13
THE STRESS–COMPENSATED CRYSTAL UNIT

INTRODUCTION

It has long been known that mechanical stresses applied to a quartz piezoid cause changes in its resonant frequency. A very simple experiment in which a circular AT-cut plate is compressed along a diameter shows that the frequency change is proportional to the applied force and depends on the direction in which the force is applied, even changing sign with the direction. Frequency changes of 100 ppm are easily produced in this way in ordinary quartz resonators.

Another well-known phenomenon is the amplitude-frequency (AF) effect in which the frequency of the resonator varies with the drive level, i.e., the power used in driving it. The frequency shift in a circular AT-cut resonator of conventional design is quite accurately proportional to the square of the driving current and hence to the power dissipated in the quartz. This frequency change is superimposed on, and is much larger than, the frequency change due to the temperature rise of the quartz. It is now recognized that the AF effect is largely, if not entirely, due to thermal stresses in the quartz as the annular ring of cooler quartz constrains the driven portion which is heated by the energy dissipated in it. The fact that the change of frequency occurs almost simultaneously with the change in drive level has led to the hypothesis that the AF effect results from variations of the elastic coefficients with amplitude. Now it can be shown that the resonant frequency of a vibrating system does depend on the amplitude if the stress/strain relationships contain nonlinear terms of odd order. But frequency changes occur in a typical AT-cut plate vibrating at constant amplitude when the central region is heated by exposing it to radiant energy from a hot source or a laser. This proves that the AF effect is not solely due directly to the increased amplitude.

It is instructive to examine the magnitude of the stress which might be expected in a typical case. To do so we may consider a 10 MHz AT-cut plate having electrodes with a diameter of 0.25 cm and a series resistance of 50 Ω. A current of 10 mA through the unit at the series-resonant frequency results in a dissipation of 5×10^{-3} W or 1.2×10^{-3} cal/sec. If this heat were absorbed in the quartz between the two electrodes, its temperature would rise at the rate of about 6°/sec. On page 00 it was shown that a temperature increase of 1°C° in a connstrained piece of quartz causes a compressional stress of 12 atms. So we may expect that the temperature rise due to the energy dissipated in the center of the quartz plate will produce very significant stresses. As the central region of the plate tries to expand against the constraining ring of cooler quartz it tends to produce a bulging effect and for this reason the phenomenon has been called the *oil-can* effect.

A striking example of the oil-can effect occurs in crystal units which are operated in fast warm-up ovens. In some of these ovens the crystal unit is brought to the operating temperature of 60 to 80°C in a period of 1 min or less by means of a heating element placed very near the crystal unit. The thermal stresses resulting from this rapid heating sometimes result in frequency excursions, or "overshoots," of several hundred parts per million during the warmup period. Attempts have been made to heat the quartz quickly by means of heating elements deposited directly on the quartz blank with similar results.

Acceleration forces including that of gravity also influence the frequency of a crystal unit. The frequency changes vary with the method of mounting the piezoid and the direction of the force. The change of frequency with position in the earth's gravitational field is called the *tip-over effect*; in AT-cut plates this is typically a few parts in 10^9 per g.

A closely related phenomenon is the frequency change associated with the stresses applied to the quartz blank by its metal electrodes.

According th EerNisse[1] frequency changes of 3 ppm may result from stresses imposed by the gold electrodes on a 5-MHz fifth-overtone AT-cut resonator. These stresses result from the different coefficients of expansion and the physical changes which occur in metal films with time.

The frequency changes in all of the foregoing phenomena are associated with changes in the elastic coefficients with stress bias. It was shown by the author (see Ref.2) in 1947 that the frequency changes which occur in rotated Y-cut plates upon irradiation to saturation with x-rays are also due to changes in the effective values of the elastic coefficients. These changes are due to internal stresses resulting from irradiation, and in those crystals showing the maximum effect they are as follows:

Maximum change in c_{14} $\sim -0.07\%$
Maximum change in c_{44} $\sim -0.03\%$
Maximum change in c_{66} $\sim -0.02\%$

These values were deduced from the frequency changes which are observed in AT-, BT- and Y-cut plates the values of which are, respectively, < 0.001 percent, -0.02 percent, and -0.01 percent. The observation that the various stress effects are strongly anisotropic suggests that some orientation might exist for which the frequency-stress effect might be zero, at least for certain kinds of stresses.

HISTORY OF THE SC-CUT

On the basis of theoreticalcalculations, EerNisse (Ref. 1) predicted that a doubly rotated quartz plate cut at the angles of $\phi = 22.5°$ and $\theta = 34.3°$ (see Fig. 11.10) should be free of frequency changes due to stress bias in the plane of the plate. In 1976 EerNisse (Ref. 3) extended the calculations to include frequency changes due to any mechanical stress bias. One important result of EerNisse's calculations was the conclusion that the frequency-stress coefficient should depend almost entirely upon the angle ϕ, while the first-order frequency-temperature

[1] See Refs. 1 and 3 at the end of this chapter.

coefficient should be determined almost entirely by the angle θ. He called the new piezoid the *SC-cut* (stress-compensated).

In 1977, Kusters et al. (Ref. 4) showed experimentally that the results predicted by EerNisse were obtained with a plate having the orientation angles $\phi = 21.93°$ and $\theta = 34.11°$. They called it the TTC-cut (thermal-transient-compensated) and pointed out its advantages as well as some of the problems associated with its production and use.

The nomenclature TS-cut [used by Holland (Refs. 12 and 13) in 1964], TTC-cut [used by Kusters (Ref. 4)], and SC-cut all refer to a plate cut at angles near $\theta = 34°$ and $\phi = 22°$; the exact angles depending on the application. We use the term SC-cut (stress-compensated), which is the one most favored at this time.

The orientation of the SC-cut is similar to that of the IT-cut introduced in 1951 by Ives and Bottom (Ref. 5). The IT-cut was developed to obtain a unit having a zero frequency-temperature coefficient at a temperature higher than that of the AT-cut. The orientation of the IT-cut is $\phi = 19°06'$ and $\theta = 34°17'$. The value of ϕ is the same as that of the (12.2) planes, thereby making x-ray orientation of the IT-cut no more difficult than that of the AT- and BT-cuts. It was not noted at the time that the frequency-stress effect is much less in the IT- than in the AT-cut, although its relative freedom from activity dips and spurious modes was pointed out.

Further theoretical treatments of, and experimental results on, the SC-cut are given in papers by Ballato et al. (Ref. 6), and Gagnepain et al., (Ref. 7).

FABRICATION OF THE SC-CUT

The problem of cutting an accurately oriented quartz plate such as an *X*-, *Y*-, or *Z*-cut in which the plane of the plate is normal to one of the crystallographic axes is relatively simple. It is somewhat more complicated if the plane of the plate is rotated about one of the axes, as with the AT- or BT-cut. It is more difficult if the plane of the plate is rotated about two of the crystallographic axes as in the IT-cut and becomes still more difficult if the line of intersection of the plate and a set of atomic planes does not lie in one of the crystallographic planes. This is the case with the SC-cut.

The X-, Y-, and Z-axes are readily identified in quartz. Cultured quartz is supplied with accurately oriented X- and Z-surfaces ground on lumbered Y-bars. In the following it is assumed that we use a Y-bar prepared in this way.

We begin by defining a rectangular coordinate system whose axes coincide with the X-, Y-, and Z-crystallographic axes, as shown in Fig. 11.10. We assume the saw to be oriented in such a way that the saw plane is normal to the Y-axis.

We now define a unit vector N which is normal to the plate having coordinates ϕ and θ (Fig. 13.1). The components of this vector are

$$\begin{pmatrix} -\cos\theta\sin\phi \\ +\cos\theta\cos\phi \\ \sin\theta \end{pmatrix}$$

The problem then is to find the angles through which the bar (or the saw) must be rotated to bring the unit vector N into coincidence with the Y axis of the coordinate system. A method for doing so has been described by Bond and Kusters (Ref. 8) and the following procedure is based upon this method.

We have shown in Chap. 2 that the matrix of Eq. (10b) performs a rotation of a vector about the Y axis and that the matrix of Eq. (10c)

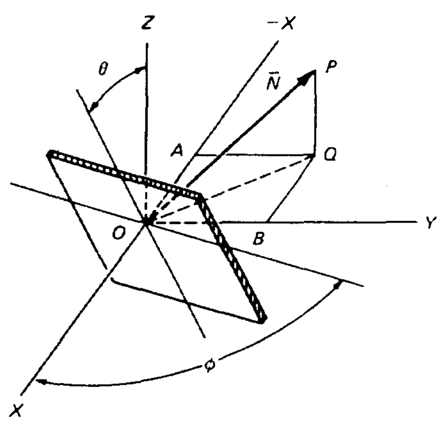

Fig. 13.1. Components of the unit vector normal to the plate. OP = N = unit vector normal to the plate. $OQ = \cos\theta$ = projection of OP on the XY plane. $OA = X$ component of N $= -\cos\theta\sin\phi$. $OB = Y$ component of N $= \cos\theta\cos\phi$. $PQ = Z$ component of N $= \sin\theta$.

performs a rotation about the Z axis. We will use these two matrices to perform the two successive rotations required to bring the vector **N** into coincidence with the Y axis. We let β be the angle by which the bar must be rotated about the Y axis and γ the angle by which it must be rotated about the Z axis. The operation for performing the two rotations is the matrix product

$$\mathbf{N}'' = \begin{pmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\beta, & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{pmatrix} \begin{pmatrix} -\cos\theta\ \sin\phi \\ \cos\theta\ \cos\phi \\ \sin\theta \end{pmatrix}$$

Carrying out the indicated operations gives the components of $\mathbf{N}''$, which are

$$\begin{aligned} N_X'' &= -\cos\theta\sin\phi\cos\beta\cos\gamma - \sin\theta\sin\beta\cos\gamma + \cos\theta\cos\phi\sin\gamma \\ N_Y'' &= +\cos\theta\sin\phi\cos\beta\sin\gamma + \sin\theta\sin\beta\sin\gamma + \cos\theta\cos\phi\cos\gamma \\ N_Z'' &= -\cos\theta\sin\phi\sin\beta + \sin\theta\cos\beta \end{aligned}$$

Now if $\mathbf{N}''$ is to be a unit vector parallel to the Y axis, then $N_X'' = N_Z'' = 0$ and $N_Y'' = 1$. Setting $N_Z'' = 0$ gives

$$\tan\beta = \frac{\tan\theta}{\sin\phi}$$

which defines the required rotation β about the Y axis.

Multiplying the equation for N_X'' by $\sin\gamma$ and the equation for N_Y'' by $\cos\gamma$ and adding the two (not forgetting that $N_Y'' = 1$), we easily get

$$\cos\gamma = \cos\theta\cos\phi$$

which defines the required rotation about the Z axis. These two equations tell us how to rotate a quartz bar from the initial position in which the coordinate and crystallographic axes coincide to bring it into the position required to saw a blank having the orientation angles ϕ and θ.

The process is quite simple. The lumbered Y-bar is first rotated about the Y axis by the angle β and held in this position by some means, while the assembly is rotated about the Z axis by the angle γ. A cut made in the XZ plane of the original coordinate system has the desired orientation.

Suppose that it is desired to cut an SC-wafer having angular coordinates

$$\theta = 34°07' = 34.117° \quad \text{and} \quad \phi = 21°56' = 21.933°$$

From the equations above,

$$\beta = 61°08' = 61.13° \quad \text{and} \quad \gamma = 39°49' = 39.83°$$

We use a lumbered Y-bar of right-hand quartz having accurately oriented surfaces normal to the X- and Z-axes. The long dimension of the bar is along the Y-axis and the intersections of the lumbered surfaces are parallel to the crystallographic Y-axis.

The bar is placed on a glass or ceramic plate with the crystallographic axes parallel to the coordinate axes and with the $+X$-axis pointing toward the operator. The top surface, which is an XY-surface, is painted for later identification. The bar is then rotated *away* from the operator by an angle of 61°08′ and held in this position by a suitable wax or plaster. The bar and the plate to which it is attached are then rotated *clockwise*, as seen from above, by the angle 39°49′ and then secured to the transfer plate or the saw table. Wafers are then sawed from the bar by making cuts perpendicular to the original position of the Y-axis. With reasonable care and currently available equipment, wafers can be cut within ± 15′ or closer to the desired orientation. If necessary the wafers can be corrected later to closer tolerances.

The wafers cut from the Y-bar have the shape of parallelograms. Two sides of the parallelogram lie in the XY-crystallographic plane and the other two in the YZ-plane. The edge of the wafer which lies in the XY-plane is identified by the previously applied paint and designated the *reference edge*. Next, two directions are identified on the face of the wafer, i.e., parallel and perpendicular to the reference edge.

The wafer is placed on the x-ray chuck with the reference edge parallel to the goniometer axis and D_θ is measured. The wafer is then rotated 90°, making the reference edge perpendicular to the goniometer axis and D_ϕ is measured. From these two measurements the values of θ and ϕ are calculated using Eqs. (111), (112), and (113). Care must be exercised to make the measurements on the correct side of the blank and with the reference edge in the correct position. Other measurements must be treated in a slightly different manner. If the x-ray equipment is set up to use the (12.2) planes (and this is highly desirable), the values of D_θ and D_ϕ will each be about 3°.

If it is necessary to measure the orientation of the circular SC-cut blanks, the direction of the reference edge must be maintained. This may be done by grinding a flat edge, accurately parallel to the reference edge, on the blank.

A more complex method for determining the orientation of doubly rotated blanks has been described by J. Clastre et al. (Ref. 11). The reader is referred to that paper for the details.

It is useful in visualizing the SC-cut wafer to make a model of a lumbered Y-bar from a piece of wood of rectangular cross section. A slab to simulate the SC-cut can be cut from the model by giving it the proper rotations and sawing a slab with a miter saw.

The necessity for making two rotations in orienting the bar for sawing SC-cut wafers may be avoided by using SC-bars especially grown for the purpose. A description of the SC-bar has been given by Ballato (Ref. 9). Such bars are quite expensive, and the advantage of requiring only a single rotation does not appear to justify the additional cost. Checking the orientation of wafers cut from the SC-bar is at least as difficult as checking those cut from Y-bars. The number of wafers and blanks obtained per kilogram of Y-bar quartz is approximately the same for both SC- and AT-cuts.

CHARACTERISTICS OF THE SC-CUT

The SC-cut was developed to obtain a resonator whose frequency would be independent of stresses in the plane of the vibrating plate resulting from thermal stresses and electrode changes. However, it has proved to have a number of other advantages over the conventional AT- and BT-cut plates. Among these are:

1. Superior warmup characteristics due to reduction of the effects of stresses associated with thermal gradients in the plate.
2. Reduced AF effect for the same reason.

3. Almost complete freedom from activity dips.
4. A flatter frequency-temperature curve, especially at the turning points.
5. Reduced tipover effect.
6. Better short-term stability due to reduced response to thermal transients.
7. Reduced aging associated with relaxation of electrode stresses.
8. Relative immunity to the effects of radiation.
9. Less aging due to changes in mounting stresses.

The added complexities of orientation and cutting are disadvantages of the SC-cut but these have not proven to be as difficult as originally perceived. Another disadvantage is that the SC-cut is very sensitive to air damping and must therefore be operated in vacuum to obtain optimum Q values.

Some differences between the SC-cut and the AT-cut may be advantageous or not depending upon the application. The inflection point of the frequency-temperature curve is about 70° higher in the SC- than in the AT-cut. The temperatures of the turning points can be varied by changing the angle θ just as in the AT-cut (see Figs. 8.3 and 8.4). However, changes are required simultaneously in the angle ϕ in order to maintain the stress compensation feature.

The SC- like the IT-cut has three thickness modes of vibration which are designated the a, b, and c modes. (These piezoids also have two low-frequency face-shear modes.) The a mode is a quasi-longitudinal mode analogous to the thickness mode of an X-cut plate. It has a frequency constant of about 3370 kHz mm and a temperature coefficient of about −52° ppm/°C. The b mode is a quasi-shear mode having a frequency constant of about 1990 kHz mm and a T_f of about −25 ppm/°C. Within the range 0 to 100°C the frequency-temperature curve of the b mode is almost linear. The c mode is also a quasi-shear mode. It is analogous to the AT cut. The frequency constant of the c mode is about 1800 kH mm and consequently the SC-cut plate is about 10 percent thicker than an AT-cut plate of the same frequency.

The relative activities of the b and c modes can be varied a bit by the design of the plate but usually they are about the same. The SC-cut plate can be excited on either mode or on both modes simultaneously. The presence of the two modes requires special attention in the design of oscillator and filter circuits employing the SC-cut plate. Both modes are accompanied by the usual family of inharmonic overtone modes.

The reason for the presence of the three thickness modes in the SC- and IT-cut resonators can be understood by considering the piezoelectric strains produced by an electric field in a quartz plate. For simplicity we consider three plates, one cut perpendicular to the X-axis, one perpendicular to the Y-axis and one at an intermediate position. The projections of these three plates on the Z-plane are shown in Fig. 13.2. In all cases the electric field is perpendicular to the plate.

In the X-cut (a) the electric field has components along the X_2 and X_3 axes as well as along X_1. The effects of these components is to produce an extensional (positive) strain along X_1 and compressional (negative) strains along the direction perpendicular to X_1. These are responsible for the piezoelectric coefficients d_{11} and $d_{12} = -d_{11}$ [see Eq. (26)].

The electric field in the Y-cut plate (b) has a component along X_1 producing a positive strain and one along X_3 causing a negative strain. There is no component along X_2. The resultant of the two longitudinal strains is a shear strain (see page 41) and the piezoelectric coefficient $d_{26} = -2d_{11}$.

The rotated blank (c) in which the applied electric field has components along all three of the X-axes experiences both longitudinal and shear strains resulting in the a and c modes. The shear strain which results in the b mode is orthogonal to that which produces the c

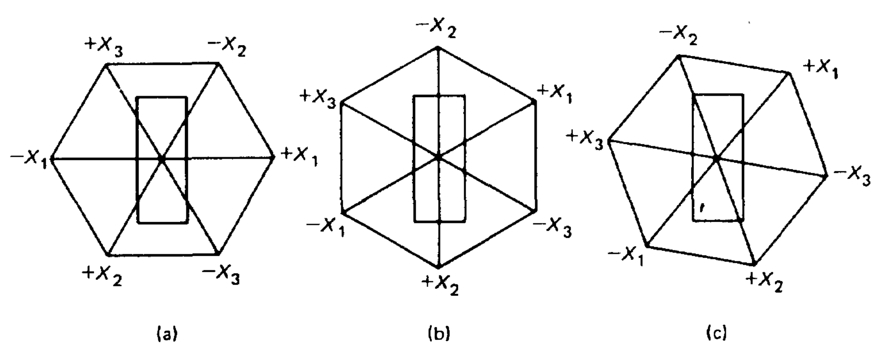

Fig. 13.2. Piezoelectric strains in X-cut, Y-cut, and rotated plates.

mode. It occurs as a result of the θ rotation and therefore would not be present in the simple model of Fig. 13.2*c*.

The reason for the sensitivity of the SC-cut to air damping can also be understood by comparing the strains produced by the electric field in the *Y*-cut plate of *b* and those induced in the rotated plate of *c*. In the *Y*-cut plate the positive strain parallel to X_1 can be resolved into components along X_2 and perpendicular to X_2. The same can be done for the negative strain along X_3. The resultant of these strains is zero in the direction perpendicular to the plate. In the rotated cut, however, the components of the strains along X_1, X_2, and X_3 do not cancel in the direction perpendicular to the plate, with the result that the surface suffers a displacement in this direction. The resulting pumping action is responsible for the air damping observed in the SC-cut.

TESTING SC-CUT UNITS

The chief difficulty in testing SC-cut crystal units is the presence of the *b* mode. A CI-meter, tuned to the frequency of the *c* mode, may have a tendency to jump to the *b* mode. Thus the CI-meter is not well adapted to making frequency-temperature tests on the SC-cut. The transmission-line method using a phase-locked loop is somewhat more reliable but, of course, more expensive and less convenient. Oscillators having narrow-band filters in the feedback network are indicated for use in test equipment for SC-cuts.

The frequency of the SC-cut is less sensitive to the rate of temperature change and therefore frequency-temperature tests can be carried out more rapidly than with units containing AT- or BT-cut plates.

Since the shape of the frequency-temperature curve in the SC-cut is largely determined by the value of the angle θ, a frequency-temperature test is adequate for determining the correct value for this angle. But the frequency-stress behavior is largely a function of the angle ϕ, and to determine the correct value for this angle, some type of frequency-stress test is required.

It is necessary to distinguish between thermal-stress sensitivity and *g* or acceleration sensitivity. Different tests are required. A simple test for thermal-stress sensitivity is to abruptly change the temperature of the crystal unit and to observe the time required for the unit to achieve frequency stability. The same result can be achieved by suddenly changing the drive level. A method (Ref. 12) has been described for testing the g sensitivity which appears to lend itself to production as well as laboratory testing. This method will now be described.

The crystal unit is rigidly attached to a vibration table calibrated for both frequency and acceleration. The frequency of the crystal unit is modulated by the acceleration-induced stresses through the frequency-stress effect. As in any FM signal, sidebands appear in the output of the crystal-controlled oscillator the frequencies of which are given by

$$f_n = f_0 \pm n\, f_m \qquad n = 0, 1, 2, 3, \ldots$$

where f_0 is the unmodulated (carrier) frequency and f_m is the modulating frequency i.e., the frequency of the vibration table. The amplitudes of the sidebands are given by $A_n = A\, J_n\, (\Delta f/f_m)$, where $\Delta f = S\, f_0\, X$. X is the acceleration and S is the fractional change in frequency per g. J_n is Bessel's function of order n.

If $(Sf_0 X/f_m) = F \ll 1$, then $J_0\,(F) \doteq 1$ and $J_1\,(F) \doteq F/2$. Therefore

$$\frac{A_1}{A_0} = \frac{(Sf_0 X)}{2f_m} \qquad \text{and} \qquad S = \left(\frac{2f_m}{Xf_0}\right)\left(\frac{A_1}{A_0}\right)$$

The ratio A_1/A_0 can conveniently be determined by the use of a spectrum analyzer with a logarithmic scale and since f_0, f_m, and X are known, the value of S is readily determined.

As an example, suppose that a 10-MHz unit is vibrated at a frequency of 500 Hz with a maximum acceleration of 100 g and that the ratio of the amplitude of the first side band to that of the carrier is 0.001. Then

$$S = \frac{2\ (500)\ 0.001}{10^7\ 100} = 10^{-9} \text{ per g}$$

The value of $F = [10^{-9}\ (100)\ 100^7]/500 = 0.002$, which is certainly small compared with unity, thus justifying the approximation used in the theory.

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