

TABLE 1
FREQUENCY TEMPERATURE BEHAVIOR OF AT, BT, CT, DT, AND RT CUTS

| CUT | ANGLE | | ^a 10 ⁻⁶ /°C | ^b 10 ⁻⁹ /(°C) ² | ^c 10 ⁻¹² /(°C) ³ |
|------------------|--------|----------|--------------------------------------|---|--|
| | ϕ | θ | | | |
| AT | 0° | 35°15' | 0 | 0.4 | 109.5 |
| AT Opt. Angle | 0° | 35°20' | -0.40 | -0.02 | 109.3 |
| BT | 0° | -49°12' | 0 | -40 | -128 |
| CT | 0° | 36°20' | 0 | -58 | -151 |
| DT | 0° | -50°40' | 0 | -17 | 52 |
| RT | 15° | -34°30' | 0 | -6 | -2 |

TABLE 2
**GENERAL EXPRESSIONS FOR THE CHRISTOFFEL STIFFNESSES Γ_{ik} ,
THE PIEZOELECTRIC STRESS CONSTANTS Ξ_i ,
AND THE DIELECTRIC PERMITTIVITY ϵ_s**

| | a_1^2 | 2 | a_3^2 | a_2a_3 | a_3a_1 | a_1a_2 |
|---------------|-----------------|-----------------|-----------------|-------------------|-------------------|-------------------|
| Γ_{11} | c_{11} | c_{66} | c_{55} | $2c_{65}$ | $2c_{51}$ | $2c_{16}$ |
| Γ_{22} | c_{66} | c_{22} | c_{44} | $2c_{24}$ | $2c_{46}$ | $2c_{62}$ |
| Γ_{33} | c_{55} | c_{44} | c_{33} | $2c_{43}$ | $2c_{35}$ | $2c_{54}$ |
| Γ_{23} | c_{65} | c_{24} | c_{43} | $c_{23} + c_{44}$ | $c_{45} + c_{63}$ | $c_{64} + c_{25}$ |
| Γ_{13} | c_{51} | c_{46} | c_{35} | $c_{45} + c_{36}$ | $c_{31} + c_{55}$ | $c_{56} + c_{41}$ |
| Γ_{12} | c_{16} | c_{62} | c_{54} | $c_{64} + c_{52}$ | $c_{56} + c_{14}$ | $c_{12} + c_{66}$ |
| Ξ_1 | e_{11} | e_{26} | e_{35} | $e_{25} + e_{36}$ | $e_{31} + e_{15}$ | $e_{16} + e_{21}$ |
| Ξ_2 | e_{16} | e_{22} | e_{34} | $e_{24} + e_{32}$ | $e_{36} + e_{14}$ | $e_{12} + e_{26}$ |
| Ξ_3 | e_{15} | e_{24} | e_{33} | $e_{23} + e_{34}$ | $e_{35} + e_{13}$ | $e_{14} + e_{25}$ |
| ϵ_s | ϵ_{11} | ϵ_{22} | ϵ_{33} | $2\epsilon_{23}$ | $2\epsilon_{13}$ | $2\epsilon_{12}$ |

TABLE 3
EXPRESSIONS FOR THE CHRISTOFFEL STIFFNESSES Γ_{ik} ,
THE PIEZOELECTRIC STRESS CONSTANTS Ξ_j AND THE
DIELECTRIC PERMITTIVITY ϵ_s FOR THE TRIGONAL CRYSTAL CLASS
 D_3 , e.g., ALPHA-QUARTZ

| | a_1^2 | a_2^2 | a_3^2 | $a_2 a_3$ | $a_3 a_1$ | $a_1 a_2$ |
|-----------------|-----------------|-----------------|-----------------|-------------------|-------------------|-------------------|
| Γ_{11} | c_{11} | c_{66} | c_{44} | $2c_{14}$ | 0 | 0 |
| Γ_{22} | c_{66} | c_{11} | c_{44} | $-2c_{14}$ | 0 | 0 |
| Γ_{33} | c_{44} | c_{44} | c_{33} | 0 | 0 | 0 |
| Γ_{23} | c_{14} | $-c_{14}$ | 0 | $c_{13} + c_{44}$ | 0 | 0 |
| Γ_{31} | 0 | 0 | 0 | 0 | $c_{13} + c_{44}$ | $2c_{14}$ |
| Γ_{12} | 0 | 0 | 0 | 0 | $2c_{14}$ | $c_{12} + c_{66}$ |
| Ξ_1 | e_{11} | $-e_{11}$ | 0 | $-e_{14}$ | 0 | 0 |
| Ξ_2 | 0 | 0 | 0 | 0 | e_{14} | $-2e_{11}$ |
| Ξ_3 | 0 | 0 | 0 | 0 | 0 | 0 |
| ϵ_{33} | ϵ_{11} | ϵ_{11} | ϵ_{33} | 0 | 0 | 0 |

TABLE 4
ELASTIC STIFFNESSES $c_{\lambda\mu}$ IN 10^9 Nm^{-2}
FOR ALPHA-QUARTZ AT 20°C

| $\lambda\mu$ | STIFFNESSES OF QUARTZ Bechmann (9) | | STIFFNESSES OF QUARTZ Mindlin and Gazis (15) |
|--------------|---------------------------------------|----------------------------|---|
| | $c_{\lambda\mu}^{E\sigma}$ | $c_{\lambda\mu}^{D\sigma}$ | $c_{\lambda\mu}$ |
| 11 | 86.74 | 87.49 | 86.75 |
| 33 | 107.2 | 107.2 | 107.2 |
| 12 | 6.99 | 6.23 | 5.95 |
| 13 | 11.91 | 11.91 | 11.91 |
| 44 | 57.94 | 57.98 | 57.8 |
| 66 | 39.88 | 40.63 | 40.4 |
| 14 | -17.91 | -18.09 | -17.8 |

TABLE 5
MEASURED AND CALCULATED FIRST-, SECOND- AND THIRD-ORDER TEMPERATURE COEFFICIENTS
OF FREQUENCY a, b, c FOR THE THICKNESS MODES A, B, AND C OF SOME SELECTED QUARTZ PLATES

TEMPERATURE COEFFICIENTS, MODE A

| | ORIENTATION | | T _(MAX) | OBSERVED VALUES | | | CALCULATED VALUES | | |
|----------|-------------|----------|--------------------|-------------------------------------|---|--|-------------------------------------|---|--|
| | ϕ | θ | °C | $a \times 10^{-6}/^{\circ}\text{C}$ | $b \times 10^{-9}/(^{\circ}\text{C})^2$ | $c \times 10^{-12}/(^{\circ}\text{C})^3$ | $a \times 10^{-6}/^{\circ}\text{C}$ | $b \times 10^{-9}/(^{\circ}\text{C})^2$ | $c \times 10^{-12}/(^{\circ}\text{C})^3$ |
| Z CUT | 10° | -28° | >-160° | -76.5 | | | -78.4 | -111.0 | -62.2 |
| | 10° | -38° | >-160° | -94.7 | | | -85.6 | -128.0 | -94.9 |
| | * 12°30' | -34° | >-160° | -81.0 | -129.0 | -58.9 | -78.6 | -118.0 | -86.5 |
| | * 15° | -33° | >-160° | -75.0 | -110.0 | +82.5 | -72.0 | -111.0 | -90.0 |
| | * 15° | -35° | >-160° | -72.0 | -88.3 | +90.3 | -73.0 | -114.0 | -94.0 |
| | * 20° | -36° | >-160° | -70.0 | -93.2 | +266.0 | -66.6 | -110.0 | -106.0 |
| | 30° | 0° | >-160° | -20.5 | -53.2 | -36.6 | -20.5 | -52.0 | -36.9 |
| | * 30° | ±10° | >-160° | -29.3 | -67.0 | -58.3 | -29.8 | -64.3 | -68.6 |
| | * 30° | ±20° | >-160° | -42.0 | -86.0 | -92.7 | -41.4 | -80.7 | -95.4 |
| | * 30° | ±30° | >-160° | -54.8 | -105.0 | -119.0 | -51.7 | -96.0 | -113.0 |

TEMPERATURE COEFFICIENTS, MODE B

| | | | | | | | | | |
|-----------|----|---------|--------|-------|-------|--------|-------|-------|--------|
| BT CUT | 0° | -40° | >+160° | +20.7 | -15.2 | -112.6 | +19.9 | -21.3 | -116.0 |
| | 0° | -45° | +125° | +10.7 | -32.0 | -128.0 | +9.1 | -31.6 | -123.0 |
| | 0° | -49°13' | +25° | 0.0 | -40.0 | -128.0 | 0.0 | -39.6 | -128.0 |
| | 0° | -50° | -10° | -1.3 | -44.4 | -130.0 | -1.3 | -42.0 | -129.0 |
| | 0° | -55° | >-160° | -13.7 | -71.7 | -171.9 | -12.3 | -51.4 | -133.0 |
| | 0° | -60° | >-160° | -24.5 | -75.0 | -134.8 | -22.8 | -60.9 | -136.0 |
| | | | | | | | | | |

*Denotes cuts for which complete data on A, B, and C Modes are available.

TABLE 5 (Continued)

TEMPERATURE COEFFICIENTS, MODE B (Continued)

| | ORIENTATION | | T _(MAX) | OBSERVED VALUES | | | CALCULATED VALUES | | |
|----|-------------|----------|--------------------|-------------------------------------|---|--|-------------------------------------|---|--|
| | ϕ | θ | °C | $a \times 10^{-6}/^{\circ}\text{C}$ | $b \times 10^{-9}/(^{\circ}\text{C})^2$ | $c \times 10^{-12}/(^{\circ}\text{C})^3$ | $a \times 10^{-6}/^{\circ}\text{C}$ | $b \times 10^{-9}/(^{\circ}\text{C})^2$ | $c \times 10^{-12}/(^{\circ}\text{C})^3$ |
| 18 | 5° | -43° | > +160° | +35.7 | -9.5 | +195.8 | +9.2 | -33.7 | -123.0 |
| | 5° | -45° | +50° | +3.2 | -44.6 | -169.6 | +5.4 | -36.5 | -125.0 |
| | 5° | -47° | +10° | -0.9 | -41.1 | -118.0 | +1.7 | -39.8 | -127.0 |
| | 5° | -48° | +10° | -1.5 | -43.1 | -122.6 | -0.17 | -41.5 | -128.0 |
| | 5° | -49° | -40° | -5.1 | -53.5 | -156.0 | -2.1 | -43.1 | -129.0 |
| | 10° | -24° | >160° | +18.2 | -34.0 | -124.0 | -0.10 | -10.2 | -39.2 |
| | 10° | -26° | >160° | +16.6 | -43.0 | -148.0 | +6.5 | +21.5 | +13.2 |
| | 10° | -30° | +140° | +10.0 | -27.0 | -103.0 | +16.0 | -33.9 | -106.0 |
| | 10° | -32° | +110° | +9.8 | -33.8 | -197.3 | +13.6 | -36.4 | -112.0 |
| | 10° | -33° | +85° | +7.5 | -39.9 | -168.5 | +12.5 | -37.3 | -114.0 |
| | 10° | -34° | +80° | +5.7 | -41.5 | -141.7 | +11.4 | -38.0 | -115.0 |
| | 10° | -35° | +75° | +5.4 | -42.9 | -50.8 | +10.3 | -38.7 | -117.0 |
| | 10° | -38° | +40° | +1.1 | -39.0 | -91.3 | +6.8 | -40.7 | -120.0 |
| | 10° | -40° | -5° | -2.4 | -41.9 | -115.4 | +4.3 | -42.2 | -122.0 |
| | 10° | -42° | -60° | -5.9 | -47.5 | -81.2 | +1.9 | -43.6 | -123.0 |
| | 12°30' | -30° | +70° | +2.22 | -27.2 | -74.95 | +8.4 | -39.0 | -100.7 |
| | 12°30' | -31° | +60° | +2.5 | -32.3 | -89.2 | +7.6 | -40.8 | -104.0 |
| | 12°30' | -32° | +50° | +2.1 | -33.8 | -88.3 | +7.9 | -41.9 | -107.0 |
| | 12°30' | -33° | +45° | +1.8 | -34.9 | -90.5 | +6.0 | -42.7 | -108.0 |
| | 12°30' | -33°30' | 0° | -1.3 | -43.4 | -118.5 | +5.6 | -43.1 | -109.0 |
| * | 12°30' | -34° | 0° | -2.1 | -44.7 | -75.6 | +5.1 | -43.5 | -110.0 |

*Denotes cuts for which complete data on A, B, and C Modes are available.

TABLE 5 (Continued)

TEMPERATURE COEFFICIENTS, MODE B (Continued)

| | ORIENTATION | | T _(MAX) | OBSERVED VALUES | | | CALCULATED VALUES | | |
|---|-------------|----------|--------------------|-------------------------------------|---|--|-------------------------------------|---|--|
| | ϕ | θ | | $a \times 10^{-6}/^{\circ}\text{C}$ | $b \times 10^{-9}/(^{\circ}\text{C})^2$ | $c \times 10^{-12}/(^{\circ}\text{C})^3$ | $a \times 10^{-6}/^{\circ}\text{C}$ | $b \times 10^{-9}/(^{\circ}\text{C})^2$ | $c \times 10^{-12}/(^{\circ}\text{C})^3$ |
| 61 | 15° | -31° | >-160° | -11.2 | -21.7 | -26.4 | +0.75 | -44.4 | -98.7 |
| | 15° | -32° | >-160° | -8.8 | -39.1 | -88.8 | +0.64 | -45.6 | -101.0 |
| | 15° | -33° | >-160° | -7.5 | -33.2 | -64.8 | +0.23 | -46.5 | -103.0 |
| | 15° | -34° | >-160° | -7.3 | -37.4 | -80.3 | -0.37 | -47.1 | -104.0 |
| | 15° | -34°30' | >-160° | -7.4 | -28.1 | -39.7 | -0.67 | -47.4 | -105.0 |
| | 15° | -35° | >-160° | -7.45 | -33.5 | -58.8 | -0.98 | -47.7 | -106.0 |
| | 15° | -35°30' | >-160° | -8.5 | -39.3 | -75.1 | -1.34 | -48.0 | -107.0 |
| | 20° | -36° | >-160° | -19.9 | -30.5 | +20.3 | -9.7 | -51.4 | -95.2 |
| | 30° | ±10° | >-160° | -14.9 | -42.5 | -86.0 | -14.7 | -33.7 | -73.6 |
| | 30° | ±20° | >-160° | -9.3 | -21.8 | -34.4 | -9.3 | -20.4 | -37.6 |
| | 30° | ±30° | >-160° | -19.1 | -24.5 | -28.9 | -18.2 | -27.0 | -33.9 |
| | 30° | ±40° | >-160° | -22.2 | -39.3 | -46.5 | -19.5 | -50.9 | -77.9 |
| | 30° | ±50° | >-160° | -28.0 | -43.9 | -33.1 | -24.2 | -58.8 | -99.2 |
| <u>TEMPERATURE COEFFICIENTS, MODE C</u> | | | | | | | | | |
| AT CUT | 0° | +50° | >-160° | -71.5 | -82.6 | | -66.2 | -67.8 | +46.8 |
| | 0° | +40° | >-160° | -17.5 | -16.0 | | -24.0 | -22.4 | +95.8 |
| Y CUT | 0° | +35°30' | 25° | 0.0 | +0.40 | +109.5 | 0.0 | -0.4 | +109.0 |
| | 0° | +10° | >+160° | +96.3 | | | +90.5 | +64.1 | +53.4 |
| | 0° | 0° | >+160° | +97.1 | | | +92.5 | +57.5 | +5.8 |
| | 0° | -10° | >+160° | +89.7 | | | +81.2 | +41.4 | -36.8 |
| | 0° | -20° | >+160° | +65.8 | | | +62.9 | +21.1 | -71.2 |

Denotes cuts for which complete data on A, B, and C Modes are available.

TABLE 5 (Continued)

TEMPERATURE COEFFICIENTS, MODE C (Continued)

| | ORIENTATION | | T _(MAX) | OBSERVED VALUES | | | CALCULATED VALUES | | |
|----|-------------|----------|--------------------|-------------------------------------|---|--|-------------------------------------|---|--|
| | ϕ | θ | °C | $a \times 10^{-6}/^{\circ}\text{C}$ | $b \times 10^{-9}/(^{\circ}\text{C})^2$ | $c \times 10^{-12}/(^{\circ}\text{C})^3$ | $a \times 10^{-6}/^{\circ}\text{C}$ | $b \times 10^{-9}/(^{\circ}\text{C})^2$ | $c \times 10^{-12}/(^{\circ}\text{C})^3$ |
| 20 | 5° | -24° | > +160° | +33.1 | -58.5 | -334.0 | +27.8 | -67.8 | -253 |
| | 5° | -25° | +15° | -0.57 | -82.2 | -60.0 | -3.2 | -63.4 | +12.0 |
| | 5° | -26° | > -160° | -6.0 | -28.3 | -66.2 | -6.4 | -28.4 | -79.2 |
| | 5° | -28° | > -160° | -1.7 | -16.4 | -51.6 | -3.0 | -20.8 | -58.6 |
| | 10° | -28° | +70° | +3.3 | -40.0 | -48.0 | -2.4 | -23.0 | -57.6 |
| | 10° | -30° | +55° | +0.8 | -11.5 | -30.0 | -1.1 | -16.0 | -40.3 |
| | 10° | -31° | +40° | +0.61 | -10.7 | -34.2 | -1.2 | -14.4 | -34.4 |
| | 10° | -32° | +30° | +0.26 | -9.8 | -32.4 | -1.5 | -13.0 | -29.3 |
| | ✓10° | -33° | -40° | -0.87 | -7.81 | -21.5 | -1.9 | -12.5 | -25.0 |
| | 10° | -34° | > -160° | -1.7 | -6.3 | -10.5 | -2.7 | -12.0 | -20.7 |
| | 10° | -35° | > -160° | -0.2 | -5.8 | -7.2 | -3.5 | -11.8 | -16.8 |
| | 10° | -36° | > -160° | -3.4 | -6.0 | -5.0 | -4.6 | -11.9 | -13.3 |
| | 10° | -38° | > -160° | -5.5 | -5.9 | +1.7 | -7.2 | -12.6 | -6.9 |
| | 10° | -40° | > -160° | -8.8 | -6.5 | +13.5 | -10.4 | -14.3 | -1.5 |
| | 10° | -48° | > -160° | -27.4 | -19.6 | +40.0 | -29.0 | -29.0 | +8.8 |
| | 12°30' | -25° | +140° | +12.1 | -35.6 | -94.0 | +13.5 | -46.6 | -110.0 |
| | 12°30' | -26° | +130° | +12.3 | -37.9 | -115.0 | +12.9 | -50.2 | -122.0 |
| | 12°30' | -27° | +130° | +12.5 | -38.6 | -98.0 | +13.2 | -69.8 | -150.0 |
| | 12°30' | -28° | +140° | +14.6 | -39.7 | -132.0 | +3.24 | -37.3 | -62.8 |
| | 12°30' | -31° | +124° | +2.7 | -10.5 | -28.0 | -0.34 | -13.9 | -29.6 |
| | 12°30' | -31°30' | +120° | +2.0 | -9.3 | -10.0 | -0.65 | -13.3 | -27.2 |
| | 12°30' | -32° | +108° | +1.9 | -8.8 | -23.1 | -0.95 | -12.8 | -24.9 |
| | 12°30' | -32°30' | +90° | +1.4 | -8.7 | -24.9 | -1.28 | -12.5 | -22.8 |
| | 12°30' | -33° | +62 | +0.8 | -7.6 | -19.6 | -1.62 | -12.1 | -20.7 |
| | 12°30' | -33°30' | 0° | -0.4 | -7.4 | -14.0 | -2.07 | -11.9 | -18.8 |
| | * 12°30' | -34° | +16° | -0.04 | -7.5 | -18.0 | -2.54 | -11.7 | -16.9 |

* Denotes cuts for which complete data on A, B, and C Modes are available.

TABLE 5 (Continued)

TEMPERATURE COEFFICIENTS, MODE C (Continued)

| ORIENTATION | | T _(MAX) | OBSERVED VALUES | | | CALCULATED VALUES | | | | |
|-------------|----------|--------------------|-----------------|-------------------------------------|---|--|-------------------------------------|---|--|-------|
| | ϕ | θ | °C | $a \times 10^{-6}/^{\circ}\text{C}$ | $b \times 10^{-9}/(^{\circ}\text{C})^2$ | $c \times 10^{-12}/(^{\circ}\text{C})^3$ | $a \times 10^{-6}/^{\circ}\text{C}$ | $b \times 10^{-9}/(^{\circ}\text{C})^2$ | $c \times 10^{-12}/(^{\circ}\text{C})^3$ | |
| 21 | RT CUT * | 15° | -30° | >160° | +13.3 | -21.1 | -47.8 | +3.2 | -15.3 | -25.8 |
| | | 15° | -31° | >160° | +11.3 | -13.7 | -39.2 | +1.2 | -13.3 | -22.4 |
| | | 15° | -32° | >160° | +6.6 | -9.7 | -19.2 | 0.00 | -12.2 | -18.8 |
| | | 15° | -33° | >160° | +4.7 | -7.2 | -17.5 | -1.1 | -11.7 | -15.3 |
| | | 15° | -34° | +154° | +1.8 | -7.16 | -1.3 | -2.29 | -11.6 | -12.1 |
| | | 15° | -34°30' | +100° | +1.3 | -7.3 | -6.6 | -2.9 | -11.55 | -10.5 |
| | | 15° | -35° | +11° | -0.2 | -7.02 | -2.6 | -3.48 | -11.5 | -8.86 |
| | | 15° | -35°30' | -15.2° | -0.6 | -7.3 | -4.7 | -3.64 | -11.7 | -7.4 |
| | IT CUT * | 17°30' | -34°30' | >+160° | +2.49 | -8.24 | -4.41 | -2.2 | -11.4 | -3.2 |
| | | 17°30' | -35° | +30° | +0.2 | -8.3 | +2.37 | -3.34 | -11.35 | -3.27 |
| | | 17°30' | -35°30' | +100° | +1.34 | -8.4 | -3.36 | -4.0 | -11.41 | -3.38 |
| | | 17°30' | -36° | -30° | -1.02 | -8.9 | +2.9 | -4.89 | -11.48 | -3.5 |
| | | 20° | +34°20' | +15° | -0.06 | -8.9 | +52.0 | -0.5 | -10.5 | +62 |
| | | 20° | -28° | +80° | +4.0 | -32.6 | -28.1 | +7.1 | -39.5 | -57.9 |
| | | 20° | -30° | +93° | +4.6 | -30.3 | -29.4 | +10.6 | -18.4 | -14.1 |
| 20° | | -32° | +149° | +7.1 | -24.9 | -19.4 | +3.7 | -11.4 | +0.02 | |
| 20° | | -34° | >160° | +7.0 | -13.8 | +0.6 | -1.2 | -11.1 | +1.6 | |
| 20° | | -36° | >160° | +2.7 | -10.5 | +15.1 | -4.9 | -11.9 | +5.2 | |
| 20° | | -37° | -6° | -0.7 | -9.9 | +13.2 | -6.8 | -12.6 | +7.1 | |
| 20° | | -38° | -115° | -4.1 | -11.45 | +12.8 | -8.7 | -13.5 | +8.9 | |
| 20° | | -39° | -165° | -6.2 | -11.32 | +19.9 | -10.7 | -14.6 | +10.6 | |

*Denotes cuts for which complete data on A, B, and C Modes are available.

TABLE 5 (Continued)

TEMPERATURE COEFFICIENTS, MODE C (Continued)

| ORIENTATION | | T _(MAX) | OBSERVED VALUES | | | CALCULATED VALUES | | |
|-------------|----------|--------------------|-------------------------------------|---|--|-------------------------------------|---|--|
| ϕ | θ | °C | $a \times 10^{-6}/^{\circ}\text{C}$ | $b \times 10^{-9}/(^{\circ}\text{C})^2$ | $c \times 10^{-12}/(^{\circ}\text{C})^3$ | $a \times 10^{-6}/^{\circ}\text{C}$ | $b \times 10^{-9}/(^{\circ}\text{C})^2$ | $c \times 10^{-12}/(^{\circ}\text{C})^3$ |
| 25° | -26° | +120° | +4.3 | -23.2 | -11.8 | +3.4 | -37.7 | -28.0 |
| 25° | -28° | +105° | +3.7 | -23.15 | -13.7 | +4.2 | -35.9 | -28.2 |
| 25° | -30° | +90° | +3.5 | -24.2 | -12.8 | +6.4 | -27.8 | -15.8 |
| 25° | -34° | >150° | +4.9 | -18.0 | -5.6 | +0.83 | -12.2 | +17.8 |
| 30° | ±10° | >+160° | +13.3 | -7.9 | +34.4 | +11.5 | -21.8 | +43.0 |
| 30° | ±20° | >+160° | +7.0 | -16.9 | -0.4 | +6.3 | -28.5 | +14.1 |
| 30° | ±30° | >+160° | +5.0 | -20.5 | -12.4 | +5.1 | -25.2 | +5.7 |
| 30° | ±32° | >+160° | +4.3 | -14.9 | +11.6 | +4.3 | -18.4 | +20.4 |
| 30° | ±34° | +60° | +0.75 | -12.96 | +17.4 | +0.26 | -14.3 | +31.5 |
| 30° | ±36° | -100° | -4.55 | -14.3 | +20.6 | -5.4 | -14.4 | +34.4 |
| 30° | ±40° | >-160° | -14.9 | -19.8 | +27.0 | -16.8 | -19.8 | +34.7 |
| 30° | ±50° | >-160° | -46.4 | -50.1 | +44.5 | -46.2 | -47.0 | +21.6 |
| 30° | ±60° | >-160° | -72.8 | -101.7 | | -73.1 | -80.0 | -14.4 |

TABLE 6
VALUES FOR $T_{c_{44}}^{(1)}$, $T_{c_{66}}^{(1)}$, $T_{c_{14}}^{(1)}$ IN $10^{-6}/^{\circ}\text{C}$ USING TWO
DIFFERENT ZERO ANGLES OF THE AT AND BT CUTS

| | ZERO ANGLE AT: $\theta = 35^{\circ}15'$ BT: $\theta = -49^{\circ}13'$ | ZERO ANGLE AT: $\theta = 35^{\circ}22'$ BT: $\theta = -49^{\circ}40'$ | Δ PERCENT |
|--------------------|--|--|----------------------------|
| $T_{c_{44}}^{(1)}$ | -177.4 | -175.6 | 1.02 |
| $T_{c_{66}}^{(1)}$ | 177.6 | 179.3 | 0.96 |
| $T_{c_{14}}^{(1)}$ | 101.3 | 103.9 | 2.55 |

TABLE 7
VALUES FOR THE FIRST-ORDER TEMPERATURE COEFFICIENTS OF THE
STIFFNESSES FOR ALPHA-QUARTZ IN $10^{-6}/^{\circ}\text{C}$

| | BECHMANN [5] 1934 | MASON [19] 1940 | ATANASOFF [20] AND HART 1941 | MASON [22] 1951 | GPO [23] RESEARCH REPORT No. 13524 1951 | KOGA [24] et al. 1958 | NEW VALUES 1961 |
|-----------------|----------------------|--------------------|------------------------------------|--------------------|--|-----------------------------|--------------------|
| | | $T_o = 20^{\circ}$ | $T_o = 35^{\circ}$ | $T_o = 50^{\circ}$ | $T_o = 20^{\circ}$ | $T_o = 20^{\circ}$ | $T_o = 25^{\circ}$ |
| $Tc_{11}^{(1)}$ | -48 | -54 | -49.7 | -53.5 | — | -44.3 | -48.5 |
| $Tc_{33}^{(1)}$ | -208 | -251 | -213 | -165 | — | -188 | -160 |
| $Tc_{12}^{(1)}$ | -2115 | -2350 | -3000 | -3030 | — | -2930 | -3000 |
| $Tc_{13}^{(1)}$ | -530 | -687 | -580 | -510 | — | -492 | -550 |
| $Tc_{44}^{(1)}$ | -151 | -160 | -169 | -171 | -169 | -172 | -177 |
| $Tc_{66}^{(1)}$ | 144 | 161 | 170.1 | 168 | 168 | 180 | 178 |
| $Tc_{14}^{(1)}$ | 82 | 96 | 107 | 90 | 94 | 98 | 101 |

TABLE 8
VALUES FOR THE SECOND-ORDER TEMPERATURE COEFFICIENTS OF THE
STIFFNESSES FOR ALPHA-QUARTZ IN $10^{-9}/(^{\circ}\text{C})^2$

| | MASON [22] 1951 | GPO [23] RESEARCH REPORT No. 13524 1951 | KOGA, et al. [24] 1958 | NEW VALUES 1961 |
|--------------------|----------------------------|--|----------------------------|----------------------------|
| | $T_o = 50^{\circ}\text{C}$ | $T_o = 20^{\circ}\text{C}$ | $T_o = 20^{\circ}\text{C}$ | $T_o = 25^{\circ}\text{C}$ |
| $T_{c_{11}}^{(2)}$ | -75 | ———— | -407 | -107 |
| $T_{c_{33}}^{(2)}$ | -187 | ———— | -1412 | -275 |
| $T_{c_{12}}^{(2)}$ | -1500 | ———— | -7245 | -3050 |
| $T_{c_{13}}^{(2)}$ | -2000 | ———— | -596 | -1150 |
| $T_{c_{44}}^{(2)}$ | -212 | -233 | -225 | -216 |
| $T_{c_{66}}^{(2)}$ | -5 | 193 | 201 | 118 |
| $T_{c_{14}}^{(2)}$ | -270 | -82 | -13 | -48 |

TABLE 9
VALUES FOR THE THIRD-ORDER TEMPERATURE COEFFICIENTS
OF THE STIFFNESSES FOR ALPHA-QUARTZ IN $10^{-12}/(^{\circ}\text{C})^3$

| | MASON [22] 1951 | KOGA et al [24] 1958 | NEW VALUES 1961 |
|--------------------|----------------------------|----------------------------|----------------------------|
| | $T_o = 50^{\circ}\text{C}$ | $T_o = 20^{\circ}\text{C}$ | $T_o = 25^{\circ}\text{C}$ |
| $T_{c_{11}}^{(3)}$ | -15 | -371 | -70 |
| $T_{c_{33}}^{(3)}$ | -410 | -243 | -250 |
| $T_{c_{12}}^{(3)}$ | 1910 | 4195 | -1260 |
| $T_{c_{13}}^{(3)}$ | 600 | -5559 | -750 |
| $T_{c_{44}}^{(3)}$ | -65 | -190 | -216 |
| $T_{c_{66}}^{(3)}$ | -167 | -777 | 21 |
| $T_{c_{14}}^{(3)}$ | -630 | -625 | -590 |

TABLE 10
ELASTIC COMPLIANCES $s_{\lambda\mu}$ FOR ALPHA-QUARTZ
AT 20°C IN $10^{-12} \text{ m}^2\text{N}^{-1}$

| | COMPLIANCES OF QUARTZ According to R. Bechmann [9] | | COMPLIANCES OF QUARTZ Calculated from Mindlin and Gazis [15] Using their Values $c_{\lambda\mu}$ |
|--------------|---|----------------------------|---|
| $\lambda\mu$ | $s_{\lambda\mu}^{E\sigma}$ | $s_{\lambda\mu}^{D\sigma}$ | $s_{\lambda\mu}$ |
| 11 | 12.77 | 12.64 | 12.71 |
| 33 | 9.60 | 9.60 | 9.60 |
| 12 | -1.79 | -1.66 | -1.61 |
| 13 | -1.22 | -1.22 | -1.23 |
| 44 | 20.04 | 20.03 | 20.017 |
| 66 | 29.12 | 28.58 | 28.64 |
| 14 | 4.50 | 4.46 | 4.41 |

TABLE 11
VALUES FOR THE FIRST-ORDER TEMPERATURE COEFFICIENTS OF THE
COMPLIANCES FOR ALPHA-QUARTZ IN $10^{-6}/^{\circ}\text{C}$

| | BECHMANN [5] 1934 | GIEBE AND BLECHSCHMIDT [25] 1940 | MASON [19] 1940 | MASON [22] 1951 | GPO [23] RESEARCH REPORT No. 13524 1951 | NEW VALUES 1961 |
|------------------------|----------------------|--|--------------------|----------------------------|--|----------------------------|
| | | $T_o = 20^{\circ}\text{C}$ | | $T_o = 50^{\circ}\text{C}$ | $T_o = 20^{\circ}\text{C}$ | $T_o = 25^{\circ}\text{C}$ |
| $\text{Ts}_{11}^{(1)}$ | 11.5 | _____ | 12 | 16.5 | _____ | 15.5 |
| $\text{Ts}_{33}^{(1)}$ | 180 | _____ | 213 | 134.5 | _____ | 140 |
| $\text{Ts}_{12}^{(1)}$ | -1125 | _____ | -1265 | -1270 | _____ | -1370 |
| $\text{Ts}_{13}^{(1)}$ | -148 | _____ | -238 | -678 | _____ | -166 |
| $\text{Ts}_{44}^{(1)}$ | 175 | 175.9 | 189 | 201 | 200 | 210 |
| $\text{Ts}_{66}^{(1)}$ | -119 | -128.9 | -133.5 | -138 | -134 | -145 |
| $\text{Ts}_{14}^{(1)}$ | 113 | _____ | 123 | 139.5 | 125 | 134 |

TABLE 12
VALUES FOR THE SECOND-ORDER TEMPERATURE COEFFICIENTS OF THE
COMPLIANCES FOR ALPHA-QUARTZ IN $10^{-9}/(^{\circ}\text{C})^2$

| | AND BLECHSCHMIDT 1940 | MASON [22] 1951 | RESEARCH REPORT No. 13524 1951 | NEW VALUES 1961 |
|--------------------|-----------------------------|--------------------|--------------------------------------|--------------------|
| | | | | |
| $T_{s_{11}}^{(2)}$ | | 58.5 | | 85.3 |
| $T_{s_{33}}^{(2)}$ | | 144 | | 247 |
| $T_{s_{12}}^{(2)}$ | | -575 | | -1385 |
| $T_{s_{13}}^{(2)}$ | | -2110 | | -718 |
| $T_{s_{44}}^{(2)}$ | 298 | 200 | 272 | 262 |
| $T_{s_{66}}^{(2)}$ | -118 | -18 | -83 | -85 |
| $T_{s_{14}}^{(2)}$ | | 40 | 83 | 93 |

TABLE 13

| | | |
|--|--|--|
| | | |
| | | |
| | | |

TABLE 14
MEASURED AND CALCULATED VALUES FOR FREQUENCY CONSTANTS N AND
FIRST-, SECOND-, AND THIRD-ORDER TEMPERATURE COEFFICIENTS
FOR THE CT CUT VIBRATING IN CONTOUR MODE

| | | OBSERVED | | | CALCULATED | | |
|-------------------------------|--|----------------------|-----------------------|----------------|----------------------|-----------------------|----------------|
| | | $Y_{\theta 0^\circ}$ | $Y_{\theta 45^\circ}$ | $Y_{\theta 0}$ | $Y_{\theta 0^\circ}$ | $Y_{\theta 45^\circ}$ | $Y_{\theta 0}$ |
| (yx1) ϑ Angle | θ | 37°40' | | 36°15' | | 36°20' | |
| kc-mm | N | 3087 | 3583 | 3766 | 3145 | 3594 | 3793 |
| $10^{-6}/^\circ\text{C}$ | a | | | | | | |
| $10^{-9}/(^\circ\text{C})^2$ | b | | 0 | | | 0 | |
| $10^{-12}/(^\circ\text{C})^3$ | c | | -58 | | | -58 | |
| | | | -151 | | | -161 | |
| $10^{-6}/^\circ\text{C}$ | $\frac{\partial a}{\partial \theta}$ | | | | | 5.28 | |
| $10^{-9}/(^\circ\text{C})^2$ | $\frac{\partial b}{\partial \theta}$ | | | | | 4.75 | |
| $10^{-12}/(^\circ\text{C})^3$ | $\frac{\partial c}{\partial \theta}$ | | | | | 2.10 | |
| $^\circ\text{C}$ | T_m | | 25° | | | 25° | |
| $^\circ\text{C}$ | $\frac{\partial T_m}{\partial \theta}$ | | 45° | | | See Fig. 36 | |

TABLE 15
MEASURED AND CALCULATED VALUES FOR FREQUENCY CONSTANTS N AND
FIRST-, SECOND-, AND THIRD-ORDER TEMPERATURE COEFFICIENTS
FOR THE DT CUT VIBRATING IN CONTOUR MODE

| | | OBSERVED | | CALCULATED | | |
|---------------------------------|--|----------------------------|----------------|----------------------|-----------------------|----------------|
| | | $Y_{\theta 0^\circ}$ | $Y_{\theta O}$ | $Y_{\theta 0^\circ}$ | $Y_{\theta 45^\circ}$ | $Y_{\theta O}$ |
| (yxl) θ Angle | θ | $\rightarrow 51^\circ 50'$ | | | | |
| | N | 2073 | 2341 | 2085 | 2356 | 2486 |
| | a | 0 | | 0 | | |
| $10^{-9}/(^{\circ}\text{C})^2$ | b | -17 | | -19.4 | | |
| $10^{-12}/(^{\circ}\text{C})^3$ | c | 52 | | 75.6 | | |
| $10^{-6}/^{\circ}\text{C}$ | $\frac{\partial a}{\partial \theta}$ | | | -2.30 | | |
| $10^{-9}/(^{\circ}\text{C})^2$ | $\frac{\partial b}{\partial \theta}$ | | | -2.01 | | |
| $10^{-12}/(^{\circ}\text{C})^3$ | $\frac{\partial c}{\partial \theta}$ | | | -1.06 | | |
| $^{\circ}\text{C}$ | T_m | 25 $^{\circ}$ | | 25 $^{\circ}$ | | |
| $^{\circ}\text{C}$ | $\frac{\partial T_m}{\partial \theta}$ | -75 $^{\circ}$ | | | | |

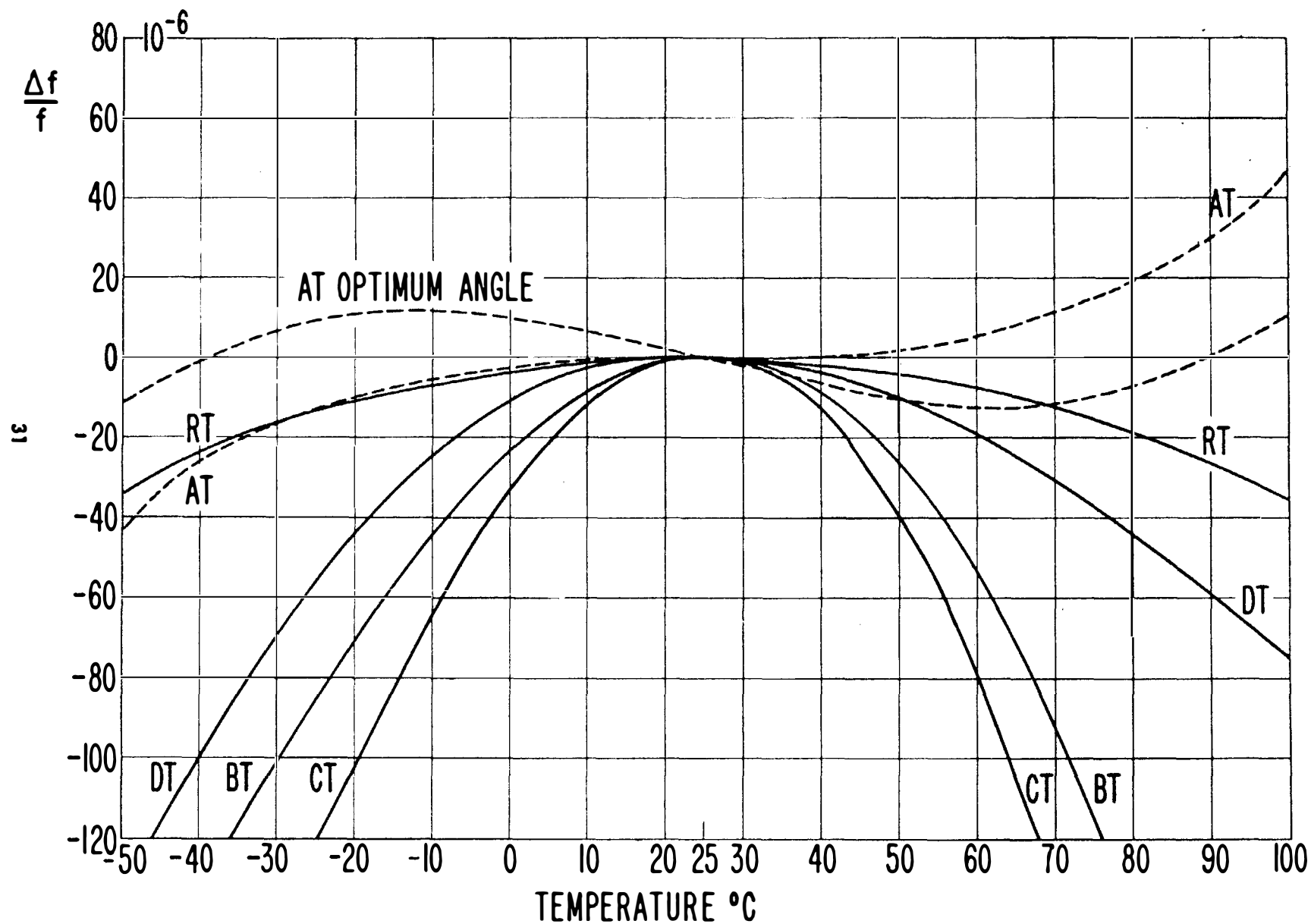


Fig. 1 Frequency temperature behavior of the AT, BT, CT, DT, and RT cuts.