APPENDIX 3

COMPLETE EXPRESSIONS FOR $\frac{\partial T_m}{\partial \theta}$ WHERE T_m IS THE MAXIMUM OR MINIMUM TEMPERATURE OF THE FREQUENCY-TEMPERATURE CHARACTERISTICS

The expression for the frequency-temperature variation of a particular mode of motion, as given in Equation (8) and Appendix 2, is used to calculate the temperatures T_μ where the frequency deviation becomes a maximum or minimum:

$$T_\mu - T_o = \Delta T = \frac{-b_o}{3c_o} \pm \sqrt{\left(\frac{b_o}{3c_o}\right)^2 - \frac{a_o}{3c_o}} , \qquad \text{(A3-1)}$$

where a_o, b_o, and c_o are the temperature coefficients of frequency in Equation (8a) taken at the temperature T_o. These coefficients are functions of the orientation of the plate and determined from Equations (9), (10), and (11).

The variation of T_μ with respect to the angle θ follows from Equation (A3-1). The sign chosen is the same as that in Equation (A3-1).

$$\pm\sqrt{b_o^2 - 3a_oc_o} \cdot \frac{\partial T_\mu}{\partial \theta} = -\frac{1}{2}\frac{\partial a_o}{\partial \theta} - \Delta T \cdot \frac{\partial b_o}{\partial \theta} + \left(\frac{a_o}{2c_o} + \frac{b_o \Delta T}{c_o}\right)\frac{\partial c_o}{\partial \theta} . \qquad \text{(A3-2)}$$

$$p \cdot b_\mu \cdot \frac{\partial T_\mu}{\partial \theta} = -\frac{a}{2}(\theta_a - \theta_c) - b\ \Delta T\ (\theta_b - \theta_c) , \qquad \text{(A3-3)}$$

where b_μ is the parabola constant b at T_μ, and

$$p = 1 + a_o\Delta T + b_o\ \Delta T^2 + c_o\ \Delta T^3, \text{ where } \Delta T \approx 1, \qquad \text{(See A2-5)}$$

further, $\theta_x = \frac{1}{x}\frac{\partial x}{\partial \theta}$ is is the first-order coefficient of the quantity x with respect to the angle θ. Equations (A3-2) and (A3-3) yield $\frac{\partial T}{\partial \theta}$ when ϕ is substituted for θ.