11

The Butler Oscillator

11.1 INTRODUCTION

The Butler oscillator, also called the Bridged-T Oscillator, is the most popular member of a family of oscillators wherein the emitter current is the crystal (or other resonator) current. The emitter current is therefore sinusoidal, and the output is relatively free of harmonic content. Also, the crystal (or other resonator) acts as a filter to reduce the noise outside the effective bandwidth of the crystals, f/Q_{op}.

The reason for the popularity of this oscillator is not the noise characteristic mentioned above, but the fact that it can function at very high frequencies because the transistor is used in its common base configuration, which, as pointed out in Section 2.5, is particularly desirable for high-frequency operation.

By the Butler oscillator is meant the oscillator, whose circuit is shown in Fig. 11.1. All the components in that circuit are physical; that is, they are all installed components.

In comparing Fig. 11.1 to Figs. 5.5*b* and 5.5*c*, one is struck by their similarity. They are identical except that the circuit of Fig. 11.1 has a crystal in series with the emitter. The principal effect of the crystal is to vary the magnitude and phase of the g_m of the transistor, as the frequency varies, by emitter local degeneration action.

One would therefore expect that the analysis of Fig. 11.1 would be similar to that of Figs. 5.5*b* and 5.5*c*. However, adapting the analysis of these circuits to the circuit of Fig. 11.1 turns out to be very tedious and yields little insight into the circuit operation. Therefore, the analysis of Section 11.2 has been developed.

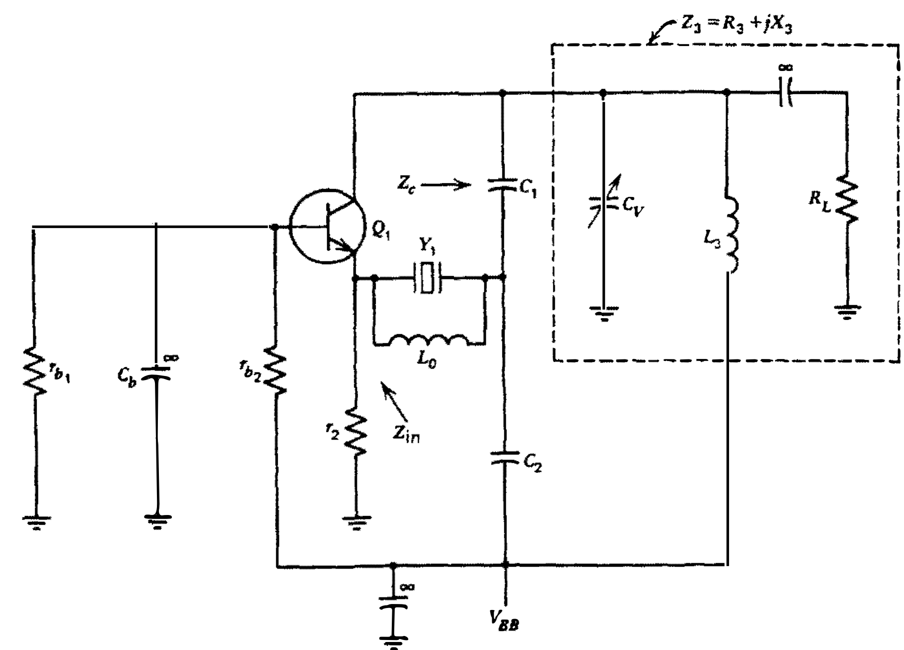


Figure 11.1 Butler oscillator.

11.2 OSCILLATOR CIRCUIT ANALYSIS

11.2.1 Introduction

One could endeavor to use two-port network theory for the oscillator, but because of the large signals existing within the oscillator, the results will not be very useful and will be very complicated.

Instead, it has been found to be more practical to use the feedback and negative resistance oscillator models combined with an approximate hybrid-PI network model for the transistor.

11.2.2 Transistor Approximations

In order to arrive at a reasonably simple oscillator treatment, it is necessary to make many approximations as to the transistor behavior, beginning with the model of Fig. 2.14 and the large-signal characteristics of Section 6.3.

Obviously these approximations are valid only for high-performance transistors of the type referenced in Section 2.5.1. Some of the approximations are questionable even for these high-performance transistors, but they must be made in order to produce a relatively simple analytical model. These approximations will cause the resultant model to be only approximate, but experience has shown it to be useful, and deviations from the desired performance can be corrected during the trimming procedure.

11.2.2.1 Assumptions and Approximations for Fig. 2.14

1 $\alpha = -1.$ (11.1)
To ensure that this assumption is valid, it is required that

$$f_T > 4fI_E \tag{11.1a}$$

2 $r_{bb'}$ is assumed zero.
3 $r_{ce} \to \infty$ (11.2)
4 C_{bet} and C_{bed} can be neglected, as it is in shunt with r_e, which is small.
5 C_{cb} can be lumped into the Z_3 network of Fig. 11.1.

11.2.2.2 Assumptions for Section 6.3

It is assumed that the material in this section is valid at high frequencies up to 200 MHz. Again, this assumption is somewhat questionable, but very useful.

11.2.3 Derivation of the Oscillatory Conditions

Figure 11.2 is the ac equivalent circuit of Fig. 11.1, but in a more general form, since it does not restrict the makeup of Z_1, Z_2, and Z_3 as is done in Fig. 11.1. The restrictions will be applied in conformance with the requirements of the oscillatory equations.

In the following analysis r_2 is neglected as being much larger than the emitter input impedance, so that Z_{IN} is the emitter input impedance.

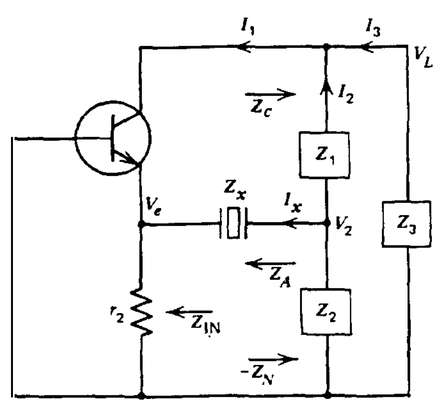


Figure 11.2 Schematic of ac circuitry of the Butler oscillator.

Let

$$Z_A = Z_{IN} + Z_x = R_A + jX_A \tag{11.3}$$

where

$$R_A = R_{IN} + R_x, \qquad X_A = X_{IN} + X_x \tag{11.3a}$$

By inspection,

$$I_2 = I_1 \frac{Z_3}{Z_3 + Z_1 + Z_2 Z_A/(Z_2 + Z_A)} \tag{11.4}$$

$$I_x = \frac{-I_2}{Z_2 + Z_A} Z_2 \tag{11.5}$$

$$= \frac{-I_1 Z_3}{Z_3 + Z_1 + Z_2 Z_A/(Z_2 + Z_A)} \cdot \frac{Z_2}{Z_2 + Z_A} \qquad \text{from Eq. (11.4)} \tag{11.5a}$$

$$= \frac{-I_1 Z_3 Z_2}{(Z_3 + Z_1)(Z_2 + Z_A) + Z_2 Z_A} \tag{11.5b}$$

but at equilibrium oscillation,

$$I_1 = \alpha I_x$$

$$= -I_x \qquad \text{from Eq. (11.1)}$$

Therefore, from Eqs. (11.5b) and (2.82)

$$(Z_3 + Z_1)(Z_2 + Z_A) + Z_2 Z_A = Z_3 Z_2 \tag{11.6}$$

or

$$Z_A(Z_1 + Z_2 + Z_3) = -Z_1 Z_2 \tag{11.7}$$

Let

$$Z_1 + Z_2 + Z_3 = Z_s = R_s + jX_s \tag{11.8}$$

then Eq. (11.7) becomes

$$Z_A Z_s = -Z_1 Z_2 \tag{11.9}$$

solving for Z_A

$$Z_A = \frac{-Z_1 Z_2}{Z_s} = \frac{-Z_1 Z_2}{R_s + jX_s} \tag{11.10}$$

If Z_1 and Z_2 are specified to be $-jX_1$ and $-jX_2$, respectively, as in Fig. 11.1, Eq. (11.10) becomes

$$Z_A = \frac{-(-X_1 X_2)}{R_s + jX_s} \tag{11.11}$$

It should be noted that Eq. (11.11) will not change if Z_1 and Z_2 are both made loss-free inductors. The necessary and sufficient condition is that both X_1 and X_2 have the same sign. If Z_1 and Z_2 are lossy inductors, the equation will remain essentially the same, but will be slightly more complicated.

Equation (11.11) is now separated into its real and imaginary parts,

$$R_A = \overbrace{\frac{-(-X_1 X_2)}{R_s^2 + X_s^2} R_s}^{-R_N} \tag{11.12}$$

$$X_A = \underbrace{\frac{-X_1 X_2}{R_s^2 + X_s^2}}_{-X_N} X_s \tag{11.13}$$

Equation (11.12) contains the amplitude-determining information, Eq. (11.13) contains the frequency-determining information.

For convenience in analysis, the right-hand term in Eq. (11.12) is designated as $-R_N$. Similarly, the right-hand term of Eq. (11.13) is designated as $-X_N$. Together, they form $Z_N = R_N + jX_N$. At equilibrium, Z_A is constrained to be equal to $-Z_N$.

Equations (11.12) and (11.13) are called the *oscillatory equations* or the *equations for oscillation* and much can be learned from the study of these equations.

Also, for the case where Z_1 and Z_2 are loss-free capacitors, from Eqs. (11.8), (11.12), and (11.13),

$$R_s = R_3 \tag{11.14}$$

$$X_s = X_3 - (X_1 + X_2) \tag{11.15}$$

R_N will be recognized as the negative resistance R_g in the negative resistance oscillator model discussed in Section 1.3.1. It is seen, from that section, that oscillation cannot exist when $R_A > R_N$. The oscillation amplitude increases as long as $R_A < -R_N$. The oscillation is at amplitude equilibrium when $R_A = -R_N$.

An important consequence of the foregoing is that, since X_s, R_s, X_1, and X_2 are quantities independent of the amplitude, R_A must increase as the amplitude increases in order to attain amplitude equilibrium.

Frequency Stability Relationships

From Eq. (11.3a)

$$X_A = X_{\mathrm{IN}} + X_x$$

If X_{IN} is assumed constant, then

$$\partial X_A = \partial X_x$$

which is a measure of the frequency stability of the oscillator since ∂f is almost proportional to ∂X_x.

For any particular independent parameter, U, it is desirable to make $\partial X_A / \partial U \to 0$ to achieve minimum frequency change with variation of that parameter.

11.2.4 Relationships between Z_s and the Physical Components of Fig. 11.1

L_3 and C_V together are equivalent to a variable inductor having a reactance

$$X_L = \frac{2\pi f L_3}{1 - (2\pi f)^2 \times 10^{-6} L_3 C_v} \tag{11.16}$$

where C_V is designed so that

$$(2\pi f)^2 \times 10^{-6} L_3 C_v < 1 \tag{11.16a}$$

It is interesting to note from Eq. (11.16) that X_L increases as C_V increases. As shown in Fig. 11.1, Z_3 consists of X_L and R_L in parallel: or

$$Z_3 = \frac{R_L}{1 + (R_L/X_L)^2} + j\frac{X_L}{1 + (X_L/R_L)^2} \tag{11.17}$$

and from Eq. (11.8)

$$Z_s = \underbrace{\frac{R_L}{1 + (R_L/X_L)^2}}_{R_s} + \underbrace{j\left[\frac{X_L}{1 + (X_L/R_L)^2} - (X_1 + X_2)\right]}_{X_s} \tag{11.18}$$

or

$$Z_s \approx \overbrace{\frac{X_L^2}{R_L}}^{R_s} + j\overbrace{[X_L - (X_1 + X_2)]}^{X_s} \tag{11.18a}$$

when $R_L \gtrapprox 5X_L$.

It should be noted that Eq. (11.18a) introduces only a small error in R_s but may introduce a fairly substantial error in X_s since X_s, which is a relatively small quantity, is the difference of two large quantities.

For any given oscillator, X_1, X_2, and R_L are constants. X_L depends upon the tuning of C_V. Therefore, R_s and X_s are not independent quantities and for every value of X_s there is a corresponding value of R_s as determined by Eq. (11.18).

11.2.5 Calculation of Z_c, V_L, V_e, V_2, P_T, P_A, and P_x

From Fig. 11.2

$$Z_c = \frac{(Z_1 + Z_2Z_A/(Z_2 + Z_A))Z_3}{Z_3 + Z_1 + Z_2Z_A/(Z_2 + Z_A)}$$

$$= \frac{Z_1Z_2 + (Z_1 + Z_2)Z_A}{Z_2}$$

from Eq. (11.7) and

$$Z_c = \frac{Z_A}{Z_2}(Z_1 + Z_2 - Z_s)$$

from Eq. (11.10) or

$$Z_c = \frac{-Z_AZ_3}{Z_2} \tag{11.19}$$

from Eq. (11.8) and

$$V_L = -I_1Z_c = -I_1\left(\frac{-Z_AZ_3}{Z_2}\right)$$

$$= -I_x\frac{Z_AZ_3}{Z_2} \tag{11.20}$$

from Eqs. (2.83) and (11.19).

$$V_e = I_x Z_{IN} \tag{11.21}$$

$$V_2 = I_x Z_A = -V_L \frac{Z_2}{Z_3} \tag{11.22}$$

Let P_T be the total power dissipated

$$P_T = P_L + P_A \tag{11.23}$$

Then

$$10^3 P_T = I_x^2 \mathrm{Re}\left(\frac{-Z_A Z_3}{Z_2}\right) \tag{11.24}$$

$$P_L = P_T - P_A$$

$$= P_T - I_x^2 R_A \times 10^{-3} \tag{11.25}$$

11.2.6 Relationships for the Special Case Where $X_A = 0$, $Z_1 = -jX_1$, and $Z_2 = -jX_2$

In this case the crystal is operating at series resonance if Z_{IN} is real; if not, $X_x = -X_{IN}$. Also $Z_A = R_A$.

From Eqs. (11.11) and (11.18)

$$X_s = 0 \tag{11.26}$$

at which point

$$X_L = (X_1 + X_2)\left[1 + \left(\frac{X_1 + X_2}{R_L}\right)^2\right] \tag{11.27}$$

$$\approx X_1 + X_2 \tag{11.27a}$$

also,

$$R_s = R_3 = \frac{X_1 X_2}{R_A} \tag{11.28}$$

From Eq. (11.15)

$$X_3 = X_1 + X_2 \tag{11.29}$$

From Eq. (11.22)

$$V_2 = I_x R_A \tag{11.30}$$

From Eq. (11.24)

$$10^3 P_T = I_x^2 \mathrm{Re}\left(\frac{-Z_A Z_3}{-jX_2}\right) \tag{11.31}$$

$$= I_x^2 \frac{X_1 + X_2}{X_2} R_A \tag{11.32}$$

from Eq. (11.29) or

$$10^3 P_T = I_x^2 R_A + I_x^2 \frac{X_1}{X_2} R_A \tag{11.33}$$

The first term is $P_A \times 10^3$.

Therefore, from Eq. (11.25),

$$10^3 P_L = I_x^2 R_A \frac{X_1}{X_2} \tag{11.34}$$

In order to simplify the calculations without, however, incurring large errors, let

$$R_A \geqq 5X_2 \tag{11.35}$$

Then

$$V_L \approx \frac{X_1 + X_2}{X_2} V_2 \tag{11.36}$$

Let

$$\frac{X_1}{X_2} = n \tag{11.37}$$

Then

$$X_1 = nX_2 \tag{11.38}$$

and

$$V_L \approx (n + 1) V_2 \tag{11.39}$$

and

$$P_L = nP_A \tag{11.40}$$

from Eq. (11.34). Obviously,

$$10^3 P_L = \frac{V_L^2}{R_L} \tag{11.41}$$

$$R_L = \frac{[(n+1)I_x R_A]^2}{10^3 P_L} \tag{11.42}$$

from Eqs. (11.39), (11.41), and (11.30).

11.2.7 The Concept of the R_A Gain Factor, A_{L_0}

Under small-signal conditions R_A will have a value

$$R_{A_0} = R_x + R_{\mathrm{IN}_0} \tag{11.43}$$

This is measured with the o properly terminated. When the loop is closed, R_A will

$$R_A = R_x + R_{\mathrm{IN}} \tag{11.44}$$

R_{IN} will, of course, increase ude of oscillation.

The ratio A_{L_0} is defined

$$A_{L_0} = \frac{R_A}{R_{A_0}} \tag{11.45}$$

or

$$R_A = A_{L_0}(R_{\mathrm{IN}_0} + R_x) \tag{11.45a}$$

11.2.8 Limiting

In this oscillator also, there are two types of limiting.

1 R_{IN} limiting
2 Collector base limiting, used only when V_L is large.

11.2.8.1 R_{IN} Limiting (See Section 6.3.3)

In this case the crystal current I_x is fixed by the dc emitter current I_E in accordance with Eq. (6.23). R_{IN} will then assume the value required to satisfy Eq. (11.12).

To ensure correct limiting,

$$V_E = V_{BB} - 1.7V_L - 1700 \tag{11.46}$$

or when V_E calculates to be $> V_{BB}/2$, then

$$V_E = \frac{V_{BB}}{2} \tag{11.46a}$$

11.2.8.2 Collector Base Limiting

This type of limiting is similar to that described in Section 6.2.4. However, modifications must be made to the material in that section because of the sinusoidal i_E.

In this form of limiting:

I_E is determined by Eq. (2.97).

$$V_E = V_{BB} - 1.4V_L \tag{11.47}$$

as V_e is usually very small and can be neglected.

11.2.8.3 Type of Limiting Used in the Algorithms for the Butler Oscillator

R_{IN} limiting is used in the circuits designed in the algorithms. This is done because of the superior high-frequency transistor performance. Also, when properly designed, the output is almost independent of the crystal resistance, as the output is dependent upon I_x which is essentially fixed by the emitter current I_E, as stated in Eq. (6.23).

11.2.9 Calculation of C_b in Fig. 11.1

The function of C_b is to effectively connect the base to ground at *ac*. This is calculated by making its reactance much smaller than that of the emitter to ground, which is $R_A - R_{df}$; then

$$C_b \gg \frac{1.59 \times 10^6}{(R_A - R_{df})f} \tag{11.48}$$

11.2.10 Calculation of L_0 in Fig. 11.1

The function of L_0 is to neutralize the C_0', which is equal to the sum of the C_0 of the crystal and the parallel wiring capacitance. L_0 is necessary at high frequencies where C_0' would tend to short out the crystal action and oscillations uncontrolled by the crystal may be permitted. It is recommended that L_0 be supplied at $f \geqq 50$ MHz; the value of L_0 is

$$\frac{10^6}{C_0'(6.28f)^2} \tag{11.49}$$

11.2.11 Calculation of the Effect of the Q of L_3 in Fig. 11.1

Often L_3 has a power loss that is appreciable compared to that in the load R_L and therefore should be taken into account.

Assume that L_3 has a $Q = Q_{L_3}$. Then the associated equivalent parallel resistance

$$r_{L_3} = Q_{L_3} X_{L_3} \tag{11.50}$$

This resistance is in shunt \ ɔower loss is

$$P_L \cdot \frac{R_L}{r_{L_3}} \tag{11.51}$$

The total power dissipated

$$P_L' = P_L\left(1 + \frac{R_L}{r_{L_3}}\right)$$

$$= P_L\left(1 + \frac{R_L}{Q_{L_3} X_{L_3}}\right) \tag{11.52}$$

from Eq. (11.50).

11.2.12 Calculation of the Oscillator Operating Q

Obviously

$$Q_{op} = \frac{Q_x \cdot R_{df}}{-R_N} \tag{11.53}$$

where R_N is the negative resist tive resistance oscillator model. For the case where $X_A = 0$,

$$-R_N = R_A \tag{11.54}$$

Therefore, in this case

$$Q_{op} \approx \frac{Q_x \cdot R_{df}}{R_A} \tag{11.55}$$

11.2.13 Calculation of R_N, X_N, and X_c in Terms of the Physical Component Values

From Eqs. (11.12) and (11.18)

$$-R_N \approx \frac{X_1 X_2 \dfrac{R_L}{1 + (R_L/X_L)^2}}{\left[\dfrac{R_L}{1 + (R_L/X_L)^2}\right]^2 + \left[\dfrac{X_L}{1 + (X_L/R_L)^2} - (X_1 + X_2)\right]^2} \tag{11.56}$$

From Eqs. (11.13) and (11.18)

$$-X_N \approx \frac{-X_1 X_2 \left(\dfrac{X_L}{1 + (X_L/R_L)^2} - (X_1 + X_2)\right)}{\left[\dfrac{R_L}{1 + (R_L/X_L)^2}\right]^2 + \left[\dfrac{X_L}{1 + (X_L/R_L)^2} - (X_1 + X_2)\right]^2} \tag{11.57}$$

From Eqs. (11.19) and (11.17)

$$|Z_c| = \frac{\sqrt{(R_N^2 + X_N^2)}\, R_L X_L}{X_2 \sqrt{R_L^2 + X_L^2}} \tag{11.58}$$

11.2.14 Discussion of Design Procedures

In order to arrive at quantitative design procedures which are reasonably simple, it is necessary to make additional assumptions that are often incorrect. One assumption is that the transistor input impedance Z_{IN} is independent of the load in the collector circuit. Furthermore, the R_{IN} is calculable as described in Section 2.5.2.2. As implied in Eq. (2.85) and explained in Section 11.4, the latter assumptions are valid only when the effects of C_{ce} can be neglected, which is very often not true at high frequencies. The net result is that X_{IN} may be substantial, and more importantly, it varies with the tuning of the collector load. In addition, R_{IN} is also caused to vary when the collector load changes. Taking into account these variations in any design procedure renders the procedure incredibly complex. It was therefore opted to neglect these variations and to make the above-mentioned assumptions. However, an attempt will be made to explain the consequences of these assumptions during the development of the design procedures. In doing so, these assumptions will be denoted as assumptions $A \equiv \text{Asmp}\,A$.

It is noteworthy that most of the above variations in R_{IN} can be rendered harmless by shunting the emitter with a low-value resistor, or capacitor, but insufficient loop gain may result.

At this point, it is worthwhile examining the influence of the above assumptions upon the theory developed thus far.

The basic oscillatory equations (11.12) and (11.13) are correct since they were derived without Asmp A. Also, Z_N can be rendered easily calculable by making $X_{c_2} \ll R_A$ as stated in Eq. (11.35), so that the contribution of Z_A to Z_N becomes very small. It should be noted, as stated previously, that once Z_N is known, Z_A can be determined from Eqs. (11.12) and (11.13).

All other equations involving Z_A are equally correct. However, the contributions of the transistor and the crystal to Z_A are somewhat indeterminate, and these are now examined in detail. Z_A is known from Eqs. (11.12) and (11.13). Therefore, R_A is known. R_{IN} is known because $R_{IN} = R_A - R_x$, and R_x is known. X_A is known from Eq. (11.13).

$X_A = X_{IN} + X_x$, but the relative contributions of X_{IN} and X_x are unknown because of the dependence of X_{IN} upon the collector load as described above. Also, with X_x unknown, little is known about the frequency stability because it is directly related to the behavior of X_x.

Another important aspect of Asmp A is that R_{IN_0}, and hence A_{L_0}, are assumed to be known. In fact, R_{IN_0} is not known. Again, this is very unfortunate because oscillations can only exist when $R_N > R_{IN_0}$, and therefore one does not know when oscillations will cease.

As a result of the above, Sections 11.2.3.1 and 11.2.7 are meaningless except in special situations which will be described later. Although quantitative information on X_x and R_{IN_0} cannot be obtained from the design procedures in the general case, some useful qualitative information will be supplied, as will be demonstrated later.

Two design procedures will now be developed. Both procedures will include Asmp A. The designs obtained from these procedures will be somewhat incorrect, as pointed out above, but the required performance will be obtained from these designs combined with the trimming procedures.

In the development of these design procedures, additional assumptions may be made to simplify the execution of the procedure. However, errors introduced by these additional assumptions and the true performance can be determined by means of the exact relationships derived in Section 11.2.

For convenience, the substance of Asmp A is repeated here:

1 Z_{IN} is independent of the collector load.
2 X_{IN} = constant, and is equivalent to a simple linear reactance.
3 $$R_{\mathrm{IN}_0} = \frac{26}{I_E} \tag{2.87}$$
neglecting the 1 Ω and $r_{bb'}$.

11.3 THE $X_A = 0$ DESIGN PROCEDURE

11.3.1 Introduction

The Butler oscillator is primarily used because of its ability to operate at much higher frequencies than the other oscillators thus far discussed. Accordingly, one is forced to accept its shortcomings, such as low operating Q.

One of the most popular designs requires that C_1, C_2, and X_3 be tuned to resonance at the operating frequency f. This implies that $X_A = 0$, and for the case where $X_{\mathrm{IN}} = 0$, the crystal is operating at series resonance. This operating state is experimentally attained by replacing the crystal with an equivalent resistor and tuning the oscillator so that the resulting operating frequency approximates the crystal frequency. (See, for example, page 94 of Ref. 1.10.) This operating mode has the advantage that its amplitude stability is optimized since the negative resistance term, and hence R_A, in Eq. (11.12), is maximized. Also, the amplitude is maximized, which is a convenient condition to observe.

It will be seen later that this mode of operation does not have the maximum frequency stability. However, because of its popularity, an algorithm for the design of the Butler oscillator operating in this mode, called the $X_A = 0$ mode, is developed using the theory presented in Section 11.2, including the $X_A = 0$ relationships of Section 11.2.6.

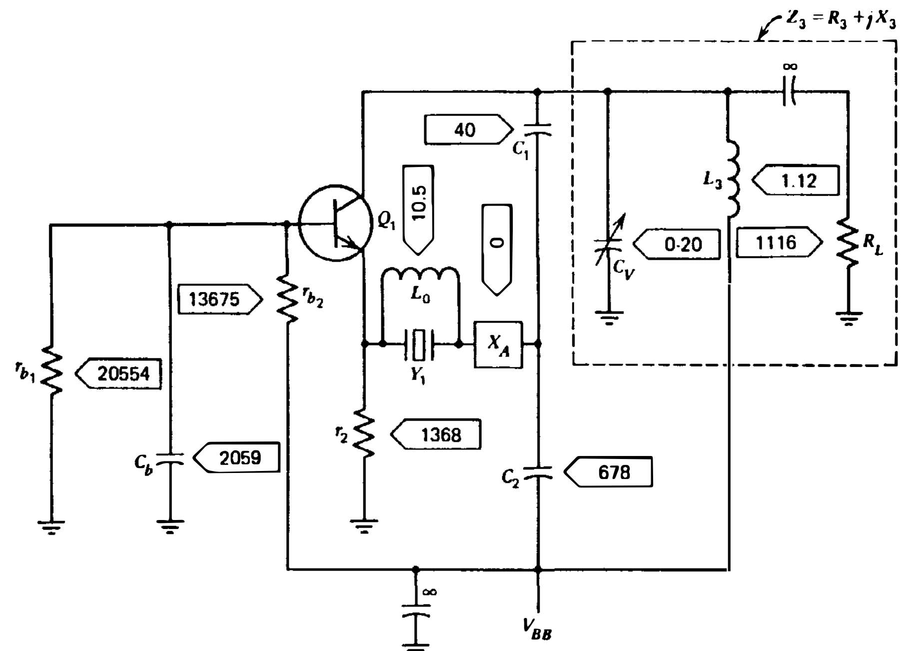
11.3.2 The $X_A = 0$ Algorithm

This algorithm is principally prepared using the relationships in Section 1.2.6. It is presented in Algorithm 12.7.

The user has to supply the information in Sections a, b, c, and d. It will be noted that items c7 to c10 are not in the algorithm, but checks should be made after the design to ensure that they are not exceeded. Also, because of the approximations in the algorithms, the influence of Items c4 to c6 must be included in the trimming procedure.

11.3.3 The $X_A = 0$ Design Example

Design Example 11.1 is the design example computed from the algorithm. The ratio $n = C_2/C_1$ is very large because of the large P_L/P_x. The Q_{L_3} has been chosen to be sufficiently high so that the losses of L_3 contribute little to P_L'.



Design Example 11.1 The $X_A = 0$ Butler oscillator.

All units in

℧ MHz Ω mW pF μH
mA, mV dc or rms

	Item	Value
Oscillator Performance	f	20
	P_l	5
	P_x	0.1
	V_{BB}	10,000
Principal Crystal Data	R_{df}	20
	I_x	2.24
	Cut	AT
	N	3
	C_0'	6
Transistor Data	β_o	30
	f_T	700
	BV_{ce}	15,000
	C_{cb}	---
	C_{bet}	—
	C_{ce}	—
	P_{dis}	
	Type	2N918
Circuit Parameters	A_{Lo}	2
	Q_{L_3}	100
	R_A/X_2	5
Calculated Data	I_{BB}	3.49
	$\|Z_c\|$	1038
	R_N	56.6
	X_A	0.7
	X_l	218.5

Some of the important performance characteristics of the example are plotted in Fig. 11.3. The data were calculated from Eqs. (11.56) to (11.58). (See Section 11.4.1 for the significance of X_L.)

The following useful information may be gathered from this figure:

1 From Fig. 11.3*b* it is seen that the circuit will oscillate for values of $X_L \approx 180$ to $250\ \Omega$ since $R_N > [(R_{\mathrm{IN}_0} + R_{df}) = 29]\ \Omega$ for these values of X_L. Also, from the same figure, the intersection of the curve with the $R_N = 38\ \Omega$ line determines the X_L region where $|Z_c|$, plotted in Fig. 11.3*c*, is proportional to $|V_L|$. This is so because $|V_L| = I_x|Z_c|$ and, in this region, $[R_N = R_{df} + R_{\mathrm{IN}}] \geqq [(R_{df} + 2R_{\mathrm{IN}_0}) = 38]\ \Omega$, and, as shown in Fig. 2.15, I_x remains essentially constant at $I_E/\sqrt{2}$ when $R_{\mathrm{IN}} \geqq 2R_{\mathrm{IN}_0}$. Outside of this region, V_L varies much more rapidly than $|Z_c|$.

2 Figure 11.3*a* shows that at the design value of X_L the slope of the curve is maximum. This means that the design is *very poor with respect to frequency stability* as any variation in X_L will cause a relatively large shift in X_A, which is directly related to the frequency. (For further discussion of Fig. 11.3*a*, see Section 11.4.2.)

3 Figure 11.3*d* shows that the frequency stability of this design, with respect to variations in R_L, is quite good.

It should be noted that the above curves are theoretical. In practice, the curves will be similar, but skewed because of the effects of C_{ce}. (See Section 11.4.4.)

11.3.4 Trimming for Algorithm 12.7

See Algorithm Figure 12.7.

11.3.4.1 Introduction

See Section 7.5.3.1.

11.3.4.2 Basis of the Trimming Procedure

The trimming is based upon the following relationships.

1 $$P_x = \frac{I_x^2 R_{df}}{1000}$$

2 $$I_x = \frac{I_E}{\sqrt{2}}$$

from Eq. (6.23)

3 $$P_L = \left(\frac{n}{1+n}\right)^2 \frac{R_L I_x^2}{1000}$$

from Eqs. (11.40), (11.56), and (11.57 and assuming $(R_L/X_L)^2 \gg 1$.

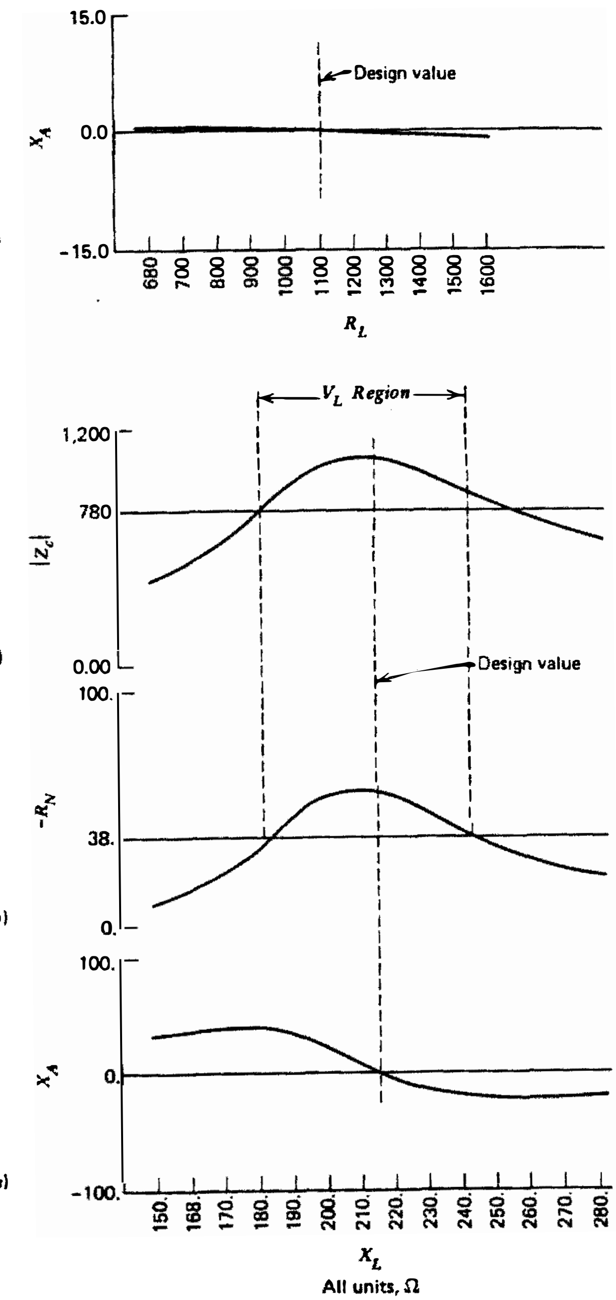


Figure 11.3 Performance characteristics of Design Example 11.1. (a) X_A versus X_L. (b) R_N versus X_L. (c) $|Z_c|$ versus X_L. (d) X_a versus R_L.

11.3.4.3 Typical Trimming Steps

The following describes the action required to increase a given characteristic. Obviously, the opposite action decreases the same characteristic.

11.3.4.3.1 Crystal Power P_x

Relation 1 states that to increase P_x, I_x must be increased. Relation 2 states that to increase I_x, I_E must be increased by adjusting r_2, r_{b_2}, and r_{b_1} as necessary. Relation 3 also states that the output power is also increased as I_x increases.

11.3.4.3.2 Output Power P_L

Relation 3 states that increasing I_x, n, and R_L will increase P_L. However, increasing I_x will increase P_x and if it is desired to maintain P_x constant, only n and R_L may be increased. Increasing R_L will also increase V_L so that readjustment of r_2, r_{b_1}, and r_{b_2} may be necessary to obtain the proper limiting conditions. It should be noted that increasing R_L reduces the share of the total generated power delivered to the external load. In addition, if n or R_L is changed, X_L must be readjusted for maximum output.

11.4 THEORY AND DESIGN OF THE STABLE BUTLER OSCILLATOR

11.4.1 Introduction

Section 11.2.3.1 states that for any independent parameter U, $\partial X_A/\partial U$ should approach zero to achieve minimum frequency change with variation of U.

It is desirable, if possible, to select a physical parameter U_1, the effective value of which is influenced by variations in the other physical parameters in the circuit. It follows that if $\partial X_A/\partial U_1 \to 0$, variations in the other parameters will also have a small effect upon the frequency. It is then sufficient to investigate only the behavior of $\partial X/\partial U_1$ to determine the frequency stability of the oscillator. If $\partial X_A/\partial U_1$ proves to be small, it follows that $\partial X_A/\partial U_n$ will also be small.

It is seen from Eqs. (11.13) and (11.18) that a suitable parameter for U_1 is X_L, which is a direct strong function of C_v and L_3. In addition, the effective value of X_L is a function of the transistor parameters. Equation (11.18) also states that variations in $X_1 + X_2$ can be accounted for by equivalent variations in X_L.

The only parameter which is independent of X_L is R_L, which must be treated separately. It will be seen later that $\partial X_A/\partial X_L$ can be much greater than $\partial X_A/\partial R_L$.

In view of the above, an investigation will be made of the behavior of $\partial X_A/\partial X_L$ and $\partial X_A/\partial R_L$. The results of the investigation will be used to prepare an algorithm for circuits in which $\partial X_A/\partial X_L$ is small. Oscillators in which $\partial X_A/\partial X_L$ is small are called *stable oscillators*.

11.4.2 Basis of the Stable Oscillator Designs

Figure 11.3*a* shows that there are two values of X_L at which $\partial X_A/\partial X_L \rightarrow 0$. One of these points at which X_A is plus is called the X_{A+} point. The point, at larger X_L, where X_A is minus, is called the X_{A-} point. Similarly, the value of X_L at these points is called X_{L+} and X_{L-}.

The X_{A+} point has been experimentally observed and often the oscillator is set to this point by first tuning to the maximum output point (see Fig. 11.3*c*) in the direction of increasing X_L and then backing up a bit until the output is about 3 dB less. However, the reason for the increased stability has not been understood.

The X_{A-} point is difficult to observe experimentally because the value of R_{IN_0} is so large at this point, for the reasons discussed in Section 11.4.4, that unless the oscillator is specifically designed to operate at this point it ceases oscillating before the point is reached.

An analysis, which is very long and tedious and therefore not repeated here, shows that these points exist when $|Z_c|$, in Fig. 11.1, $\approx 0.7R_L$. The values of X_L for these points are computed below.

11.4.3 The Stable Butler Oscillator Relationships

Assume that the circuit is so designed that Z_A is very large compared to X_2. Z_A therefore contributes very little to Z_c. With this assumption,

$$|Z_c| \approx R_L \| X_p = 0.7R_L \tag{11.59}$$

where X_p is the parallel combination of $X_1 + X_2$ and X_L.

Obviously, when $|X_c| \approx 0.7R_L$

$$X_p \approx R_L \tag{11.60}$$

and

$$\theta_{Z_c} = 45° \tag{11.60a}$$

Therefore,

$$R_L \approx |X_p| = \left| \frac{X_L(X_1 + X_2)}{X_L - (X_1 + X_2)} \right| \tag{11.61}$$

Solving Eq. (11.61) for

$$X_{L+} = \left| \frac{R_L(X_1 + X_2)}{R_L + (X_1 + X_2)} \right| \tag{11.62}$$

and

$$X_{L-} = \left| \frac{R_L(X_1 + X_2)}{R_L - (X_1 + X_2)} \right| \tag{11.63}$$

R_L is now computed as follows:

$$1000P_L = \frac{(I_x|Z_c|)^2}{R_L} \tag{11.64}$$

$$\approx \frac{I_x^2(0.7R_L)^2}{R_L}$$

from Eq. (11.59), so that

$$R_L \approx \frac{2000P_L}{I_x^2} \tag{11.65}$$

Continuing the assumption that Z_A is very large compared to X_2, then, in Fig. 11.2,

$$V_2 \approx \frac{X_2}{X_2 + X_1} V_L \approx \frac{X_2}{X_2 + X_1} I_x Z_c \tag{11.66}$$

and

$$I_x \approx \frac{V_2}{Z_A} \equiv \frac{X_2}{X_2 + X_1} \frac{I_x Z_c}{Z_A} \tag{11.67}$$

from which

$$Z_A \approx \frac{X_2 Z_c}{X_2 + X_1} \tag{11.68}$$

From Eq. (11.60a)

$$|\theta_{Z_c}| \approx 45°$$

Therefore, from Eq. (11.68)

$$|\theta_{Z_A}| \approx 45° \tag{11.69}$$

or

$$R_A \approx X_A \approx 0.7|Z_A| \tag{11.70}$$

Let

$$X_1 = nX_2 \tag{11.71}$$

Then, from Eqs. (11.68), (11.70), and (11.71),

$$1.4R_A \approx \frac{|Z_c|}{1 + n} \tag{11.72}$$

Solving for n,

$$n = \frac{0.7|Z_c|}{R_A} - 1$$

which becomes from Eq. (11.59)

$$n \approx \frac{0.5R_L}{R_A} - 1 \tag{11.73}$$

Final Relationships for X_{L+} and X_{L-}

From Eqs. (11.62), (11.63), and (11.73)

$$X_{L+} \approx \frac{R_L}{2R_A/X_2 + 1} \tag{11.74}$$

$$X_{L-} \approx \frac{R_L}{2R_A/X_2 - 1} \tag{11.75}$$

and

$$\frac{X_{L-}}{X_{L+}} = \frac{2R_A/X_2 + 1}{2R_A/X_2 - 1} \tag{11.76}$$

which increases as R_A/X_2 decreases. For example,

$$\frac{R_A}{X_2} = 5, \qquad \frac{X_{L-}}{X_{L+}} = 1.11$$

$$\frac{R_A}{X_2} = 2.5, \qquad \frac{X_{L-}}{X_{L+}} = 1.5$$

As will be demonstrated later, it is desirable to have larger values of X_{L-}/X_{L+} because the slope of the X_A versus X_L curves then generally decreases. Therefore, C_1 and C_2 may decrease to the point where they may be unrealizably small at very high frequencies. In addition, the transistor will contribute more to X_L, which is undesirable.

11.4.4 The Effect of C_{ce}

As pointed out in Eq. (2.85)

$$Y_{\mathrm{IN}} = y_{11b} - \frac{y_{12b}y_{21b}}{y_{22b} + Y_L} \tag{2.85}$$

at the usual frequency of operation,

$$y_{22b} \rightarrow 0$$

and

$$y_{11b} \approx -y_{21b}$$

therefore

$$\left[1 \;.\; \frac{y_{12b}}{}\right] \tag{11.77}$$

For $Y_L = G_L + jB_L$,

$$Y_{\mathrm{IN}} \approx y_{11b}\left[1 + \frac{y_{12b}[G_L - jB_L]}{G_L^2 + B_L^2}\right] \tag{11.78}$$

$$y_{12b} \approx -j|y_{12b}| \tag{11.79}$$

$$Y_{\mathrm{IN}} \approx y_{11b}\left[1 - \frac{|y_{12b}|B_L}{G_L^2 + B_L^2} - j\frac{|y_{12b}|G_L}{G_L^2 + B_L^2}\right] \tag{11.80}$$

X_{L+}, the sign of B_L = $-$

X_{L-}, the sign of B_L = $+$

Therefore

Y_{IN} tends to increase for X_{L+}
decrease for X_{L-}

or

R_{IN} tends to decrease for X_{L+}
increase for X_{L-}

Therefore, the oscillator will cease operation sooner as X_L increases on the X_{L-} side.

It should be noted that the above analysis is for the small-signal case. Its quantitative validity for the large signals present in oscillators is therefore questionable, but it is certainly qualitatively valid. If Y_{12b} (including the contributions of the layout) is known, it is worthwhile calculating the effect as described above. If it turns out to be very small, it is in order to design for the X_{L-} point because of its superior frequency stability.

11.4.5 The Stable Oscillator Design Algorithm

This algorithm is prepared for the X_{L+} point because of the reasons explained in Section 11.4. However, the algorithm also lays the groundwork for the design procedure for the X_{L-} point.

All the equations used in preparing the algorithm have already been derived except for the relationship for Q_{op}, which is

$$Q_{op} \approx Q_x \frac{R_{df}}{-R_N} \tag{11.81}$$

The algorithm is presented in Algorithm 12.8. It gives in detail the design procedure for the X_{L+} point and indicates how the design procedure is modified for the X_{L-} point. Since crystals at very high frequencies are normally rated for operation at series resonance, the algorithm includes inductor, L_A, in Step 23, which permits the crystal to operate at series resonance and facilitates the correlation of the crystal specification with the circuit operation. For X_{L-}, X_A will consist of a capacitor.

11.4.6 The Stable Oscillator Design Example

Design Example 11.2 is the design example computed from the algorithm at X_{L+}. This design example is based upon the assumption that $X_2 = R_A/5$. The ratio $n = C_2/C_1$ is large because of the large P_L/P_x.

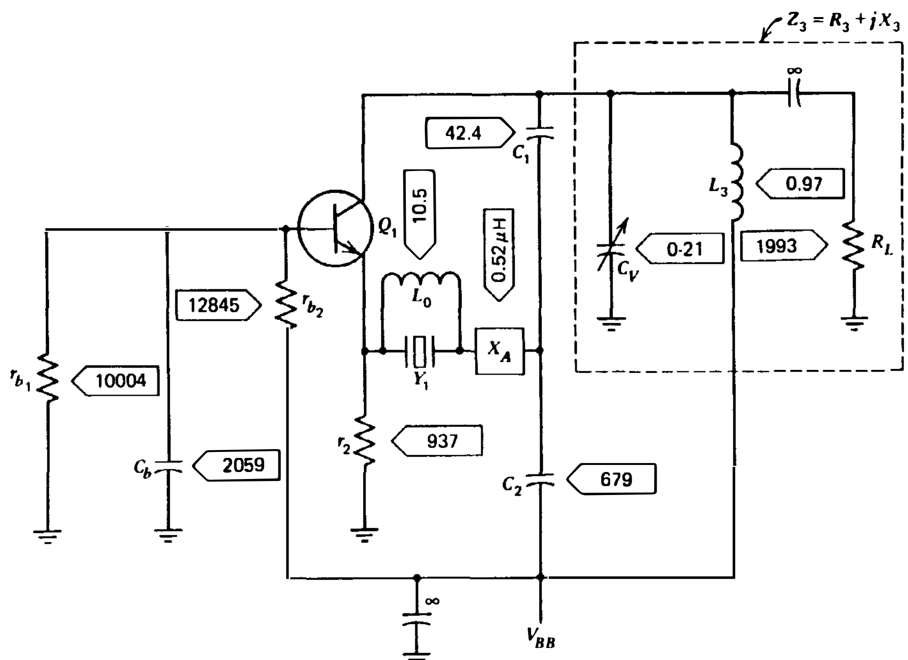
Some of the important performance characteristics of Design Example 11.2 are plotted in Fig. 11.4. The data was calculated from precise Eqs. (11.56) to (11.58). (For the explanation of the V_L region see Section 11.3.3, item 1.)

The following useful information may be gathered from this figure:

1 From Fig. 11.4*b* it is seen that the circuit will oscillate for values of $X_L \approx 170$ to $240\ \Omega$.

2 From Fig. 11.4*c*, it is seen that V_L at X_{L+} and X_{L-} is about 3 dB below the peak.

3 From Fig. 11.4*d*, it is seen that the frequency stability versus R_L of this design is considerably worse than the design of Fig. 11.3*d*. This, however, is



Design Example 11.2 The stable Butler oscillator.

All units in

Ω MHz Ω mW pF µH
mA, mV dc or rms

	Item	Value
Oscillator Performance	f	20
	P_L	5
	V_{BB}	10000
Principal Crystal Data	R_{df}	20
	I_x	2.24
	Cut	AT
	N	3
	C_0'	6
Transistor Data	β_o	30
	f_T	700
	BV_{ce}	15000
	C_{cb}	—
	C_{bet}	—
	C_{ce}	—
	P_{dis}	
	Type	2N918
Circuit Parameters	A_{L_0}	2
	Q_{L_3}	100
	R_A/X_2	5
Calculated Data	I_{BB}	3.65
	$\|Z_c\|$	1324
	R_N	54.8
	X_A	66.2
	X_L	181

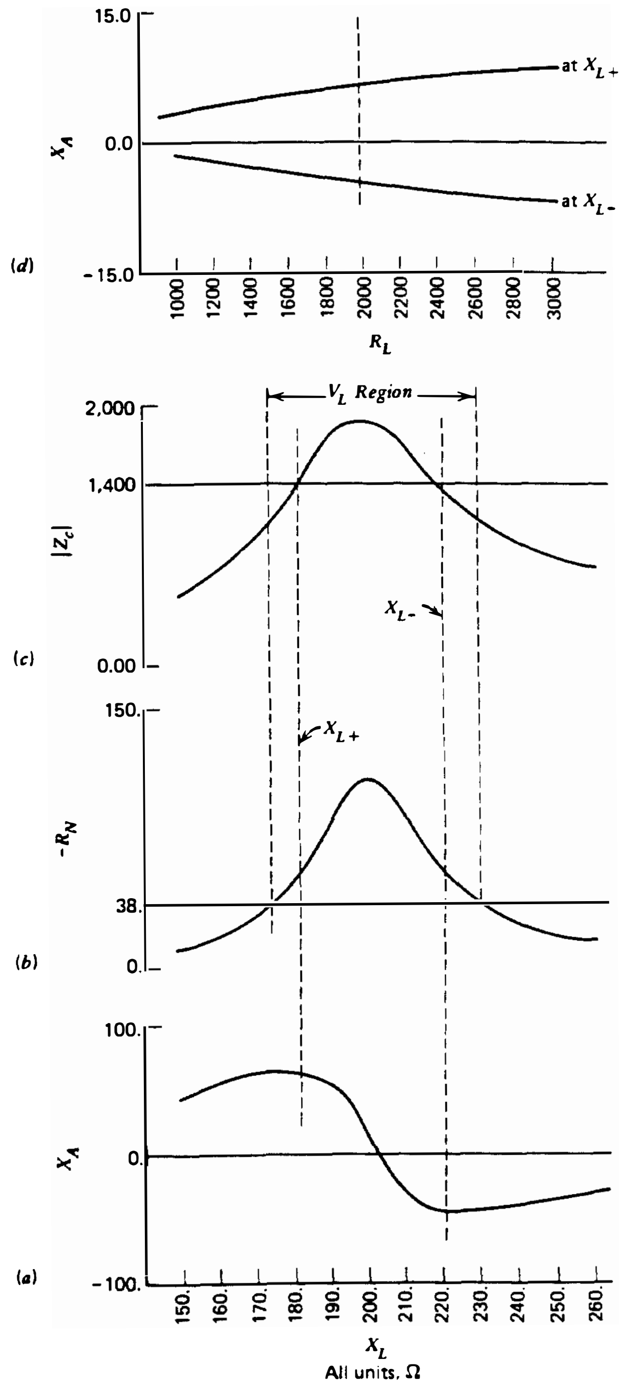


Figure 11.4 Performance characteristics of Design Example 11.2. (*a*) X_A versus X_L. (*b*) R_N versus X_L. (*c*) $|Z_c|$ versus X_L. (*d*) X_A versus R_L.

not important in those cases where the oscillator feeds an amplifier and R_L is therefore relatively constant.

4 Figure 11.4*a* shows that the slope of the curve is indeed 0 at X_{L+} and X_{L-} as called for by the design procedure. The figure also shows that the curve is flat for only relatively small ranges of X_L. It is desirable that the X_L range of X_A flatness be increased. A clue as to how to achieve this increase is obtained by noting that the ratio of X_{L-}/X_{L+} is rather small, and a larger ratio may be necessary.

Equation (11.76) states that a larger ratio may be obtained by decreasing R_A/X_2.

In the design of Fig. 11.4, $R_A/X_2 = 5$. A second design was therefore made with $R_A/X_2 = 2.5$.

The circuit values are given in Design Example 11.3 and the performance is that in Fig. 11.5. It will be noted that the X_A versus X_L curves are considerably flatter. However, this design has the disadvantage that the X_L values are approximately double the values of those in Fig. 11.4 and may be unrealizable at the higher frequencies.

Both Figs. 11.4 and 11.5 show that the slope of the X_A versus X_L curve is considerably smaller at X_{L-} than at X_{L+}, and therefore the X_{L-} point is preferable except for the C_{ce} effects discussed in Section 11.4.4.

Also, both Figs. 11.4 and 11.5 prove that the approximate design procedure derived in Section 11.4.3 yields results which are consistent with exact Eqs. (11.56) to (11.58), except for the value of $|X_A|$, which is 15% higher than the design value at X_{L+} and 15% lower at X_{L-}.

Again, the curves of Figs. 11.4 and 11.5 are theoretical. In practice, the curves will be similar but skewed because of the effect of C_{ce} (see Section 11.4.4).

11.4.7 Trimming for Algorithm 12.8

11.4.7.1 Introduction

See Section 7.5.3.1.

11.4.7.2 Basis of the Trimming Procedure

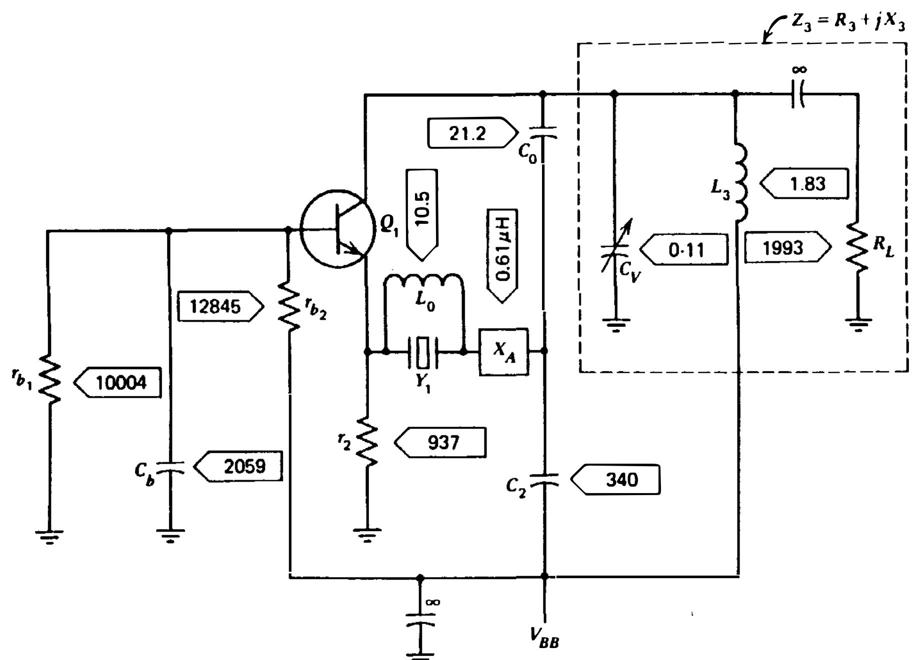
1
$$P_x = \frac{I_x^2 R_{df}}{1000}$$

2
$$I_x = \frac{I_E}{\sqrt{2}}$$

from Eq. (6.23)

3
$$P_L = \frac{I_x^2 R_L}{2000}$$

from Eq. (11.65)



Design Example 11.3 The stable Butler oscillator.

All units in

Ω MHz Ω mW pF μH
mA, mV dc or rms

	Item	Value
Oscillator Performance	f	20
	P_L	5
	P_s	0.1
	V_{BB}	10,000
Principal Crystal Data	R_{df}	20
	I_s	2.24
	Cut	AT
	N	3
	C_0'	6
Transistor Data	β_o	30
	f_T	600
	BV_{ce}	15,000
	C_{cb}	—
	C_{bet}	—
	C_{ce}	—
	P_{dis}	
	Type	2N918
Circuit Parameters	A_{L_0}	2
	Q_{L_3}	100
	R_A/X_2	2.5
Calculated Data	I_{BB}	3.65
	$\|Z_c\|$	1328
	R_N	55.3
	X_A	77.1
	X_L	332

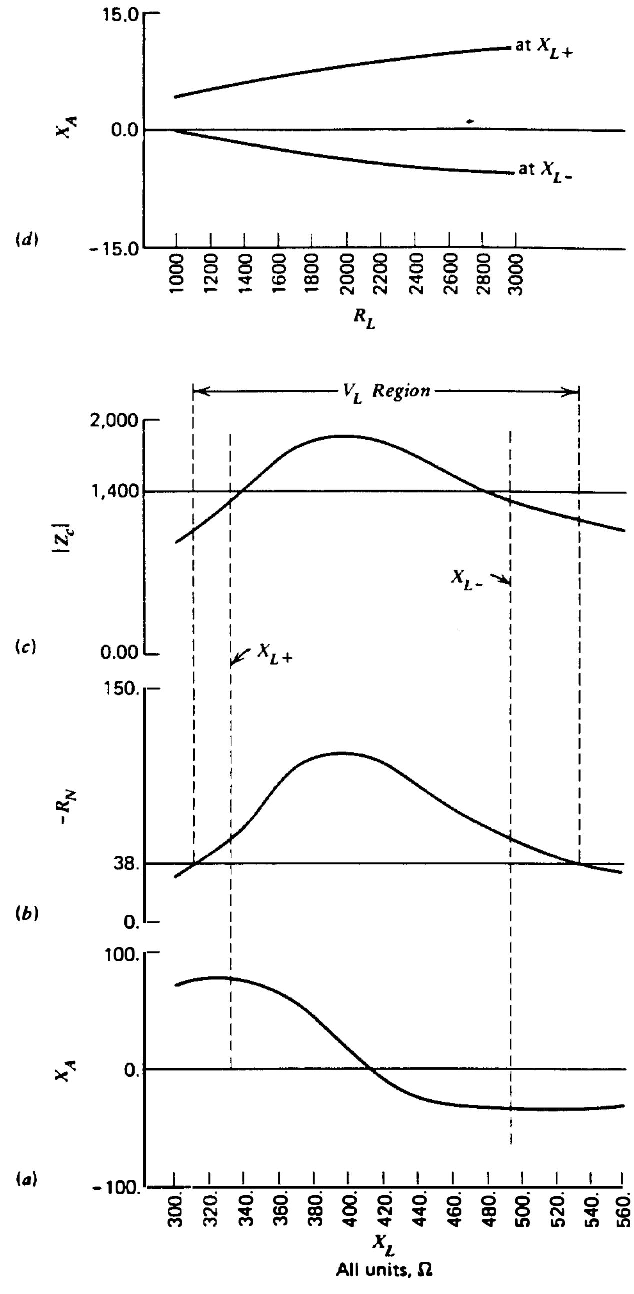


Figure 11.5 Performance characteristics of Design Example 11.3. (a) X_A versus X_L. (b) R_N versus X_L. (c) $|Z_c|$ versus X_L. (d) X_A versus R_L.

3b
$$n_2 = \frac{R_{L_2}}{R_{L_1}}(n_1 + 1) - 1$$
from Eq. (11.73)

3c
$$X_{L+_2} = \frac{R_{L_2}}{R_{L_1}} X_{L+_1}$$
from Eq. (11.74)

For the significance of the subscripts 1 and 2, see Section 11.4.7.3.2.

11.4.7.3 Typical Trimming Steps

The following describes the action required to increase a given characteristic. Obviously, the opposite action describes the same characteristic.

11.4.7.3.1 Crystal Power P_x

Relation 1 states that to increase P_x, I_x must be increased. Relation 2 states that to increase I_x, I_E must be increased by adjusting r_2, r_b, and r_{b_2} as necessary. Also, Relation 3a states that the output power also increases as I_x increases.

11.4.7.3.2 Output Power P_L

Relation 3a states that increasing I_x and R_L increases P_L. However, increasing I_x will increase P_x and, if one wants to maintain P_x constant, only R_L may be increased. At the same time, if R_L is changed from R_{L_1} to R_{L_2} n must be changed from n_1 to n_2 as per Section 11.4.7.2, relation 3b. X_{L_+} must be changed from X_{L+_1} to X_{L+_2} as per Section 11.4.7.2, relation 3c. X_{L+} may also be adjusted experimentally by setting it to maximum P_L and then backing off 3 dB down.