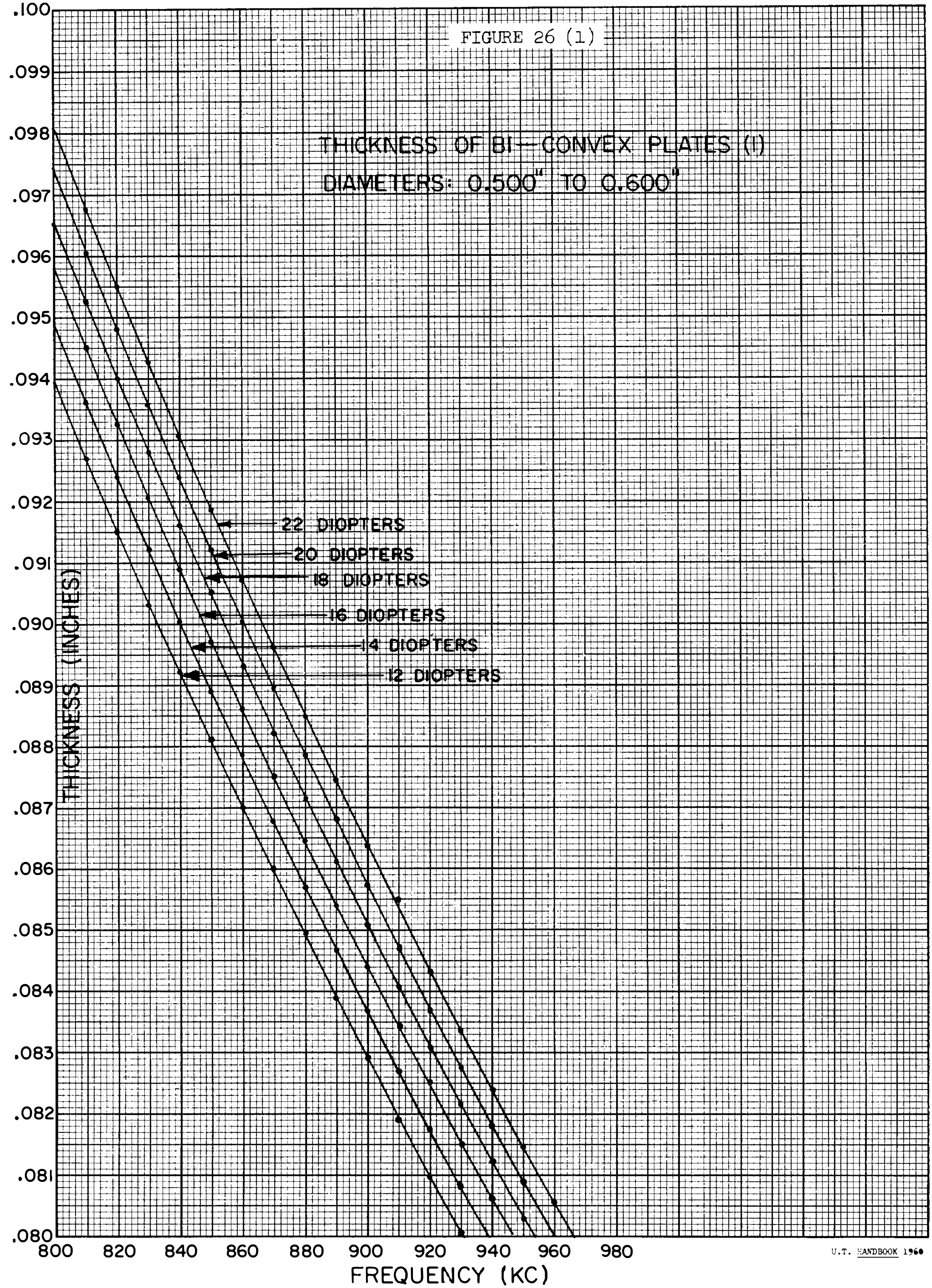


FIGURE 26 (1)
THICKNESS OF BI-CONVEX PLATES (I)
DIAMETERS: 0.500" TO 0.600"
22 DIOPTERS
20 DIOPTERS
18 DIOPTERS
16 DIOPTERS
14 DIOPTERS
12 DIOPTERS
THICKNESS (INCHES)
.100
.099
.098
.097
.096
.095
.094
.093
.092
.091
.090
.089
.088
.087
.086
.085
.084
.083
.082
.081
.080
800
820
840
860
880
900
920
940
960
980
FREQUENCY (KC)

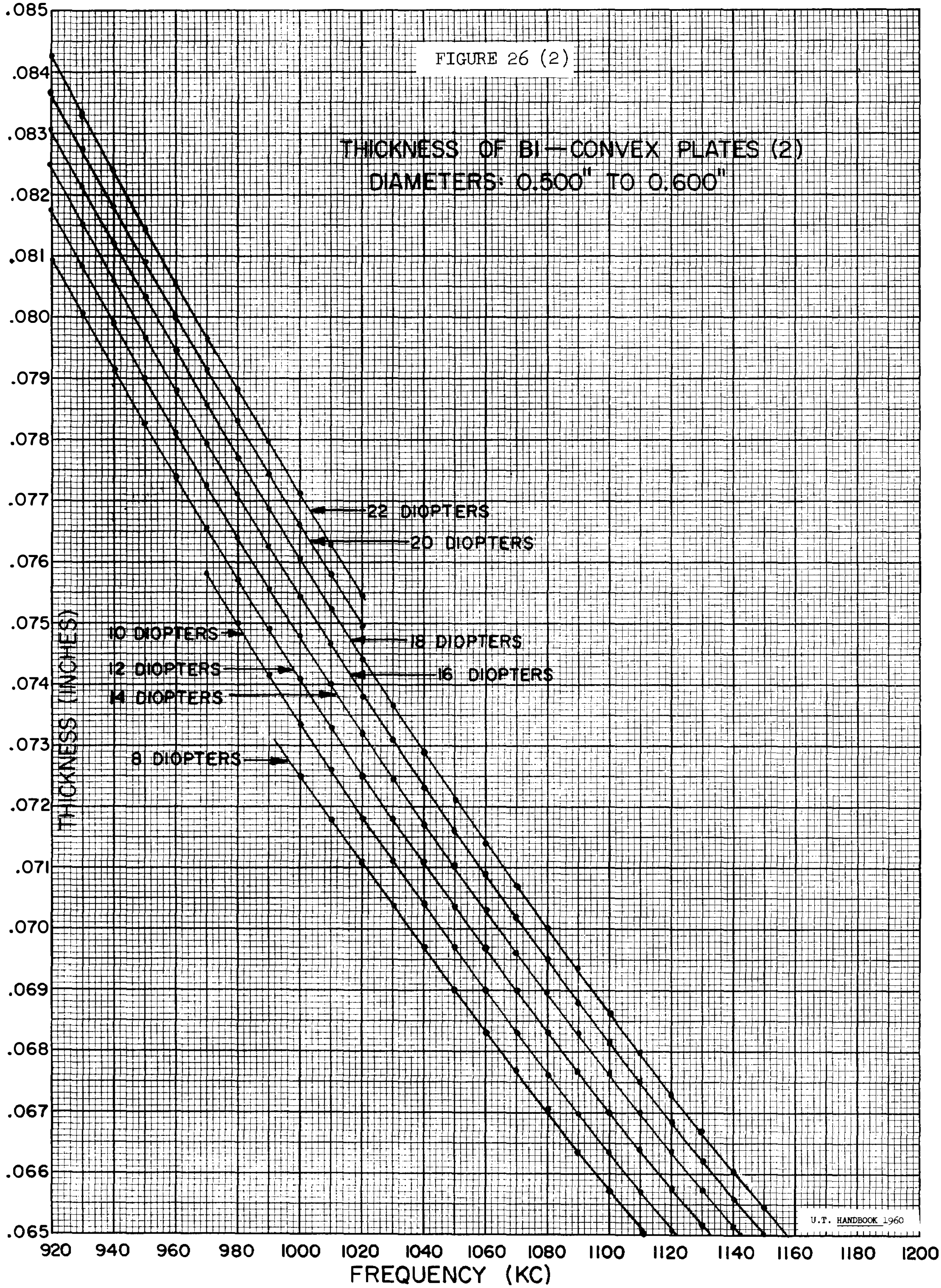


FIGURE 26 (2)
THICKNESS OF BI-CONVEX PLATES (2)
DIAMETERS: 0.500" TO 0.600"
22 DIOPTERS
20 DIOPTERS
18 DIOPTERS
16 DIOPTERS
10 DIOPTERS
12 DIOPTERS
14 DIOPTERS
8 DIOPTERS
THICKNESS (INCHES)
.085
.084
.083
.082
.081
.080
.079
.078
.077
.076
.075
.074
.073
.072
.071
.070
.069
.068
.067
.066
.065
920
940
960
980
1000
1020
1040
1060
1080
1100
1120
1140
1160
1180
1200
FREQUENCY (KC)
U.T. HANDBOOK 1960

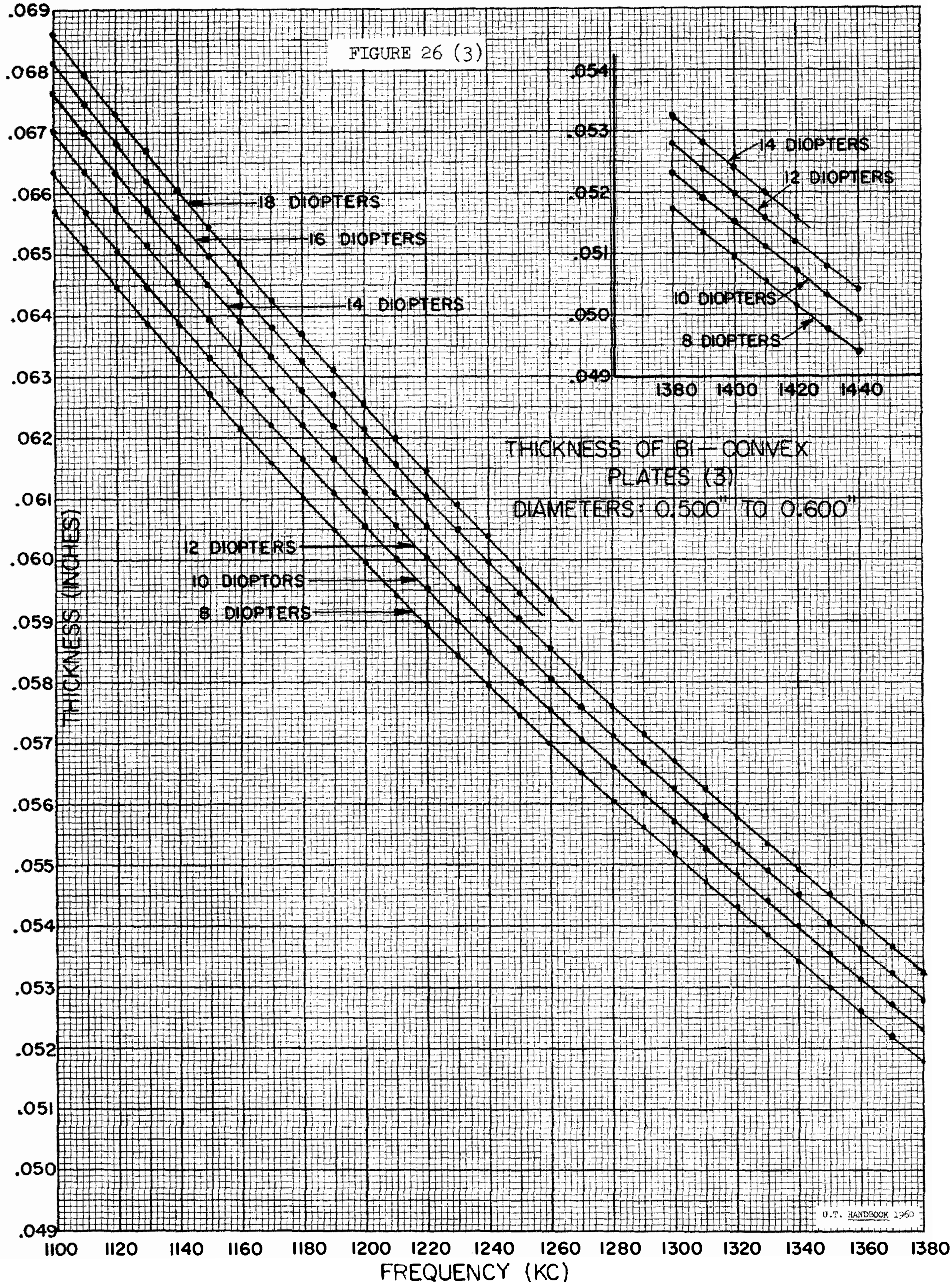


FIGURE 26 (3)
THICKNESS OF BI — CONVEX PLATES (3)
DIAMETERS: 0.500" TO 0.600"
18 DIOPTERS
16 DIOPTERS
14 DIOPTERS
12 DIOPTERS
10 DIOPTORS
8 DIOPTERS
14 DIOPTERS
12 DIOPTERS
10 DIOPTERS
8 DIOPTERS
.054
.053
.052
.051
.050
.049
1380
1400
1420
1440
.069
.068
.067
.066
.065
.064
.063
.062
.061
.060
.059
.058
.057
.056
.055
.054
.053
.052
.051
.050
.049
1100
1120
1140
1160
1180
1200
1220
1240
1260
1280
1300
1320
1340
1360
1380
THICKNESS (INCHES)
FREQUENCY (KC)
U.T. HANDBOOK 1960

SHAPE AND CONTOUR

For convenience, plates will be understood to belong to one of two categories:

(1) Uncontoured plates are plates with no deliberate and no known and significant departure from parallelism. That is, the two major surfaces are equidistant from each other at all points. The adjective flat is avoided for reasons which appear in Appendix II, Geometrical Measurements.

(2) Contoured is used loosely to cover the entire range of shapes in which the two major surfaces are not parallel, including all kinds of deliberate or significant bevels.

Uncontoured Plates

All lapping machines, and particularly lapping machines of the eccentric (pin) type tend to produce some contour, the plates being thinner at the edges. The necessity for taking special measures to minimize this tendency depends upon how great the tendency is, how irregular it is, and how high the frequency is. If, for example, a bad lapping process is not only resulting in thin edges but also in wedginess (one side thicker than the other), it may be necessary to take corrective measures for plates intended to operate on relatively low frequencies. Most current lapping processes, employing pin laps only for the finishing of very thin plates, and using carefully conditioned lapping plates on all machines, produce a degree of parallelism which is satisfactory enough that no intensive investigation of the subject has been found necessary below about 60 mc on the fifth harmonic.

From 60 mc to 100 mc on the fifth harmonic, the parallelism of the major surfaces becomes important. The center of the quartz plate should not be much more than 20 millionths of an inch thicker than any point near the circumference. For details see Appendix II, Geometrical Measurements.

Contoured Plates

Contouring is a means of restricting the active area of a resonator plate in order to decrease the loss of energy to the mounting supports and in order to control coupled modes of vibration when, for practical reasons, the diameter to thickness ratio of the plate has to be small.

Contour design is complicated by the two separate functions which contouring performs and by the wide variety of kinds of contour which are possible. The most commonly employed contours are the plano-convex, the double convex, and the bevelled. Step contours (two different curvatures) are also used occasionally. The double convex and the plano-convex contour designs involve the following variables: frequency, plate diameter, electrode diameter, and curvature. To these variables, the bevelled design adds plateau diameter, which greatly complicates the problem of establishing designs.

Contouring may also be used to influence the inharmonic overtones.

The minimum value of the diameter to thickness ratio for uncontoured plates depends upon a number of factors: quality required, temperature range, type of mount, and specifications with reference to the effects of coupled modes and inharmonic modes. Considering chiefly the type of specification usually given for volume production units, such as the CR-18, the minimum diameter to thickness ratio for good, uncontoured units is approximately 67.* This assumes that the uncontoured units are really uncontoured, as from a good planetary lapping process.

The problem of designing a plano-convex or a bi-convex plate is essentially one of determining a suitable combination of plate diameter and surface curvature for a particular frequency. For plates having a diameter to thickness ratio of about 20 or greater, neither the diameter nor the curvature is highly critical and hence designs can be specified for the whole frequency range involved employing only a limited number of plate diameters and curvatures. As the diameter to thickness ratio decreases below 20, these parameters become more critical, and resonators differing in frequency by only a few percent may require a different combination of plate diameter and curvature.

The following graph is useful for finding the minimum safe frequencies for uncontoured plates of various diameters. It is sufficiently accurate for use with such boundary values as $d/t < 67$ and $d/t > 20$ as given in this discussion.

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* As an alternative to contour, and especially as an alternative to isolated and critical designs, some manufacturers still employ carefully dimensioned rectangular plates. Elaborate tables of safe dimensions have been prepared.

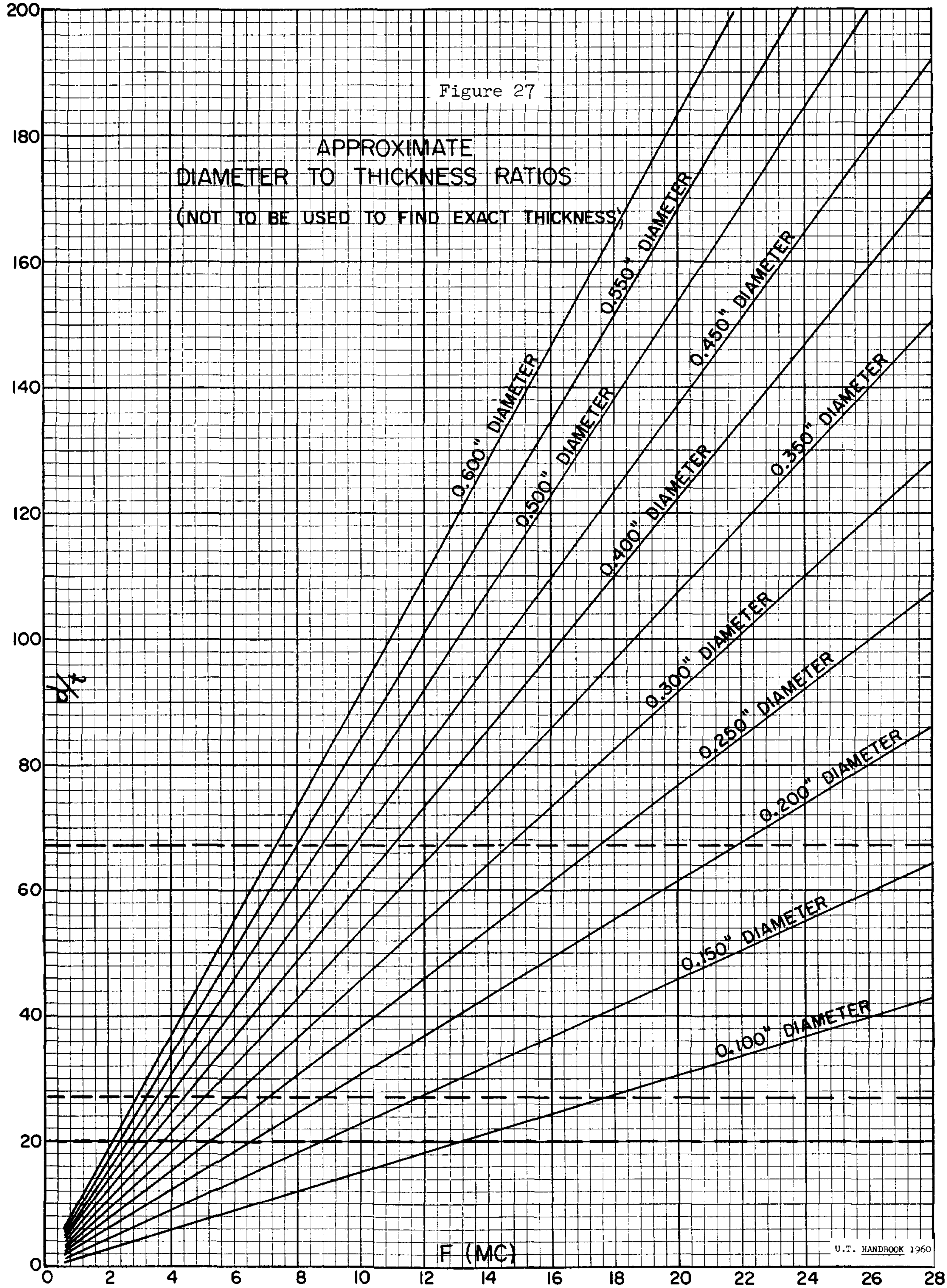
Figure 27
APPROXIMATE
DIAMETER TO THICKNESS RATIOS
(NOT TO BE USED TO FIND EXACT THICKNESS)
0.600" DIAMETER
0.550" DIAMETER
0.500" DIAMETER
0.450" DIAMETER
0.400" DIAMETER
0.350" DIAMETER
0.300" DIAMETER
0.250" DIAMETER
0.200" DIAMETER
0.150" DIAMETER
0.100" DIAMETER
d/t
F (MC)
0
20
40
60
80
100
120
140
160
180
200
2
4
6
8
10
12
14
16
18
20
22
24
26
28
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Kinds of Contour and Nomenclature

The symbols PP, PsP, etc., are not, as yet, in standard use, but are suggested as a convenience. P means plane or flat and S means a spherical curvature. If the letter is a capital, it refers to the center of the plate. If it is not followed by a small letter, it refers to the entire surface (one side) of the plate. If a small letter follows, it indicates the shape of the edges. By using a series of letters for different types of curvature, almost any shaped circular plate can be described. Thus E_pP_e could mean a plate with an elleptical contour in the center and flat edges on one side and a flat center and elliptical edges on the other side -- a rather unlikely shape.

Units of Curvature and Useful Equations

The curvature of a quartz plate is usually expressed in millimeters or inches radius of curvature, or in diopters. The diopter is an optical unit, defined in terms of focal length, and therefore involving the refractive index of the material of the lens. The chief manufacturers of spherical laps, which are described in diopter values, adhere to an arbitrary, fixed value, as given in the conversion equations below. It has been suggested that the quartz resonator plate industry should, since it is not concerned with focus, use the reciprocal meter instead of the diopter.

Radius of Curvature and Diopters

r = radius of curvature

$$r(\text{inches}) = \frac{20.866}{\text{diopters}}$$

$$r(\text{mm}) = \frac{530}{\text{diopters}}$$

$$\text{Diopters} = \frac{20.866}{r(\text{inches})} = \frac{530}{r(\text{mm})}.$$

Reciprocal Meters

$$\text{The reciprocal meter} = \frac{1000}{r(\text{mm})} = \frac{\text{diopters}}{0.530}.$$

FIGURE 28

NOMENCLATURE FOR CONTOURS

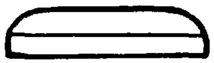
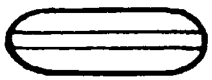
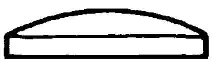
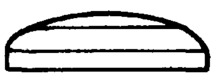
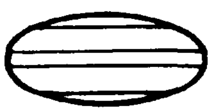
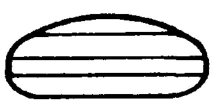
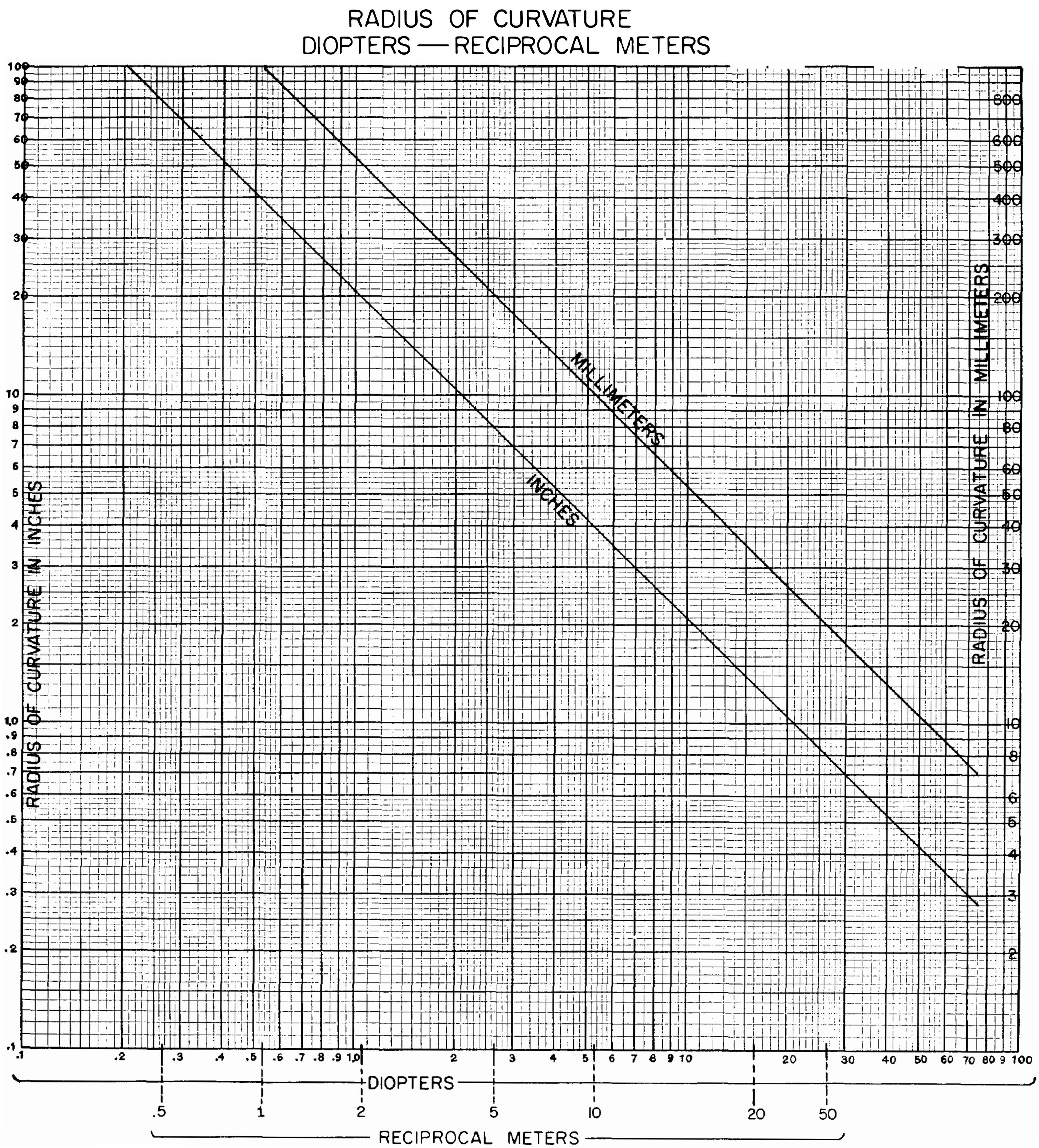
DIAGRAM	TOP CENTER	TOP EDGE	BOTTOM CENTER	BOTTOM EDGE	NAME	SYMBOL
	flat	flat	flat	flat	uncontoured,parallel	PP
	flat	spherical	flat	flat	plano-bevel	PsP
	flat	spherical	flat	convex, spherical	double bevel	PsPs
	spherical	same as center	flat	flat	plano-convex	SP
	convex, spherical	same as center	convex, spherical	same as center	double-convex	SS
	convex, spherical	convex,spherical but greater curvature than center	flat	flat	plano-step	SsP
	convex, spherical	convex, spherical but greater curvature than center	convex, spherical	convex, spherical but greater curvature than center	double-convex with double bevel	SsSs
	convex, spherical	convex, spherical but greater curvature than center	flat	convex, spherical	plano-convex with double bevel	SsPs

FIGURE 29


RADIUS OF CURVATURE
DIOPTERS — RECIPROCAL METERS
RADIUS OF CURVATURE IN INCHES
RADIUS OF CURVATURE IN MILLIMETERS
MILLIMETERS
INCHES
100 90 80 70 60 50 40 30 20 10 9 8 7 6 5 4 3 2 1.0 .9 .8 .7 .6 .5 .4 .3 .2 .1
800 600 500 400 300 200 100 80 60 50 40 30 20 10 8 6 5 4 3 2
.1 .2 .3 .4 .5 .6 .7 .8 .9 1.0 2 3 4 5 6 7 8 9 10 20 30 40 50 60 70 80 9 100
DIOPTERS
.5 1 2 5 10 20 50
RECIPROCAL METERS

<u>Relative Curvature</u>

Relative curvature, (t/r) is a useful quantity when it is desired to transfer proportions.

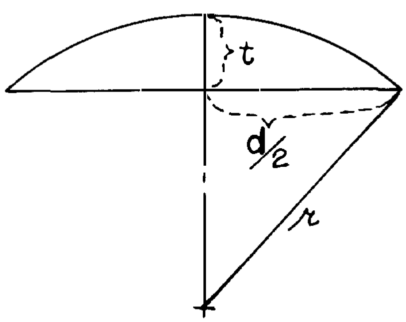
$$t/r = \frac{t(\text{inches})\ \text{diopters}}{20.86} = \frac{t(\text{mm})\ \text{diopters}}{530}$$

where t is the thickness of the quartz plate and r is the radius of curvature of the surface.

<u>Maximum and Minimum Equations -- Geometrical Limits</u>

The figure shows the maximum curvature (minimum radius of curvature, r) which can be put upon a blank with diameter d, and thickness t.

Plano-Convex

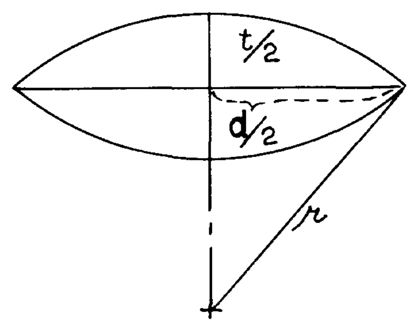


Minimum $r = \sqrt{(r-t)^2 + (\frac{d}{2})^2} = \frac{d^2 + 4t^2}{8t}$.

Maximum curvature in diopters is $\frac{166.928t}{d^2 + 4t^2}$ when d and t are in inches, and $\frac{4240t}{d^2 + 4t^2}$ when d and t are in millimeters.

Maximum $t/r = \frac{8}{(d/t)^2 + 4}$

Bi-Convex



Minimum $r = \sqrt{(r - \frac{t}{2})^2 + (\frac{d}{2})^2} = \frac{d^2 + t^2}{4t}$.

Maximum curvature in diopters is $\frac{83.464t}{d^2 + t^2}$ when d and t are in inches, and $\frac{2120t}{d^2 + t^2}$ when d and t are in millimeters.

Maximum $t/r = \frac{4}{(d/t)^2 + 1}$

Practical Considerations Influencing Choice of Type of Contour

Three types of contour are in most common use: plano-convex(SP), bevelled (PsP and PsPs), and bi-convex (SS).

The plano-convex contour is relatively easy to control in production, including processing without change of the orientation angle. It is also relatively easy to design for d/t ratios greater than about 27.

The bevelled type is difficult to design because it introduces an additional variable, the plateau diameter. In practice, most bevelled designs have proved critical, and therefore difficult to control in production.

The bi-convex type is difficult to control in production. In particular, it is difficult to maintain the orientation angle, and difficult to control the symmetry of the contour. Lack of precise symmetry is a serious factor in the production of resonator plates for filter use.

Definition of Contour Design and the Principle of Similarity

A good contour design should provide a low resistance resulting from low mounting losses, and there should be no abrupt increases in resistance with temperature (activity dips), throughout the temperature range to which the design applies. Tolerances should be reasonably wide.

If the diameter to thickness ratio is not too small, it is possible by varying the physical parameters of a plate to define relatively large safe areas at some specific frequency. For example, at 4 mc it has been found that if the plate has a diameter to thickness ratio greater than about 27 and a plano-convex curvature between 1.75 and 2.5 diopters, it will have a low resistance and no significant fluctuations in resistance over an extended temperature range (-75°C to +150°C). This meets all the requirements for an excellent, non-critical design for 4 mc.

When such an isolated design is found, it may be possible to use it over a range of frequencies by following the similarity principle. The principle of similarity* states that if two homogeneous, isotropic bodies have the same shape, elastic constants, and density,

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* Lord Rayleigh, *Theory of Sound*.

and if all the linear dimensions of each have a constant ratio, then the periods of vibration of the two bodies will have the same ratio as their dimensions. To apply this principle to the design of quartz resonators we need only add the requirement that the two bodies have the same crystallographic orientation.

Figure 30 facilitates the use of the principle of similarity for the transfer of a known good design for some specific frequency to another frequency. For example, it says that if a blank 0.300" in diameter with a contour of 3 diopters works satisfactorily at 6 mc, at 3 mc a blank 0.600" in diameter with a contour of 1.5 diopters should be satisfactory.

Practical Designs for Specific Frequency and Diameter Ranges

1. For diameters no greater than 0.600" and frequencies down to 3 mc ($d/t > 27$).

The lower limits of diameter and frequency are determined by the fact that the minimum diameter to thickness ratio for this particular design is about 27. The principle of similarity is applied not only to find the optimum values, but also to find the safe tolerances for each parameter.

The confirmed results are shown in Figures 31 and 32. The contour is plano-convex.

(In Figure 32 "maximum plate diameter" is the geometrical maximum. See equations on page 62.)

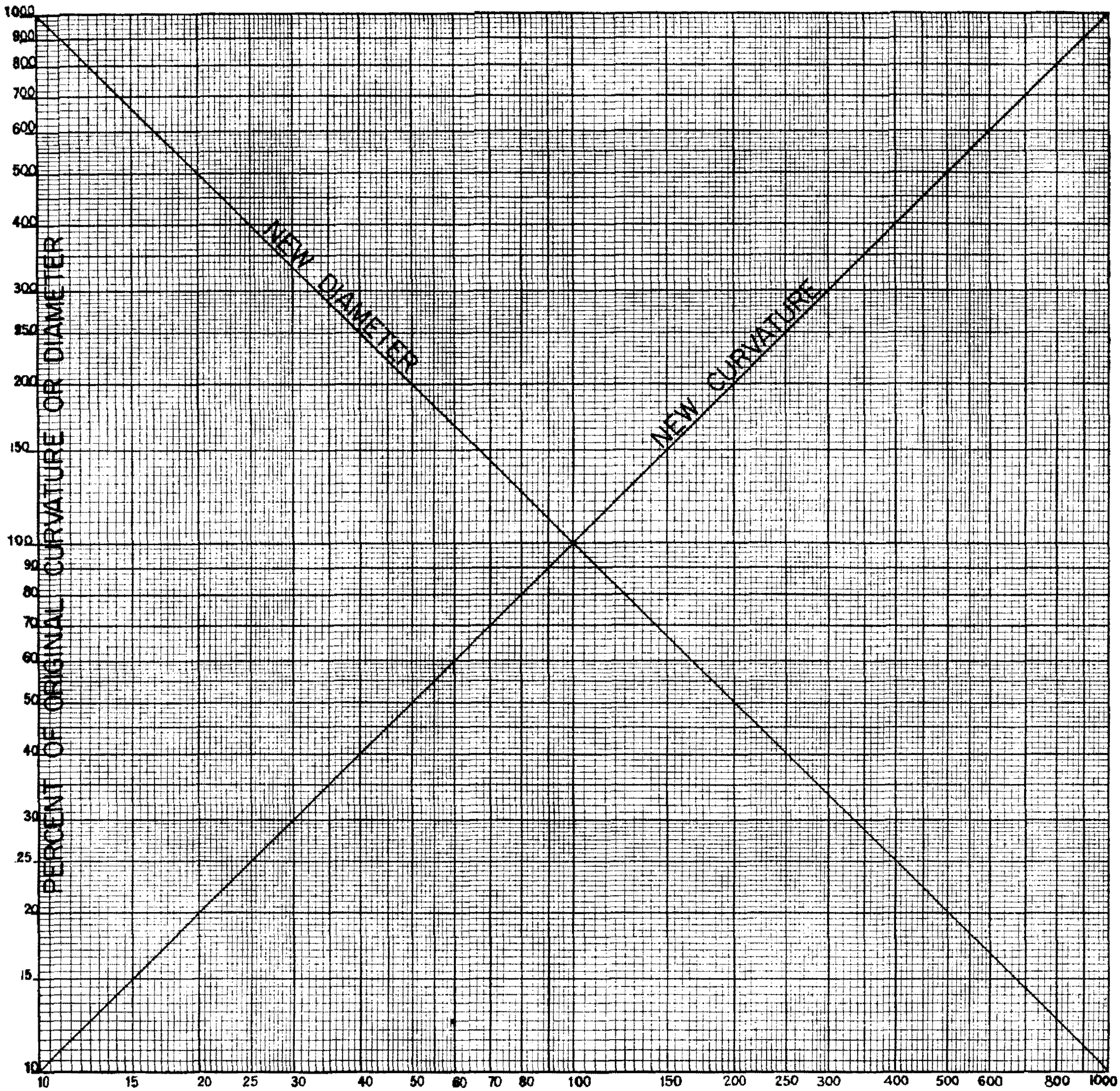
Let us assume that we wish to produce resonators at three frequencies in this range, 3.5 mc, 4 mc, and 6 mc. We read the graphs as follows:

Frequency	Curvature in Diopters	Minimum Plate Diameter
3.5	1.5 - 2.2	0.510"
4	1.75 - 2.5	0.450"
6	2.6 - 3.7	0.300"

Obviously one convenient diameter such as 0.550", 0.575", or 0.600" will serve for all three. For the 3.5 and 4 mc units we can use a curvature of two diopters, and for the 6 mc units, a curvature of three diopters.

FIGURE 30

APPLICATION OF SIMILARITY PRINCIPLE TO DETERMINE EQUIVALENT PLANO—CONVEX OR BI—CONVEX DESIGN AT A DIFFERENT FREQUENCY.



PERCENT OF ORIGINAL FREQUENCY

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FIGURE 31

DESIGN PARAMETERS — PLANO CONVEX ($\frac{d}{t}$>27)

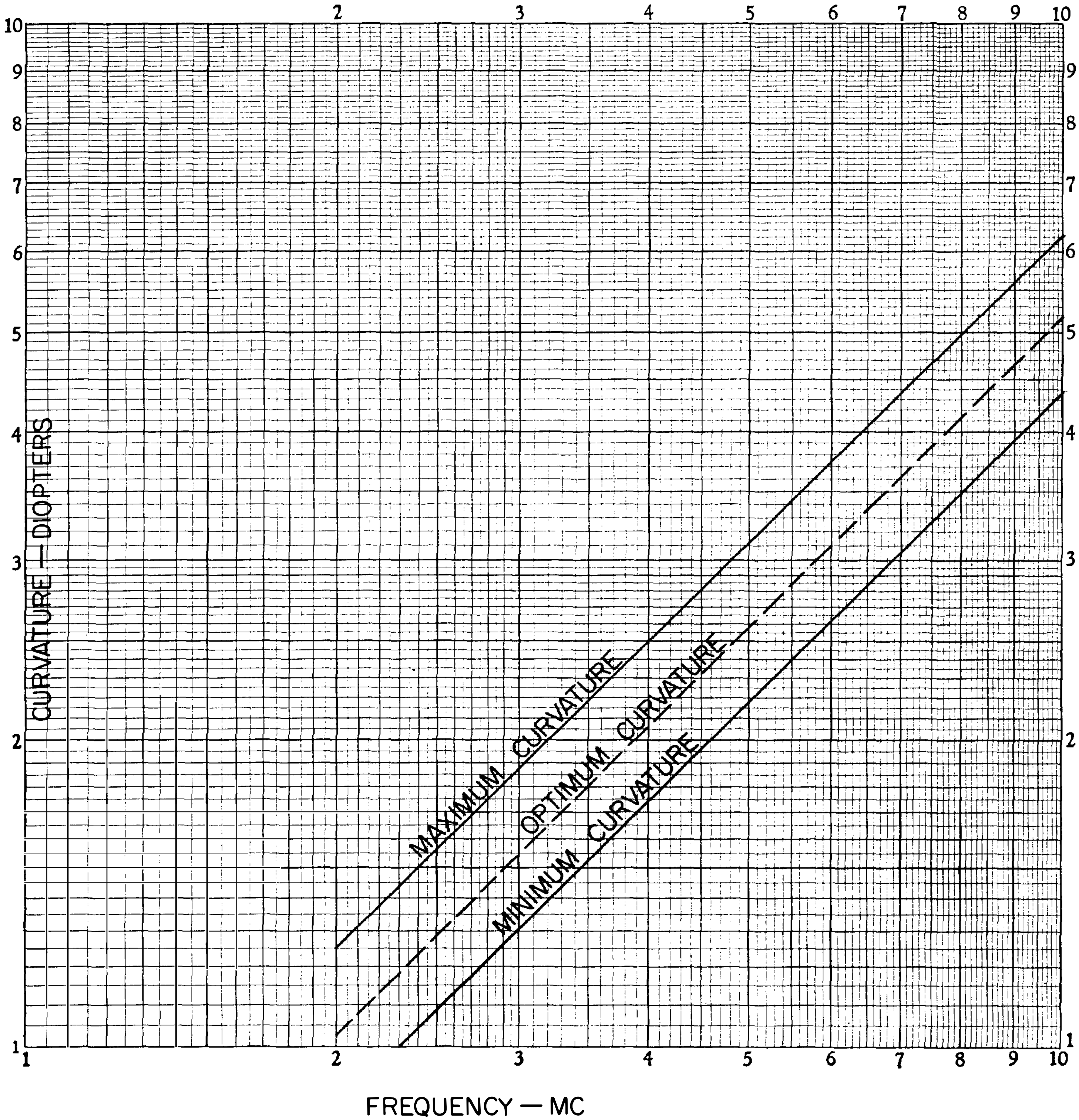
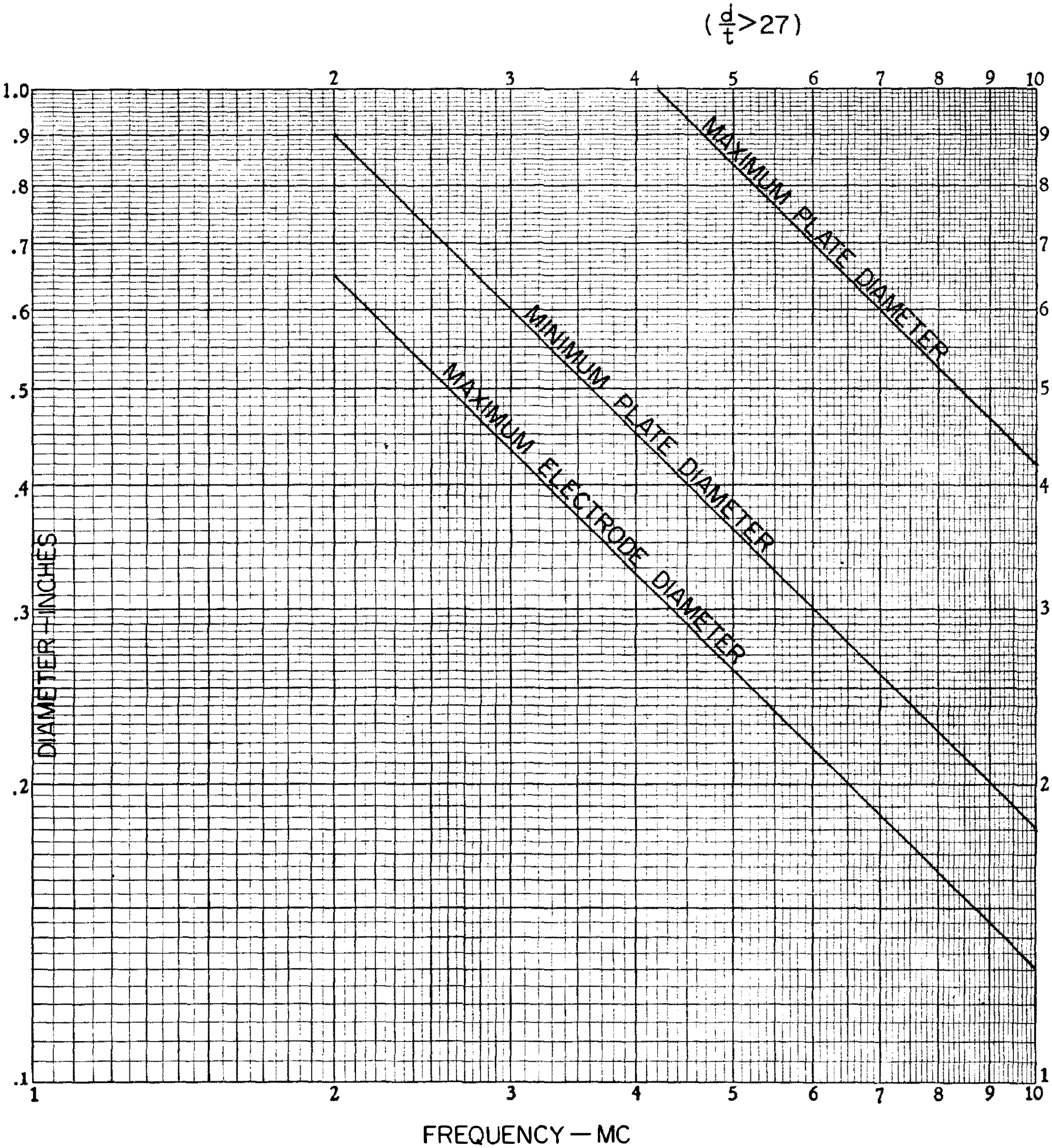


FIGURE 32


(d/t > 27)
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
DIAMETER—INCHES
1
2
3
4
5
6
7
8
9
10
FREQUENCY — MC
MAXIMUM PLATE DIAMETER
MINIMUM PLATE DIAMETER
MAXIMUM ELECTRODE DIAMETER

2. For diameters no greater than 0.600" and frequencies down to 2 mc (d/t > 17).

The design given in (1) above could be used at 2 mc if it were possible to use a plate over 0.900" in diameter. Since 0.600" is a common maximum convenient size for frequently specified holders, it is necessary to apply the principle of similarity to define another design. This is done in Figure 33. This graph says that from 5 mc down to 1.9 mc a plano-convex curvature of slightly more than five diopters can be used on plates no smaller than 0.600", and that we have various other alternatives; for example, at 2 mc a plate 0.550" in diameter with a curvature of six diopters would be marginal.

It will be noted that Figure 33 overlaps Figures 31 and 32. For example, at 3.25 mc, using Figures 31 and 32, we would select a diameter between 0.550" and 0.600" and a curvature of 1.75 diopters. For the same 3.25 mc frequency, using Figure 33, we would select a curvature between 3.5 and 5.5 diopters and any convenient diameter between 0.500" and 0.600". This means that the principle of similarity is being applied to two different sets of proportions, the second set requiring much more curvature, and permitting us to go a megacycle farther down in frequency before a diameter greater than 0.600" is required.

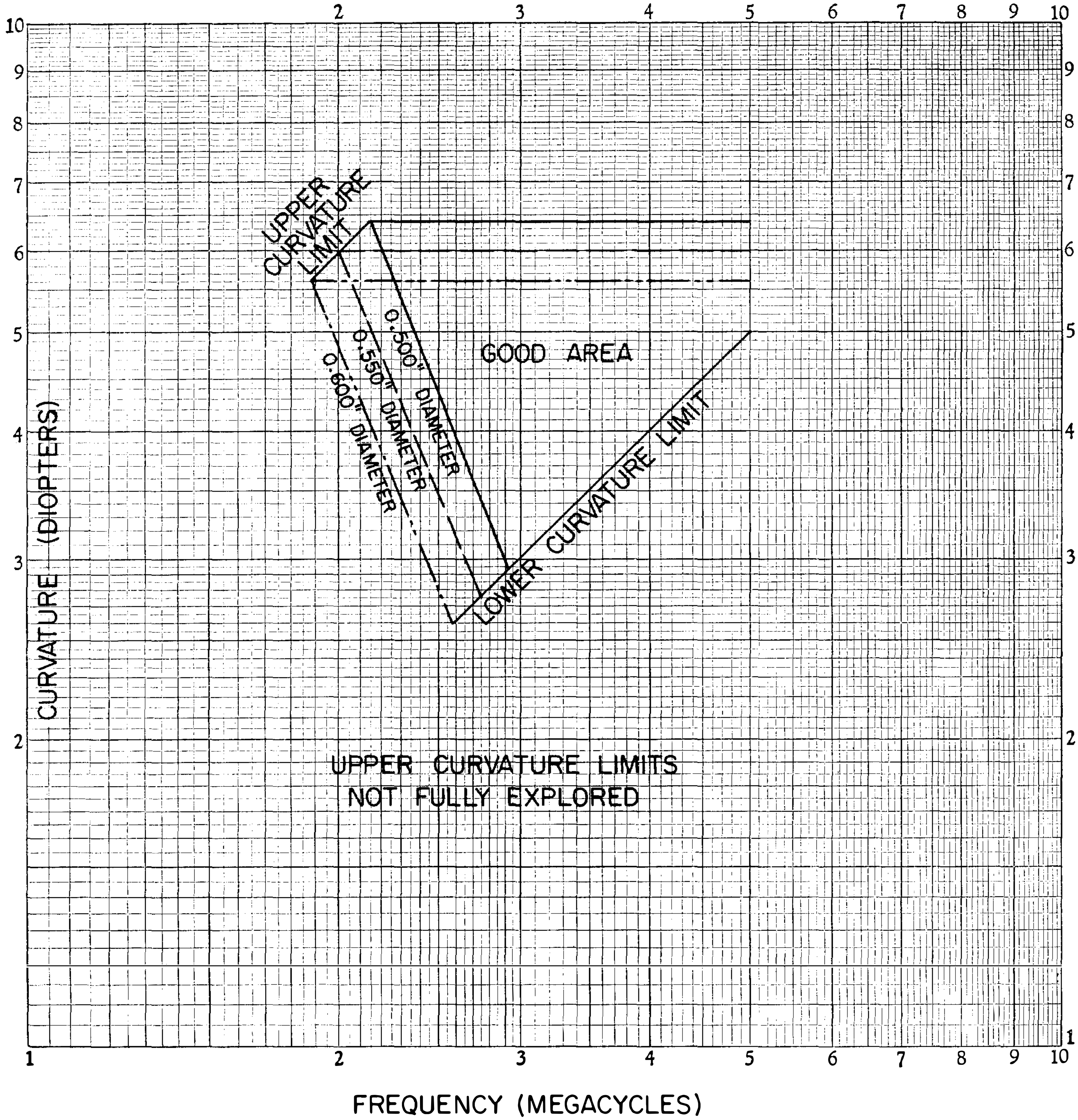
3. Transfer of Designs to Higher Frequencies and Smaller Diameters

According to the principle of similarity, a design can be transferred to a smaller diameter providing that all parameters are altered accordingly; that is the diameter to thickness ratio must be preserved, and if a smaller diameter is used, the thickness must be less and the frequency higher. For example, we wish to make 4 mc units in a holder which will accomodate a plate with a maximum diameter of 0.350". From Figure 27 we learn that our diameter to thickness ratio will be about 21 and that this is the ratio for an 0.500" plate at 2.8 mc. So we turn to Figure 33 and find that at 2.8 mc for an 0.500" plate we have a range of curvatures from 3.3 to 6.4 diopters. But 4 mc is 143% of 2.8 mc. So, turning to Figure 30 we find our curvature for a 4 mc, 0.350" plate to be between 5 and 9 diopters (round numbers).

As the design is scaled down (smaller diameter, thinner plate) the resistance of the resonators will increase.

FIGURE 33

DESIGN FOR PLANO—CONVEX PLATES
DIAMETER TO THICKNESS RATIO 17 TO 27



4. Designs for Diameter to Thickness Ratios less than 17

It will have been noted that although the design represented by Figure 33 results in extending the frequency range downward only 1 megacycle, the representation in graphical form has become quite complex. For smaller diameter to thickness ratios, the parameters become much more critical, and graphical presentation is impractical.

Tables 2 and 3 give a series of recommended designs for bi-convex plates from 800 kc to 1500 kc and for plano-convex plates from 1400 kc to 3000 kc. In all cases the plate is to be mounted on the Z' axis at two points. The electrode size is not critical but should be at least 0.300" in diameter. The first six bi-convex designs (800-850 kc) are of the so called "racetrack" shape in which flat surfaces are ground normal to the Z' axis in order to limit the mounting axis dimension to no more then 0.600 inches.

TABLE 2 (1)

- RECOMMENDED BI-CONVEX PLATE DESIGNS -

FREQUENCY KILOCYCLES	DIAMETER INCHES	Z' WIDTH INCHES	CURVATURE DIOPTERS
800	.640	.595	17.00
810	.630	.590	17.25
820	.620	.580	17.25
830	.615	.575	17.50
840	.605	.565	17.75
850	.600	.560	18.00
850	.600		16.00
860	.595		16.25
870	.585		16.50
880	.580		16.50
890	.575		16.75
900	.565		17.00
900	.600		14.00
910	.595		14.25
920	.585		14.25
930	.580		14.50
940	.575		14.50
950	.570		14.75
960	.560		15.00
970	.555		15.00
980	.550		15.25
990	.545		15.50
1000	.540		15.50

TABLE 2 (2)

RECOMMENDED BI-CONVEX PLATE DESIGNS (cont'd)

FREQUENCY KILOCYCLES	DIAMETER INCHES	CURVATURE DIOPTERS
1000	.550	15.00
1020	.540	15.25
1040	.530	15.50
1060	.520	16.00
1080	.510	16.25
1100	.500	16.50
1120	.590	10.25
1140	.580	10.50
1160	.570	10.75
1180	.560	10.75
1200	.550	11.00
1225	.595	7.50
1250	.585	7.50
1275	.570	7.75
1300	.560	7.75
1325	.550	8.00
1350	.540	8.25
1375	.530	8.25
1400	.520	8.50
1400	.550	12.00
1425	.540	12.25
1450	.530	12.50
1475	.520	12.75
1500	.515	12.75

TABLE 3 (1)

RECOMMENDED PLANO-CONVEX PLATE DESIGNS

FREQUENCY KILOCYCLES	DIAMETER INCHES	CURVATURE DIOPTERS
1400	.600	14.00
1425	.590	14.25
1450	.580	14.50
1475	.570	14.75
1500	.560	15.00
1525	.550	15.25
1550	.540	15.50
1575	.535	15.75
1600	.525	16.00
1625	.515	16.25
1650	.510	16.50
1675	.500	16.75
1700	.585	17.00
1725	.575	17.25
1750	.565	17.50
1775	.560	17.75
1800	.550	18.00
1825	.545	18.25
1850	.535	18.50
1875	.530	18.75

TABLE 3 (2)

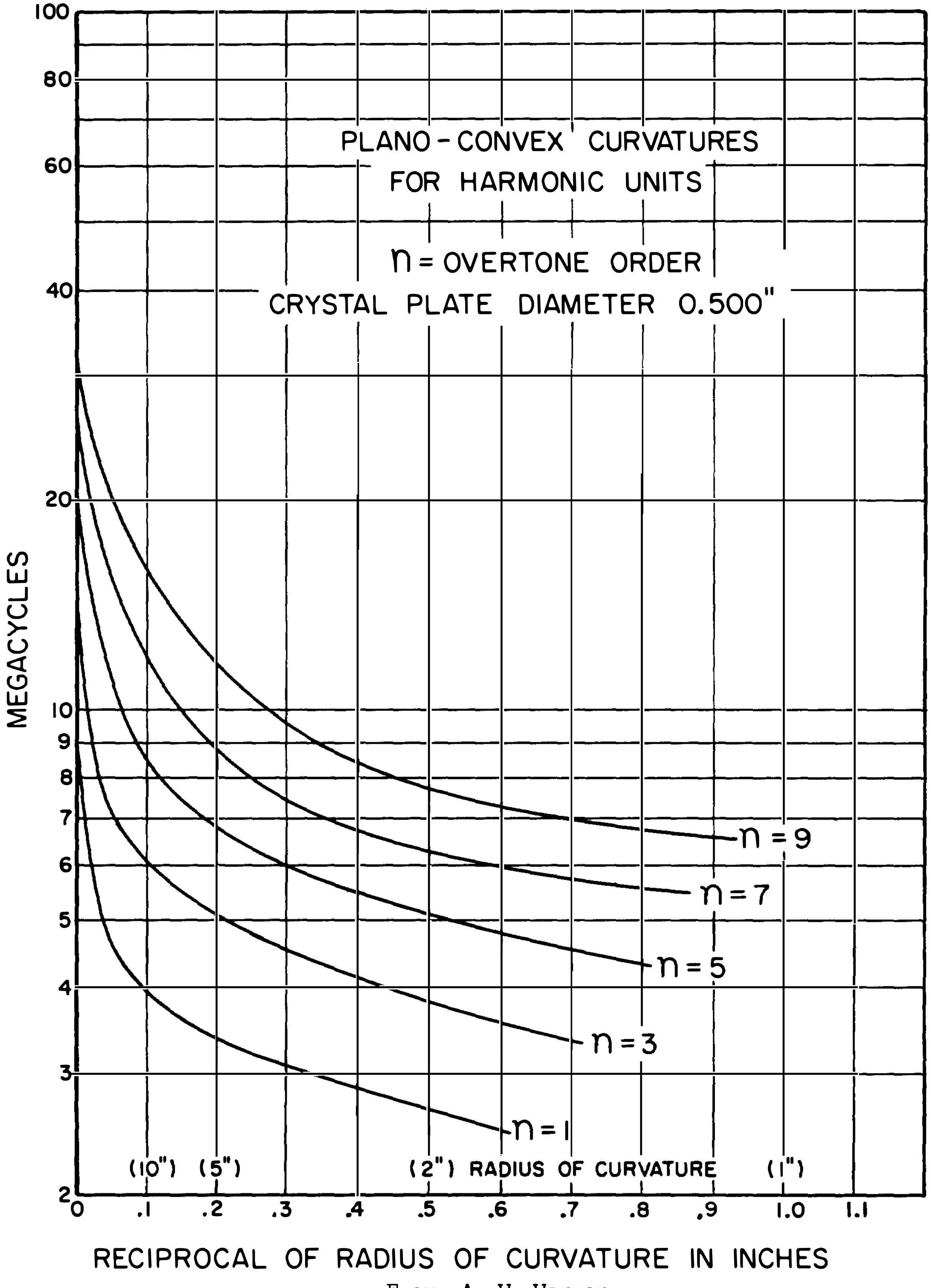
RECOMMENDED PLANO-CONVEX PLATE DESIGNS (cont'd)

FREQUENCY KILOCYCLES	DIAMETER INCHES	CURVATURE DIOPTERS
1900	.520	19.00
1925	.515	19.25
1950	.510	19.50
1975	.500	19.75
2000	.600	12.00
2050	.585	12.25
2100	.570	12.50
2150	.560	13.00
2200	.545	13.25
2250-2500	.550	5.00
2500-2750	.550	4.50
2750-3000	.550	4.00

Contour Designs for Harmonic Units

Because most uses of quartz resonator units on other than their fundamental frequency have been at relatively high frequencies where the diameter to thickness ratios were quite large, there is very little data on the contouring of plates intended for harmonic operation. The problem arises, however, in connection with two kinds of resonators: very high precision resonators at comparatively low frequencies, and extremely small resonators at higher frequencies. The first category has been extensively investigated by the Bell Telephone Laboratories (see page 35 above) for a few special frequencies. Figure 34 represents a general exploration of the subject, and is valuable, at least as a starting point for further investigations made in terms of particular specifications. For example, below 5 mc some of the curvatures indicated may not result in a sufficient reduction of activity dips unless some adjustments are made.

FIGURE 34



From A. W. Warner,
<u>Proc. I.R.E.</u>
Sept. 1952,
p. 1033