PART I

THE QUARTZ RESONATOR

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In recent years the growing use of radio for communication has resulted in a shortage of space in the radio spectrum. To alleviate this situation, the assigned frequencies are being crowded closer together. The result is to impose more strict requirements upon the accuracy with which the transmitter must adhere to its assigned frequency, and, of course, equally stringent requirements upon the receiving equipment, including the accuracy of the tuned circuits in the receiver.

The problem of maintaining a constant frequency in an oscillating system usually involves incorporating an element into the system which has a natural period of oscillation that is relatively independent of amplitude, temperature, etc. The best known example of this is the pendulum of a clock or the hairspring and balance wheel in a watch. If the driving force maintaining the oscillation varies, it will result in a change in the amplitude of oscillation, but since the period of the pendulum is practically independent of the amplitude, the frequency remains essentially constant.

The frequency controlling element most commonly used in vacuum tube oscillators is the "LC" circuit, consisting of an inductor and a capacitor either in series or parallel and constituting what is known as a "tuned" circuit. The tuned circuit controls the frequency of the oscillator by virtue of the fact that near its resonant frequency, the impedance of the circuit varies rapidly with frequency. The ability of the tuned circuit to maintain a constant frequency of oscillation is limited, however, by variations in its resonant frequency with temperature, vibration, aging of components, etc., and by the fact that the electrical resistance of its components limits the "Q" that can be obtained. The greater the "Q" of the tuned circuit, the less the resonant frequency of the oscillator will be affected by variations in vacuum tube characteristics, power supply voltage, etc.

The resonant frequency stability of tuned circuits can be considerably improved if great care is exercised in their design and construction and if added refinements such as temperature control are employed, but this results in components that are both large and expensive. Even with excellent design, the highest Q's obtainable are of the order of several hundred.

Many of the deficiencies of the tuned circuit can be overcome by employing a mechanical vibrating element that is driven by an electrical circuit. Among such "electro-mechanical" resonators that have been successfully employed are electrically driven tuning forks, magneto-striction resonators, and piezoelectric resonators. By proper choice of the material composing the resonating element, the change in resonant frequency of the tuning fork or magneto-striction resonator with temperature can be minimized. Q's of several thousand can be obtained with these resonators. The piezoelectric quartz resonator, however, is much superior to the other electro-mechanical resonators both in stability with change in temperature and in Q. Q's of several hundred thousand are quite common.

Basically, a piezoelectric resonator consists of a suitably cut and shaped piece of piezoelectric material (called a piezoid) with associated electrodes. The electric field produced when a potential is applied to the electrodes causes the piezoid to distort in some manner, depending upon the crystallographic orientation of the piezoid and the electrode configuration. Removing the electric field allows the piezoid to return to its original shape. The actual distortion produced is quite small. If an alternating potential is applied to the electrodes, the piezoid will vibrate at the frequency of the alternating potential. The impedance of the resonator, however, will be quite high unless the frequency of the applied voltage coincides with a natural resonant frequency of the piezoid. The very rapid change in impedance near the resonant frequency enables the piezoelectric resonator to control the frequency of a vacuum tube oscillator.

The Piezoelectric Effect

The piezoelectric effect has been known since the work of Pierre and Jacques Curie in 1880, who found that certain crystals when compressed in certain directions relative to the crystallographic axes develop electrical charges on their surface. *Piezo* comes from a Greek work meaning "to press." Other types of stresses are effective in producing electrical charges in some crystals, and the term is applied in general to the production of electrical charges by the application of mechanical stresses to crystals.

Although the Curies studied the phenomenon rather extensively and constructed a few simple devices utilizing the effect, few workers became interested in the phenomenon for nearly forty years. One exception would perhaps be made for the German crystallographer Voigt, who considered the phenomenon is his great book on crystal physics, *Lehrbuch der Kristall Physik*, which he published in 1910.

During World War I, Langevin, again in France, used plates cut from quartz crystals to generate and receive sound waves in water for use in detecting submarines and other submerged objects. His work was the origin of modern sonar and ultrasonics. In the course of some similar experiments in 1918, Cady observed that a vibrating crystal, in the neighborhood of a mechanically resonant frequency, had a reactive effect upon the driving circuit. From this observation has come the piezoelectric resonator which is used, not only as a frequency stabilizing device, but also as a filter and electromechanical transducer.

Quartz

In his Lehrbuch, Voigt listed the classes of crystals which exhibit the piezoelectric effect and established criteria for the existence of the phenomenon. He showed that piezoelectricity is not a particularly rare phenomenon since it appears in 20 of the thirty-two possible classes of crystals. What is rare, however, is a crystal having suitable mechanical and thermal properties as well as exhibiting the piezoelectric effect. Of all the crystals which have been examined, quartz stands completely alone in meeting all the requirements for a satisfactory material for use in the communication art.

Alpha quartz is one of the crystalline forms of silicon dioxide. Above 573°C another crystalline structure, beta quartz, is formed. The limiting temperature for the use of alpha quartz is, therefore, 573°C. Beta quartz is not useable for oscillator plates. Hereafter the word quartz will refer only to alpha quartz.

Although quartz crystals are among the finest examples of large crystals found in nature, they are often far from perfect. Furthermore the external shape or condition of a given specimen provides little or no indication of the usability of the material for piezoelectric purposes.

Quartz crystals are usually transparent, although discolored specimens are often found. The discolorations in rose quartz, amethyst, etc. are due to extremely small concentrations of impurity atoms in the crystal. The discoloration in smoky quartz, which ranges from a light yellow to black, is due to the exposure of quartz crystals containing certain imperfections, to gamma radiation. The smokiness can be removed by heating the crystal to a temperature of about 400°C for a few minutes. Heating and cooling must be done slowly in order to avoid shattering the crystal. Insofar as is known at present, smokiness has no effect on the piezoelectric properties, but since the smokiness is known to be associated with the effect of radiation on crystals containing minute amounts of impurities, it is possible that effects might be noticeable in units designed to meet extremely rigid specifications.

A "faced" crystal is one having one or more identifiable natural faces. The presence of one or more natural faces facilitates orientation and mounting for sawing, but methods are available whereby quartz crystals having no identifiable faces may be utilized. Much excellent quartz is obtained from river beds where abrasion has removed all traces of the natural faces from the crystals.

Quartz is an enantiomorphic crystal in that it occurs in two different forms called right-handed and left-handed. The effect of this difference is to reverse the arrangement of the natural faces of the crystal so that one is the mirror image of the other (Figure 1). If the natural faces are not all present, the handedness of a crystal can be determined by the use of polarized light.

Specimens of right and left hand quartz are equally usable for piezoelectric purposes but cognizance must be taken of the hand of the quartz in fabricating piezoids. A crystal in which both right and left quartz are present is said to be optically twinned. In general the boundary between a region of right quartz and a region of left quartz must not be included in a piezoid since for a given exciting field the motion of the two parts is different. Optical twinning can be detected either by polarized light or by etching. If the twin boundary is located advantageously, the crystal may be sawed approximately along the twin boundary and the two portions fabricated separately.

A more troublesome defect is Dauphiné or electrical twinning. This defect consists of a localized reversal of the polarity of the X-axis. The boundary between regions of electrically twinned quartz must be excluded from a given piezoid. Electrical twinning is readily discernible on sawed or lapped wafers after etching.

Although both electrical and optical twinning are revealed by etching, the two are easily distinguishable. The boundaries between portions of an optical twin are straight lines parallel to prism faces. Optical twinning is often confined to regions near the surface of large crystals. The boundary between portions of an electrical twin do not follow any systematic pattern but tend to wander at random.

Numerous other defects exist in quartz crystals. Such defects as cracks, veils, bubbles, needles, inclusions, specks, etc., can be seen by immersing the crystal in a fluid having an index of refraction roughly equal to that of quartz, and illuminating it with a strong light. Some of these defects may be tolerable in certain types of units but for others quartz of the highest possible quality is required.

Quartz is found at several places on the earth, such as Madagascar, Australia, India, the Appalachian Mountains, Arkansas, the Lake Superior region, and the Ural Mountains. A certain amount of Madagascar quartz is used in Europe, but the main source of supply of radio grade quartz for the Western World is Brazil,where two extremes are fortunately combined, extremely rich deposits and extremely cheap labor. The known quartz supplies on the North American continent would not be worth mining even were the labor rates here a small fraction of what they are.

Only a very small fraction of the Brazilian quartz which is mined is shipped, and still less is actually sold to the manufacturer of oscillator plates, for before it reaches the manufacturer rough visual inspection for extreme discoloration, fractures, optical twinning, etc., has eliminated much of the material. Even after this rough inspection, a large part of the material is not usable. Of the total weight of quartz -- good, milky, bull quartz, etc. -- which is mined in Brazil, it is estimated that only 1 pound in 10,000 reaches the manufacturer of quartz resonator plates.*

Specifications and methods of grading natural quartz for piezoelectric applications are given in MIL-C-15729A (18 October 1951). Pertinent parts of this specification are reproduced in Appendix III of this Handbook.

Scientists in Italy and Germany, before and during World War II, expended much effort toward the development of a process for the artifical growing of quartz crystals. They were not successful in creating a practical production process, but after the war the subject was undertaken under government sponsorship, chiefly, in this country. The program was notably successful, and by 1959 cultured quartz was available on the market at prices which, taking into account elimination of waste, and saving in tooling and labor, made it competitive with natural quartz in an increasing number of applications.

There are different methods of growing quartz, different pressures and temperatures, and growth in different directions is favored by differently oriented and dimensioned seed plates. Most of the cultured quartz which has been made so far, for reasons which are not yet perfectly understood, has a lower Q (greater internal loss) than natural quartz. This difference has, however, not been certainly

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* For further detail on natural quartz, see Nos.5 and 6, Vol. 30 (1945) of The American Minerologist, and W. D. Johnston and R. D. Butler, "Quartz Crystal in Brazil," Bulletin of the Geological Society of America, Vol.57, No.7 (July, 1946) pp. 602-649.

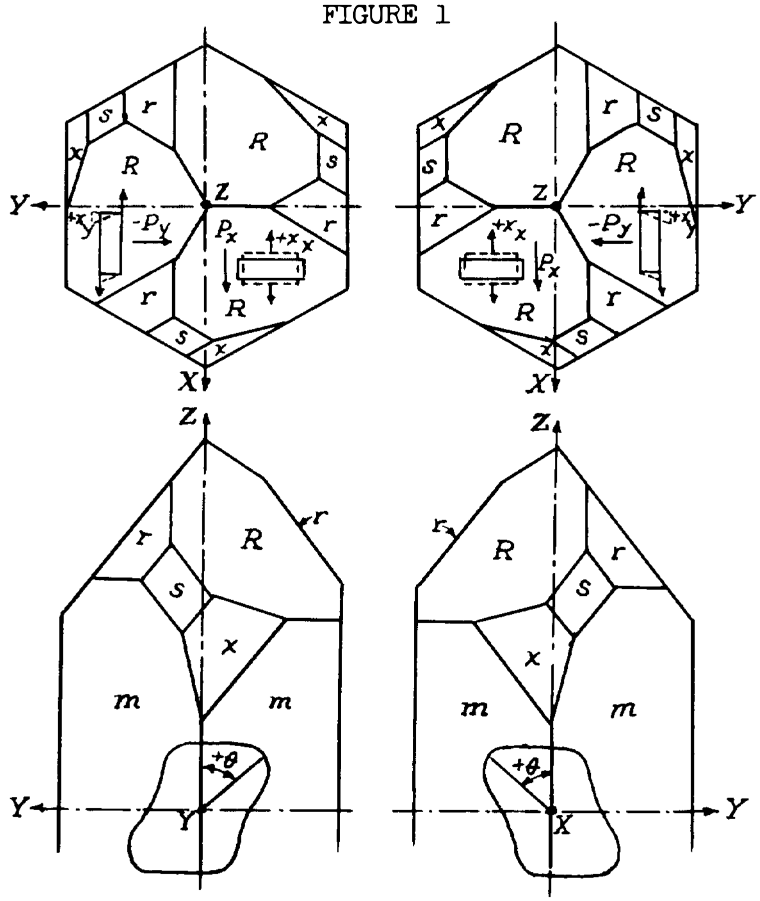
detected except in a few specially designed types of resonators with very high Q, or those operated at very low temperatures. Therefore, good cultured quartz may be used for all units now produced in quantity without any difference in the design of the units, and without the anticipation of any difference in the performance characteristics between units made from natural and those made from cultured quartz.

The Crystallography of Quartz

The crystallographer places quartz in Class 18 of the thirty two classes into which all crystals may be grouped. It is described as a trigonal, holoaxial, enantiomorphic crystal, meaning that the crystal has three fold symmetry about one axis, and that it may exist in either of two forms -- right or left hand.

A drawing showing the form of the idealized quartz crystal is given in Figure 1. The distinction between right and left quartz is

FIGURE 1



LEFT- AND RIGHT-QUARTZ, SHOWING STRAINS X_x AND X_y, WITH ACCOMPANYING POLARIZATIONS P_x AND P_y, ALSO SHOWING THE POSITIVE SENSE OF THE ANGLE OF ROTATION θ.

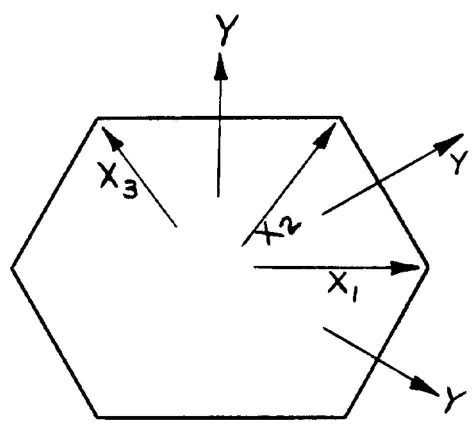
By permission from Piezoelectricity, by G. Cady, Copyright, 1946, Mc Graw-Hill Book Company Inc.

indicated by the position of the natural faces marked s and x. It should be pointed out, however, that these faces are not often found on actual crystals. In fact, in many pieces of quartz which are quite suitable for use in fabricating resonators, none of the faces is easily distinguishable. In such crystals, the internal arrangement of the atoms which comprise the crystal must be determined by the use of polarized light and X-rays.

A cross section of a quartz crystal is shown in Figure 2. In ideal form the cross section is a regular hexagon although, in most crystals the sides of the hexagon are not of equal length. In all cases, however, opposite sides of the hexagon are parallel since the surfaces are parallel to the planes of atoms within the crystal.

A direction parallel to a line bisecting the angle between the prism faces is called an X-axis. It should be noted that an axis in a crystal is not a line but a direction. There are three X-axes in quartz as shown in Figure 2. An X-axis is called a polar axis

FIGURE 2



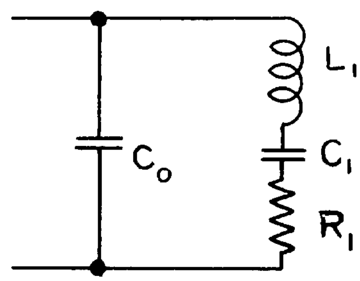
because if the crystal is subjected to a tensile stress in the direction of an X-axis, electrical polarization is produced. The directions normal to the prism faces are called Y-axes or sometimes "mechanical axes." The direction normal to the cross section is called the optic-axis or the Z-axis.

Electrical Characteristics of a Quartz Resonator

Near its resonant frequency a quartz crystal unit behaves as if it were an electrical circuit of the type shown in Figure 3.

FIGURE 3

EQUIVALENT CIRCUIT OF QUARTZ RESONATOR*



The capacitance C_o is simply the capacitance of the capacitor formed by the electrodes on the quartz plate plus any capacitance added by the lead wires and the holder. It may be measured like any other capacitor by bridge or substitution methods provided the measurement is made at a frequency which differs by a few per cent from any of the resonant frequencies of the piezoid.

If an alternating electrical potential having a frequency equal to or very near that of one of the mechanical resonance frequencies is impressed on the terminals of the unit, the piezoid is set into vibration through the piezoelectric effect.

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* In this Handbook, where harmonic orders are frequently being discussed, L_1, C_1, and R_1 are normally used only when the resonator is assumed to be operating on its fundamental frequency. If the fundamental, or any harmonic is meant, the notation is L_n, C_n, R_n, or a specific harmonic order is indicated by L_3, C_3, R_3. Wherever necessary, C_e is substituted for C_o, meaning the capacitance between the electrodes, or the total capacitance, C_o less the holder and mount capacitance. In the case of standard military units in HC-6/U and HC-18/U holders, with a specified maximum shunt capacitance of 7 μμf, C_h amounts to approximately 10% of the total shunt capacitance. In the case of units with small electrodes, such as some filter units and extreme miniatures, the percentage can be very much greater. See also the discussion of "Metallic Resistance" under Electrical Measurements. R_1, R_3, etc. as used in this Handbook normally refer to the resistance as measured at series resonance, and therefore, include the "metallic" resistance of the leads, the bonds, and the electrodes.

The electric current flowing in the circuit including the crystal unit is composed of two components, the displacement current into the capacitance C_o and the piezoelectric current through the branch L_1, C_1, R_1.

The L_1, C_1, R_1 branch is known as the "motional arm," and its components as the "motional inductance," "motional capacitance," and "motional" or "series resistance." "Dynamic" is also used to designate these components of the equivalent circuit.

At the frequency of mechanical resonance the piezoelectric current is much larger than the displacement current and the reactances of L_1 and C_1 are equal and opposite. Therefore, the crystal unit appears to be a resistance R_1 in parallel with the capacitance C_o. At frequencies below that of mechanical resonance, the reactance of the motional arm is negative, presenting a capacitive reactance which is in parallel with that of C_o. Above the frequency of mechanical resonance, the reactance of the motional arm is positive, and the motional arm thus has inductive reactance which is in parallel with the capacitive reactance of C_o. At a certain frequency the inductive reactance of the series arm is exactly equal to the capacitive reactance of C_o, and the combination forms a parallel resonant circuit.

Figure 4 shows various conditions as the frequency is varied.* X is the reactance of the motional arm. The point at which it is zero is designated as f_s, or the series resonant frequency. Since, however, R is increasing through this point, there will be a slightly lower frequency at which the total impedance of the unit is at a minimum. This is designated f_m, the frequency at minimum impedance. As indicated above, at and near the mechanical resonant frequency of the crystal the displacement current through C_o is very small, that is, the reactance of C_o is so great that it can ordinarily be ignored at least at low frequencies. If, however, it is not ignored, we must distinguish another resonant frequency, f_r, where the total reactance, X_e, including that of C_o, is zero. This is called the resonant frequency.

As the frequency is increased above the resonant frequency, the reactance of the motional arm, and of the unit as a whole, becomes increasingly inductive until its reactance approaches the value of the reactance of the C_o arm, when the total reactance of the unit again decreases to 0 at f_a, the anti-resonant frequency. At the frequency f_p, the resistance of the crystal unit is a maximum, while at f_n the total impedance is a maximum.

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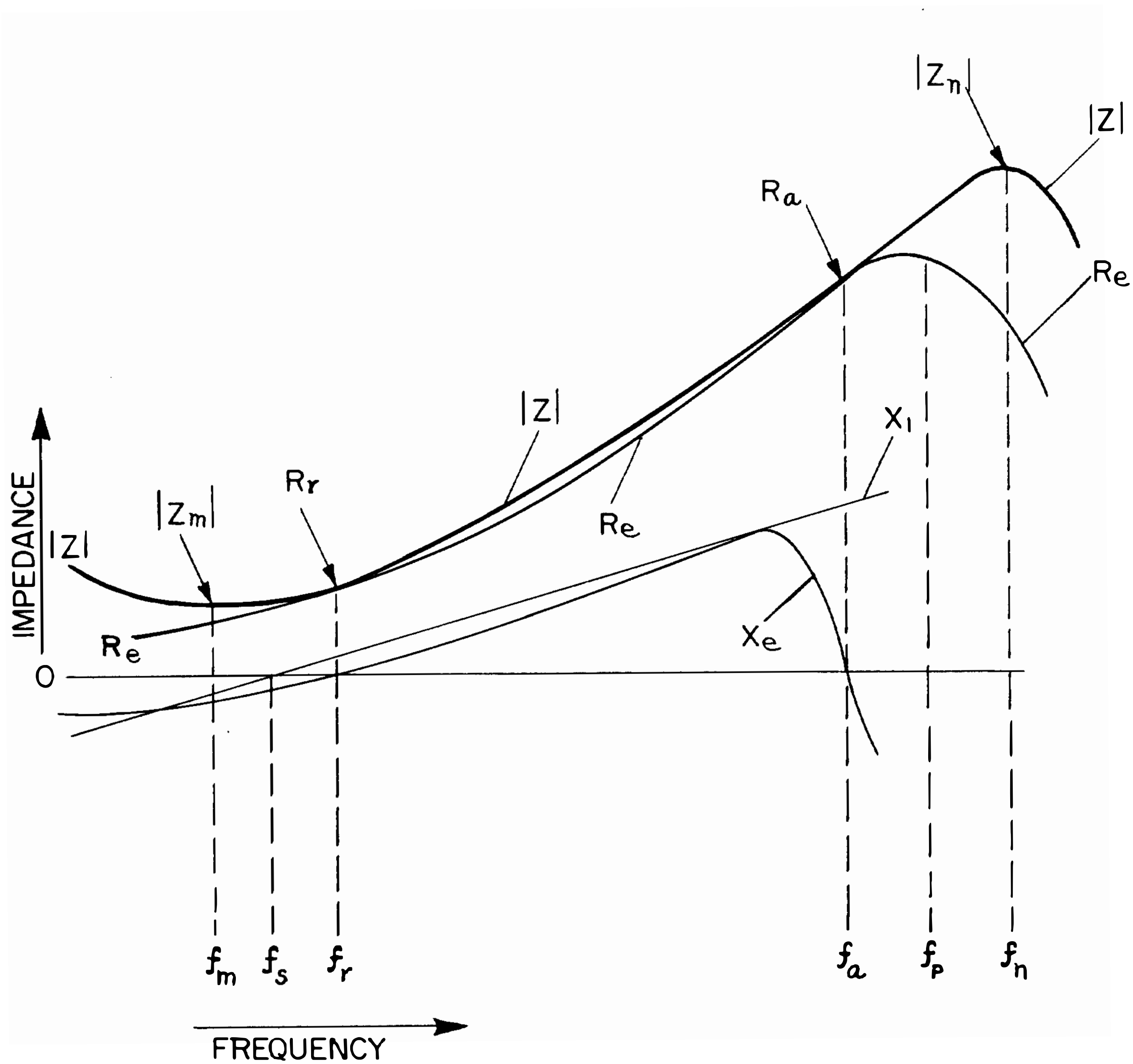
* The figure is from, and the text is an abridgement of <u>I.R.E. Standards On Piezoelectric Crystals -- The Piezoelectric Vibrator: Definitions and Methods of Measurement (1957)</u>.

FIGURE 4


|Zn|
|Z|
Ra
Re
|Z|
X1
Rr
|Zm|
Re
|Z|
IMPEDANCE
Xe
Re
O
fm
fs
fr
fa
fp
fn
FREQUENCY

At the resonant frequency (f_r), which is very nearly equal to the mechanical resonance frequency of the piezoid (f_s), the impedance of the crystal unit is a pure resistance having a value which is relatively low as is characteristic of a series resonant circuit. At the anti-resonance frequency, which is a few hundreths of one percent higher, the impedance is again a pure resistance but a very high resistance. Between the two frequencies the reactance of the crystal unit is inductive, whereas above and below the range the reactance is capacitive.

The actual values of the equivalent circuit constants of a quartz resonator are quite different from those that can be obtained with inductors and capacitors. For example, a commercial "High Q" LC circuit has the following parameters:

Frequency -------------	10 megacycles
Inductance ------------	.01 millihenries
Capacitance -----------	25 micro-micro farads
Resistance ------------	4 ohms
Q --------------------	157

By contrast, a typical quartz resonator has the following parameters:

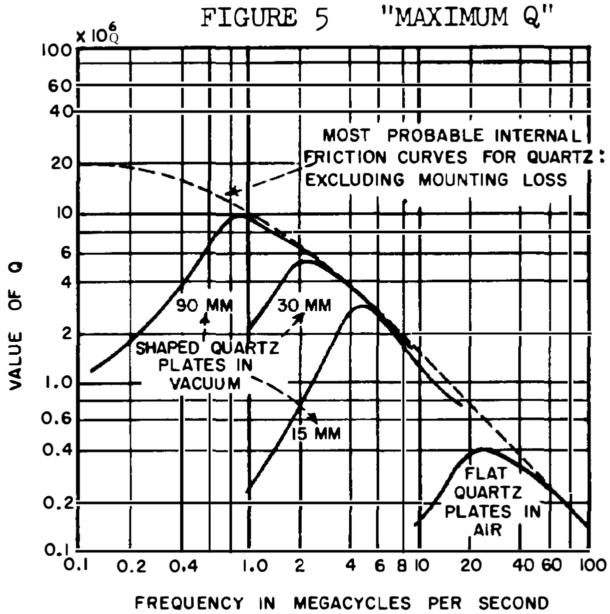
Frequency -------------	10 megacycles
Inductance ------------	9.2 millihenries
Capacitance -----------	.028 micro-micro farads
Resistance -----------	3.8 ohms
Q --------------------	152,000

The enormously greater reactance of the quartz resonator is responsible for the enormously higher Q. Q values in the millions can be achieved with specially designed quartz resonators.

In the absence of all frictional or damping forces the Q would, of course, be infinite. Damping in the piezoelectric resonator is due to internal friction within the quartz itself, surface losses associated with friction at the surface, mounting losses, and atmospheric damping. The relative importance of each of the factors depends upon the size and type of piezoid, method of mounting, surface preparation, atmosphere, and possibly upon the quality of the quartz itself.

The maximum Q is of course obtained when all of the damping losses are minimized. The mounting losses in AT* and BT* plates are minimized by making the diameter/thickness ratio as large as possible, or by contouring, or both. In this way the edges of the plate to which the suspending wires are attached are rendered as nearly free of vibration as possible. Atmospheric damping results from the viscous forces in the surrounding gas. This type of damping may be reduced by surrounding the plate with hydrogen or helium, and minimized by evacuating the holder.

FIGURE 5 "MAXIMUM Q"



VALUES OF Q FOR VARIOUS—SIZED, CONTOURED, AT—CUT SHEAR VIBRATING CRYSTALS, AND THE MOST PROBABLE Q VS FREQUENCY FOR QUARTZ ALONE.

WARREN P. MASON,
IN IRE TRANSACTIONS ON INSTRUMENTATION,
VOL. I—7 (DEC. 1958), No. 384, P. 189.

Surface losses are due to friction at surface cracks and in the metallic film which forms the electrodes. Any viscous material such as a film of oil or grease on the surface results in serious

* The terms AT and BT are explained later.

degradation of the Q. Even particles of dust or any loose material on the surface results in the dissipation of energy and consequent reduction of Q. To achieve the maximum Q, the surfaces of the plate must be carefully prepared by etching or polishing and sometimes by both. The metallic films must be properly applied and must adhere firmly, and the surface of the piezoid must be absolutely free of all viscous matter.

The internal losses within the quartz itself are thought to be frequency dependent, increasing with the frequency. Consequently the maximum possible Q of the piezoelectric resonator decreases as the frequency increases. It is known that the internal losses are temperature dependent especially in some synthetic quartz at low temperatures. Figure 5 shows the value of Q for AT plates of various frequencies and sizes. The nature of the curve suggests that a maximum Q for any given frequency is determined by the internal losses within the crystalline material itself.

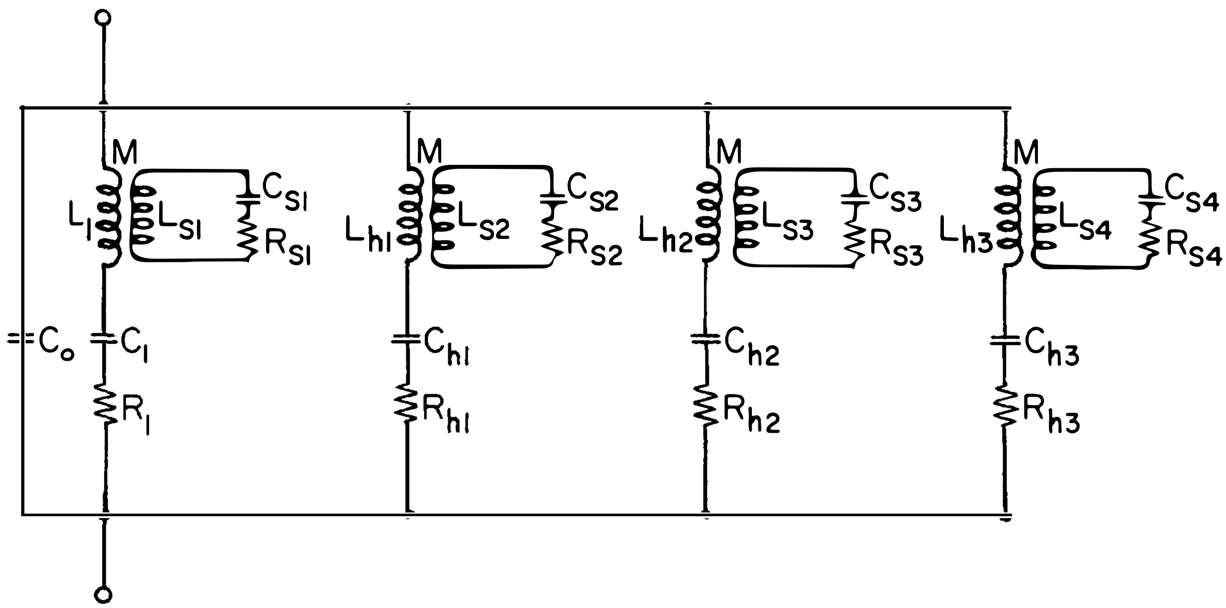
No other device combines the high Q, small size, ruggedness, low cost and low frequency-temperature coefficient of the quartz crystal unit.

Modes of Motion

Quartz plates are capable of many modes of motion. For the various modes there are different frequency-dimensional relationships and frequency-temperature characteristics. Consequently, in a single resonator unit, more than one mode of motion usually has to be considered. In some cases other modes can be excited electrically. In other cases the other modes are excited mechanically by the modes which are excited electrically.

Figure 6 represents the various modes as an equivalent circuit.

FIGURE 6 *



C_o: holder and electrode capacitance
L_1, C_1, R_1 : parameters of fundamental thickness shear
L_{hn}, C_{hn}, R_{hn} : parameters of other thickness shears
L_{sn}, C_{sn}, R_{sn} : any mechanical or electrically coupled modes
M : coefficient of coupling

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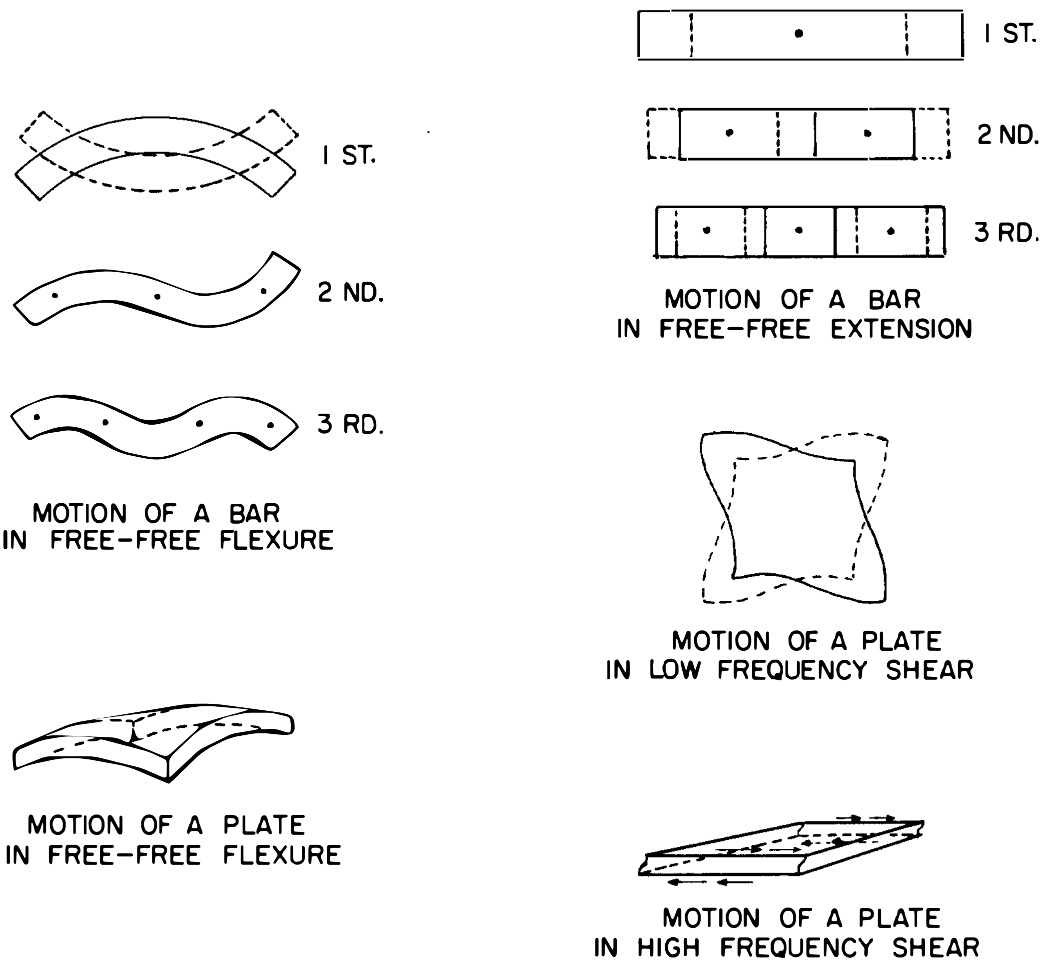
* Based on equivalent circuit by H. Ekstein, Parasitic Vibrations in Quartz Oscillator Plates, Report 11, July 1, 1947, Project 90-408A, Armour Research Foundation for U. S. Squier Signal Laboratory.

Three of the more important modes of motion are flexure, extensional, and shear, and there are two important shears, face shear (low frequency), and thickness shear (high frequency).

FIGURE 7 *



MOTION OF A BAR IN FREE-FREE FLEXURE

MOTION OF A BAR IN FREE-FREE EXTENSION

MOTION OF A PLATE IN LOW FREQUENCY SHEAR

MOTION OF A PLATE IN FREE-FREE FLEXURE

MOTION OF A PLATE IN HIGH FREQUENCY SHEAR

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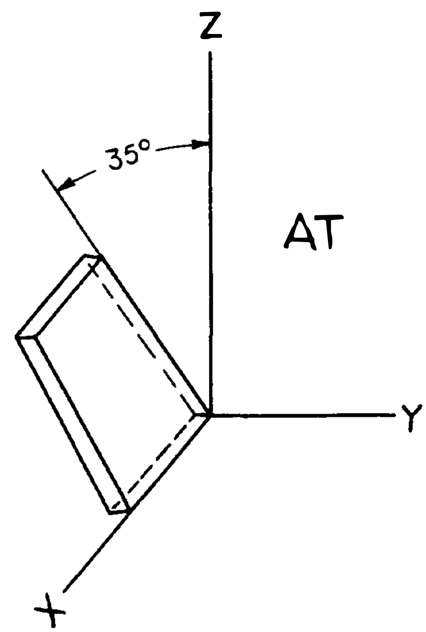
* From Raymond A. Heising, Quartz Crystals for Electrical Circuits. Copyright 1946, D. Van Nostrand Co., Inc., Princeton, New Jersey, Chapter VI, which see for a more thorough treatment of the entire subject.

For frequencies above about 800 kc, only the thickness shear mode of vibration is commonly used, and of the two orientations, BT and AT, the AT (Figure 8) is now the most commonly used.

FIGURE 8



This Handbook is limited to a consideration of the AT cut plate.

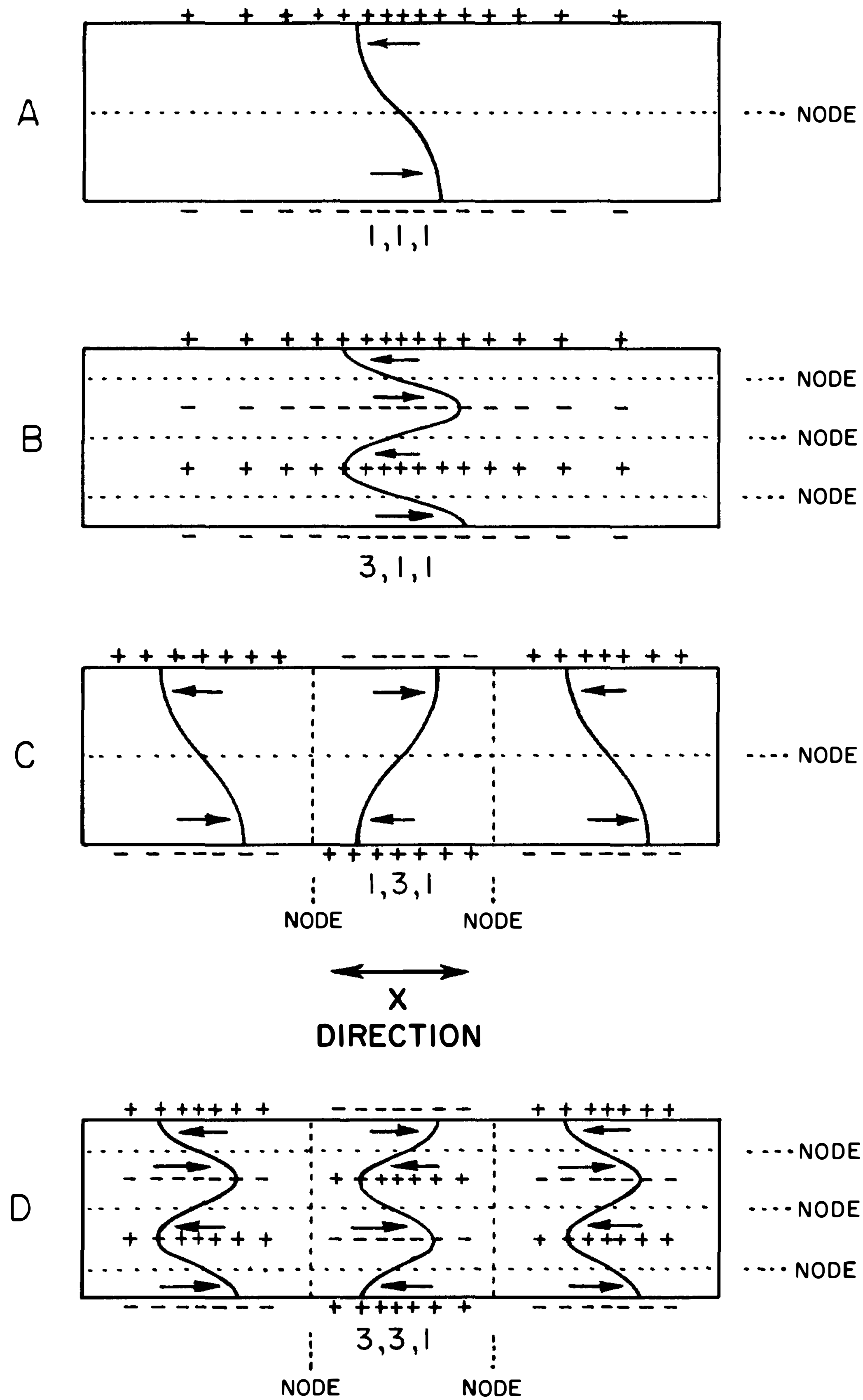
The harmonics and "inharmonics" of thickness shear vibrations can be partially described in a 3 digit system. The first digit indicates the number of half waves through the thickness of the plate. The second digit represents the number of loops along the X-axis. The third digit represents the number of loops along the Z-axis. The modes 1,1,1; 3,1,1; 5,1,1; 7,1,1; represent the fundamental, and the third, fifth, and seventh harmonics. The resonant frequencies are near but usually not exact odd intergral multiples of the fundamental, f 3,1,1, being nearly 3 x f 1,1,1, etc. The motion of the plate and the associated electrical polarity may be represented as in Figure 9A and 9B. When vibrating in the 1,1,1 mode the opposite major surfaces of the plate are of opposite polarity. The strain varies sinusoidally in the thickness direction. The thickness of the plate is one half wave length. It is assumed that the plane midway between and parallel to the major surfaces(of an uncontoured plate) is a nodal plane. When vibrating in the 3,1,1 mode the thickness of the plate is three half wave lengths. There are three nodal planes. The polarity signs are introduced in the figure to show that the plate cannot be excited in an even ordered harmonic such as 2,1,1, since the polarity of the two major surfaces would be the same at any given instant.

The modes described by a digit in the second or third place other than 1 will be called inharmonics. (It is misleading to call them "spurious" modes because they are as regularly present as 3,1,1; 5,1,1; etc.) They occur in such combinations as: 1,3,1; 1,1,3; 1,5,1; 1,1,5; 3,3,1; 3,1,3; 3,5,1; 3,1,5 etc. Even order modes (1,2,1; 1,4,1 .....) should not occur in a perfectly symetrical plate, but they are often observed.

Figure 9C represents the third inharmonic of the fundamental, and 9D represents the third inharmonic of the third harmonic. In each case there are two nodal planes normal to the major surfaces, in addition to the one nodal plane parallel to the major surfaces of the fundamental and the three nodal planes parallel to the major surfaces of the third harmonic. The inharmonic division is along a lateral dimension.

THICKNESS SHEAR MODES CROSS SECTIONS

FIGURE 9



Figures 10 and 11 show "powder patterns" obtained by sprinkling oscillator plates with powder and allowing the vibration of the plates to shake the powder away from the areas of greatest mechanical vibration. Such patterns are easily obtained on the surfaces of relatively low frequency, contoured plates.

Full sets of inharmonics occur, such as 1,1,3; 1,3,1; 3,1,3; 3,3,1; 3,1,5; 3,5,1; 3,1,7; 3,7,1; 5,1,3 etc. There are, in addition, regular appearances of inharmonic modes which cannot be described by the 3 digit system. For example, two sets of 3's, 5's, or 7's etc., which are not along either the X' or Z' axis.

It has been shown that the inharmonic frequencies of a thick isotropic plate, vibrating in thickness shear, can be calculated from the equation of motion for such a plate. A complete solution for the inharmonic modes of the anisotropic quartz plate is not available, but observation shows that the inharmonics of any particular thickness shear oscillator plate tend to be in regular sequences.

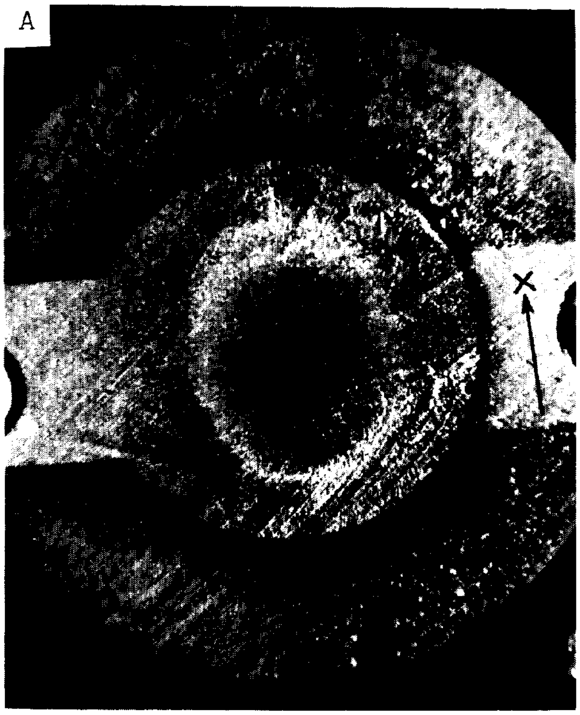
Three examples are given in Figure 12. They are chosen to illustrate that the sequences of frequencies appear to be regular, and in no way resemble the chaotic effects to be expected if they were due to irregularities in the resonator plate. They are also chosen to illustrate that different types of apparently regular sequences are observed, and that no generalization can be made until the subject is better understood. The frequencies of the set of 3,n,1 modes of a contoured plate appear to form a linear sequence, $[f\ 3,n,1 - f\ 3,1,1 = a(n - 1)]$, but the set of 1,n,1 modes of another contoured plate are slightly less linear, and the set of oblique modes of another thick harmonic unit, while appearing to fall on a smooth curve, are still less linear.

The inharmonic modes are usually of lower amplitude than the main modes. They have the same cubic type of frequency-temperature characteristic.

FIGURE 10

POWDER PATTERNS OF FUNDAMENTAL
AND LOW ORDER INHARMONICS

(Diameter 0.550", Plano-Convex, 4 Diopter Contour)



1,1,1 3,995 KC

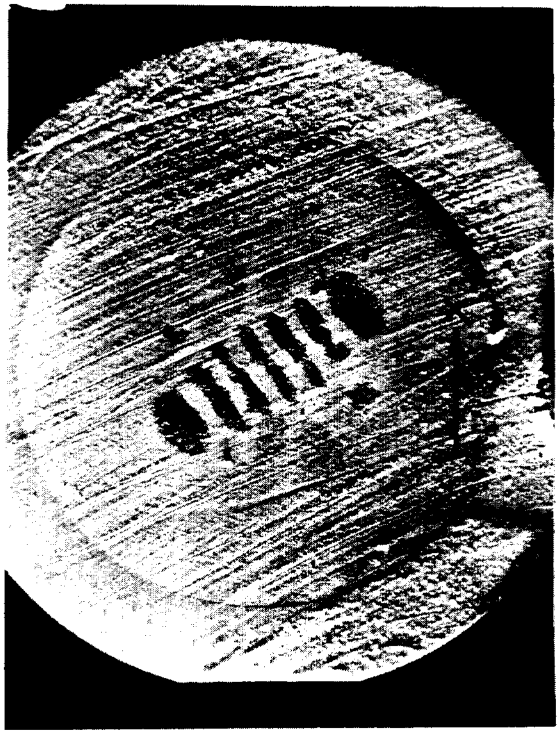
1,1,3 4,202 KC

1,3,1 4,244.5 KC

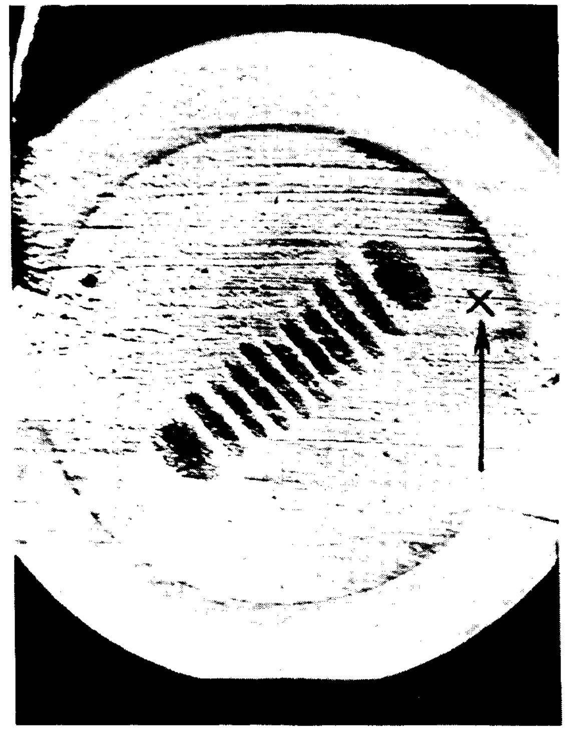
1,5,1 4,485 KC

FIGURE 11

POWDER PATTERNS OF PART OF AN "OBLIQUE" SEQUENCE



3,0,0,7 15.3846 MC



3,0,0,11 15.7076 MC

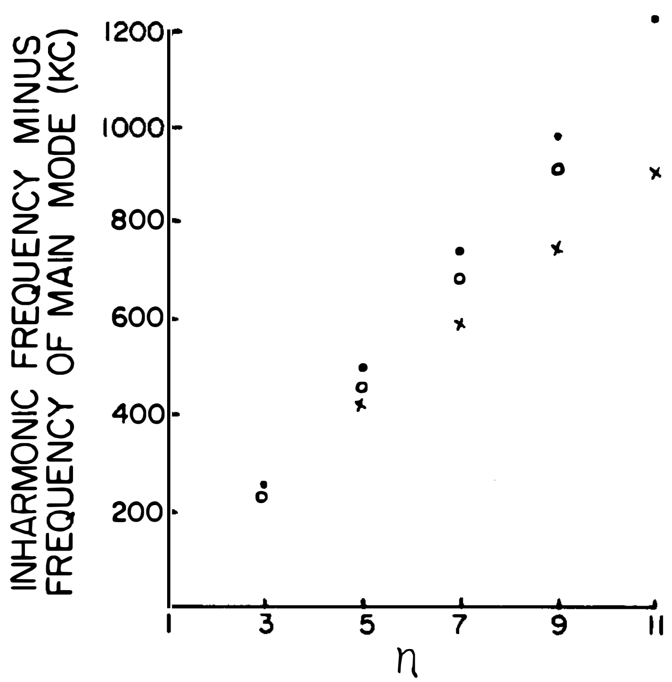
3,0,0,17 16.1622 MC

Three of 13 "oblique" modes which occur in two sequences, identified on a 3rd harmonic plate operating at about 14.8791 mc on 3,1,1. (Diameter 0.550", electrode diameter 0.400", plano-convex, 2 diopters.) In addition to several relatively complex modes, 27 simple modes in orderly sequences were identified on this plate: the fundamental; the two sequences 3,1,n and 3,n,1, from 3,1,3 through 3,1,15; the two "oblique" sequences from the 5th through the 15th order, plus the one of the 17th order shown.

FIGURE 12

EXAMPLES OF INHARMONIC MODE SEQUENCES

Examples show that spacing of inharmonic frequencies tends to be regular. No quantitative relation between the three different cases is to be inferred from the figure.



•-3,n,1, modes of a 3rd harmonic, 15 mc unit, plano-convex contour, 4 diopters.

o- 1,n,1, modes of a 4 mc unit, plano-convex contour, 2 diopters.

x- Set of oblique modes of a 3rd harmonic, 15 mc unit, plano-convex contour, 2 diopters.

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The phase reversals which create the harmonics (3,1,1; 5,1,1; etc.) occur along what might be called the "main" frequency determining dimension, the thickness, and the frequencies are approximately odd multiples of the fundamental (1,1,1). The phase reversals which create the inharmonics (n,1,3; n,1,5; etc.) occur along what might be called "minor" frequency determining dimensions (axes of the major surfaces), and their frequencies are near the main (n,1,1) frequency. In the case of most plates, which have parallel sides or are convex, all inharmonic frequencies are higher than the fundamental or harmonic, that is f 1,1,3 > f 1,1,1 and f 3,1,3 > f 3,1,1 etc.

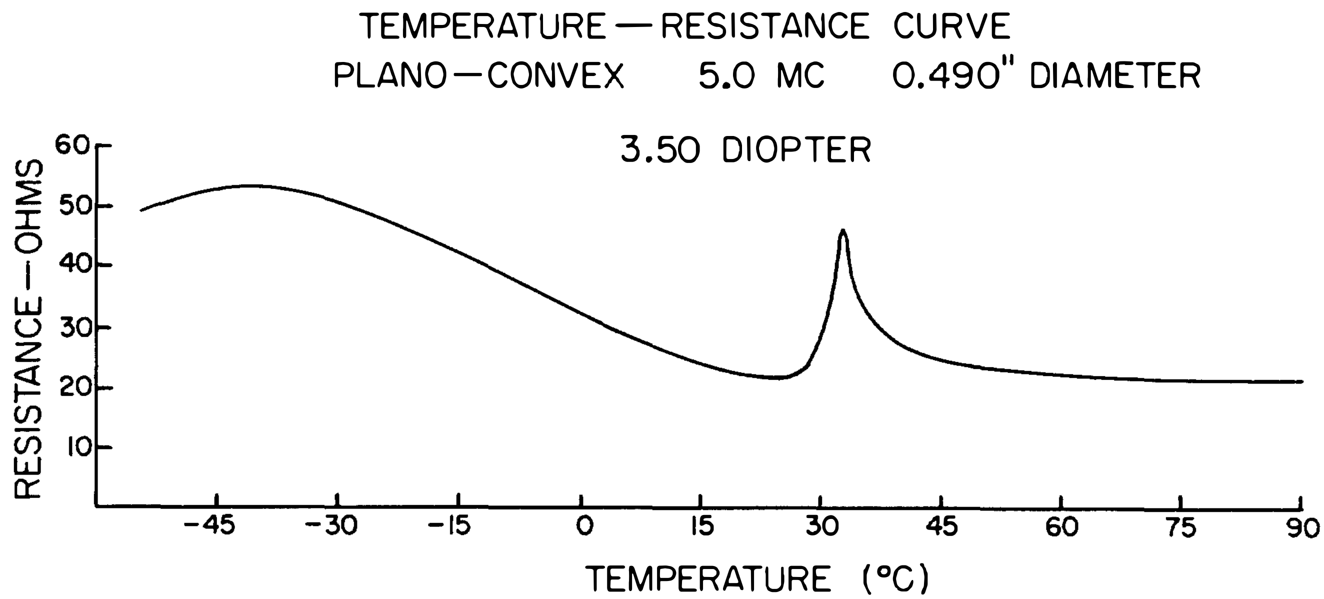
The possible modes of vibration, as we have seen, include in addition to the main thickness shear (1,1,1) and its harmonics (3,1,1; 5,1,1....), the inharmonic thickness shears (1,1,3; 1,3,1; 1,1,5; 3,1,5 etc.) and other than thickness shear vibrations such as face shear, flexure, extensional, etc. In nearly all cases the quartz resonator of the AT type is intended for a use which makes it desirable that only the fundamental of the thickness shear and its harmonics be operative. Consequently all other modes are sometimes lumped together under the name "unwanted"modes.

Vibration in a quartz resonator can be maintained either by piezoelectric excitation supplied by an external electric generator, or by mechanical excitation. The mechanical excitation may be from another mode of vibration, the exciting vibration being maintained piezoelectrically. This the family of thickness-shear vibrations (1,1,1; 3,1,1; 1,1,3....) is driven electrically by an electrical field parallel to the thickness of the resonator. Such a field cannot directly excite certain other modes, such as the flexure; but the flexure can be excited mechanically by the thickness shear vibration. Such a mechanically excited mode is called a "coupled" mode.

The important coupled modes as well as other than thickness shear modes such as face shear which can be electrically excited in a thickness shear resonator, are of interest here because of their effect upon the electrical properties of the "wanted" thickness shear. Such "unwanted" modes (which are not thickness shears) occur in AT resonators at much lower frequencies than the frequency of the thickness-shear, and they have much larger temperature coefficients of frequency. Consequently, at some temperature the "wanted" thickness-shear mode may coincide in frequency with a high order of an electrically excited face-shear mode or with a mechanically coupled flexural or torsional vibration. If coupling occurs, energy is lost through the coupling, resulting in an increase in equivalent resistance.

Since the non-thickness-shear modes have relatively large temperature coefficients of frequency, it is only at certain temperatures that their frequencies coincide sufficiently with the frequency of the thickness-shear mode for coupling to occur. The result is that as the temperature of a thickness-shear resonator is varied there will be more or less sharp increases in resistance at the temperature points where the frequencies of some order of the various interfering modes approach the resonator's main frequency close enough for coupling to occur. These are commonly called "activity dips." They are accompanied by shifts in frequency. Figure 13 shows two activity dips, a low temperature dip which extends over a wide temperature range, and a second, sharper dip at +33°C. Dips can be sharper, and much greater in amplitude.

FIGURE 13



The major frequency determining parameters of the non-thickness-shear modes are the lateral dimensions. In common practice most AT resonators, from 800 kc to 150 mc are made in a quite narrow range of diameters, approximately 0.250" to 0.600". Consequently the fundamental frequencies of the coupled and other non-thickness-shear modes remain quite low, and the order of the unwanted mode which is coupled to a very high frequency thickness-shear is quite high, with a consequent reduction in coupling. The result is that difficulty with interference from the low frequency modes is greatest at comparatively low frequencies of the wanted thickness-shear mode, very roughly beginning as an acute problem when the diameter to thickness ratio reaches some figure below 100 and greater than 50 and increasing inversely to the diameter to thickness ratio.

"Activity dips," resembling those created at lower frequencies by coupled modes, are observed in very high frequency resonators, but unlike the "dips" resulting from coupled modes, they appear to be little effected by the geometry of the plate. It is generally assumed, therefore, that they have a different origin. They are more often observed when very high frequency harmonic plates are plated with heavy, gold electrodes, than when aluminum electrodes are used.

Control of the low-frequency coupled and electrically excited modes can take the form of reducing the amplitude or the coupling of the unwanted mode, or of moving the frequencies of the unwanted modes so that their frequencies do not coincide with the frequency of the wanted mode throughout the desired temperature range. These effects can be achieved to some extent by controlling the lateral dimensions of the plate, but the tendency recently has been to rely more upon shaping the plate with bevels or full contours.

The situation with the various thickness-shear modes is quite different in all respects from that encountered with the coupled and electrically excited low frequency (non-thickness-shear) modes. The inharmonics do not cause activity dips. The temperature coefficient of frequency of all of the thickness-shear modes is of the same, approximately cubic, form. The actual value of this coefficient, for any one angle varies according to the harmonic or inharmonic order, but the general form of the frequency vs. temperature curve is the same for the fundamental and for all harmonics and inharmonics, and within the range of practically useful units, the temperature coefficient of frequency remains much smaller than for the low frequency modes. Moreover, in contrast to the non-thickness-shear modes, which tend to be weakly coupled and of negligible importance at very high frequencies, the inharmonics are present at all frequencies. Although there are many possible inharmonic modes it is usually only two or three which are troublesome. There are three special cases in which the inharmonics can cause difficulty:

(1) A selective, tuned crystal oscillator is tuned toward the frequency of the crystal resonator from a frequency higher than that of the resonator's n,1,1 mode. If there is a strong, low order, inharmonic, such as n,1,3, it is possible in either automatic or manual tuning to tune to n,1,3, instead of n,1,1, resulting in a frequency error and in higher equivalent resistance. It is, therefore, preferred practice to tune such an oscillator from a frequency setting lower than that of n,1,1.

(2) A quartz resonator is operating at different temperatures, at some of which the high order frequency of a low frequency mode coincides with the n,1,1, frequency, causing the equivalent resistance of the n,1,1, mode to increase. If there is a strong nearby inharmonic, such as n,1,3, or n,3,1, the increase in the resistance of n,1,1, may cause the oscillator to jump to the inharmonic. If the inharmonic is strong and near in frequency to n,1,1, the "activity dip" caused by the coupled mode need not be large enough to be objectionable in itself in order for it to cause such a frequency jump.

(3) When used as a passive element in a bandpass filter network, a quartz resonator is usually required to exhibit positive reactance within a single, specific frequency range. If one or more inharmonics is too close to the main mode, and is also capable of developing positive reactance, the filter attenuation characteristic will have excessive ripples in the pass-band skirts.

Inharmonics, being natural modes of vibration of a thickness-shear resonator, cannot be eliminated. They can be controlled in amplitude and in frequency separation from the main mode by varying the size and shape of the quartz plate and of the electrodes. In general, increasing the curvature of the plate increases the frequency difference between the main mode and a given inharmonic. Electrode diameter and curvature also effect the amplitude of the inharmonics. It is also possible to "damp" the inharmonics by various means which have less damping effect upon the n,1,1, mode than upon the inharmonics.