9
COUPLED MODES AND ACTIVITY DIPS

ACTIVITY

The amplitude of oscillation of a crystal-controlled oscillator is usually determined, at least partially, by the characteristics of the crystal unit. If the oscillator is required to develop a certain minimum signal voltage, and if the equivalent resistance of the crystal unit is too high, the crystal unit may fail at one or more temperatures to oscillate at the specified amplitude. The grid current of the oscillator tube (or its solid-state counterpart) provides a simple measure of the amplitude of oscillation. The amplitude of oscillation, in a given circuit, under specified conditions, is often termed the *activity*. Crystal units were formerly specified in terms of activity, but since the invention of the CI-meter, the equivalent resistance has been used instead of activity since the resistance is characteristic of the crystal unit whereas the activity depends on both the crystal unit and the associated circuit.

ACTIVITY DIPS

Quite often the activity of a crystal unit decreases abruptly or even drops to zero at a certain temperature. Such a decrease is called an *activity dip*. Since it is much easier to monitor a quantity such as activity than to monitor the equivalent resistance, the activity is still often used in testing crystal units. Correlation between the two quantities is easily established. All the motional parameters of the equivalent circuit undergo changes near the temperature at which the activity dip occurs. The equivalent resistance increases (decreasing the amplitude of oscillation) and the Q decreases while L, C, and their product, which determines the resonant frequency, deviate from their normal values.

Figure 9.1 shows the variation of activity and frequency with temperature of a 5-MHz AT-cut plate. An activity dip occurs at a temperature of about 50°C in this particular unit.

The activity and frequency of a crystal unit may vary abruptly at some temperature for several different reasons. A unit encapsulated in an atmosphere containing water vapor usually displays a decrease in activity and frequency as the temperature of the crystal unit is raised through the dew point. Condensation of water on the cold blank produces both loading and damping, resulting in the observed changes. This phenomenon has been used as a dew-point indicator. Contaminants such as solder flux and poorly adhering plating sometimes also produce similar perturbations of the activity and frequency at certain temperatures. Piezoids made from cultured quartz sometimes show reduced activity at low temperatures, but this is not a true activity dip and in any case is not very important, since the quality of cultured quartz has been greatly improved with experience in production.

True activity dips are caused by coupling between the desired mode of vibration and some other mode of vibration which happens to have the same frequency at that temperature. We have already discussed some of the modes of vibration which can be excited in an

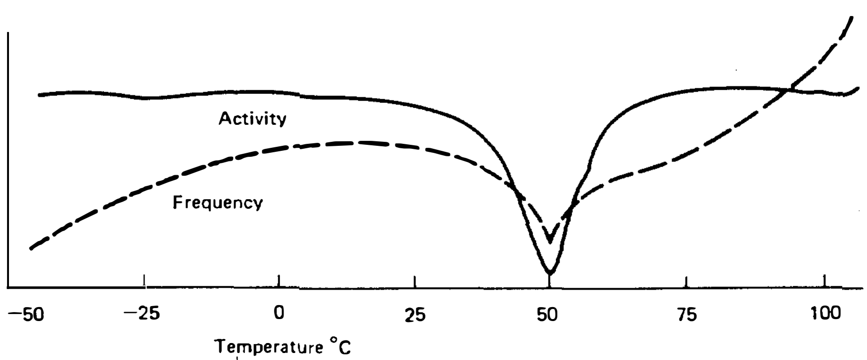

Fig. 9.1. Frequency and activity of a 5-MHz AT-cut plate vs. the operating temperature (experimental).

elastic solid. In addition to the inharmonic modes of the thickness-shear vibration, we also have the face-shear mode and all its overtones; longitudinal vibrations which may be excited under certain conditions; and, perhaps most serious of all, flexure modes which sometimes occur. In addition to the various modes of vibration of the quartz plate, vibrations of various kinds may be excited in the mounting structures, and in low-frequency piezoids, even air resonances may occur. When one of these modes is excited energy is thereby dissipated, thus increasing the equivalent resistance R and decreasing Q. The mass and/or stiffness added to the vibrating system causes changes in the frequency of resonance. The interaction between the two modes of vibration is called *coupling.*

COUPLING IN VIBRATING SYSTEMS

Coupling between mechanical or electric oscillating systems is a common physical phenomenon. Figure 9.2 shows a simple experiment for demonstrating the principles involved. Two simple pendulums, P_1, and P_2, consisting of metal spheres and fine threads are supported from a tightly stretched horizontal cord. When one of the pendulums is set into vibration, part of its energy is transferred to the other, thereby setting it into motion. If the lengths of the two pendulums are adjusted so that their frequencies are equal, all the energy of the first pendulum will be transferred to the second pendulum after a few cycles. If the frequency of the second pendulum differs from that of the first, little or no motion is excited in the second pendulum.

If the motion of one of the pendulums is heavily damped, by submerging it in water or some viscous liquid, and if the resonant frequencies are the same, the vibration of the first pendulum is quickly

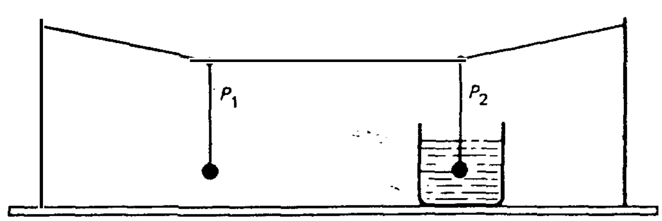

Fig. 9.2. Coupled pendulums with P_2 damped.

damped out. If the frequencies are quite different, little energy is transferred to the damped pendulum and therefore the motion of the first is only slightly affected.

The coupling in quartz crystal units may be either mechanical or electrical or a combination of the two. We first look at examples of coupling in which the electric field excites other modes of vibration in addition to the desired mode. Then we examine some examples of mechanical coupling and finally some of the more complex cases in which both are involved.

COUPLING BY COMMON ELECTRIC EXCITATION

To become familiar with the principles involved we first consider the simple case of an X-cut plate with the electrodes producing an electric field in the X-direction. The piezoelectric strain matrix for quartz shows [see Eq. (26)] that an electric field in the X-direction produces three types of strain. They are a longitudinal strain in the X- or thickness direction, a longitudinal strain in the Y-direction, and a shear strain in the YZ-plane, i.e., about the X-axis. If the electric field is due to an alternating current of the correct frequency, the piezoid may be set into vibration in one or more of the following ways. A longitudinal wave may be excited in the thickness direction excited through the piezoelectric coefficient d_{11}. A longitudinal wave may be excited in the Y-direction through d_{12}. A face-shear vibration may be set up in the plate (in the YZ-plane) through piezoelectric coefficient d_{14}. Harmonic and inharmonic overtone modes of each of the modes may also be excited, making the number of modes which can be excited very large and distributing them over a large range of frequencies. The temperature-frequency coefficients of the modes are unequal, since they are controlled by different elastic coefficients, so the possibility exists that the frequencies of two or more modes may become equal at certain temperatures. An activity dip occurs when the frequency of one of the other modes coincides with that of the desired mode. The crystal unit behaves electrically like the circuit of Fig. 9.3, where we may consider the parameters L' and C' to be temperature-dependent.

Consider a Y-cut plate with electrodes producing a field in the Y-direction. The piezoelectric matrix for quartz, Eq. (26), shows that two types of strain are produced by the field in the Y-direction. The

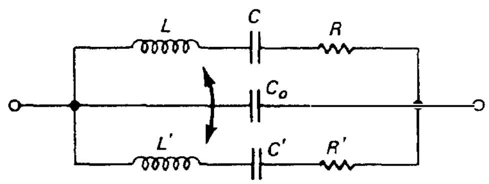

Fig. 9.3. Equivalent circuit of two modes excited by the same electric field. Coupling exists between L and L'.

coefficient $d_{25} = d_{14}$ relates a shear strain in the XZ-plane with the field in the Y direction. If the field varies at the correct frequency, the plate may be set into vibration in one of its many face-shear modes.

The coefficient $d_{26} = -2d_{11}$ relates a shear strain in the XY-plane to the field in the Y-direction. Again, if the field is produced by an alternating current of the proper frequency, a shear wave is excited which travels in the thickness direction of the plate. This is, of course, the mode of motion employed in the AT- and BT-cut plates. This mode also has many harmonic and inharmonic overtones which may be excited, as we have previously shown. The temperature coefficients of all these modes is substantially zero.

If the normal to the plate is not exactly parallel to the Y-axis, the electric field has a component in either the X- or Z-direction or both. Fortunately, the Z-component does not excite any piezoelectric strain, but a component in the X-direction may excite any of the modes which can be excited in an X-cut plate. Since an AT- or BT-cut plate is simply a Y-cut plate rotated about the X-axis, any field component in the X-direction is capable of exciting these same modes in these plates. For this reason it is important in fabricating AT- and BT-cut plates to be sure that the normal to the plate lies in the YZ-plane.

COUPLING BY MECHANICAL OR ELASTIC EXCITATION

In addition to the many modes of vibration which can be excited by the same electric field, other modes can be excited through a common strain. As an example of such a mechanically coupled system we may consider a bar of quartz with its rectangular cross section in the XZ-

plane and its length in the Y-direction. Many frequency standards were formerly made in this way. The field is applied in the X-direction and the bar is set into longitudinal vibration in the Y-direction through the coefficient d_{12}. The resulting extensional strain, y_y, produces extensional stresses in the X- and Z-directions through the stiffness coefficients c_{12} and c_{32} of Eqs. (13 and 13*b*), and a shear stress in the YZ-plane through coefficient c_{42} (which is equal to $-c_{14}$).

We consider first the extensional stress in the Z-direction produced through the elastic coefficient c_{32}. Although it is impossible to excite a longitudinal vibration in the Z-direction in quartz by an electric field, such a vibration can be excited by coupling to the extensional vibration in the Y-direction through the cross-coupling coefficient c_{32}. If the two frequencies are compatible, a standing wave in the Z-direction may be excited by mechanical coupling with the basic vibration consisting of a standing wave in the Y-direction.

In the same way, the extensional stress in the X-direction, X_x, produced through the coefficient d_{11} may excite longitudinal vibration in the Y-direction. This driving force is in addition to that produced by the electric field which is, of course, able to excite the same motion.

Finally, the Y_z stress produced through the coefficient c_{42} can produce a face-shear resonance if the frequencies are compatible and if the boundary conditions permit. This mode, like the previous one, can also be excited electrically so that the mechanical coupling is added to the electric coupling.

A particularly interesting and important example of coupling is that of the coupling between the face-shear modes and the thickness-shear modes in the Y-family of quartz crystal units. It is somewhat easier to think first of a Y-cut plate and later extend the ideas to the AT- and BT-cut plates. The thickness-shear mode of the Y-cut involves an x_y strain, but from Eq. (13*b*) it follows that an x_y strain produces a $-Z_x$ stress through the cross-coupling coefficient c_{56}. Stress Z_x produces strain z_x, which is the strain involved in the face-shear mode of vibration. The result is that the face-shear vibration is excited by the thickness-shear vibration through the cross-coupling coefficient c_{56}; and if the conditions are correct, resonance may occur. The mechanical coupling between the thickness- and face-shear modes thus augments the electric coupling which exists since both modes may also be excited by the same electric field. The result is that the two modes are very strongly coupled and the frequency constant is perturbed in the manner shown in Fig. 9.4.

We consider next an AT-cut plate having lateral dimensions 1.0 × 1.0 cm and thickness 0.60 mm. The thickness-shear frequency is approximately 2.8 MHz and the frequency of the 11 face-shear mode is about 300 kHz. The frequency of the 99 overtone of the face-shear vibration is very nearly equal to that of the thickness-shear mode, and since the frequency of the face-shear mode changes with temperature, the two may become equal at some point within the specified temperature range.

The coupling between the thickness- and face-shear modes in the AT-cut plate is provided by the cross-coupling coefficient c'_{56}, which may be calculated by the same method used earlier to calculate c'_{66}. The result is found to be

$$c'_{56} = c_{14}\,(c^2 - s^2) - (c_{66} - c_{44})\,sc$$

which becomes zero for two angles,

$$\theta = 31°46' \text{ and } \theta = -58°15'$$

Plates cut at these angles are designated the AC- and BC-cuts, respectively. The C signifies that the coupling is zero. The AC-cut has been used as a thermometer. It has a T_f of +20 ppm/°C and its freedom from coupling to face-shear modes makes it relatively free from frequency perturbations.

The value of c'_{56} is small at the orientation of the AT-cut, which is only 3.5° from the AC-cut. Consequently, the coupling between the thickness-shear and face-shear modes is small. The coupling be-

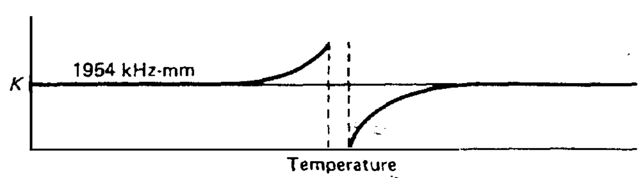

Fig. 9.4. Perturbation of the frequency constant by coupled modes in the *Y*-cut plate.

tween the two modes is greater in the BT-cut, which is over 9° from the BC-cut. It is unfortunate that the angles for zero coupling and zero T_f are not the same.

VIBRATIONS IN MOUNTING STRUCTURES

The piezoid must always be supported in some manner. Unless the mounting structure can be attached at points which are nodes of motion, the mounting structure becomes a part of the vibrating system. This phenomenon is particularly troublesome in CT- and DT-cut piezoids vibrating in the face-shear mode. The supporting wires are usually attached at the nodal points at the center of the plates. If the point of attachment is not exactly at the nodal point, the supporting wire becomes a part of the vibrating system and at certain temperatures may be set into resonance, thereby absorbing energy and causing an activity dip.

In AT-cut plates the vibrating area is usually confined to the center of the plate by making the electrodes considerably smaller than the plate. The zone around the edge may then be used to support the plate. If the lateral dimensions of the plate are large compared with its thickness, little difficulty is encountered with damping due to energy transferred to the mounting structure. If the ratio of lateral dimensions to thickness is less than about 20, it is difficult to find nodal points at which to support the plate. Points which are nodal points at one temperature become antinodes at another temperature; the mounting structure is set into vibration and an activity dip results.

COUPLING THROUGH COMMON BOUNDARY CONDITIONS

A transverse wave traveling in the direction of a lateral dimension of a plate is called a flexure mode. These modes are often excited in thickness-shear plates such as the AT-cut plate because of the similarity between the strains in the two modes. Figure 9.5 shows the type of strains involved in the thickness-shear vibration of an AT-cut plate and a flexure mode in the same plate. The similarity of the strains at the ends is immediately apparent. The flexure modes are very easily excited in rectangular AT-cut plates, especially if the length of the plate in the X-direction is less than 20 to 30 times the thickness.

Fig. 9.5. Edge view of the strains in an AT-cut plate (*a*) vibrating in thickness shear and (*b*) vibrating in its first even flexure mode.

Some of the modes of motion of a bar or plate vibrating in flexure are shown in Fig. 9.6. Only the even-order modes are strongly coupled. Neither the strains at the boundaries nor the dynamical balance is conducive to coupling between the thickness-shear and odd-order flexure modes.

The thickness-shear mode tends to excite a transverse wave which travels in the X-direction in the blank. The frequency of the thickness-shear vibration is substantially independent of the temperature but that of the flexure mode is not, so that at some temperature the frequency of the thickness-shear mode is equal to that of one of the flexure modes and the plate is set into resonance with severe effects on both the frequency and the activity of the plate. A dramatic demonstration of the coupling to the flexure mode may be made by using an AT-cut plate having dimensions of 15 to 20 mm and a frequency of 2 to 3 MHz. The rectangular plate is mounted between parallel electrodes with a small air gap and excited at its resonant frequency in a circuit which drives it strongly. By adjusting the length in the X-direction, or the temperature, or both, conditions for coupling to one of the even-order flexure modes can be established.

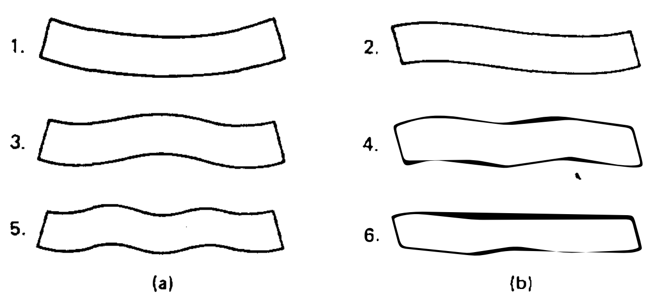

Fig. 9.6. Edge view of the displacement in plates vibrating in (*a*) odd flexure modes and (*b*) in even flexure modes.

If the space between the electrode and the quartz blank is observed in a dark room while the blank is strongly excited, a glow discharge may be observed in the form of parallel lines perpendicular to the X-direction. The strain is apparently sufficient at the antinodes of the flexure to produce voltages great enough to produce ionization of the air and the glow which results. If operated in this manner for a little time, the electrodes will be pitted with the pits outlining the position of the nodes and antinodes of the flexure motion.

Flexure modes are most troublesome in rectangular plates and at low frequencies. This is one of the reasons for preferring circular plates over rectangular plates in the fabrication of AT-cut piezoids.

It is possible to utilize the coupling between the flexure modes and the thickness-shear mode to obtain excellent temperature coefficients over a limited range of temperature. The thickness constant K is changed by the strong coupling between the two modes and if the lateral dimensions of the plate are less than about 10 times the thickness, the change may easily be ± 10 percent. Such plates are very difficult to mount, so their application is limited.

EFFECTS OF TEMPERATURE

Because of the various types of coupling which may exist, a crystal unit may be an excellent resonator at one temperature and quite inactive at another temperature. Sometimes the frequency "skips" from one value to another as the temperature is changed. The reason for this behavior is to be found in the different frequency-temperature coefficients of the various modes. The piezoid is usually designed to make the value of T_f as low as possible, but it is not possible to make T_f for all of the other modes simultaneously also zero. Thus at some temperature the frequency of another mode may become equal to that of the resonator and the activity and frequency are thereby altered.

The inharmonic modes of the thickness shear do not contribute to the problem of activity dips. They have the same frequency-temperature coefficient as the main mode and therefore cannot interfere. It does happen sometimes that as the activity of the main mode is degraded by an interfering mode, the oscillator frequency "jumps" to that of one of the inharmonic modes to return to the main mode, at another temperature.

DIMENSIONING

The frequency of the main mode in quartz crystal units employing AT- and BT-cut plates is determined by the thickness of the plate. The frequencies of most of the interfering modes are determined by the lateral dimensions. Therefore, the lateral dimensions must be tightly controlled to avoid coupling between desired and undesired modes of motion. The process of adjusting the lateral dimensions to avoid coupled modes is called *dimensioning.* Crystal units made with square or rectangular blanks must be dimensioned very carefully to avoid activity dips and to obtain a reasonable production yield.

Most of the coupled modes are much less severe in circular than in rectangular blanks. Consequently, most crystal units in the higher-frequency range are made with circular blanks. The face-shear mode, however, can still be excited in a circular blank; and in applications in which the entire blank is excited by the electric field, the diameter of the blank becomes important. Such modes cannot exist if only the center portion of the blank is excited, because the active region is surrounded by a zone of inactive quartz. If the width of this zone is 20 or more times the thickness of the blank, it also provides an inactive region for supporting the blank.

LOW DIMENSIONAL RATIOS

An infinitely long bar or an infinitely large plate has only a single mode of vibration and hence activity dips due to coupled modes would never appear. In actual plates the severity of the activity-dip problem depends upon the lateral/thickness dimensional ratio and becomes rapidly more severe as the ratio drops below about 20. Thus the activity-dip problem may be quite insignificant in a 10-MHz AT-cut plate having a diameter of 0.75 cm in which d/e = 45 and very troublesome in a 2.5-MHz unit in which the AT-cut blank has a diameter of 1.00 cm and a dimensional ratio of 15.

The use of AT-cut blanks in which d/e is less than about 20 should be avoided where possible to avoid the difficulty of producing units of this kind which are free of activity dips over an extended frequency range and, of course, the unavoidable economic cost.

The new-SC-cut (Chap. 13) is relatively free of activity dips and for this reason may find application in certain cases where activity dips are a serious problem with AT-cut plates.

Another consequence of a low *d/e* ratio in an AT-cut plate is that the product of the frequency and the thickness is not a constant (page 135). This is due to the coupling between the thickness-shear mode and low-order overtones of other modes which strongly perturbs the vibration frequency. The frequency-temperature curve is distorted for the same reason. The familiar "cubic" curve of the high-frequency AT-cut resonator may be entirely unrecognizable in a low-frequency unit having a *d/e* ratio of 10 or less.

If units must be designed with *d/e* ratios less than about 20, it must be expected that much effort will be required to find a suitable design and much care will be required to hold the required tolerances in production.