Part 4
Crystal oscillators and filters

9 Quartz crystal oscillators

9.1 INTRODUCTION

An electronic oscillator can be characterized as a device for producing ac power at a specific frequency from a dc power source. *Harmonic* oscillators are further characterized by the presence somewhere within the device, not necessarily at the output terminals, of a voltage or current with a nominally sinusoidal waveform. A simple model of a harmonic oscillator consists of a (linear) amplifier and a passive feedback network connected as in Fig. 9.1. When the output from the feedback network is fed back to the input of the amplifier with the correct amplitude and phase, then sustained oscillations can occur. If the amplifier has a voltage gain A and the feedback network has a voltage ratio β, then the necessary condition for sustained oscillation is $A\beta = 1$.

Both A and β will generally be complex functions of the angular frequency ω, so that the complex relation $A\beta = 1$ represents two real conditions. If A, β are written $|A|\exp(j\phi_A)$ and $|\beta|\exp(j\phi_\beta)$ so that ϕ_A and ϕ_β represent the phase shifts through the amplifier and feedback network respectively, then the two conditions for oscillation are

$$\phi_A + \phi_\beta = 2N\pi \tag{9.1}$$

$$|A||\beta| = 1 \tag{9.2}$$

Equation (9.1), with N an arbitrary integer, simply states that the total phase shift around the oscillator must be an integral multiple of 2π or 360°,

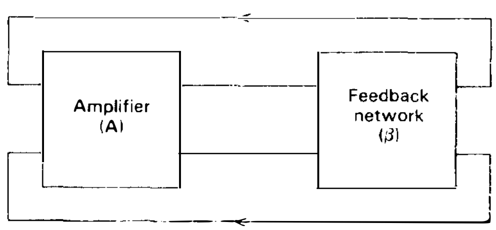

Fig. 9.1 Feedback oscillator schematic.

and Eqn (9.2) requires that the magnitude of the amplifier gain must be sufficient to compensate for the loss in the feedback network.

These conditions are those required for steady-state oscillations. When the amplifier is first turned on, then the only signal present will be white noise, mainly due to the active devices in the circuit. In this initial state, the conditions necessary for the build-up of oscillations at a particular frequency ω are that the phase condition Eqn (9.1) be satisfied, and that the loop gain $|A|\,|\beta|$ be *greater* than 1. In order for the amplitude of the oscillations not to increase without limit, it is then necessary to assume that the voltage gain A of the amplifier decreases with amplitude, so that at some finite signal level the gain condition Eqn (9.2) is satisfied.

These basic considerations are common to all types of harmonic oscillator, including crystal oscillators, and there are excellent texts available dealing with the elaborations necessary in specific cases. In particular, the recent books by Frerking (1978), Matthys (1983) and Parzen (1983) can be recommended for the details of crystal oscillator design. The review articles by Smith and Frerking in *Precision Frequency Control* (Gerber and Ballato, 1985) should also be consulted for detailed references, particularly in the context of applications requiring the highest stability. For present purposes, it is sufficient to consider an elementary small-signal model of a single transistor oscillator to illustrate the commonly used circuit configurations and their differences and similarities.

9.2 ELEMENTARY CIRCUITS

Figure 9.2 shows in schematic form a basic single transistor oscillator, with all power supply and biasing circuits omitted for clarity. Replacing the transistor by its small-signal hybrid-pi equivalent circuit leads to Fig. 9.3, and absorbing the passive equivalent circuit elements into the impedances Z_1, Z_2

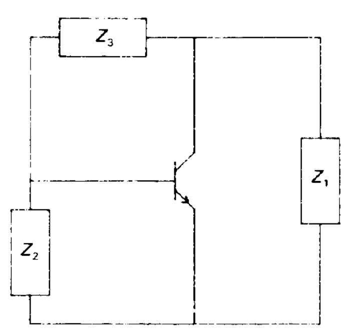

Fig. 9.2 Single transistor oscillator.

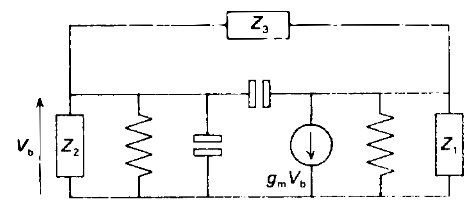

Fig. 9.3 Single transistor oscillator equivalent circuit.

and Z_3 finally leads to the circuit of Fig. 9.4. If I_1 and I_2 are the currents through Z_1 and Z_2, respectively, then by inspection

$$I_1 Z_1 - I_2(Z_2 + Z_3) = 0 \tag{9.3}$$

$$-g_m V_b = I_1 + I_2 \tag{9.4}$$

$$V_b = I_2 Z_2 \tag{9.5}$$

where g_m is the transconductance of the transistor and V_b the base-emitter signal voltage.

Substituting Eqn (9.5) into Eqn (9.4) leads to

$$I_1 + I_2(1 + g_m Z_2) = 0 \tag{9.6}$$

E(constitute a pair of simultaneous equations for the cu e condition for non-trivial solutions to exist is that th)efficients vanishes, that is, that

$$Z + g_m Z_1 Z_2 = 0 \tag{9.7}$$

where Z has been written for $Z_1 + Z_2 + Z_3$.

Equation (9.7) takes on a particularly simple form when the reactive parts of Z_1 and Z_2 are much larger than their resistive parts, that is when Z_1, Z_2 can be replaced by jX_1 and jX_2. Then writing Z_3 as $R_3 + jX_3$ and separating the real and imaginary parts of Eqn (9.7) gives

$$X_1 + X_2 + X_3 = 0 \tag{9.8}$$

$$g_m = R_3/X_1X_2 \tag{9.9}$$

Equation (9.8) determines the frequency of oscillation as the series resonance

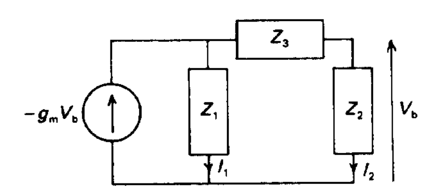

Fig. 9.4 Simplified equivalent circuit.

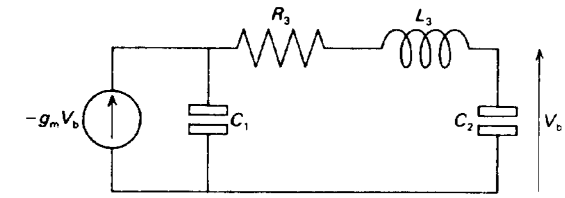

Fig. 9.5 Pierce type circuit.

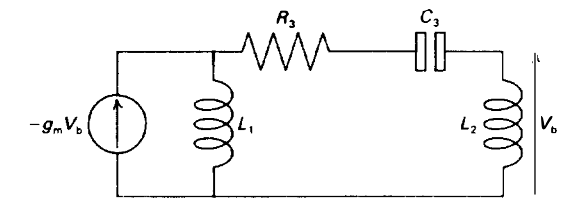

Fig. 9.6 Miller type circuit.

frequency of the loop Z_1, Z_2 and Z_3, and Eqn (9.9) determines the transconductance g_m required to maintain the oscillation. As expected, the larger the value of R_3, the larger is the required value of g_m.

It further follows from Eqn (9.9) that since g_m and R_3 are necessarily positive, so too is the product X_1X_2. Therefore X_1 and X_2 must both have the same sign, and it then follows from Eqn (9.8) that X_3 must have the opposite sign. Consequently, the circuit of Fig. 9.4 has the two possible realizations shown in Figs. 9.5 and 9.6, where it is to be understood that the L, C and R elements shown are to be interpreted as the effective values of what in real oscillators would be combinations of circuit elements. Both realizations have been utilized in crystal oscillators, with that of Fig. 9.5 leading to the commonly used *Pierce*, *Colpitts* and *Clapp* circuits, whereas that of Fig. 9.6 leads to the now rarely used *Miller* circuit, which will not be further considered. The Pierce, Colpitts and Clapp circuits, although sharing the same small-signal ac equivalent circuit, differ in the location of the ac ground point. Consequently, their actual physical realizations and the details of their design and performance differ markedly. In the Pierce circuit, the emitter is at ac ground, while in the Colpitts and Clapp the ground point is at the collector and base respectively. Figures 9.7, 9.8 and 9.9 show the elementary schematics for the three circuits.

The Pierce circuit is generally preferred to the Colpitts on the grounds of ease of design and superior frequency stability (Frerking, 1978). However, the fact that in the Colpitts one side of the crystal is grounded makes this circuit very popular where a number of crystals are to be switched in and out of the circuit.

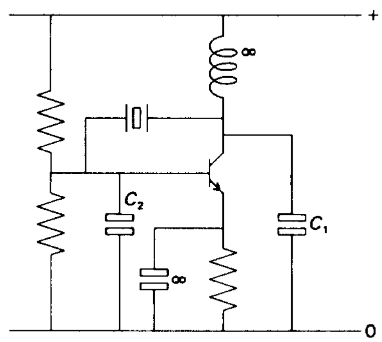

Fig. 9.7 Pierce os

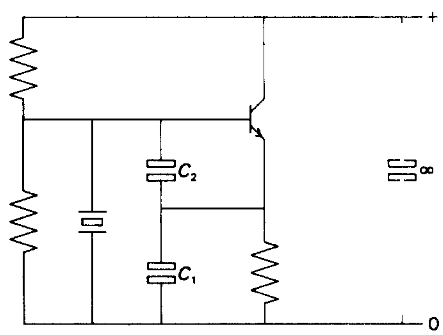

Fig. 9.8 Colpitts c

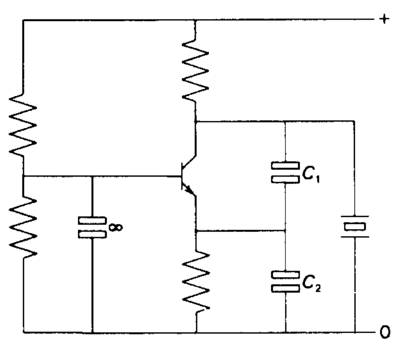

Fig. 9.9 Clapp oscillator.

9.3 FREQUENCY STABILITY: CIRCUIT CONSIDERATIONS

The primary characteristic of crystal oscillators as compared to other types of electronic oscillator is their superior frequency stability. This results in the first instance from the characteristics of the crystal itself, but nevertheless the sustaining circuit must be carefully designed in order not to degrade the performance of the crystal. This section outlines some of the relevant considerations in the choice of the circuit elements, and the following section covers the key characteristics of the crystal itself.

If the transistor transconductance g_m is assumed to be real and the impedances Z, Z_1, Z_2 and Z_3 are written $R + jX$, $R_1 + jX_1$, etc., then the fundamental condition for oscillation Eqn (9.7) can be separated into its real and imaginary parts

$$\left.\begin{aligned} R + g_m(R_1R_2 - X_1X_2) &= 0 \\ X + g_m(X_1R_2 + X_2R_1) &= 0 \end{aligned}\right\} \quad (9.10)$$

Eliminating g_m leads to the relation

$$X/R = (X_1R_2 + X_2R_1)/(R_1R_2 - X_1X_2) \quad (9.11)$$

Equation (9.11) can be regarded as the determinant of the frequency of oscillation, through the frequency dependence of the various reactances and resistances appearing in the equation. For crystal oscillators of the Pierce family, the crystal impedance is contained in the terms X and R on the left side. Consequently, *in the neighbourhood of the crystal's resonance frequency*, the left side of Eqn (9.11) is strongly dependent on frequency, whereas the right side is to a good approximation independent of frequency. Writing $F(\omega)$ for X/R and F_0 for the right side of Eqn (9.11), then leads to the frequency equation

$$F(\omega) = F_0 \quad (9.12)$$

In the Pierce case, the reactances X_1 and X_2 are both negative, so that assuming R_1 and R_2 to be comparatively small the quantity F_0 is small and positive. By definition, F is the ratio of X to R, and is therefore the tangent of the phase angle ϕ of the loop impedance Z, which is in turn made up of the crystal impedance Z_3 in series with the capacitive impedances Z_1 and Z_2. Consequently Z consists of the crystal in series with an effective load capacitance of reactance $X_1 + X_2$, damped by the resistive components R_1 and R_2. Equation (9.12) then demands that the circuit oscillate at a frequency such that the phase ϕ has a small positive value.

Supposing now that some change in the circuit elements of the oscillator (other than the crystal itself) causes a small variation in the equivalent impedances Z_1 and Z_2 and consequently a small change dF_0 in F_0, then it is evident from Eqn (9.12) that for oscillations to be maintained there must be a corresponding change dF in $F(\omega)$. If the accompanying change in frequency is $d\omega$, then $dF = (dF/d\omega)d\omega$ and

$$d\omega = dF_0/(dF/d\omega) \quad (9.13)$$

Clearly then, to minimize the frequency changes due to incidental changes in the oscillator circuit, the quantity ($dF/d\omega$) must be maximized.

As shown in Section 6.4, a crystal with a series load capacitor is equivalent to a crystal with no load capacitor but with modified parameter values. Also, for a crystal with a good figure of merit M (Section 6.3.1), a resistance in series can be regarded as effectively adding to the motional arm resistance of the crystal. The loop impedance Z is then just equivalent to that of a crystal with parameters given in terms of the actual crystal impedance Z_3 and the impedance Z_1 and Z_2 by

$$\begin{aligned} C'_0 &= C_0 C_L/(C_0 + C_L) \\ C'_m &= C_m/K^2 \\ R'_m &= K^2 R_m + R_1 + R_2 \\ Q' &= QK^2 R_m/(K^2 R_m + R_1 + R_2) \end{aligned} \tag{9.14}$$

where C_L is the effective load capacitance; C_0, C_m and R_m are, respectively, the shunt and motional capacitances and the motional resistance of the crystal; and the primed parameters are the effective values in the circuit. The multiplier K is given by

$$K = 1 + C_0/C_L$$

and C_L is

$$C_L = -1/\{\omega(X_1 + X_2)\}$$

with ω the nominal oscillation frequency.

As shown in Section 6.3.3, the phase angle ϕ of the admittance of a crystal unit is given in terms of the figure of merit M and the normalized frequency variable x by

$$\tan(\phi) = (1 + x^2M^2 - xM^2)/M$$

and its rate of change with frequency by

$$d(\tan(\phi))/d\omega = 2(Q/\omega_s)(2x - 1)$$

In the present case, the phase angle of the loop *impedance* Z is therefore given by

$$F = \tan(\phi) = -(1 + x^2M^2 - xM^2)/M \tag{9.15}$$

and the rate of change with frequency by

$$dF/d\omega = -2(Q/\omega_s)(2x - 1) \tag{9.16}$$

where in Eqns (9.15) and (9.16) the parameters M, Q and x are all appropriate to the effective crystal parameters defined in Eqn (9.14). In particular, the Q factor appearing in Eqn (9.16) is the effective Q defined in the fourth of Eqns (9.14).

Now the condition for oscillation, Eqn (9.12), implies that F be small and positive. From the discussion in Section 6.3.2, it then follows that the normalized frequency x must be in the vicinity of one of the zero phase frequencies $x = M^{-2}$ or $x = 1 - M^{-2}$. Since in the latter case the effective crystal resistance is much higher than in the former (Section 6.3.7), it follows that x must in fact have a value slightly above M^{-2}, so that F is approximately $(xM^2 - 1)/M$. Then to maximize the magnitude of $dF/d\omega$ in Eqn (9.16), it is clear that on the one hand the effective Q must be as large as possible, but on the other hand x must be kept as close as possible to the limiting value of M^{-2}. The latter condition is equivalent to the condition that the value of F and hence of F_0 be minimized.

For a given crystal Q, it follows from the fourth of Eqns (9.14) that to prevent the effective Q from being significantly degraded by the circuit, the resistive components R_1 and R_2 of the impedances Z_1 and Z_2 must be kept very much less than the effective motional resistance of the crystal K^2R_m. Assuming that R_1 and R_2 have been reduced to a minimum, the second condition for maximizing the frequency stability implies that the reactive components X_1 and X_2 have to be chosen so that F_0 is minimized. Noting that the total reactance $X_1 + X_2$ is effectively fixed by the need to operate the crystal on a specified load reactance, the minimization of F_0 has to be subject to this constraint, so that any change dX_1 in X_1 has to be accompanied by an equal and opposite change $dX_2 = -dX_1$ in X_2. For small R_1 and R_2, F_0 reduces to

$$F_0 = -R_2/X_2 - R_1/X_1$$

so that

$$\begin{aligned} dF_0/dX_1 &= (R_2/X_2^2)(dX_2/dX_1) + R_1/X_1^2 \\ &= R_1/X_1^2 - R_2/X_2^2 \end{aligned} \tag{9.17}$$

The extreme value of F_0 occurs when dF_0/dX_1 vanishes, that is when

$$(X_1/X_2)^2 = R_1/R_2 \tag{9.18}$$

If as is frequently more convenient the impedance Z_1, Z_2 are expressed in terms of the equivalent admittances $G_1 + jB_1$ and $G_2 + jB_2$, then this expression can be rewritten as

$$(B_1/B_2)^2 = G_1/G_2 \tag{9.19}$$

Either of Eqns (9.18) or (9.19) then allow the optimum choice to be made for the ratio of the reactive components of Z_1 and Z_2.

9.4 FREQUENCY STABILITY: CRYSTAL CHARACTERISTICS

As shown in Section 9.3, an appropriate choice of the circuit elements in a crystal oscillator can minimize the effect of slight changes in the elements on the output frequency of the unit. On the other hand, slight changes in the crystal frequency itself will be reflected directly in changes in the output frequency. Such instabilities can be classified under three main headings: the *short-term* frequency stability, the *long-term* frequency stability or *ageing*, and the *temperature* stability.

Short-term stability refers to the variation in output frequency of the oscillator observed in successive measurements. The usual measure of this quantity is the standard deviation of a number of frequency readings, each reading being effectively the average frequency over a period of time known as the *sampling interval* τ. The standard deviation is then found to depend on the length of the sampling interval, as shown schematically in Fig. 9.10. The different regions of the typical curve shown can be associated with different statistical noise processes, but the further association of these processes with actual physical mechanisms in the resonator or the sustaining circuit is still a subject of active research (Gerber and Ballato, 1985). There appears to be a confirmed correlation between resonator Q and that component of the frequency instability usually described as '1/f' or 'flicker' noise, but other than this only the general statement that short-term stability appears to depend on such process-dependent factors as surface finish, cleanliness, and electrode adhesion can be made. Since these factors also influence such non-linear phenomena as the dependence of crystal frequency and resistance upon drive level, it is sometimes possible to correlate these effects with observed short-term frequency stabilities and use them to select crystals that will have improved short-term behaviour. It is also empirically established that where short-term stability is of importance, the crystal unit should be operated at a relatively high drive level. Unfortunately, this conflicts with the need to operate at very low levels of drive to achieve the best possible long-term ageing behaviour, discussed next.

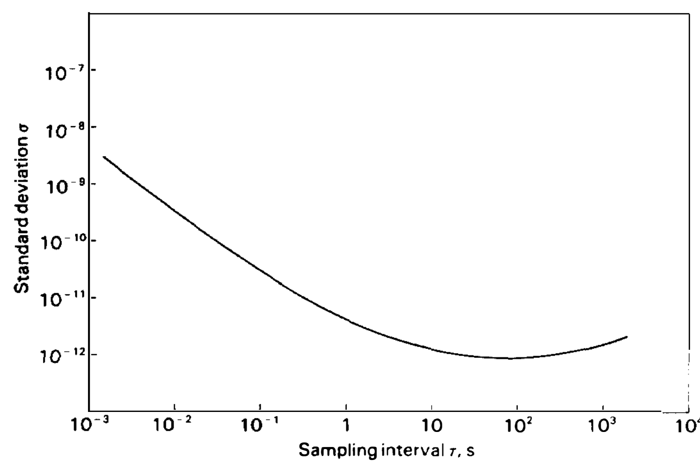

Fig. 9.10 Short term frequency stability.

The long-term stability of oscillators or crystal units refers to their frequency drift over periods of months or years, and is usually expressed as ppm per year or parts in 10^{-8} per day. In a properly designed oscillator, where the effects of component variations have been minimized, the long-term stability is almost entirely that of the crystal unit, which has been discussed in Chapter 7.

As in the case of the long-term stability, in a properly designed oscillator the frequency–temperature stability will be that of the component crystal. Exceptions to this rule would occur if, for example, inductors were used to extend the frequency adjustment range of the oscillator without due attention to their temperature coefficients, or if the crystal were operated at a load capacitance much different from the specified nominal value. The frequency–temperature characteristics of the AT-cut resonator family are discussed in Chapter 7, and the techniques of temperature compensation and control, used to obtain oscillators with superior temperature stabilities to those that can be obtained from the crystal alone, are briefly discussed in the following section.

9.5 TYPES OF CRYSTAL OSCILLATOR

Crystal oscillators are available for a wide range of applications, with stabilities ranging from 0.1% for simple timing functions, as in microprocessor clocks, to 5×10^{-11} for secondary frequency standards or reference frequency sources for satellite navigation systems. Prices correspondingly range from a few pounds to many thousands of pounds for the most sophisticated devices.

Despite the great variety of available oscillator types, it is useful to categorize crystal oscillators into four main groups. These are *simple packaged oscillators, temperature compensated oscillators, oven controlled oscillators*, and *voltage controlled oscillators*. These groupings overlap, for instance in such cases as voltage controlled temperature compensated crystal oscillators, and take no account of the differences that may exist between oscillators in the same group in respect of such factors as output waveform and level, power supply requirements, provision of auxiliary outputs at multiples or sub-multiples of the nominal frequency, and so on. Nevertheless the classification is useful for purposes of description, and follows that of the IEC Publication 679-2, *Quartz crystal controlled oscillators, Part 2: Guide to the use of quartz crystal controlled oscillators* (IEC, 1981).

9.5.1 Simple packaged crystal oscillators (SPXOs)

SPXOs are crystal oscillators in which no provision is made either for temperature compensation or temperature control, so that the frequency–temperature stability is essentially that of the crystal itself. The AT-cut crystal is generally adopted for use in such oscillators, with frequency division being used to obtain output frequencies below the practical AT manufacturing range. As discussed in Chapter 7, the characteristics of the AT-cut are such that good frequency stability can be achieved over wide temperature ranges, with ± 10 ppm over a temperature range of −20 to +70°C being typical. An exception to this general rule is the recent announcement (Ochiai *et al.*, 1986) of packaged oscillators employing the miniature GT cut crystal described in Section 1.4.6. These units are claimed to have stabilities of ± 2.5 ppm over a temperature range of − 30 to + 70°C, but are so far restricted to a frequency range of 1 to 3 MHz.

By far the most common application of SPXOs is in providing timing signals in digital electronic equipment. Such oscillators are termed *crystal clock oscillators* and are produced in high volume in standard dual-in-line packages, with outputs tailored to be directly compatible with TTL or CMOS logic circuits. Clock oscillators are produced in the freuency range up to 70 MHz, with some recent devices extending the upper limit to 100 MHz by using fifth overtone crystal units (Oita, 1986). The manufacture of clock oscillators makes full use of hybrid circuit techniques. The active components in the maintaining circuit are integrated on a single IC chip and the crystal blank is mounted on pillars on the substrate to allow direct adjusment of the crystal (and hence the oscillator) frequency. The need for a separate package for the crystal is eliminated by using resistance-welding techniques to provide a hermetically sealed enclosure for the complete oscillator assembly. Typical stabilities for this type of clock oscillator are ± 50 or ± 100 ppm over a temperature range of 0 to 70°C.

9.5.2 Temperature compensated crystal oscillators (TCXOs)

The AT-cut quartz resonator provides excellent frequency stability with respect to temperature variations as compared to most alternatives. For example, over an operating temperature range of − 10° to + 60°C, a stability of ± 5 ppm can be routinely achieved. With additional care and some penalty in cost, the temperature range can be marginally extended or, alternatively, the stability marginally improved. To obtain further significant improvements, however, necessitates a different approach. For example, to obtain a stability of ± 5 ppm over the temperature range − 30° to + 60°C is not practically possible with an unaided AT-cut crystal, even though it is theoretically feasible. There are two main approaches to the problem of improving upon the stabilities available from an unaided crystal. In the first, to be considered in the following section, the crystal and possibly the oscillator circuit itself are placed in a temperature-controlled environment provided by some sort of oven. In the second, temperature sensitive elements are introduced into the oscillator circuit in such a way that the variation in crystal frequency with temperature is counterbalanced by variations in the effective load reactance of the crystal.

The load reactance is usually a capacitor with a value in the region of 20 pF. 'Active' compensation techniques utilize a varactor diode to provide the load capacitor. Variation of the bias voltage applied to the varactor results in a variation of load capacitance and hence a corresponding variation in the crystal frequency. 'Passive' compensation techniques realize the crystal load capacitance as a network of capacitors, resistors and thermistors whose reactive component displays the desired temperature behaviour.

The essence of the active method of compensation is to tailor the varactor bias circuit so that the bias voltage produced as a function of temperature is precisely that required to produce frequency changes in the crystal equal and opposite to those induced by the temperature variations themselves. Active compensation techniques can be further classified as analog or digital, depending on the method adopted to generate the required bias voltage. In the classical, analog, approach the bias circuit is made up of a network of resistors and temperature-dependent elements. The latter are most commonly thermistors, but other approaches have been tried. In the digital approach, the bias voltages required at several discrete points across the temperature range are stored in a PROM. The signal from a suitable temperature-sensing device is then converted to digital form and used to address the PROM, with the bias voltage then being applied to the varactor diode as in the analog case.

It is generally the case that the active techniques provide better performance, but at some cost in terms of additional power consumed in the bias circuit and in additional components. Using analog techniques, about the best that can be achieved is a stability of 0.5 ppm over a temperature range of -30 to $+80°C$, with the limiting factors being the ageing of the crystal and also the presence of perturbations in the frequency–temperature characteristic of the crystal. As usual, there is a 'trade-off' between temperature range and achievable stability. With digital techniques, there is in principle no limit to the degree of compensation possible aside from the ageing of the crystal and the repeatability of its frequency–temperature characteristic, but in practice there are limitations owing to the finite precision of the digital devices used. Stabilities of better than 0.1 ppm over wide temperature ranges have been reported (Frerking, 1978). However, it is presently the case that digitally compensated TCXOs remain very much more expensive than their analog counterparts, and are much less readily available.

9.5.3 Oven controlled crystal oscillators (OCXOs)

TXCOs provide an excellent solution to the problem of providing a medium to high precision frequency source in a wide range of applications, particularly those where constraints on size and power consumption effectively rule out the use of oven controlled oscillators. A typical application would be as a reference frequency source in a synthesized portable or mobile radiotelephone. Nevertheless, for the highest precision requirements, the use of an oven to provide a stable operating temperature remains mandatory, despite the disadvantages mentioned of increased size and power consumption.

The ovens used in OCXOs can generally be classified as either 'on/off' or 'proportionally controlled' types. In the former, a sensor, such as a bimetallic strip or a mercury-contact thermometer, is used to detect changes in the oven temperature and switch the oven heater on or off as required. This has the advantage of simplicity, but on the other hand leads to relatively large variations of the oven temperature between the on and off cycles, and additionally the set point temperature is liable to drift over long periods of time. Consequently for precision oscillators, the second type of oven control is preferred, in which the oven temperature is continuously monitored and the heater current adjusted according to the difference between the actual oven temperature and the nominal set point temperature. This is usually achieved by using a temperature-sensitive resistance in a bridge circuit and using the output from the bridge to control the heater current.

For the very highest precision, elaborate double ovens are used, with the crystal and essential parts of the oscillator circuit contained within the inner oven, and the oven control and power supply circuits contained in the outer oven. This allows at least an order of magnitude improvement in the temperature stability of the inner oven to be achieved as compared to a single oven. Typical values are 0.5 to 0.01 °C stability for a single oven, and 0.001 °C for a double oven, when operated over wide ambient temperature ranges (cf. Frerking in Gerber and Ballato, 1985).

To take full advantage of these temperature stabilities, the oven operating temperature has to be carefully matched to the turnover temperature of the crystal employed. As already discussed in Section 2.6, it is in this area that the newer doubly rotated crystal cuts have been found to have significant advantages over the AT-cut. Firstly, the slope of the frequency–temperature characteristic in the neighbourhood of the turnover point is smaller, making the error resulting from a mismatch of oven temperature and turnover point less critical, but more importantly the transient frequency changes resulting from the temperature cycling of the oven are also much reduced in the doubly rotated cuts.

9.5.4 Voltage controlled crystal oscillators (VCXOs)

VCXOs are oscillators whose output frequency is capable of being controlled by an external voltage. Typically, such oscillators would be used in phase-locked loop applications, or to allow frequency modulation of the output signal. The frequency variation is usually obtained by varying the bias voltage on a varactor diode in series with the crystal, just as in a TCXO, but with the differences that the control voltage has to be supplied by the user, and usually a rather larger range of adjustment is required than in a TCXO.

This second factor, together with the need to achieve a linear relationship betweeen control voltage and output frequency, implies that often more sophisticated networks than a simple connection of the varactor in series with the crystal are required. The simple formula for the shift in crystal frequency with load capacitance (Eqn 6.2) shows the inherent non-linearity involved with large changes in load capacitance value. Fortunately the non-linearity of the voltage–capacitance characteristic of typical varactor diodes tends to compensate for the non-linearity of the crystal 'pulling' characteristic, so that the overall linearity of the voltage–frequency characteristic is considerably better than that of either the crystal or the varactor considered separately. These points, together with the advantages and disadvantages of using inductors to increase the range of frequency adjustment are discussed in detail by Neubig (1979). In any case, however, it should be noted that in increasing the adjustment range of the output frequency, whether by introducing inductors or by increasing the 'pullability' of the crystal itself, the inherent stability of the oscillator itself is correspondingly decreased.