Appendix H

Large-Signal Transistor Parameters with Emitter Degeneration

The emitter current of a transistor in the common-emitter configuration has a characteristic similar to that shown in Figure H-1. If a significant amount of emitter degeneration is used, the curve may be approximated by a straight line as shown. The slope of this line is $1/(r_e + R_f)_0$ where r_e is the small-signal transistor emitter resistance calculated at the mean emitter current I_e, and R_f is the external emitter degeneration resistance. Thus the emitter current is given approximately by

$$i_e = \frac{E \sin \omega t + E_0}{(r_e + R_f)_0} \qquad (E \sin \omega t + E_0) \geqslant 0 \tag{H-1}$$

$$i_e = 0 \qquad (E \sin \omega t + E_0) < 0. \tag{H-2}$$

The collector current is given by $i_c = \alpha i_e$. Normally α is very near unity and the collector is taken to be equal to the emitter current for this analysis.

The effective conduction angle is 2θ, and the voltage E_0 is given by

$$E_0 = -E \cos \theta. \tag{H-3}$$

Thus we may write

$$i_e = \frac{E \sin \omega t - E \cos \theta}{(r_e + R_f)_0} \qquad \left(\frac{\pi}{2} - \theta\right) \leqslant \omega t \leqslant \left(\frac{\pi}{2} + \theta\right) \tag{H-4}$$

$$i_e = 0 \qquad \text{elsewhere.}$$

Using Fourier analysis, we may represent i_e by an infinite series of the form

$$i = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t, \tag{H-5}$$

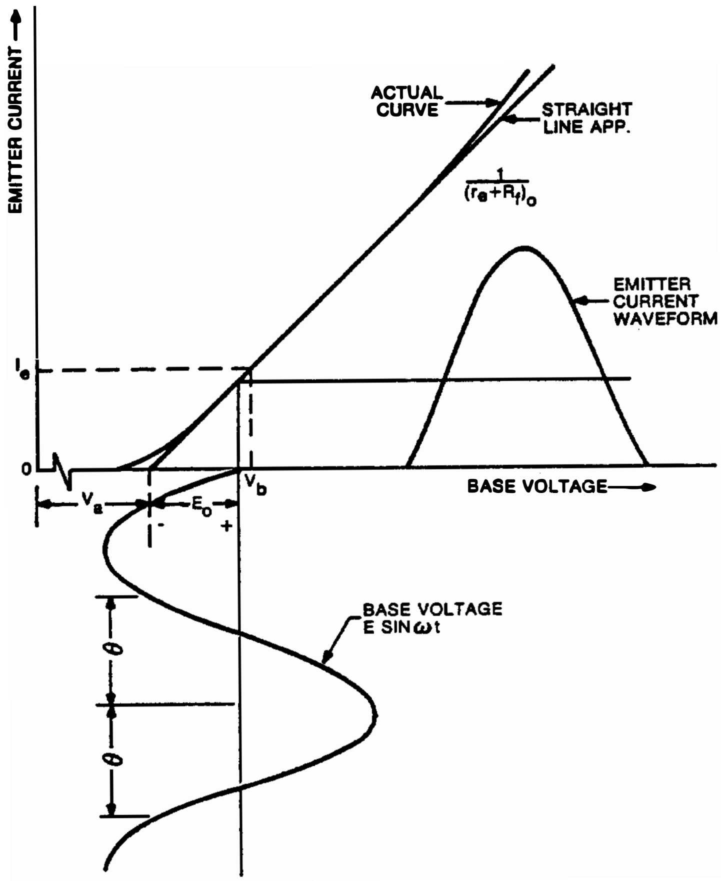

Figure H-1. Input characteristic of transistor.

where the a_n and b_n terms are defined by the equations

$$a_n = \frac{1}{\pi}\int_0^{2\pi} i_e \cos n\phi \, d\phi \tag{H-6}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} i_e \sin n\phi \, d\phi, \tag{H-7}$$

where $\phi = \omega t$. Let us first find the dc value of the current given by $a_0/2$.

$$a_0 = \frac{1}{\pi}\int_{(\pi/2)-\theta}^{(\pi/2)+\theta} \frac{E \sin\phi - E\cos\theta}{(r_e + R_f)_0}\, d\phi$$

$$= \frac{2E(\sin\theta - \theta\cos\theta)}{(R_f + r_e)\pi}. \tag{H-8}$$

Thus the dc emitter current is given by

$$I_e(\text{mean}) = \frac{E}{(r_e + R_f)_0\pi}(\sin\theta - \theta\cos\theta). \tag{H-9}$$

The fundamental component of current is given by the a_1 and b_1 terms, which are

$$a_1 = \frac{1}{\pi}\int_{(\pi/2)-\theta}^{(\pi/2)+\theta} \frac{E\sin\phi - E\cos\theta}{(r_e + R_f)_0}\cos\phi\, d\phi = 0 \tag{H-10}$$

$$b_1 = \frac{1}{\pi}\int_{(\pi/2)-\theta}^{(\pi/2)+\theta} \frac{E\sin\phi - E\cos\theta}{(r_e + R_f)_0}\sin\phi\, d\phi$$

$$= \frac{E(\theta - \frac{1}{2}\sin 2\theta)}{\pi(r_e + R_f)_0}. \tag{H-11}$$

The second harmonic component is given by the a_2 and b_2 terms, which are:

$$a_2 = \frac{1}{\pi}\int_{(\pi/2)-\theta}^{(\pi/2)+\theta} \frac{E\sin\phi - E\cos\theta}{(r_e + R_f)_0}\cos 2\phi\, d\phi$$

$$= \frac{E(-\frac{1}{3}\sin 3\theta - \sin\theta + \cos\theta\sin 2\theta)}{\pi(r_e + R_f)_0} \tag{H-12}$$

$$b_2 = \frac{1}{\pi}\int_{(\pi/2)-\theta}^{(\pi/2)+\theta} \frac{E\sin\phi - E\cos\theta}{(r_e + R_f)_0}\sin 2\phi\, d\phi = 0. \tag{H-13}$$

The third harmonic component is given by the a_3 and b_3 terms, which are

$$a_3 = \frac{1}{\pi}\int_{(\pi/2)-\theta}^{(\pi/2)+\theta} \frac{E\sin\phi - E\cos\theta}{(r_e + R_f)_0}\cos 3\phi\, d\phi = 0. \tag{H-14}$$

$$b_3 = \frac{1}{\pi}\int_{(\pi/2)-\theta}^{(\pi/2)+\theta} \frac{E\sin\phi - E\cos\theta}{(r_e + R_f)_0}\sin 3\phi\, d\phi$$

$$= \frac{E(-\frac{1}{2}\sin 2\theta + \frac{2}{3}\cos\theta\sin 3\theta - \frac{1}{4}\sin 4\theta)}{\pi(r_e + R_f)_0}. \tag{H-15}$$

Substituting these terms in equation (H-5) gives the equation

$$i = \frac{E(\sin\theta - \theta\cos\theta)}{\pi(r_e + R_f)_0} + \frac{E(\theta - \frac{1}{2}\sin 2\theta)}{\pi(r_e + R_f)_0}\sin\omega t$$

$$+ \frac{E(-\frac{1}{3}\sin 3\theta - \sin\theta + \cos\theta\sin 2\theta)}{\pi(r_e + R_f)_0}\cos 2\omega t$$

$$+ \frac{E(-\frac{1}{2}\sin 2\theta + \frac{2}{3}\cos\theta\sin 3\theta - \frac{1}{4}\sin 4\theta)}{\pi(r_e + R_f)_0}\sin 3\omega t + \cdots. \tag{H-16}$$

Substituting for $(r_e + R_f)_0$ from equation (H-9),

$$i = I_e(\text{mean})\left[1 + \frac{(\theta - \frac{1}{2}\sin 2\theta)}{(\sin\theta - \theta\cos\theta)}\sin\omega t\right.$$

$$+ \frac{(-\frac{1}{3}\sin 3\theta - \sin\theta + \cos\theta\sin 2\theta)}{(\sin\theta - \theta\cos\theta)}\cos 2\omega t$$

$$\left. + \frac{(-\frac{1}{2}\sin 2\theta + \frac{2}{3}\cos\theta\sin 3\theta - \frac{1}{4}\sin 4\theta)}{(\sin\theta - \theta\cos\theta)}\sin 3\omega t + \cdots\right]. \tag{H-17}$$

Thus it can be seen that knowing the mean emitter current and the conduction angle, the fundamental and harmonic components of the collector current may be calculated. The conduction angle is found using equation (H-9). Unfortunately, θ cannot be solved for directly, but the equations can be plotted in parametric form using θ as the parameter. This was done and the results are presented in section 6.4.2.

The equivalent input impedance of the transistor is given by

$$R_{\text{in}} = \frac{(\text{fundamental component of base voltage})}{(\text{fundamental component of base current})}. \tag{H-18}$$

The base current is given by $i_e/(\beta + 1)$; therefore, from equation (H-17) we have

$$i_b(\text{fund}) = \frac{I_e(\text{mean})\,(\theta - \frac{1}{2}\sin 2\theta)}{(\beta + 1)\,(\sin\theta - \theta\cos\theta)}\sin\omega t. \tag{H-19}$$

Substituting E sin ωt for the base voltage and equation (H-19) for the base current, the input resistance may be computed from equation (H-18) as

$$R_{\text{in}} = \frac{E(\beta+1)(\sin\theta - \theta\cos\theta)\sin\omega t}{I_e(\text{mean})(\theta - \frac{1}{2}\sin 2\theta)\sin\omega t}$$
$$R_{\text{in}} = \frac{E(\beta+1)(\sin\theta - \theta\cos\theta)}{I_e(\text{mean})(\theta - \frac{1}{2}\sin 2\theta)}. \tag{H-20}$$

Substituting for I_e(mean) from equation (H-9) gives

$$R_{\text{in}} = \frac{(\beta+1)(r_e+R_f)_0\pi}{(\theta - \frac{1}{2}\sin 2\theta)}.$$

If we define

$$\frac{r_e+R_f}{(r_e+R_f)_0} = \frac{\pi}{\theta - \frac{1}{2}\sin 2\theta}, \tag{H-21}$$

then

$$R_{\text{in}} = (\beta+1)(r_e+R_f), \tag{H-22}$$

but the small-signal input resistance ($\theta = \pi$) is given by

$$R_{\text{in}0} = (\beta+1)(r_e+R_f)_0;$$

hence

$$\frac{R_{\text{in}}}{R_{\text{in}0}} = \frac{\pi}{\theta - \frac{1}{2}\sin 2\theta}. \tag{H-23}$$

The effective transconductance of the transistor is given by

$$g_m = \frac{\text{fundamental component of collector current}}{\text{fundamental component of base voltage}}. \tag{H-24}$$

Substituting the fundamental component of collector current from equation (B-17) and E sin ωt for the base voltage, we have

$$g_m = \frac{I_e(\text{mean})(\theta - \frac{1}{2}\sin 2\theta)\sin\omega t}{E(\sin\theta - \theta\cos\theta)\sin\omega t}. \tag{H-25}$$

Substituting for the mean emitter current from equation (H-9), we have

$$g_m = \frac{(\theta - \frac{1}{2}\sin 2\theta)}{(r_e+R_f)_0\pi}. \tag{H-26}$$

Again using the definition of equation (H-21), we have

$$g_m = \frac{1}{(r_e + R_f)}, \tag{H-27}$$

but the small-signal transconductance is given by

$$g_{m0} = 1/(r_e + R_f)_0;$$

therefore,

$$\frac{g_m}{g_{m0}} = \frac{\theta - \frac{1}{2}\sin 2\theta}{\pi}. \tag{H-28}$$

It is also possible to calculate the bias shift due to the presence of the signal voltage. From Figure (H-1) it can be seen that a no-signal base voltage V_b is required to establish an emitter current I_e.

With a signal present and a desired I_e(mean) equal to the no-signal I_e, a voltage $V_a + E_0$, is required. By inspection we see that $I_e(\text{mean}) \times (r_e + R_f)_0$ is the distance between the intersection of the resistance line with the X-axis and V_b. Thus $V_a = V_b - I_e(\text{mean})\,(r_e + R_f)_0$. The bias required with signal is thus given by

$$V_B = V_a + E_0 = V_b - I_e(\text{mean})\,(r_e + R_f)_0 + E_0. \tag{H-29}$$

But from the equation (H-3) we have

$$E_0 = -E\cos\theta;$$

therefore,

$$V_B = V_b - I_e(\text{mean})\,(r_e + R_f)_0 - E\cos\theta. \tag{H-30}$$

Substituting for E from equation (H-9), we have

$$V_B = V_b - I_e(\text{mean})\,(r_e + R_f)_0 \left(1 + \frac{\pi}{\tan\theta - \theta}\right). \tag{H-31}$$

The bias shift $V_B - V_b$ is therefore given by

$$V_{\text{bias}} = I_e(\text{mean})\,(r_e + R_f)_0 \left[1 + \frac{\pi}{\tan\theta - \theta}\right]. \tag{H-32}$$

Normalizing the shift to $I_e(\text{mean})\,(r_e + R_f)_0$, we have

$$\frac{V_{\text{bias}}}{I_e(\text{mean})\,(r_e + R_f)} = 1 + \frac{\pi}{\tan\theta - \theta}. \tag{H-33}$$

The pea current for $0 \leqslant \theta < \pi$ is given by

$$i(\text{pea }) = \frac{E + E_0}{(r_e + R_f)_0} = \frac{E(1 - \cos\theta)}{(r_e + R_f)_0}. \tag{H-34}$$

Thus using equation (H-9),

$$\frac{i(\text{peak})}{I_e(\text{mean})} = \frac{\pi(1 - \cos\theta)}{\sin\theta - \theta\cos\theta}. \tag{H-35}$$

For $\theta = \pi$,

$$i(\text{peak}) = I_e + \frac{E}{(r_e + R_f)_0}. \tag{H-36}$$