Appendix J

Mathematical Development of the Sideband Level versus Phase Deviation Equation

The frequency spectrum of a phase or frequency modulated signal is well known and will not be derived here. A good treatment of the subject appears in several texts.[42]

The general form of the solution is

$$e = J_0(\delta) E_c \sin \omega_c t + J_1(\delta) E_c [\sin(\omega_c + \omega_m) t - \sin(\omega_c - \omega_m) t]$$
$$+ J_2(\delta) E_c [\sin(\omega_c + 2\omega_m) t + \sin(\omega_c + 2\omega_m) t] \cdots \quad \text{(J-1)}$$

where

e = resultant modulated signal,
E_c = peak unmodulated carrier voltage,
$J_n(\delta)$ = Bessel function of the first kind and of order n,
δ = deviation ratio (for frequency modulation, $\delta = f_d/f_m$; for phase modulation, δ is the peak phase deviation),
ω_c = carrier angular frequency,
ω_m = angular frequency of modulation $\omega_m = 2\pi f_m$, and
f_d = peak frequency deviation in hertz.

Bessel functions of the first kind are given by the infinite series,

$$J_n(\delta) = \frac{\delta^n}{2^n n!} \left[1 - \frac{\delta^2}{2(2n+2)} + \frac{\delta^4}{2(4)(2n+2)(2n+4)} - \frac{\delta^6}{2(4)(6)(2n+2)(2n+4)(2n+6)} + \cdots \right]. \quad \text{(J-2)}$$

For small phase deviations ($\delta \ll 1$), only the carrier and first sideband pair are significant. The ratio of their amplitudes is given by $J_1(\delta)/J_0(\delta)$. From equation

(J-2) it can be seen that for small $\delta, J_0(\delta) \doteq 1$ and $J_1(\delta) \doteq \delta/2$. The relative sideband level is then given by $\delta/2$, where δ is in radians.

The data for Figure 4-2 is given in decibels and degrees; therefore, it is necessary to modify the result, giving the equation

$$\frac{J_1(\delta)}{J_0(\delta)} = 20 \log \frac{\theta}{2(57.3)} \text{ dB} \tag{J-3}$$

where θ is in degrees.