# APPENDIX 6
# PROBLEMS AND QUESTIONS

## CHAPTER 1

1. Calculate the change of frequency caused by the removal of one layer of atoms ($5 \times 10^{-8}$ cm) from both sides of a 10 MHz AT-cut blank whose thickness is 0.0169 cm.
2. List the properties of (alpha) quartz which makes it so valuable in the field of electronic communications.
3. The crystal unit in a watch is designed to have a frequency of $2^{15}$ Hz. If it actually has a frequency of $2^{15} + 1$, how much will the watch gain per day?

## CHAPTER 2

1. Explain why the quartz should not be subjected to thermal or mechanical stresses during the fabrication of piezoelectric resonators.
2. The ratios of the intercepts of a given plane with the $X$-, $Y$-, and $Z$-axes of a cubic crystal are 2:1:3, respectively. What are the Miller indices of the plane?
3. Show that the algebraic sum of the first three digits in the B-M indices of a plane in quartz is always zero.
4. Compute the reciprocal of the following matrix

$$A = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 0 & 7 \\ 1 & 5 & 1 \end{pmatrix}$$

5. The following gives the thermal conductivity of quartz in cal/cm sec deg.

| | $Z\parallel$ | $Z\perp$ | |
|---|---|---|---|
| -50° | 40 | 20.5 | |
| 0° | 32 | 17.0 | $\times 10^{-3}$ |
| +50° | 25.5 | 14.9 | |

Compute the thermal conductivity in the direction parallel and perpendicular to a line making an angle of 35° with the $Z$-axis at a temperature of 25°C.

## CHAPTER 3

1. A plate of quartz is cut with the following specifications: 2.00 cm $\times$ 2.00 cm $\times$ 0.0500 cm in the $X$-, $Y$-, and $Z$-directions, respectively. The blank is laid on a flat level surface and a 1-kg brass weight is placed on top of it. Assume that the weight is evenly distributed and compute (*a*) the applied stress, (*b*) the resulting strain, and (*c*) the change of thickness.
2. The coefficients of linear expansion for quartz are $14.3 \times 10^{-6}$ and $7.8 \times 10^{-6}$ per degree in the directions perpendicular and parallel to $Z$, respectively. Compute the dimension change in the quartz plate of Prob. 1 when the temperature is changed from 0 to 100°C.
3. What stress must be applied to a $Z$-cut plate of quartz to maintain the strain at zero when the temperature of the plate is changed by 1°?
4. A quartz rod having a length of 10 cm and a diameter of 0.5 cm is cut from a quartz crystal. The length of the rod is in the $X$-direction. A tensile force of 10 N (about 2.2 lb) is applied to the rod. Compute the dimensional changes which result. (Be sure to use the correct $c$ and $s$ constants.)

## CHAPTER 4

1. A circular $X$-cut plate has a radius of 1.00 cm and a thickness of 1.00 mm. The major surfaces are plated. The plate is subjected to a compressional force equal to the weight of 1 kg (9.8N). Assume that the force is evenly distributed over the surface and that the electric insulation is perfect. Compute the voltage developed across the blank. Procedure:

    1. Compute the stress.
    2. Compute the strain.
    3. Compute the polarization.
    4. Compute the capacitance.
    5. Compute the voltage.

2. A potential of 1000 V is applied to the electrodes of the quartz plate of Prob. 1. What compressional force must be simultaneously applied to avoid any change in the thickness of the plate? Procedure:

    1. Compute the electric-field intensity.
    2. Compute the piezoelectric strain (no constraint).
    3. Compute the stress required to reverse the strain.
    4. Compute the required force.

3. Verify the two equations at the top of page 62.

4. A potential difference of 1 V is applied to the electrodes of an $X$-cut quartz plate having a thickness of 0.0100 cm. How much does the thickness of the blank change? What temperature change would be required to produce the same change of thickness?

## CHAPTER 5

1. The velocity with which a wave is propagated on a string is given by $v = (T/m)^{1/2}$, where $T$ is the tension and $m$ is the mass per unit length of the string. Compute the frequencies of the fundamental and overtones of a violin string assuming reasonable values for the tension $T$, the length $L$, and $m$.
2. The wave velocity in a square drumhead is 50 m/sec. The drumhead has dimensions of 75 X 75 cm. Compute the frequencies of the fundamental and the first nine overtones. Explain why the theoretical and experimental values differ.
3. A metal rod having a length of 75 cm and a diameter of 0.5 cm is rigidly clamped at its midpoint. The velocity of a longitudinal wave in the rod is 1500 m/sec. Compute the frequencies of the tones produced by striking the bar on the end.
4. Explain why the "frequency constant" (frequency X thickness) of an actual AT-cut plate is not constant.
5. Figure 5.6 is the experimentally observed overtone spectrum of a contoured 3.1-MHz AT-cut plate having a diameter of 1.4 cm. Use Eq. (42) and calculate the overtone frequencies for an isotropic plate of the same size. The effective radius of the contoured plate is about one-half that of the plate. Arrange the calculated frequencies in series as in Fig. 5.6 and note the differences due to neglect of the effects of anisotropy.

## CHAPTER 6

1. Obtain Eq. (46) by substituting Eq. (45) into Eq. (44).
2. A circular $X$-cut blank having a diameter of 1.50 cm and a thickness of 0.3000 mm is fully plated. Find the frequency and the approximate values of $C_0$, $L_m$, and $C_m$. Discuss the errors resulting from the various approximations and assumptions.
3. Repeat the calculations of Prob. 2 with a $Y$-cut plate having the same dimensions.
4. Repeat the calculations of Prob. 2 with an AT-cut and a BT-cut having the same dimensions.

## CHAPTER 7

1. A certain crystal unit has a series-resonant frequency $f_R$ of 10.000000 MHz and a series resistance of 20 Ω. The static capacitance $C_0$ = 2.50 pF. When operated with a load capacitance of 7.5 pF the frequency $f_A$ = 10.002500 MHz. Determine the values of $L$, $C$, and $Q$ for this unit. ($L = L_m$, $C = C_m$.)
2. What will be the antiresonant frequency of the crystal unit of Prob. 1 when operated with a load capacitance of 32 pF?
3. A resistor having a phase angle of 10° is used in setting a vector voltmeter to zero phase. How much error is introduced into the measurement of the frequency of a 10-MHz unit having a $Q$ of $1 \times 10^5$ by using this resistor?
4. Two crystal units have the same series-resonant frequency $f_R$ = 10.000 000 MHz. The motional capacitances are 0.00675 pF and 0.00850 pF, respectively. Each unit is operated in oscillator $A$ in which $C_x$ = 7.5 pF and in oscillator $B$ in which $C_x$ = 10.0 pF. The static capacitances of the two units are equal to 2.5 pF. Find the frequencies at which the two crystals operate in each oscillator. (This is the principle source of oscillator correlation problems.)
5. Determine the six critical frequencies of the crystal unit of Prob. 1 for (*a*) a load capacitance of 7.5 pF and (*b*) for a load capacitance of 22.5 pF.

## CHAPTER 8

1. A quartz blank having a thickness of $0.46\overline{8}$ mm oscillates at a frequency of 5450 kHz. What is its probable orientation? How could one verify this conclusion?
2. A quartz blank having a thickness of $0.006\overline{2}$ in oscillates at a frequency of 31.55250 MHz. What is the probable orientation? How could one verify the conclusion?
3. Discuss qualitatively the factors which determine the frequency-temperature relationship in a crystal unit. Explain how the $T_f$ is minimized in the AT-cut.
4. Refer to Figs. 8.3 and 8.4 and note that the curves for AT-cut blanks cut at 35°22′ are quite different. Explain the difference.
5. A 5-MHz AT-cut unit is to be used within the temperature range 0 to 60°C. What is the best orientation for this unit? What angular tolerances are compatible with a frequency tolerance of ± 5 ppm from the room-temperature frequency?
6. If the correct value for the angle $\theta$ is substituted in Eq. (92), the value of $T_f$ can be made equal to zero. Yet the curves of Fig. 8.4 show that even if $T_f$ is zero at room temperature, it is not zero at other temperatures. Explain.

## CHAPTER 9

1. Explain the origin of activity dips (band breaks) in thickness-shear quartz resonators. What can be done to avoid them?
2. Why are activity dips less troublesome in units designed to operate in the frequency range 20 to 50 MHz than in units designed to operate in the 2 to 5 MHz range?
3. Refer to the $c$ matrix for quartz (13b, p. 45) and use it to explain why AC- and BC-cut plates exhibit fewer activity dips than do AT- and BT-cut plates.

## CHAPTER 10

1. Explain the origin of the inharmonic overtone modes in quartz resonators.
2. Explain why the frequency separation of the inharmonic modes increases as the size of the vibrating area is reduced. (No mathematics is required.)
3. Design a crystal unit to operate at 20 MHz on its fundamental mode over the temperature range -40 to +90°C having no spurs within 100 kHz of the main mode and the largest possible motional capacitance. Use an AT-cut blank.
4. What are the effects of contouring the surface(s) of an AT-cut quartz plate? What are the reasons for this behavior?
5. What is the radius of curvature of a surface generated by a lap marked 1.25 diopters?

## CHAPTER 11

1. Calculate the lattice spacing and the Bragg angle for the (12.2) planes.
2. Find the angular separation between the Cu $K\alpha_1$ and Cu $K\alpha_2$ radiations reflected from the (12.2) planes of quartz.
3. Explain why the x-ray beam reflected from an etched quartz surface is weaker than that reflected from the same lapped surface before etching.
4. If the (20.3) planes were used to check the orientation of an AT-cut plate, what goniometer readings would be observed? *Ans.*: 48°32′, 19°34′, and 35°20′.
5. Explain quantitatively why one or both of the beams reflected from the plate of Prob. 4 with the $X$-axis horizontal may not be detected.
6. The (12.2) planes are used to check the wafer sawed to make an SC-cut blank. The observed goniometer readings are $g_\theta = 37°19'$ and $g_\phi = 36°26'$. What is the orientation of the wafer? *Ans.*: $\theta = 34°10'$ and $\phi = 21°35'$.

## CHAPTER 12

1. The specifications for a certain 7500-kHz quartz plate require that it be etched by at least 0.5 $f^2$. How much must the frequency be increased by etching?

2. Show that the thickness of the plated electrode which results in a plate back of $n f^2$ is independent of the frequency $f$.
3. If a precision crystal unit has a frequency of 5.000 000 000 MHz and an aging rate of $-2 \times 10^{-10}$ per day what will its frequency be at the end of one year? How many cycles will it have lost during the year?
4. Suppose that a layer of water 2.5 Å thick is adsorbed on each side of a 10-MHz AT-cut resonator. How much will this change the frequency? *Ans.*: About 10 Hz.
5. The density of aluminum is approximately equal to that of quartz. If a film thickness of 1500 Å is required for adequate electric conductance, show that the minimum permissible plate back with aluminum on an AT-cut resonator is about $0.2 f^2$.

## CHAPTER 13

1. Calculate the angles $\gamma$ and $\beta$ by which a lumbered $Y$-bar must be rotated to saw a wafer for the IT-cut orientation.