

Introduction to Quartz Crystal Unit Design

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Van Nostrand Reinhold Electrical/Computer Science and Engineering Series

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TO
QUARTZ CRYSTAL UNIT DESIGN



Photograph of a bar of cultured quartz such as the kind produced commercially. (Courtesy of Sawyer Research Products, Inc.)

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to all of whom I am indebted.**

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INTRODUCTION TO QUARTZ CRYSTAL UNIT DESIGN, by Virgil E. Bottom

FOREWORD

Quartz crystal units have been an item of commerce for more than a half century. The applications for quartz crystals have grown from their early roots in frequency standards and in amateur radio communication to the present wide spectrum of applications encompassing many electronic products. The millions of units produced annually are used extensively in the field of telecommunications and radio communications where precise frequency control, both in transmission and reception, is essential for quality and reliability of communication. Time-keeping was one of the early applications of quartz crystal units and has continued to grow in precision and in utilization. The earliest quartz clocks evolved into national standards of lapsed time and became the interpolation clocks for astronomical observations of epoch. As more fundamental atomic clocks evolved using a resonance line of cesium, quartz units continued to play a major role in providing the fly wheel for these quantum electronic clocks. In this application, the short term frequency stability of precision quartz resonators, which at present has achieved the level of a few parts in 10^{14} for one second sample times, provides again an excellent interpolation system for short times while the cesium resonance provides the absolute definition of frequency and time for periods longer than a few seconds. With the evolution of very small, rugged, low frequency, low cost resonators with improved temperature coefficients and long term stabilities approaching a part in a million per year, quartz crystal units have become practical as the frequency controlling element in electronic wrist watches. As this occurred, quartz watches rapidly replaced mechanical escapement watches and became the largest user of quartz crystal units. Their utility in timekeeping spans the entire spectrum from the lowest cost consumer product to the most precise time standard.

In addition to the applications of quartz crystal units in timing and communication, they have found extensive applications in other industrial products. They are used as timing control elements in computers, as surface wave correlators in digital communication, as discriminators and filters in instrumentation and communication, and as sensors of temperature and pressure.

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Quartz crystal units provide frequency control over the range from a few kilohertz to a gigahertz. This frequency range of nearly a million to one is achieved by a variety of resonator designs from thickness flexure resonators through width flexure, extensional, face shear, thickness shear, and surface wave resonators. In all of these designs, the period of acoustic resonance is determined by the time of propagation of the acoustic wave through a resonant structure. At the lowest frequencies, the velocity of flexure waves is slowed through the use of very small dimensional ratios and the resonant path is extended to several centimeters. At the other extreme, the acoustic resonance dimensions are decreased into the sub-micrometer region where lithographic technologies set the upper frequency limit for surface wave devices.

Over the half century of quartz crystal unit development, there has been a steady growth in the frequency stability of resonators. The first international comparisons of frequency by W. G. Cady in the early 1930's indicated accuracies in the order of one tenth of one percent. Frequency stability of a part per million during World War II, and a part in 10^8 by 1952 were bench marks in the progress of precision. In the following decades, short term frequency stability improved to a few parts in 10^{10} by 1960, several parts in 10^{12} by 1970 and to a few parts in 10^{14} in 1980. This phenomenal improvement in stability of approximately 10 orders of magnitude, over a period of 50 years, has evolved with relentless regularity at about an order of magnitude every 5 to 7 years.

Professor Bottom provides in this concise text the concepts and principles fundamental to the developments that have made possible extensive application of quartz crystal units over a wide range of frequencies with steady progress in performance. The latter chapters, 12 and 13, deal with the significant performance improvements that have evolved in quartz resonators. These chapters relate the concepts developed under the earlier discussions of crystallography, elasticity, the piezoelectric effect, and mechanical vibrations, to the device design and processing improvements that, in turn, have led to one of the most remarkable performance improvements of any device in the field of electronics.

Professor Bottom is well qualified to write this important text. He has conducted extensive academic research on piezoelectric resonators and lectured in the discipline underlying the technology of quartz crystal units. His experience in the Signal Corps Research Laboratories, as a director of industrial research and as a consultant to quartz crystal manufacturers, provides a complete perspective relating the theoretical concepts and analysis to the practical problems of device and process design.

Professor Bottom's book is a most welcome addition to the literature in the field of frequency and time.

Dr. Donald L. Hammond, Director
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PREFACE

The quartz crystal unit is an indispensable component of modern electronic technology. Not only is it used to control and manage the frequencies in virtually all communication systems but it provides the isochronous element in most clocks, watches, computers, and microprocessors. It is a product of the discovery of the phenomenon of piezoelectricity by the Curie brothers in France just one century ago. Despite the importance, indeed the indispensability, of the piezoelectric resonator to the field of electronics only a few books have been written on the subject and none has heretofore been written with the specific objective of introducing the newcomer to the field of quartz crystal unit design. Nor has the subject of piezoelectricity found a place in the curriculum of schools of electrical engineering. Consequently few Electrical Engineering graduates have heard the word *piezoelectricity* and almost none has any familiarity with the basic principles involved. The result is a severe and growing shortage of people who are in a position to contribute to this important area of technology.

This book is presented with the hope that it will encourage some young engineers to become active in the field of applied piezoelectricity and help them to find their way into it.

Piezoelectricity is not a simple subject. The phenomenon is complex, involving the more advanced concepts of both electricity and mechanics. The applications of piezoelectricity involve almost every area of classical physics including mechanics, acoustics, wave motion, optics, electrostatics, electric circuit theory, x-rays and x-ray diffraction, and crystallography. Probably few fields of engineering or applied science require familiarity with a wider range of topics from the field of physics. The mathematical tools required include complex number theory, vector and matrix algebra, and some knowledge of partial differential equations. Fabrication and testing of quartz crystal units involve some of the most delicate and precise measurements which can be made. The product is an inexpensive component whose precision, reliability, and importance are not exceeded by any product of technology.

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The range of physical and mathematical concepts required is wider than that covered in the usual undergraduate curriculum in either physics or Electrical Engineering departments, so beginning students often have difficulty with the works of Cady and Mason. It is hoped that this book will help bridge the gap between the usual undergraduate curriculum and these important works.

This book is avowedly tutorial. Those doing research in the field should turn to other sources. The purpose of this book is to help and encourage the beginning engineer to understand the basic principles of piezoelectricity and its applications to the design of quartz crystal units. For this reason, the mathematical derivations are simplified as much as possible. Complications such as second-order effects are deliberately omitted in order not to obscure the main topic (and not because they are unimportant). One-dimensional systems are used to illustrate basic principles. Anisotropy is sometimes ignored in order to simplify the discussion. The notation is simplified as much as possible in order not to obscure the main idea. The treatment is limited in scope; many important topics are not treated at all. Among the topics omitted are monolithic filter crystals, tuning fork and other low-frequency resonators, surface-wave devices, and most second-order effects. Almost all the book deals with thickness-shear-type resonators such as the AT-cut, but the principles developed can, once understood, be extended to much wider applications.

Most of the book is suitable for study by students who have completed good intermediate courses in general physics and differential and integral calculus. Where deemed necessary, some introduction to the mathematical tools is included. Derivations are given in a bit more detail than would be done in a more advanced text. The object is to elucidate rather than to abbreviate. A set of problems and questions is included as Appendix VI to help the student check and extend his understanding of the text.

This book is the result of many years of teaching the basic principles of piezoelectricity and quartz crystal unit design to undergraduate and graduate students in formal classes and to practicing engineers in seminars held under industrial auspices. Indeed the book began as a set of notes for such a seminar. It is now presented with the hope that it may serve a wider clientele.

Abilene, Texas

Virgil E. Bottom

SYMBOLS

The writer of a work of this type is faced with a dilemma in the choice of notation. One approach is to use the symbols which have conventionally been employed to represent the numerous quantities. The other is to set up a completely consistent notation with no ambiguities. The first approach results in the use of the same symbol for different quantities. The second approach leads to proliferation of symbols and difficulty in referring to earlier works. The author has chosen the first approach, believing that in instances where a given symbol represents different quantities, the meaning will be clear to the reader from the context.

A	amplitude, area
a	distance, radius, rotation matrix
b	velocity ratio, rotation matrix
C	capacitance, motional capacitance (also C_m)
C_o	static capacitance
c	stiffness coefficient, rotation matrix
D	electric displacement
D_θ, D_ϕ	angle between atomic plane and crystal plate
d	piezoelectric strain coefficient, atomic spacing, plate diameter
E	electric-field intensity
e	piezoelectric stress coefficient, thickness, voltage, extension
F	force
f	frequency
g	goniometer reading, acceleration of gravity
h	spring stiffness
I	moment of inertia, electric current
i	current
j	$(-1)^{1/2}$
K	frequency constant
k	dielectric constant, coupling coefficient, mode index, $4\pi\epsilon_o$, $2\pi/\lambda$
L	length, inductance, motional inductance (also L_m)
l	length, mode index

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m	meter, mode index
N	Avagadro's number
n	mode index, harmonic order
P	polarization
p	dipole moment
Q	quality factor
q	electric charge, q_v space charge density
R	Resistance, motional resistance (also R_m), radius of curvature
r	damping coefficient, distance, capacitance ratio
S	frequency-stress coefficient
s	compliance coefficient
T	period, time, temperature
T_f	frequency-temperature coefficient
t	time
U	internal energy
V	electric potential
v	wave velocity
w	width
X	stress, reactance
x	strain, coordinate variable
Y	admittance
y	coordinate variable
Z	impedance
α	rotation matrix, coefficient of linear expansion
β	imaginary part of propagation constant, angle of deflection of x-ray beam
γ	complex propagation constant
δ	angle between atomic planes and surface of blank partial derivative
Δ	incremental change, $r/2Q^2$
ϵ	piezoelectric stress coefficient (also e)
ϵ_0	susceptibility of space
η	dielectric susceptibility
κ	coupling coefficient
λ	wavelength
ρ	density
σ	surface charge per unit area
θ	angle
θ_B	Bragg angle
ϕ	angle
χ	root of Bessel's function of the first kind
ψ	mechanical displacement, azimuth angle
ω	angular frequency