5

The Pierce, Colpitts, Clapp Oscillator Family

5.1 INTRODUCTION

The types of oscillators mentioned in the chapter title are members of a large family of oscillators which have essentially the identical mathematical model, but which differ in practical realization.

Figure 5.1 shows the elementary schematic diagrams of these oscillators. It will be noted that they are quite similar, but differ in two respects:

1 The method of feeding the dc power and biasing the transistor.
2 The method of delivering the RF power output to R_L.

These differences appear minor, but they seriously influence the performance and the selection of the circuit to be used in a particular application. The effects of items 1 and 2 will be considered in detail in later chapters. However, it is worthwhile mentioning at this point that, except in unusual circumstances, the Pierce is preferred over the Colpitts, and the Clapp is rarely used for crystal oscillators.

Figure 5.2 is the composite schematic showing the ac portion of the three oscillators of Fig. 5.1, including the load R_L. The table indicates the oscillator node which when connected to the datum converts the oscillator of Fig. 5.2 into that of Fig. 5.1. The datum is normally the familiar *ground* point or plane. The connection of the appropriate node to ground has important effects upon the following:

1 The manner in which the dc power is fed to the oscillator and the consequent loading effects on the ac circuitry.

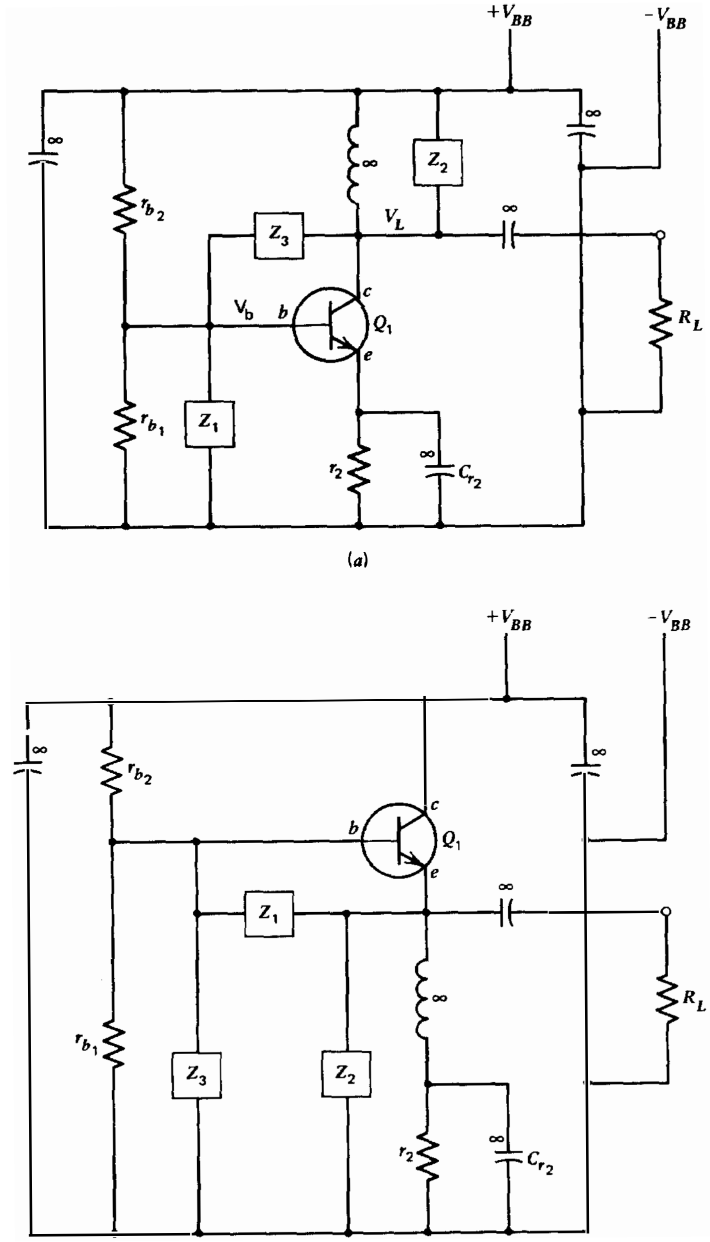
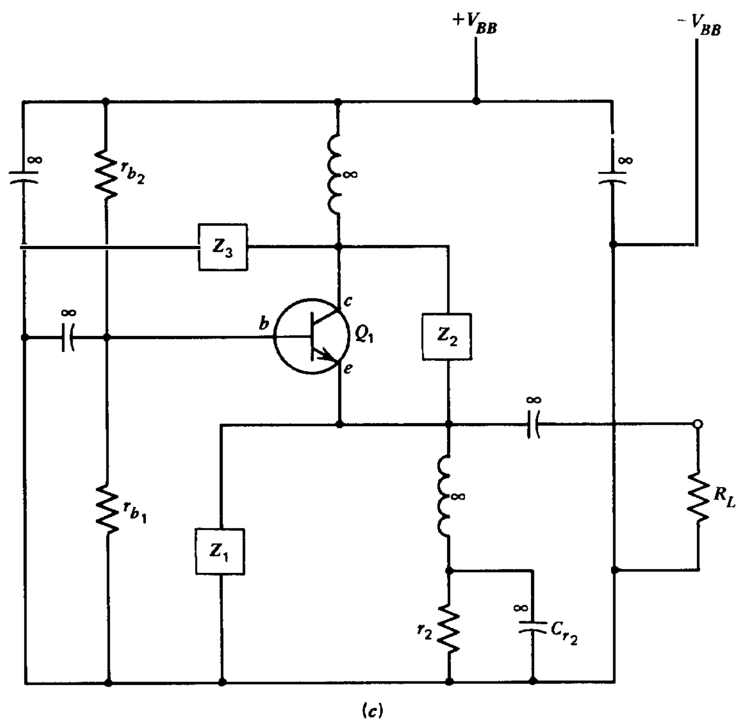


Figure 5.1 Schematics of the Pierce family of oscillators. (*a*) Pierce. (*b*) Colpitts. (*c*) Clapp.



(c)

Figure 5.1 (Continued).

Oscillator Type	Datum Connected to
P (Pierce)	e
CO (Colpitts)	c
CL (Clapp)	b

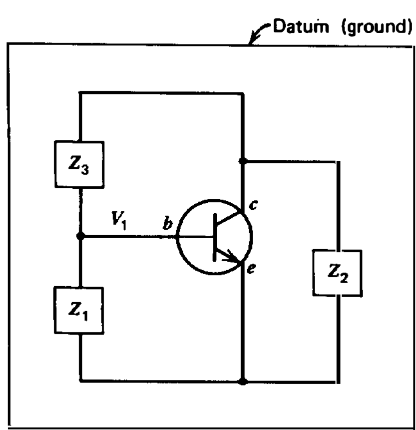


Figure 5.2 ac representation of the Pierce family of oscillators.

2 The manner in which the output power is fed to the external load.
3 The distribution of the stray elements to the ground plane which has important effects, particularly at high frequency.

5.2 THE IDEALIZED OSCILLATOR

In Fig. 5.2 the transistor immittances have been incorporated into Z_1, Z_2, and Z_3, so that $Y_{be} \to 0$. Following the procedure outlined in Section 1.3.2 and Fig. 1.16*b*, and assuming g_m is real, then

$$V_1 = -g_{m_0}V_1 \cdot \frac{Z_2(Z_3 + Z_1)}{Z_1 + Z_2 + Z_3} \cdot \frac{Z_1}{Z_3 + Z_1}$$

$$= -g_{m_0}V_1 \frac{Z_1 Z_2}{Z_1 + Z_2 + Z_3}$$

and

$$A_{L_0} = -g_{m_0} \frac{Z_1 Z_2}{Z_1 + Z_2 + Z_3} \tag{5.1}$$

writing

$$Z_1 + Z_2 + Z_3 = Z_s \tag{5.2}$$

Eq. (5.1) becomes

$$A_{L_0} = -g_{m_0} \frac{Z_1 Z_2}{Z_s} \tag{5.1a}$$

To start oscillatior

$$\left[|A_{L_0}| = g_{m_0} \frac{|Z_1||Z_2|}{Z_s} \right] > 1 \tag{5.3}$$

and

$$\theta_{Z_1} + \theta_{Z_2} - \theta_{Z_s} \approx \pi \tag{5.4}$$

or

$$\theta_{Z_s} \approx \theta_{Z_1} + \theta_{Z_2} - \pi \tag{5.4a}$$

At steady-state osc

$$A_L = 1$$

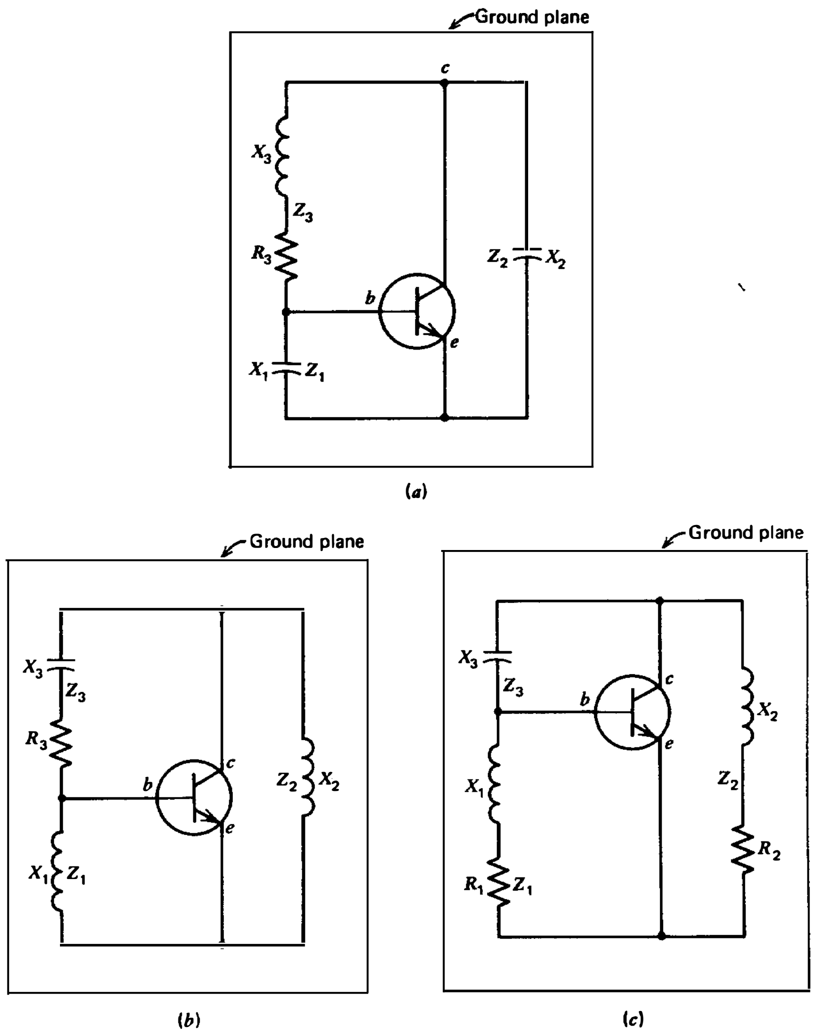


Figure 5.3 The idealized oscillator (a) X_1 and X_2, capacitive. (b) X_1 and X_2, inductive. (c) Practical equivalent of (b).

so that Eq. (5.3) becomes

$$|Z_s| = g_m|Z_1||Z_2| \tag{5.5}$$

If it is assumed that Z_1 and Z_2 are purely reactive and if Z_1 and Z_2 are both inductive or capacitive, then from Eq. (5.4a)

$$\theta_{Z_s} = 0 \qquad \text{or} \qquad 2n\pi \qquad \text{which is equivalent to } 0 \tag{5.6}$$

where n is an integer.

If it is assumed that Z_1 is inductive and Z_2 is capacitive or vice versa, then $\theta_{Z_s} = -\pi$, which is impossible since Z_1, Z_2, and Z_3 are real passive elements.

It is therefore seen that Z_1 and Z_2 must *both* be either inductive or capacitive and Z_s must be resistive, and the frequency of oscillation will be that at which

$$X_3 + (X_1 + X_2) = 0 \tag{5.7}$$

Figure 5.2 therefore, for the idealized case described above, becomes Fig. 5.3*a* or 5.3*b*.

It is evident that Fig. 5.3*b* is somewhat impractical because it is impossible to make lossless inductors, but the basic reasoning will not be rendered invalid if X_1 and X_2 are somewhat lossy.

A more practical realization of Fig. 5.3*b* would be that shown in Fig. 5.3*c*.

Going back to Fig. 5.2 recalls that Figs. 5.3*a* and 5.3*c* each represents three possible oscillator configurations so that now there are a total of six configurations. Which one should be used depends upon the application, and a few examples are now considered.

5.3 EXAMPLES OF IDEALIZED OSCILLATOR APPLICATIONS

5.3.1 Wide Range Local Oscillator for Receiver

In this case the frequency is tunable over a wide range by a single variable capacitor. The most practical arrangement is that one side of that capacitor be grounded. Looking at Figs. 5.3*b* and 5.3*c*, it is immediately seen that the only circuits satisfying the requirements are Fig. 3*c* with collector or base grounded; that is, the Colpitts or Clapp oscillator.

Figure 5.4*a* is the oscillator in the Colpitts configuration. Figure 5.4*b* is the same oscillator in the Clapp configuration.

In both circuits X_1 and X_2 can be replaced by a single tapped coil.

The above example is useful for demonstrating the application of the theory developed in Section 5.2.

5.3.2 Example of Narrow Range Local Oscillator for Receiver (in the FM Band); Also Using a Grounded Variable Capacitor

Obviously this can be accomplished by the same circuits described in Section 5.3.1. However, it is desired to demonstrate some additional possible circuits using the Fig. 5.3*a* configuration. This is made possible by the fact that a fixed inductor in parallel with a variable capacitor is equivalent to a variable inductor as shown in Fig. 5.5*a* provided that $X_c > X_L$. By using this fact, the circuits of Fig. 5.5 can be obtained from the configurations of Fig. 5.3*a*.

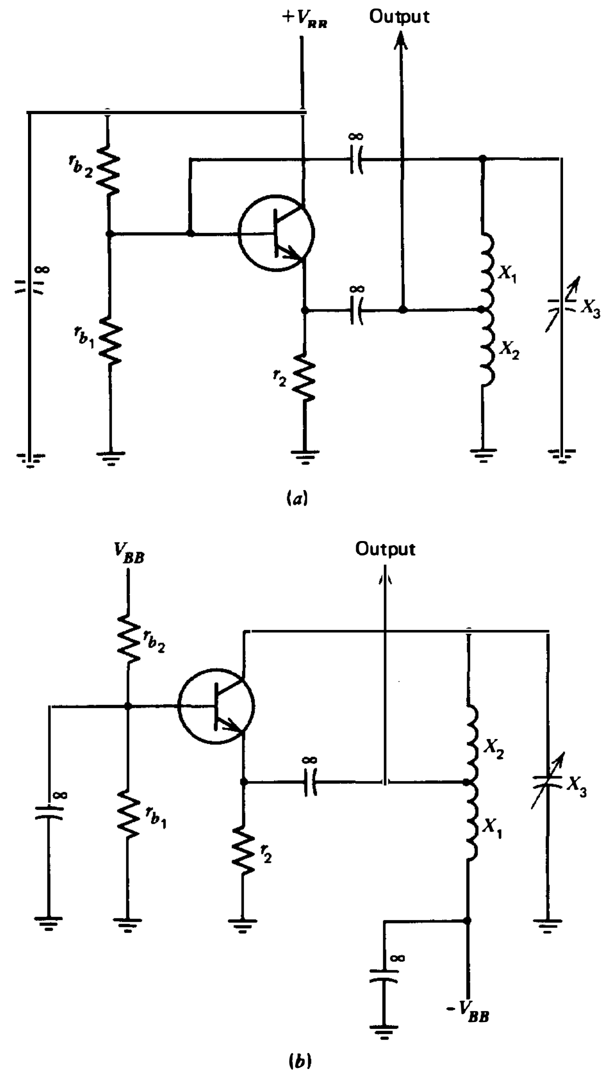


Figure 5.4 Wide-range local oscillators. (*a*) Colpitts (sometimes called Hartley). (*b*) Clapp.

5.3.3 Stable Fixed Frequency *LC* Oscillator

Because it is easier to obtain high Q capacitors than high Q inductors, the choice of oscillator configuration is the Pierce version of Fig. 5.3*a*. The resulting schematic is shown in Fig. 5.6.

For maximum stability, X_1 and X_2 should be small compared to the transistor capacitive reactances. The Q of X_3 should be as high as possible, and the circuit should be designed so that the loaded Q is almost equal to Q_{X_3}.

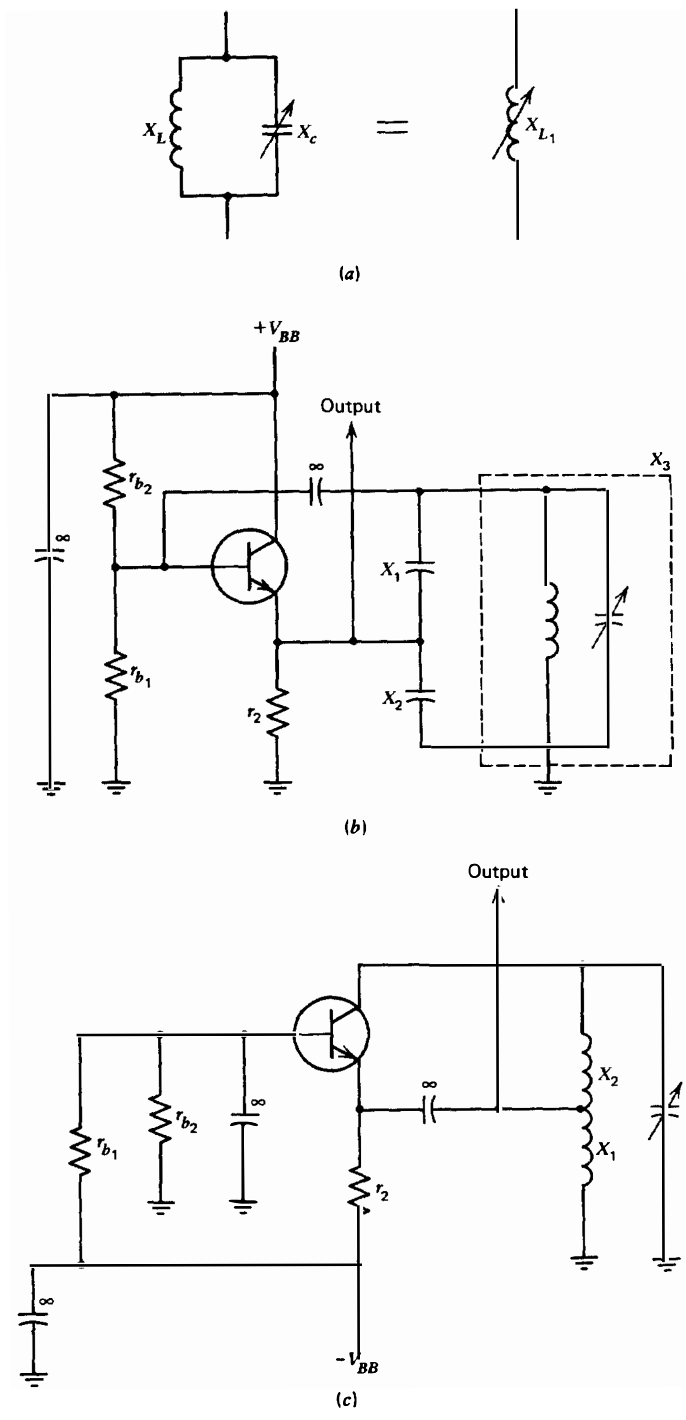


Figure 5.5 Narrow range local oscillators. (*a*) Synthesis of variable inductor. (*b*) Colpitts oscillator. (*c*) Clapp oscillator.

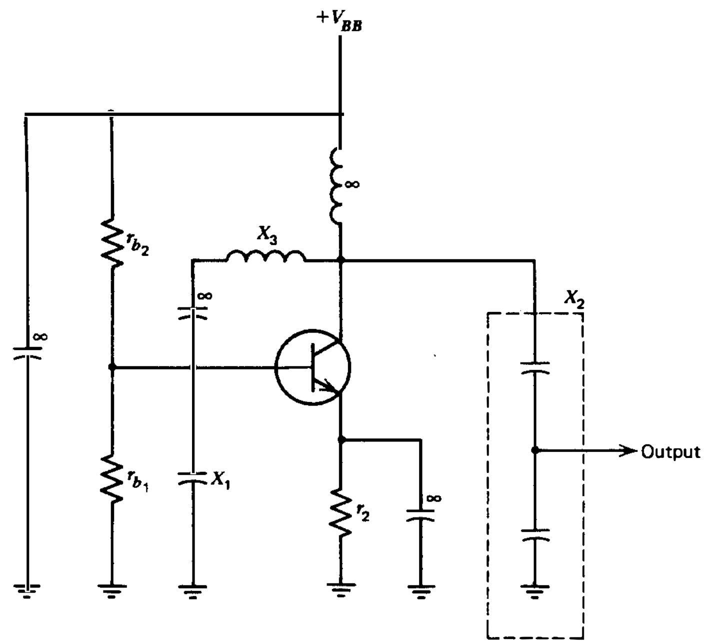


Figure 5.6 Stable Pierce oscillator.

The output power is shown as being extracted from X_2, but occasionally it is extracted from X_3 as in induction welding machinery.

In this connection, it is very important to note that the main factor in determining the stability of the oscillator is the *stability of the components*. In the past, and also at present, excessive attention was and is being paid to maximizing the Q of the oscillator circuit, in the mistaken notion that high Q always results in high stability. It can be experimentally demonstrated that operating Q's as low as 4 can exist in LC oscillators having stabilities better than 10^{-6}. This subject is discussed further in Chapter 17.

5.3.4 Crystal Oscillator with the Crystal Network Operating in the Inductive Region

In this case the crystal can be Z_3 in Fig. 5.3*a* or (Z_1 or Z_2) in Fig. 5.3*c*. In actual practice, the configuration in Fig. 5.3*c* is rarely, if ever, used because of the greater number of inductors and the greater difficulty of supplying the necessary biasing conditions. It is noteworthy, for historical reasons, that the circuit, wherein the crystal is Z_2 in Fig. 5.3*c*, is called the Miller oscillator.

Replacing Z_1, Z_2, and Z_3 in Figs. 5.1 and 5.3*a* with the appropriate components, the circuits of Fig. 5.7 result.

Examining Fig. 5.7, it will be noted that the Clapp circuit is more complicated and is therefore rarely used. The Pierce circuit is more stable since the stray elements to ground are across X_1 and Z_2, which may be relatively low

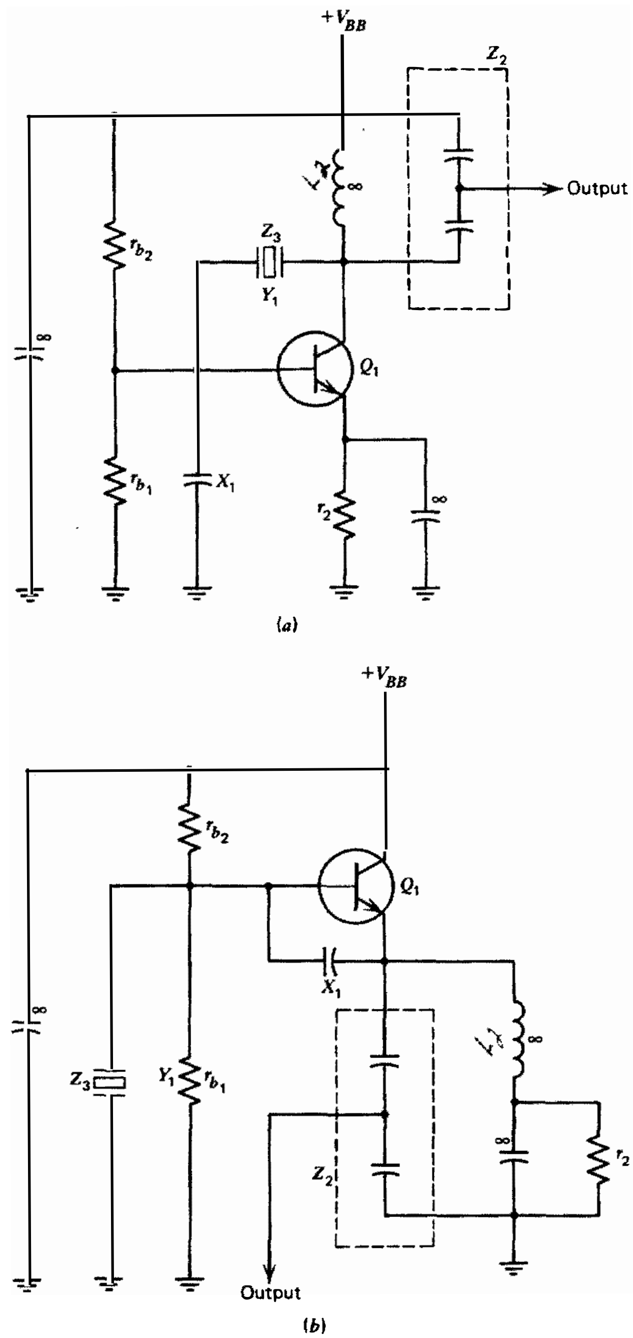


Figure 5.7 Crystal oscillators. (*a*) Pierce. (*b*) Colpitts. (*c*) Clapp with positive power supply. (*d*) Clapp with negative power supply.

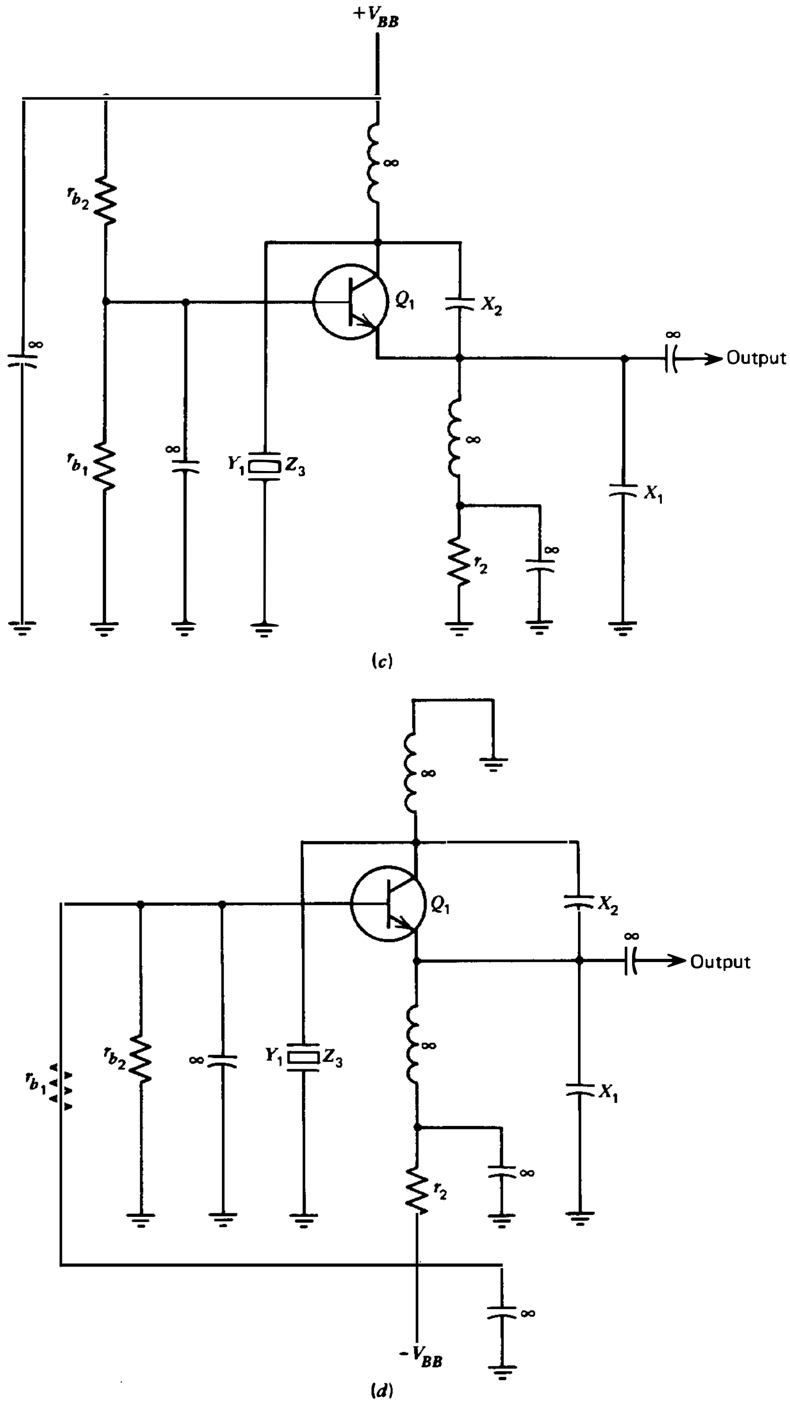


Figure 5.7 (Continued).

impedances, while in the Colpitts circuit the stray elements to ground are across the crystal, which is very sensitive. On the other hand, there are many cases where it is desired to ground one side of the crystal, and for that reason the Colpitts circuit is used. Also, the bias circuit composed of r_{b_1} and r_{b_2}, forming r_b, is across X_1 in the Pierce circuit and therefore usually has a negligible effect. In the Colpitts circuit, r_b is across the crystal which deteriorates the circuit performance. It will also be appreciated that the Colpitts circuit is more difficult to design because of these reasons. An important variation of the Colpitts circuit called the semi-isolated Colpitts oscillator is discussed in detail in Chapter 10.

5.3.5 Crystal Oscillators with the Crystal in the Zero or Negative Reactance Region

Because the $\partial X/\partial f$ of crystals in the inductive region is higher than that in the zero or negative reactance region, oscillators of this design are relatively rare. However, there are some such designs and among those are the following:

1 In VCOs where a low $\partial X/\partial f$ is desirable in order to facilitate the ease of *pushing* the frequency.

2 The standard antiresonant crystals are rated in government and industrial specifications up to about 30 MHz. Above 30 MHz only standard series resonant crystals are available. To use these crystals at their rated frequency, it is necessary for them to operate at zero reactance.

It is evident that when an inductor is connected in series with the crystal, the resulting combination can be considered a crystal operating in the inductive region and the circuits described in Section 5.3.4 can then be used. It is important that the inductor not be made too large, as spurious oscillations may result. In this application, the Pierce circuit usually works best.

5.4 EFFECT OF THE PHASE ANGLE OF g_m IN THE IDEALIZED OSCILLATOR

Repeating Eq. (5.1a)

$$A_{L_0} = \frac{-g_{m_0} Z_1 Z_2}{Z_s}$$

and at steady-state oscillation, making $Z_1 = -jZ_1$, $Z_2 = -jX_2$

$$Z_s = g_m X_1 X_2$$

from which

$$|Z_s| = |g_m| X_1 X_2 \tag{5.8}$$

or

$$|g_m| = \frac{|Z_s|}{X_1 X_2} \tag{5.8a}$$

and

$$\theta_{Z_s} = \theta_{g_m} \equiv \theta \tag{5.9}$$

but

$$\theta_{Z_s} = \cos^{-1} \frac{R_s}{|Z_s|} \tag{5.10}$$

or

$$|Z_s| = \frac{R_s}{\cos \theta} \tag{5.10a}$$

so that Eq. (5.8a) becomes

$$|g_m| = \frac{R_s}{X_1 X_2 \cos \theta} \tag{5.11}$$

Denote the value of g_m, for which $\theta_{g_m} = 0$, as $g_{m(\theta=0)}$ then from Eq. (5.7)

$$g_{m(\theta=0)} = \frac{R_s}{X_1 X_2} \tag{5.12}$$

From Eqs. (5.11) and (5.12)

$$\frac{|g_m|}{g_{m(\theta=0)}} = \frac{1}{\cos \theta} \tag{5.13}$$

which states that the amplitude of g_m must increase in accordance with Eq. (5.13).

For example, if $\theta = 20°$, then

$$|g_m| = 1.064 g_{m(\theta=0)}$$

which is not a very large increase.

The phase angle of g_m will also introduce a frequency shift which can be computed thus:
from Eq. (5.2)

$$Z_s = \{R_3 + j[X_3 - (X_1 + X_2)]\} \tag{5.2a}$$

when $\theta = 0°$, $\quad X_3 = X_1 + X_2.$

If Z_3 is a crystal, the impedance of which is

$$R_1\left(1 + 2jQ_x\frac{\Delta f}{f}\right) \tag{5.14}$$

then for $\theta = 0°$,

$$2R_1Q_x\left|\frac{\Delta f}{f}\right|_{\theta=0} = X_1 + X_2 \tag{5.15}$$

For $\theta \neq 0°$, from Eqs. (5.10a) and (5.2a),

$$\frac{R_1}{\cos\theta} = |Z_s| = \sqrt{R_1^2 + \left(\left(2R_1Q_x\frac{\Delta f}{f} - (X_1 + X_2)\right)\right)^2} \tag{5.16}$$

and, from Eq. (5.15)

$$\frac{R_1}{\cos\theta} = \sqrt{R_1^2 + \left(\frac{2R_1Q_x}{f}(\Delta f - \Delta f_{(\theta=0)})\right)^2} \tag{5.17}$$

Solving Eq. (5.17) for $(\Delta f - \Delta f_{(\theta=0)})/f$

$$\frac{\Delta f - \Delta f_{(\theta=0)}}{f} = \frac{\sqrt{(1/\cos^2\theta_{g_m} - 1)}}{2Q_x} = \frac{\tan\theta_{g_m}}{2Q_x} \tag{5.18}$$

Again for $\theta = 20°$

$$\frac{\Delta f - \Delta f_{(\theta=0)}}{f} = \frac{0.18}{Q_x} \tag{5.19}$$

For example, for $Q_x = 100{,}000$

$\theta_{g_m} = 20°$ will result in a fractional frequency shift of 1.8×10^{-6} (5.20)

In the real oscillator, R_1 should be replaced by $R_1 + (R_T - R_1)$, where R_T is the total circuit loss resistance and Q_x should be replaced by Q_{op}. In that case, the frequency shift will be R_T/R_1 times the shift calculated above.

5.5 THE REAL OSCILLATOR

5.5.1 Introduction

In this section the theory of the real oscillator is developed, taking into account all the losses present in the oscillator circuit. As explained previously, all the circuit configurations and circuit types of this chapter have the same mathematical model. Therefore, it is sufficient to analyze one particular configuration and circuit type knowing, *a priori*, that the theory will apply equally well to all other configurations and types. It is advisable to choose the particular oscillator on the basis of its popularity and its ease of design. The circuit chosen is the Colpitts, shown in Figs. 5.1*a* and 5.3*a*, because of its considerable popularity and its greatest difficulty of design. The theory can be then applied to the other popular types with rather obvious simplifications and additions, as will be done in Chapters 7 to 10.

Figure 5.8 is a schematic diagram of the Colpitts oscillator of the type to be analyzed. Figure 5.9 is the schematic diagram of the ac portion of Fig. 5.8. The differences between Figs. 5.8 and 5.9 are:

1 Figure 5.8 shows the circuit as it is physically constructed. Each component shown on this figure is a physical component and the design problem consists of determining these physical components. Capacitors shown as ∞ are meant to be capacitors, the reactance of which are essentially 0 at the operating frequency f.

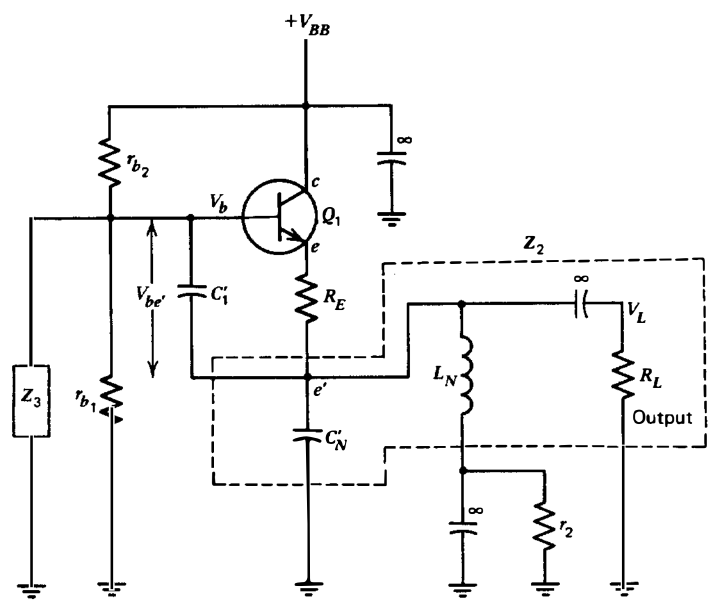
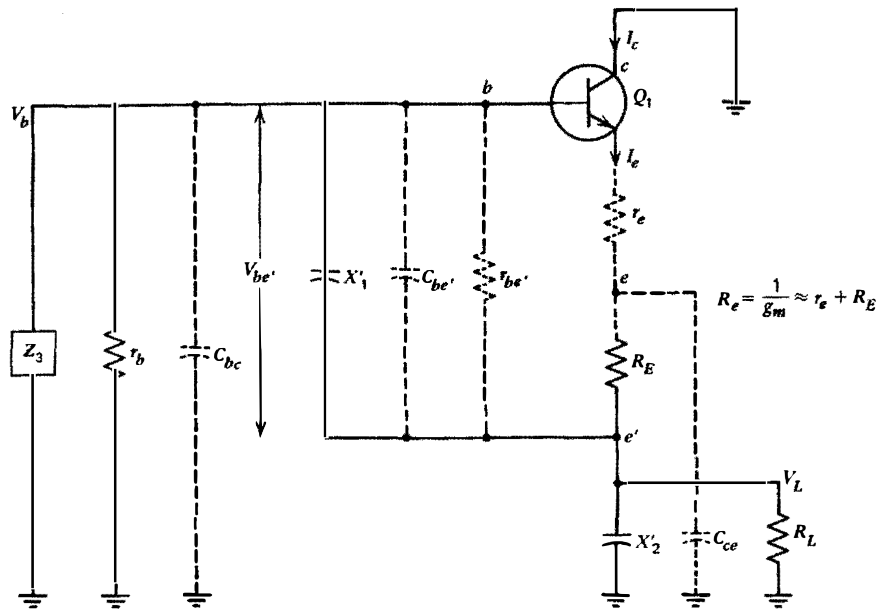
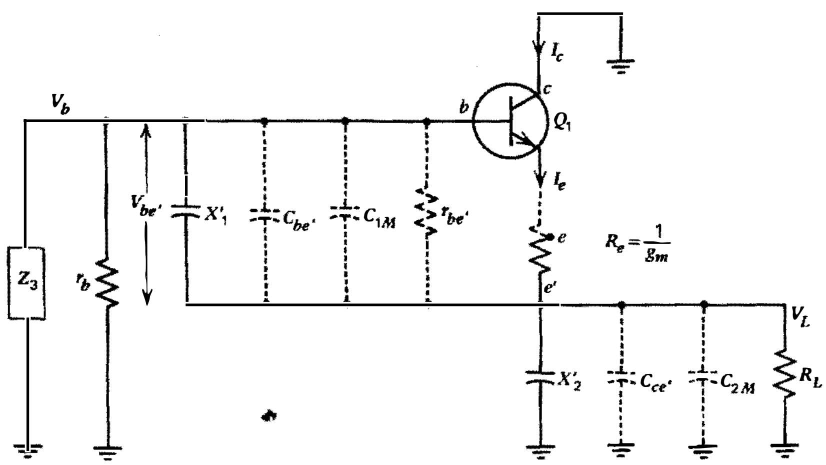


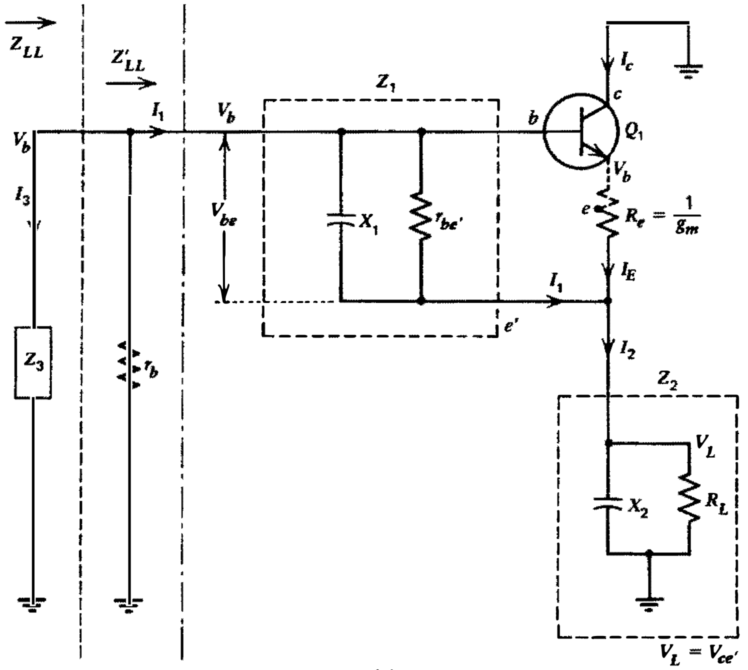
Figure 5.8 Detailed schematic of the Colpitts oscillator.





(b)

Figure 5.9 Colpitts oscillator. (a) Detailed schematic of the ac circuitry. (b) Modified schematic, Step 1. (c) Modified schematic, Step 2.



(c)

Figure 5.9 (Continued).

2 The power supply is not shown in Fig. 5.9.

3 Figure 5.9a includes the parameters of the transistor shown as dashed lines, using the Hybrid-PI model.

4 X_1' in Fig. 5.9 is the reactance of C_1' of Fig. 5.8 at the operating frequency f.

5 X_2' in Fig. 5.9 is the combined reactance of C_N' and L_N of Fig. 5.8 at f, or

$$X_2' = \frac{1}{2\pi f C_N' - 1/2\pi f L_N} \tag{5.21}$$

5.5.2 Assumptions and Conditions

1 The values of all immittances, voltages, and currents are those at the operating frequency f.

2 V_{be} is sinusoidal. (5.22)

3 g_m is real, so that $\theta_{g_m} = 0$. (5.23)

4 $g_m = \dfrac{I_e}{V_{be'}}$ (5.23a)

5 r_{ce} is either infinite or lumped into R_L.

6 β is fairly large, so that $i_C \approx i_E$. (5.24)

7 The effect of $r_{bb'}$ is negligible.

8 r_b is the combination of r_{b_1} and r_{b_2}

$$r_b = \frac{1}{1/r_{b_1} + 1/r_{b_2}} \tag{5.25}$$

5.5.3 Calculation of the Oscillatory Equations of the Circuit of Fig. 5.9*a*

1 The first step is to allocate C_{bc} in Fig. 5.9*a*, to portions in parallel with X_1' and X_2 in the manner described in Section 5.5.4.2.5. This results in Fig. 5.9*b*.

2 The second step is to combine X_1' and X_2' with its parallel capacitors to form X_1 and X_2. This results in Fig. 5.9*c*. Obviously

$$X_1 = \frac{1}{2\pi f\left((2\pi f X_1')^{-1} + C_{be'} + C_{1M}\right)} \tag{5.26}$$

$$X_2 = \frac{1}{2\pi f\left((2\pi f X_2')^{-1} + C_{ce'} + C_{2M}\right)} \tag{5.27}$$

3 In Fig. 5.9*c* we obtain from Eq. (1.8a), assuming $R_L > 5X_2$, $r_{be'} > 5X_1$,

$$Z_1 = \underbrace{\frac{X_1^2}{r_{be'}}}_{R_{\text{in}}} - jX_1 \tag{5.28}$$

$$= R_{\text{in}} + \underline{X}_1 \tag{5.28a}$$

$$Z_2 = \underbrace{\frac{X_2^2}{R_L}}_{R_2} - jX_2 \tag{5.29}$$

$$= R_2 + \underline{X}_2 \tag{5.29a}$$

(See Section 1.2.1.5 for the $\underline{X}$ notation.)

4 Z'_{LL} is now calculated

$$Z'_{LL} = \frac{V_b}{I_1} \tag{5.30}$$

By definition of R_e, R_e and Z_1 are effectively in parallel as shown in Fig. 5.9*c*, where the voltage at the base and intrinsic emitter are both V_b,

$$V_L = V_b \frac{Z_2}{Z_1 \| R_e + Z_2} \tag{5.31}$$

where $Z_1 \| R_e$ denotes the parallel combination of Z_1 and R_e

$$\frac{Z_1 R_e}{Z_1 + R_e} \tag{5.32}$$

By inspection

$$I_1 = \frac{V_b - V_L}{Z_1} \tag{5.33}$$

$$= \frac{V_b}{Z_1}\left[1 - \frac{Z_2}{Z_1 \| R_e + Z_2}\right] \tag{5.34}$$

from Eq. (5.31);

$$= \frac{V_b}{Z_1}\left(\frac{Z_1 \| R_e}{Z_1 \| R_e + Z_2}\right) \tag{5.34a}$$

Therefore, from Eq. (5.30)

$$Z'_{LL} = Z_1\left(1 + \frac{Z_2}{Z_1 \| R_e}\right) \tag{5.35}$$

$$= Z_1 + Z_2 + \frac{Z_1 Z_2}{R_e} \tag{5.35a}$$

from Eq. (5.32)

$$\left.\begin{aligned} &\text{and defining } Z_1 = R_{\text{in}} + jX_1 \\ &\text{for other } Z_n,\ Z_n = R_n + jX_n \end{aligned}\right\} \tag{5.35b}$$

$$Z'_{LL} = (\underline{X}_1 + \underline{X}_2) + R_{\text{in}} + R_2 + g_m(R_{\text{in}} + \underline{X}_1)(R_2 + \underline{X}_2) \tag{5.36}$$

from Eqs. (5.28a) and (5.29a) or

$$Z'_{LL} = \underbrace{(1 + g_m R_{\text{in}})\underline{X}_2 + (1 + g_m R_2)\underline{X}_1}_{\underline{X}_a} + \underbrace{R_{\text{in}} + R_2 + g_m \underline{X}_1 \underline{X}_2 + g_m R_{\text{in}} R_2}_{R_a} \tag{5.36a}$$

and $Z_{LL} = r_b \| Z'_{LL}$ is now calculated to be

$$Z_{LL} \approx \underline{X}_a + R_a + \frac{(\underline{X}_a)^2}{r_b} \tag{5.37}$$

assuming

$$|\underline{X}_a| > 5R_a,$$

$$r_b > 5|X_a|$$

Converting Eq. (5.37) to $Z = R + jX$ notation,

$$Z_{LL} = -j((1 + g_m R_{\text{in}})X_2 + (1 + g_m R_2)X_1) + R_{\text{in}} + R_2 + g_m R_{\text{in}} R_2$$

$$-g_m X_1 X_2 + \frac{[(1 + g_m R_{\text{in}})X_2 + (1 + g_m R_2)X_1]^2}{r_b} \tag{5.37a}$$

Noting that

$$g_m R_{in} \ll 1$$

$$q_m R_2 \ll 1$$

$$X_1 X_2 \gg R_{in} R_2$$

and substituting for R_{in} and R_2 from Eqs. (5.28) and (5.29)

$$Z_{LL} = -j[(1 + g_m R_2)X_1 + (1 + g_m R_{\text{in}})X_2] + \frac{X_1^2}{r_{be'}}$$

$$+ \frac{X_2^2}{R_L} + \frac{(X_1 + X_2)^2}{r_b} - g_m X_1 X_2 \tag{5.38}$$

It is interesting to note that $-g_m X_1 X_2$ may be interpreted as the negative resistance without which oscillations cannot exist.

5 For the circuit in Fig. 5.9*c* to oscillate, Eqs. (1.47) and (1.48) state that

$$Z_3 = R_3 + jX_3 = -Z_{LL} \tag{5.39}$$

Substituting for Z_{LL} and separating the resulting equation into real and imaginary parts

$$R_T = R_3 + \underbrace{\frac{X_1^2}{r_{be'}}}_{R_{\text{in}}} + \underbrace{\frac{X_2^2}{R_L}}_{R_2} + \underbrace{\frac{(X_1 + X_2)^2}{r_b}}_{R_b} = g_m X_1 X_2 \tag{5.40}$$

and

$$X_3 = -\left[(1 + g_m R_2)X_1 + (1 + g_m R_{in})X_2\right] \tag{5.41}$$

Solving for g_m in Eq. (5.40)

$$g_m = \frac{R_T}{X_1 X_2} \tag{5.42}$$

Equation (5.41) becomes, from Eq. (5.42),

$$X_3 = -\left[\left(1 + \frac{R_2 R_T}{X_1 X_2}\right)X_1 + \left(1 + \frac{R_{in} R_T}{X_1 X_2}\right)X_2\right] \tag{5.43}$$

$$= -(X_1 + X_2)\left(1 + \frac{R_T(R_2 X_1 + R_{in} X_2)}{X_1 X_2 (X_1 + X_2)}\right) \tag{5.43a}$$

$$= -(X_1 + X_2)(1 + \varepsilon) \tag{5.44}$$

where

$$\varepsilon = \frac{R_T(R_2 X_1 + R_{in} X_2)}{X_1 X_2 (X_1 + X_2)} \tag{5.45}$$

and is usually less than 0.01, so Eq. (5.43) becomes

$$X_3 \approx -(X_1 + X_2) \tag{5.46}$$

As has been shown previously, g_m is a function of amplitude. Therefore Eq. (5.40) can be used to determine the amplitude of oscillation. Equation (5.46) can be used to determine the frequency of operation and Eq. (5.45) can be used to determine the effect of small variations in the parameters upon the frequency.

5.5.4 Derivation of the Circuit Design Equations

5.5.4.1 Introduction

The derivation is made for the Colpitts circuit of Fig. 5.9*c* because, as previously explained, of its relative popularity and greater difficulty of design. As will be seen later, the derivation is equally valid for all the other circuit types. For experimental verification of the theory presented here, it is preferable to use the Pierce circuit because the e' point is then at ac ground potential and $V_{be'}$, the most important ac voltage in the circuit, can then be easily measured.

The derivation is based upon the concept that the most important quantities are the Z_3 current, I_3, and the output power, P_L. All other circuit parameters are derived from these quantities.

5.5.4.2 Derivation of the Basic Theory

5.5.4.2.1 Calculation of Output Power

The output power is given by

$$1000P_L = \frac{V_L^2}{R_L} \tag{5.47}$$

5.5.4.2.2 Calculation of the Currents

Assuming $Z_3 \ll r_b$,

$$\left(1 + \frac{Z_3}{r_b}\right)I_3 = -I_1 \approx I_3 \tag{5.48}$$

$$V_b = I_3 Z_3 = I_3(R_3 + jX_3) \tag{5.49}$$

and

$$|V_b| = |I_3|\sqrt{R_3^2 + (X_3)^2} \tag{5.50}$$

from Eq. (5.46)

$$= I_3\sqrt{R_3^2 + (X_1 + X_2)^2} \tag{5.50a}$$

$$V_L = V_b - I_1 Z_1 \tag{5.51}$$

from Eq. (5.48)

$$= V_b + I_3 Z_1\left(1 + \frac{Z_3}{r_b}\right) \tag{5.51a}$$

$$V_{be'} = I_1 Z_1 \approx I_3 Z_1 \tag{5.52}$$

and

assuming $|Z_1| \approx |X_1|$

$$|V_{be'}| \approx |I_3||X_1| \tag{5.52a}$$

$$V_L = I_2 Z_2 \tag{5.53}$$

From Eqs. (5.49), (5.51a), and (5.53) one obtains

$$I_2 = \frac{I_3}{Z_2}\left[Z_3 + Z_1\left(1 + \frac{Z_3}{r_b}\right)\right] \tag{5.54}$$

$$I_e = I_2 - I_1 \tag{5.55}$$

$$= \frac{I_3}{Z_2}\left[Z_3 + Z_1\left(1 + \frac{Z_3}{r_b}\right)\right] + I_3\left(1 + \frac{Z_3}{r_b}\right) \tag{5.56}$$

from Eqs. (5.48) and (5.54)

$$= \frac{I_3}{Z_2}\left[Z_1 + Z_2 + Z_3 + \frac{Z_3(Z_1 + Z_2)}{r_b} \right] \tag{5.56a}$$

or from Eq. (5.46) and approximations

$$I_e \approx \frac{I_3}{Z_2}\left[R_{\text{in}} + R_2 + R_3 + \underbrace{\frac{(X_1 + X_2)^2}{r_b}}_{R_b} \right] \tag{5.57}$$

$$\approx \frac{I_3}{Z_2} R_T \tag{5.58}$$

from Eqs. (5.28), (5.29), and (5.40); and

$$\frac{|I_3|}{|I_e|} \approx \frac{|X_2|}{R_T} \equiv a_i \tag{5.58a}$$

assuming $|X_2| \approx |Z_2|$. Also

$$I_2 = \frac{I_e}{R_T}[R_T - Z_2]$$

from Eqs. (5.48), (5.55), and (5.58)

$$= I_e \frac{R_{df} + R_{\text{in}} + R_b - jX_2}{R_T} \tag{5.59}$$

and

$$|I_2| = |I_e| \frac{\sqrt{(R_{df} + R_{\text{in}} + R_b)^2 + X_2^2}}{R_T} \tag{5.59a}$$

5.5.4.2.3 Calculation of X_2

Let

$$R_p = R_3 + R_{\text{in}} + \underbrace{\frac{X_1^2}{r_b}}_{R_{bp}} \tag{5.60}$$

and let

$$\left(R_2 = \frac{X_2^2}{R_L}\right) \to 0 \tag{5.61}$$

Occasionally, Eq. (5.61) is true, r_b and R_p are known, and it is desired to calculate X_2. Then, from Eq. (5.58a)

$$a_i = \frac{X_2}{R_p + (2X_1X_2 + X_2^2)/r_b} \tag{5.62}$$

solving for X_2

$$X_2 \approx \frac{a_i R_p}{1 - 2a_i X_1/r_b} \approx a_i R_p \left[1 + \frac{2a_i X_1}{r_b}\right] \tag{5.63}$$

because $2a_i X_1/r_b \ll 1$.

5.5.4.2.4 Calculation of V_L

$$V_L = I_2 Z_2 = I_3\left[Z_3 + Z_1\left(1 + \frac{Z_3}{r_b}\right)\right] \tag{5.64}$$

from Eq. (5.54) so that

$$V_L = I_3\left[R_3 + R_{\text{in}} + j(X_3 - X_1) + \frac{Z_1 Z_3}{r_b}\right] \tag{5.64a}$$

$$= I_3\left[R_3 + R_{\text{in}} + \frac{X_1(X_1 + X_2)}{r_b} + jX_2\right] \tag{5.65}$$

from Eq. (5.46) and approximations; and

$$V_L = I_3\sqrt{\left(R_{\text{in}} + R_3 + \frac{X_1(X_1 + X_2)}{r_b}\right)^2 + X_2^2} \tag{5.66}$$

so that

$$\frac{|V_b|}{|V_L|} = \frac{\sqrt{R_3^2 + (X_1 + X_2)^2}}{\sqrt{\left(R_{\text{in}} + R_3 + \frac{X_1(X_1 + X_2)}{r_b}\right)^2 + X_2^2}} \tag{5.67}$$

from Eqs. (5.50a) and (5.66). If the resistance components are neglected

$$\frac{|V_b|}{|V_L|} \approx \frac{X_1 + X_2}{X_2} \tag{5.67a}$$

Similarly, from Eqs. (5.52) and (5.56)

$$\frac{|V_{be'}|}{V_L|} = \sqrt{\frac{R_{\text{in}}^2 + X_l^2}{\left(R_{\text{in}} + R_3 + \dfrac{X_1(X_1 + X_2)}{r_b}\right)^2 + X_2^2}} \tag{5.68}$$

$$\approx \frac{X_1}{X_2} \tag{5.68a}$$

when the resistance components are neglected.

5.5.4.2.5 Calculation of the Approximate Contribution of C_{bc}

In Section 5.4.3, C_{cb} was replaced by C_{1M} and C_{2M} as shown in Fig. 5.9*b*. The manner in which C_{1M} and C_{2M} are calculated will be shown in this section.

The general procedure consists of partitioning C_{cb} into C_{1M} and C_{2M}, connected as shown in Fig. 5.10.

The potential at the junction of C_{1M} and C_{2M} is the same as that of the emitter e', V_L. This fact gives the clue as to how C_{1M} and C_{2M} are calculated.

From Fig. 5.10 it is seen that

$$X_{C_{1M}} + X_{C_{2M}} = X_{cb} \tag{5.69}$$

Let

$$\frac{V_{be'}}{V_L} = M_M \tag{5.70}$$

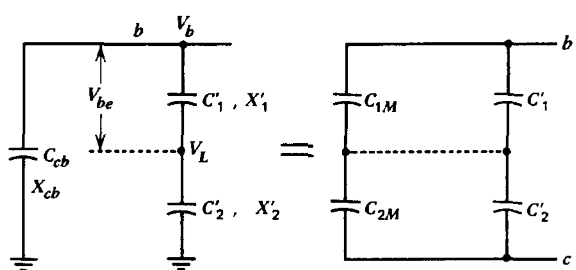


Figure 5.10 Computation of C_{1M} and C_{2M}.

then

$$X_{C_{1M}} = M_M X_{C_{2M}} \tag{5.71}$$

from which

$$X_{C_{2M}} = \frac{X_{C_{cb}}}{1 + M_M} \tag{5.72}$$

and

$$X_{C_{1M}} = \frac{MX_{C_{cb}}}{1 + M_M} = \frac{X_{C_{cb}}}{1 + 1/M_M} \tag{5.73}$$

From Eq. (5.72),

$$C_{2M} = C_{cb}(1 + M_M) \tag{5.74}$$

and from Eq. (5.73),

$$C_{1M} = C_{cb}\left(1 + \frac{1}{M_M}\right) \tag{5.75}$$

The above analysis assumes that $V_{be'}$ and V_L are in phase, which is not strictly true. The computation therefore is only an approximation. Also, the possible absence of phase coherence introduces positive and negative resistive components which may, at times, be significant. Furthermore, the analysis only accounts for the Miller effect contributions of C_{cb} to C_1 and C_2 and it does not consider all of the direct transfer of energy between the collector and the base by C_{cb}.

5.5.5 Calculation of $C_{ce'}$

It is obvious that when $R_E = 0$, $C_{ce'} = C_{ce}$. When R_E is not zero, the potential at point e almost equals the potential at point e', because of the limiting action; therefore, $C_{ce'} \approx C_{ce}$.

5.5.6 Calculation of Oscillator Operating Q, Q_{op}

The circuit Q_{op} due to the loss in Z_3 can be shown to be

$$Q_{Z_3} = \frac{f}{2R_3} \frac{\partial[(X_3 - (X_1 + X_2))]_{\mathrm{OL}}}{\partial f} \tag{5.76}$$

provided that no additional losses exist.

The total circuit Q is deteriorated by the presence of the other circuit losses. The total equivalent circuit resistance is shown to be R_T in Eq. (5.40).

Therefore,

$$Q_{op} = \frac{f}{2R_T} \frac{\partial[X_3 - (X_1 + X_2)]_{OL}}{\partial f} \tag{5.77}$$

For a crystal oscillator, from Eq. (3.28),

$$Q_{op} \approx Q_x \cdot \frac{R_{df}}{R_T} \tag{5.77a}$$

since $\partial(X_1 + X_2)/\partial f \approx 0$ for the crystal operating frequency range.

Equations (5.76) to (5.77a) require clarification.

$[X_3 - (X_1 + X_2)]_{OL}$ signifies the open loop value of $[X_3 - (X_1 + X_2)]$ as measured in a manner similar to that described in Section 1.3.2. When the loop is closed, the circuit functions as the desired oscillator, $[X_3 - (X_1 + X_2)] \to 0$, and therefore its differential also approaches zero. However, the value of $[X_3 - (X_1 + X_2)]_{OL}$ as well as that of R_T must be calculated at the signal amplitudes that exist in the oscillating mode, as these values may be strong functions of the signal amplitudes.

In a well-designed high-performance crystal oscillator, Q_{op} in Eq. (5.77a) is a weak function of *df* since R_{df} is by far the major constituent of R_T.

See also Section 7.7.2.

5.6 THE THEORY APPLIED TO CRYSTAL OSCILLATORS

5.6.1 Introduction

All the theory developed in this chapter applies completely to the crystal oscillator. In this case, Z_3 is the crystal resonator network, I_3 is the crystal current I_x, and R_3 is the effective crystal resistance R_{df}. However, the theory requires several additions to include the circuit modifications necessary to ensure that the circuit operates at the crystal correct overtone and mode.

At this point, it cannot be stressed too highly that the circuit designer should have a full knowledge of the various crystal responses and their magnitude, gathered from measurements on a representative sampling of the entire production run of the crystals, before proceeding with the design of the circuit for that run of crystals. Too often circuits have been unnecessarily complicated by assuming responses that, in fact, do not exist. For example, normally one would expect crystals to have a stronger response at the fundamental frequency than at the third overtone; so the designer includes a third overtone selector circuit. However, it is possible to build crystals which have much stronger third overtone than fundamental responses. If the designer had investigated and discovered that this condition exists for the run of the crystals to be used, the third overtone mode selector could have been eliminated, with the consequent improvement in performance, reduction in cost, and greater reliability.

The approach adopted in this book is that the circuit is designed for frequency, frequency stability, power output, crystal resistance, and crystal power, all of which are prespecified.

5.6.2 Crystal Fundamental Wide Frequency Range Oscillator Circuits

These oscillators may be the Pierce and Colpitts circuits shown in Figs. 5.7*a* and 5.7*b*. However, oscillators using the circuits over wide frequency ranges will have wide variations in frequency stability, power output, and crystal power drive. If the designer considers these variations acceptable, he should design the circuits at several extreme conditions, for example, lowest and highest frequency, lowest and highest power output, lowest and highest crystal resistance, and come up with a compromise circuit. The theory in this book will help the designer predict the oscillator behavior under all these varying conditions.

In view of the fact that wide range fundamental oscillators are in substantial use, some observations will therefore be made about them.

In the circuits of Fig. 5.7 the loop gain, for a constant crystal resistance

$$A_L \propto \frac{1}{f^2} \tag{5.78}$$

in accordance with Eq. (5.40), since both

$$X_1, X_2 \propto \frac{1}{f} \tag{5.79}$$

Therefore, the circuit will oscillate at the lowest frequency which will provide an R_T sufficient to satisfy Eq. (5.40). It is recommended that a margin of safety of 3 be provided; for example,

$$\frac{R_{T_1}}{R_{T_N}} = \frac{3}{N^2} \tag{5.80}$$

where N is the overtone order.

For example, R_T at the fundamental should not exceed $(1/3)R_T$ at the third overtone, $3R_T/25$ at the fifth overtone, and so on. Also, L_1 in Fig. 5.7 should be large enough so that its reactance will be higher than the capacitive reactance it parallels at the fundamental frequency.

5.6.3 Fundamental Oscillator Circuits with Prespecified Parameters and Performance Objectives

The design of this oscillator is fully covered by the theory already presented.

5.6.4 Overtone Oscillator Circuits (See Chapter 3)

As described in Sections 5.6.1 and 5.6.2, the oscillator will tend to operate at the crystal fundamental frequency unless steps are taken to prevent it by providing an overtone selector circuit. The selector may take one of two forms.

1 If the third overtone is desired, the selector may be a series resonant circuit tuned to the fundamental and connected between the base and emitter or the collector and emitter. This type of selector is much less desirable than type 2 since, from a practical point of view, it will require more components and since it is useful only for rejecting the fundamental.
2 The second type of selector is a parallel circuit, tuned to a frequency below f_N, the desired overtone, and above the next lower overtone and connected between base and emitter or collector and emitter. Usually, it is connected between collector and emitter because, in practice, it requires fewer additional components. The following analysis will therefore be made on the basis of the connection of the parallel tuned circuit between collector and emitter, but it should be noted that exactly the same analysis applies to the other connection.

Section 5.2 and Fig. 5.3*a* show that if X_1 is capacitive, X_2 must also be capacitive. If X_2 is inductive, then oscillations cannot take place. Suppose X_2 is the parallel tuned circuit shown in Fig. 5.11*a*, the reactance characteristics of which are shown in Fig. 5.11*b*. In Fig. 5.11*b*, X is inductive from $f \to 0$ to $f = f_s$. Above f_s, X is capacitive and oscillations will then be possible.

If f_s is set below f_N, which is the desired overtone, and above f_{N-2}, which is the nearest lower overtone, then oscillations can take place at f_N and above, provided the loop gain is sufficient. It will also be noted from Fig. 5.11*b* that above f_s $|X|$ continually decreases as f increases, thus decreasing the loop gain.

Figures 5.7*a* and 5.7*b* show oscillators originally designed for fundamental operation. To convert them to overtone operation, all that is necessary is to adjust the values of L_2 and the capacitors so that it resonates at f_s and has the value X_2 at f_N. To aid in calculating the necessary values of L_N and C_N, the following relations have been formulated: let

$$s = \frac{f_s}{f_N} \tag{5.81}$$

For $N = 1$, fundamental operation,

$$s^2 = 0.2 \tag{5.82a}$$

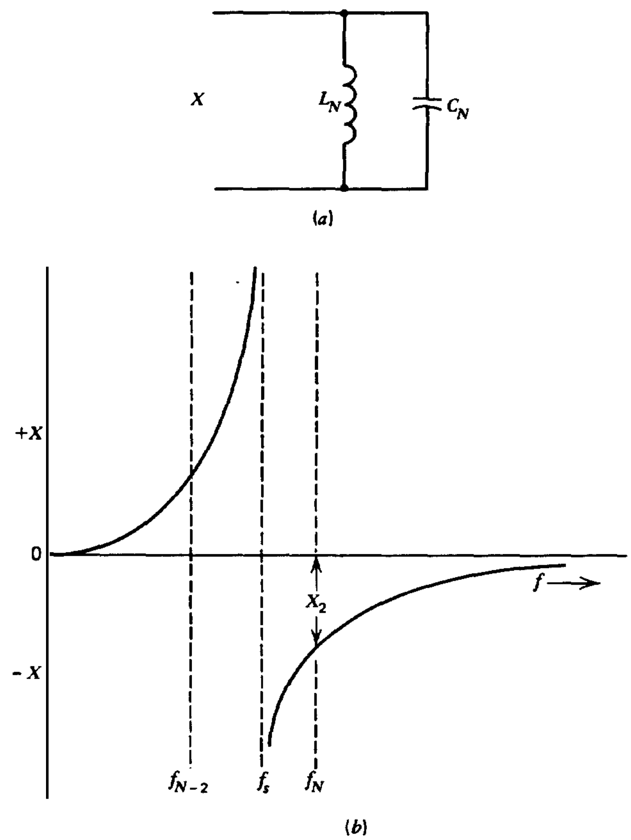


Figure 5.11 Reactance of parallel tuned circuit. (*a*) Schematic. (*b*) Reactance versus f.

For $N \neq 1$, overtone operation,

$$s = 1 - \frac{1.5}{N} \tag{5.82b}$$

$$X_{C_N} = (1 - s^2) X_2 \tag{5.83}$$

$$C_N = \frac{159{,}000}{X_{C_N} f}, \qquad C \text{ in pF}, f \text{ in MHz} \tag{5.84}$$

$$X_{L_N} = \frac{X_{C_N}}{s^2} \tag{5.85}$$

$$L_N = \frac{X_{L_N}}{2\pi f}, \qquad L_N \text{ in } \mu\text{H}, \tag{5.85a}$$

It should be noted that for fundamental operation L_N acts only as a dc path, and f_s has been chosen to make L_N a reasonably small value. Also, f_s has been so fixed by Eq. (5.82) that it is close to f_{N-2} than to f_N. This is done to reduce the value of $\partial X/\partial f$ at f_N in order to increase the stability of this circuit.

When the crystal cut is doubly rotated such as the SC-cut, additional modes are present, as described in Chapter 3. This requires that s of Eq. (5.82) be selected so that f_s is comfortably above the nearest lower strong mode. In Section 5.6.5.2, it will be shown that, for the third and fifth overtone SC, s should be 0.75. Therefore, Eq. (5.82b) should be modified to, for SC cuts,

$$S = 0.75 \qquad \text{for } N = 3, 5 \tag{5.82c}$$

5.6.5 Crystal Mode Selection Circuits (See Chapter 3)

Assuming $f_b/f_c = 1.088$.

5.6.5.1 Introduction

As mentioned in Section 5.6.4, doubly rotated crystals have additional strong modes. When these crystals are used, the circuit must be modified so that oscillations at any of these additional undesired modes do not take place. The treatment now being developed applies to the SC-cut crystal at the first, third, and fifth overtones, but the necessary modifications to the treatment for cuts having different modes will be obvious to the reader.

5.6.5.2 Basic Requirements

It can be seen from Fig. 3.17 that, for third overtone operation, the desired mode is $c3$ at f_{c3}. Adjacent are modes $a1$ at $f_{a1} \approx 0.63 f_{c3}$ and $b3$ at $f_{b3} \approx 1.088 f_{c3}$. Similarly for fifth overtone operation, the desired mode is $c5$ at f_{c5}. Adjacent are modes $b3$ at $f_{b3} \approx 0.64 f_{c5}$ and $b5$ at $f_{b5} \approx 1.088 f_{c5}$.

Therefore, for both third overtone and fifth overtone operation, the circuit should be designed so that oscillations can take place only between $0.75 f_c$ and $1.088 f_c$. It is not difficult to suppress the oscillation below $0.75 f_c$ as discussed in Section 5.5.3, but it is much more difficult to suppress the oscillation above f_c because the f_b nearest strong response, which is very often stronger than the f_c response, is so close to f_c. To be safe, it might be desirable to make the upper limit of oscillation $1.06 f$ rather than $1.088 f_c$, but this will result in sharper slope of the reactance curve and greater instability.

In Section 5.6.3 it was shown that the circuitry to suppress the oscillations below f_c by choosing a proper value of s, which, as stated above, should be 0.75. Additional circuitry is necessary to suppress the oscillation above f_c. The additional circuitry may take several different forms. One form is shown in Fig. 5.12.

In Section 5.2 it was shown that X_1 and X_2 must both be inductive or capacitive in order to oscillate. If X_2 takes the form shown in Fig. 5.12a and X_1 takes the form of Fig. 5.12b, then Fig. 5.12c shows that X_1 and X_2 are never

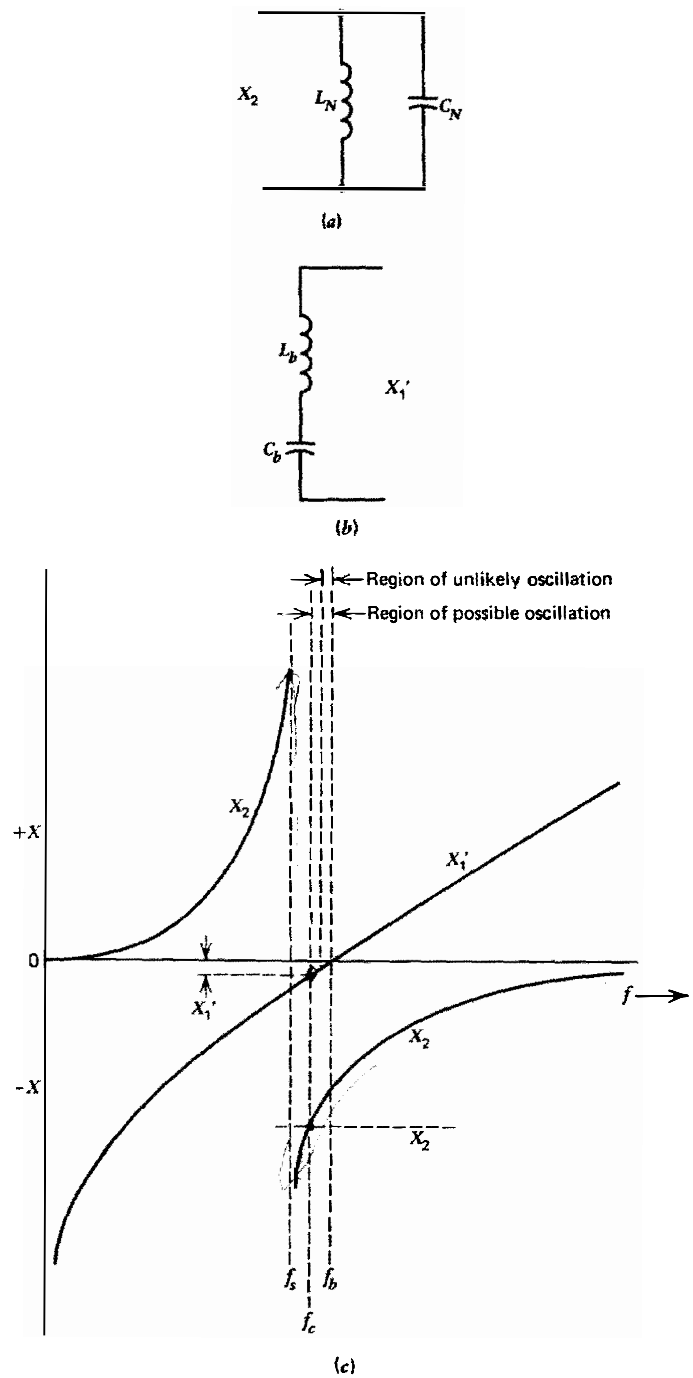


Figure 5.12 Two network type mode selection circuit. (*a*) Schematic of X_2. (*b*) Schematic of X_1'. (*c*) Reactance versus f of X_2 and X_1'.

both inductive, but are both capacitive within a narrow frequency region. Therefore, oscillations will take place within that region provided the loop gain is sufficiently high.

5.6.5.3 The Design of X_1

The design of X_2 was described in Section 5.6.4. The design of X_1' is now given for the SC-cut crystal, where $f_b = 1.088 f_c$.

$$C_b = 0.156 C_1' \tag{5.86}$$

where C_1' is the capacity of X_1' in Fig. 5.9*a*.

$$L_b = \frac{10^6}{C_b (6.84 f)^2} \quad \mu\text{H} \tag{5.87}$$

It will be noted that C_b is about one-sixth of C', and sometimes it is so small that it is not practical. In that case, if X_2' is much smaller than X_1', the networks are reversed. Often, this solution also is not practical, particularly for low crystal currents. In that case the high-reactance element remains a single capacitor and the low-reactance element is replaced by the networks shown in Fig. 5.13, which provide the combined rejection functions of X_1 and X_2 of Fig. 5.12.

5.6.5.4 The Design of the Single Overtone and Mode Selector Circuit

In the cases where this circuit is used, it usually takes the place of X_2 or the equivalent C_N and L_N in Fig. 5.8.

The reactance curves of Fig. 5.13*c* apply to both the networks of Fig. 5.13*a* and 5.13*b*. The choice of which network to use depends upon the practicability of the components making up the network.

Since X_1 is a capacitive reactance, Fig. 5.13*c* shows that oscillation is possible only in the frequency region between f_s and f_c because of the low value of X_2 which causes a low loop gain above f_c.

1 The design of network form 1,

$$X_{L_N} = \frac{(f/f_s)^2 - 1}{(f/f_b)^2 - 1} X_2 \tag{5.88}$$

or

$$X_{L_N} = -5.01 X_2 \tag{5.88a}$$

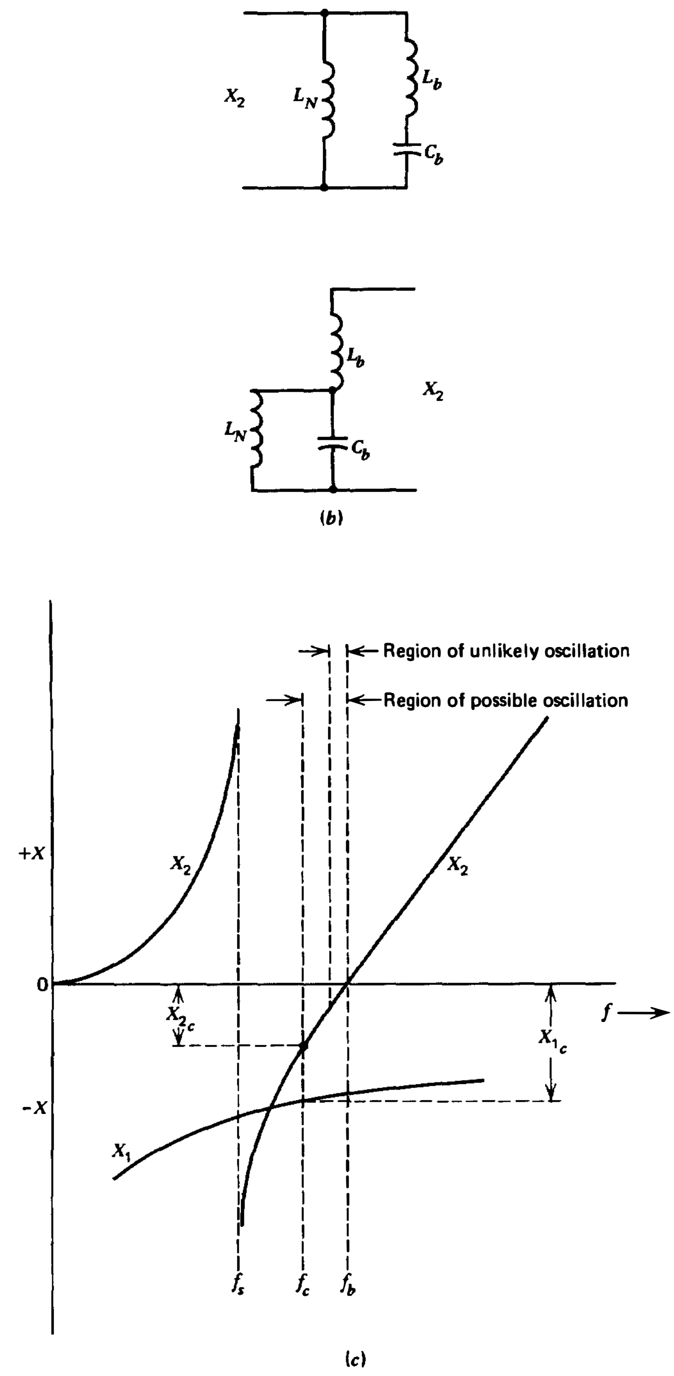


Figure 5.13 The single network overtone and mode selector (a) Schematic of form 1 network. (b) Schematic of form 2 network. (c) Reactance versus f.

for

$$\frac{f_s}{f} = 0.75, \qquad \frac{f_b}{f_c} = 1.088, \qquad f_c = f \tag{5.89}$$

and

$$L_N = \frac{X_{L_N}}{2\pi f} \tag{5.90}$$

$$L_b = \frac{L_N}{(f_b/f_s)^2 - 1} \tag{5.91}$$

$$= 0.905 L_N \tag{5.91a}$$

for the same conditions.

$$X_{C_b} = \left(\frac{f_b}{f_c}\right)^2 X_{L_b} \tag{5.92}$$

$$= 5.37 X_2 \tag{5.92a}$$

for the same conditions and

$$C_b = \frac{159{,}000}{X_{C_b} f} \tag{5.93}$$

2 The design of network form 2,

$$X_{L_N} = \frac{X_2}{\left(1 - (f_c/f_s)^2\right)^{-1} + \left((f_b/f_s)^2 - 1\right)^{-1}} \tag{5.94}$$

$$= -2.629 X_2 \tag{5.94a}$$

for the conditions of Eq. (5.89) and

$$L_N = \frac{X_{L_N}}{2\pi f} \tag{5.95}$$

$$L_b = \frac{L_N}{(f_b/f_s)^2 - 1} \tag{5.96}$$

$$= 0.905 L_N \tag{5.96a}$$

Table 5.1 Mode Selection Networks for $f_b/f_c = 1.088$

Network Form	$\frac{X_{L_N}}{X_2}$	$\frac{L_b}{L_N}$	$\frac{X_{C_b}}{X_2}$
1	−5.01	0.905	5.37
2	−2.62	0.905	1.478

for the same conditions

$$X_{C_b} = -X_{L_N}\left(\frac{f_s}{f_c}\right)^2 \tag{5.97}$$

$$= 1.478X_2 \tag{5.97a}$$

for the same conditions, and

$$C_b = \frac{159{,}000}{X_{C_b}f} \tag{5.98}$$

3 Comparison of network forms 1 and 2.

From Table 5.1 it will be noted that components of network form 2 are of considerably lower impedance levels, especially the capacitor. This can be very useful in improving the stability of many circuits. As a rule, network form 1 is preferable when X_{C_2} and/or f are low, while network form 2 is preferable when X_{C_2} and/or f are high.

It is very important to note that the form 1 and form 2 networks are valid only when $C_2' \gg C_{ce'} + C_{2M}$, in Fig. 5.9*b*. This is usually true for C_{ce}. However, there are circuits where C_{2M}, which is the Miller effect capacitance, is comparable in value to C_2': see, for example, Sections 10.2.3 and 10.2.4, and Design Example 10.1 in Chapter 10. In those circuits, the form 1 network may still be used provided $C_{ce'} + C_{2M}$ is lumped into L_N at f_c. The form 2 network should not be used as it is too complicated to compensate for large values of C_{2M}. In any event, oscillator circuits having large values of C_{2M} should be avoided as they tend to be unstable and difficult to manufacture, as pointed out in Chapter 10.

5.7 LOAD IMPEDANCE TRANSFORMATION

Section 1.4.2(2) states that one important function of the oscillator is to transform the oscillator load impedance, R_L, into the actual load impedance,

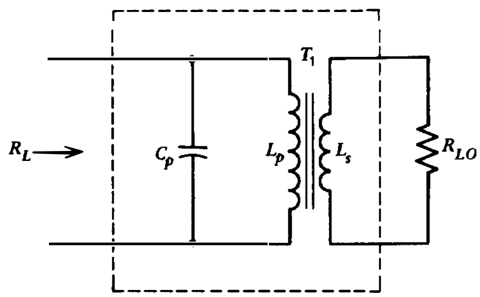


Figure 5.14 Transformer coupling network.

R_{LO}, that the oscillator will feed. In this section, some circuits for the transformation are briefly described.

5.7.1 The Transformer Coupling Network

Figure 5.14 shows the transformer coupling network. T_1 is the main component and usually consists of a toroid having a turns ratio

$$n \approx \sqrt{\frac{R_L}{R_{LO}}} \tag{5.99}$$

Care must be taken that the windings are tightly coupled, otherwise Eq. (5.99) will be incorrect.

$$2\pi f L_p \text{ is usually made about } R_L/10 \tag{5.100}$$

producing a working Q of about 10

$$C_p = \frac{25{,}300}{f^2 L_p} \tag{5.101}$$

5.7.2 Capacitive Divider Coupling Networks

The network is shown in Fig. 5.15, where

$$\frac{R_L}{R_{LO}} \approx \left(\frac{C_1 + C_2}{C_1}\right)^2 \tag{5.102}$$

$$L_p = \frac{25{,}300}{f^2 C_1 C_2/(C_1 + C_2)} \tag{5.103}$$

$$\frac{159{,}000}{fC_2} < \frac{R_{LO}}{5} \tag{5.104}$$

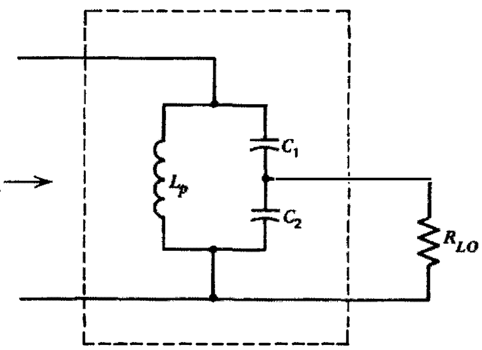


Figure 5.15 Capacitive divider coupling network.

5.7.3 Capacitive Coupling Network

This network is the simplest and is shown in Fig. 5.16.

$$\frac{159{,}000}{fC_1} = \sqrt{R_L R_{LO}} \tag{5.105}$$

$$\frac{159{,}000}{fC_1} > 5R_{LO} \tag{5.106}$$

$$L \approx \frac{25{,}300}{f^2 C_1} \tag{5.107}$$

It has the disadvantage that it increases the harmonic content of the output.

A similar circuit is where C_1 and L are interchanged. This circuit has the advantage that the output harmonic content is reduced.

5.7.4 PI Coupling Network (See Fig. 5.17)

The simplified approximate design is

$$\frac{R_L}{R_{LO}} = \left(\frac{C_2}{C_1}\right)^2 \tag{5.108}$$

$$L = \frac{25{,}300}{f^2 C_1 C_2/(C_1 + C_2)} \tag{5.109}$$

$$\frac{159{,}000}{fC_2} < \frac{R_{LO}}{5} \tag{5.110}$$

This circuit has the advantage that it reduces the harmonic content.

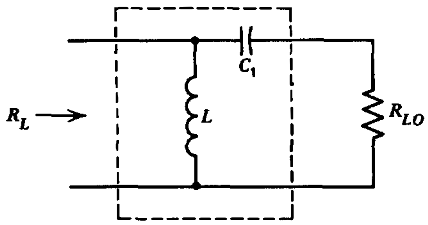


Figure 5.16 Capacitive coupling network.

5.7.5 General Remarks

In the above circuits X is in Ω, L is in μH, C is in pF, and f is in MHz. Many of the components in the above circuits may be combined with components already in the oscillator circuitry so that the coupling networks will actually require fewer components than shown.

5.8 f_T AND β_o REQUIREMENTS

In Section 5.5.2, assumption and condition 6 states that the design in this chapter is based upon the condition that the effect of $r_{bb'}$ is negligible. This condition imposes values for f_T and β_o.

5.8.1 Minimum Value of f_T

It is easily shown from Eqs. (2.63) and (2.75) that $X_{C_{be'}}$, which is the reactance of $C_{be'}$ in Fig. 5.9a, is

$$X_{C_{be'}} = \frac{f_T}{g_m f} \tag{5.111}$$

Assuming that, for this discussion,

$$C_{bed} \gg C_{bet}$$

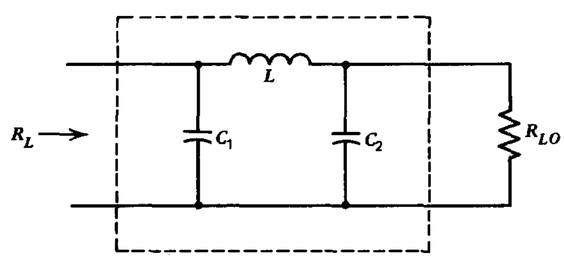


Figure 5.17 PI coupling network.

if

$$X_{C_{be}} > 10_{r_{bb'}} \tag{5.112}$$

$r_{bb'}$ will have a negligible effect.

Assuming that $r_{bb'} = 60\ \Omega$, which is pessimistic, and solving for f_T

$$f_T > 600 g_m f \tag{5.113}$$

5.8.2 Minimum Value of β_o

Again, it is easily shown for $r_{be'}$ in Fig. 5.9*a*

$$r_{be'} = \frac{\beta_o}{g_m} \tag{5.114}$$

if

$$r_{be'} > 20 r_{bb'} \tag{5.115}$$

$r_{bb'}$ will have a negligible effect.

Solving for β_o and assuming $r_{bb'} = 60\ \Omega$

$$\beta_o > 1200 g_m \tag{5.116}$$

It should be noted that if β_o does not satisfy Eq. (5.116), it can be compensated for by a relationship similar to Eq. (2.53).

$$g_m = \frac{1}{1/g'_m + r_{bb'}/\beta_o} \tag{5.117}$$

or else it can be compensated for in the trimming procedure, if necessary.

5.9 FREQUENCY STABILITY ANALYSIS

5.9.1 Introduction

As stated in Eq. (5.46), the oscillator will operate at the frequency at which, to a first approximation,

$$X_3 = -(X_1 + X_2) \tag{5.46}$$

Any factor that causes one of the quantities in Eq. (5.46) to change will cause a change in frequency. It is therefore appropriate to investigate more closely what these quantities are and what may cause frequency changes. For the purpose of analysis

$$X_3 = F(f) \tag{5.118}$$

and must be described as invariable with respect to all parameters except f. X_3 may also have an inherent frequency, as defined at specified conditions. The overall circuit frequency stability is deteriorated by X_1 and X_2, and one of the major goals of good circuit design procedure is to minimize this deterioration.

From Eqs. (5.46) and (5.118) one obtains

$$\Delta f = \frac{-\Delta X_1 - \Delta X_2}{dX_3/df} \tag{5.119}$$

$$= \frac{-\Delta(X_1 + X_2)}{dX_3/df} \tag{5.119a}$$

also,

$$\Delta X_n = \sum_{m=1}^{m} \frac{\partial X_n}{\partial U_m} \Delta U_m \tag{5.120}$$

where $m = 1, 2$, and so on (5.121)
and where U_m could be any parameter, such as temperature, frequency, time, voltage, and so on.

X_1 and X_2 are contributed by the circuit and are determined by the power output, power in Z_3, power conversion efficiency, transistor characteristics, and output load characteristics. In general, strong efforts are made to make X_1 and X_2 small so that the transistor and load will make only small contributions to the total X_1 and X_2 and, therefore, can produce only small variations in X_1 and X_2. However, there are limits as to how small X_1 and X_3 can be made. In addition, X_3 according to Eq. (5.46) also becomes smaller as $X_1 + X_2$ decreases and this becomes undesirable.

5.9.2 The Stabilizing Capacitor C_L'

In order to reduce the effect of $\Delta(X_1 + X_2)$ on the frequency, a stabilizing capacitor is inserted as shown in Fig. 5.18. This capacitor has the following characteristics:

1 It is of the most stable type.
2 Its value is as small as is consistent with the stray elements across it, which should be negligible.
3 Its value is that required to make X_3 the correct value.

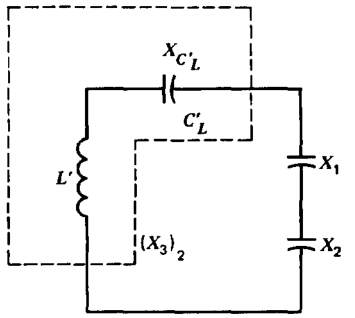


Figure 5.18 Circuit with stabilizing capacitor C'_L.

Equation (5.46) now becomes

$$(X_3)_2 = X'_3 + X_{C'_L} = -(X_1 + X_2) \tag{5.122}$$

where

$$X'_3 = X_3 - X_{C'_L} \tag{5.123}$$

and Eq. (5.119a) becomes

$$\Delta f = \frac{-\Delta(X_1 + X_2)}{d(X'_3 + X_{C'_L})/df} \tag{5.124}$$

If $(X_3)_1$ is an inductor, L_3, then

$$\frac{d(X_3)_1}{df} = (2\pi L_3) = \frac{-(X_1 + X_2)}{f} \tag{5.125}$$

If $(X_3)_2$ is the network of Fig. 5.18,

$$\frac{d(X_3)_2}{df} = 2\pi L'_3 + \frac{1}{2\pi C'_L f^2}$$

$$= \frac{-(X_1 + X_2 + 2X_{C'_L})}{f} \tag{5.126}$$

from Eq. (5.122) and it will be noted that

$$a_{dx} = \frac{d(X_3)_1/df}{d(X_3)_2/df} = 1 + \frac{2X_{C'_L}}{X_1 + X_2} \tag{5.127}$$

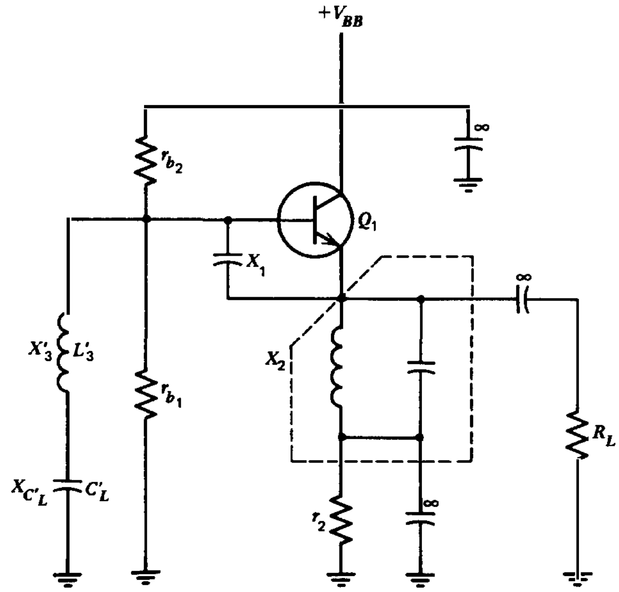


Figure 5.19 The Clapp–Gouriet oscillator (Colpitts version).

which means that

$$\frac{\Delta f_2}{\Delta f_1} = \frac{1}{a_{dx}} \tag{5.128}$$

which may be a substantial improvement in the stability.

This technique of reducing Δf is used in the Clapp–Gouriet oscillator shown in Fig. 5.19.

If X_3 is a crystal, C'_L has no significant effect in stabilizing the frequency, but it does control the operating reactance of the crystal as determined by Eq. (5.122). The calculation of the required value of C'_L is presented in Section 7.2.6.4.

5.9.3 Stability Analysis in Crystal Oscillators

From Equation (3.19)

$$\frac{dX_3}{df} \approx \frac{1}{\pi C_1 f^2} = \frac{2X_{C_1}}{f} \tag{5.129}$$

so that

$$\frac{\Delta f}{f} = \frac{-\Delta(X_1 + X_2)}{2X_{C_1}} \tag{5.130}$$

so that the crystal C_1 should be minimum in order for the circuit to have minimum effect upon the frequency stability.

It should be noted that Eq. (5.129) is only approximate since it does not take into account the effect of C_0, which increases the value of dX_3/df as f approaches the crystal antiresonant frequency f_a. One can take advantage of this effect, but it is unwise to operate too close to f_a because X_3 then becomes a critical function of the stray circuit elements across the crystal.

5.9.4 Causes of Variations in X_1 and X_2

Figure 5.9*b* shows the composition of X_1 and X_2. In this figure, the components, shown in solid lines, are physical wired-in components, while those shown in dashed lines are contributions of the transistor.

Variations in the capacitative elements will of course produce directly variations in X_1 and X_2, while variations in the resistive elements will indirectly produce variations in X_1 or X_2 in accordance with Eq. (1.8). One cause of variation is aging.

Obviously, all the components are temperature sensitive, and temperature changes will produce changes in the component values. Some of the components, particularly the transistor capacitances, are voltage sensitive so that changes in power supply voltages will produce changes in these components.

Again $C_{be'}$ and $r_{be'}$ are transistor current sensitive, so that changes in the transistor characteristic or changes in the power supply which cause the transistor current to change will also cause $C_{be'}$ and $r_{be'}$ to change. Another aspect of this effect is discussed in Section 7.3.2.

R_L is determined by conditions external to the oscillator. If R_L is changed for any reason, X_2 will change, which in turn will cause a frequency shift, Δf. The magnitude of Δf is determined by ΔR_L and the rest of the circuit. $\Delta f/f$ for a specified ΔR_L is one measure of the goodness of the oscillator and is sometimes called the oscillator isolation.