Appendix G
Large-Signal Transistor Parameters

In general, a transistor can be thought of as being made up of intrinsic and extrinsic elements. The extrinsic elements in general result from resistance, capacitance, or inductance in the leads connected to the semiconductor material. The bulk resistances of the semiconductor material also give rise to elements which to a first approximation may be lumped into the extrinsic elements. Thus, the transistor may be represented as shown in Figure G-1.

In many applications, particularly when the emitter current is low and the frequency considerably below f_t, the circuit behavior is not appreciably affected by the extrinsic elements. It is then possible to consider only the behavior of the intrinsic transistor, which is done for this analysis.

Referring to Figure G-1, it is known that the intrinsic base voltage is given by

$$V_{b'} = \frac{\lambda KT}{q} \ln \left[1 + \frac{I_e - \alpha_I I_c}{I_{\text{ed}}}\right]. \tag{G-1}$$

Most of the symbols have been previously defined in this book; therefore, a complete explanation is not given here. Since λ is usually near unity and to sim-

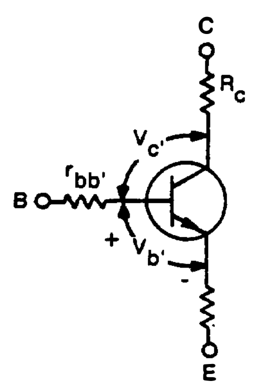

Figure G-1. Transistor model showing extrinsic elements.

plify the terminology, λ is taken to be unity for the analysis. It may be reinserted later if desired by replacing K by λK in the equations.

The term α_I is the inverted current gain I_e/I_c with the collector acting as emitter. I_{ed} is the diffusion saturation current. It should be pointed out that I_{ed} is very much a function of temperature and, in general, doubles about every 10°C. This effect results in a larger change in $V_{b'}$ with temperature than the q/KT term. In the active region the collector current is given by

$$I_c = \alpha_N I_e \tag{G-2}$$

where α_N is the normal forward emitter-to-collector current gain. This equation is used in equation (G-1) in both the active region and for large signals in the cutoff region during part of each cycle. While equation (G-2) does not necessarily hold during cutoff, the collector current is so small during that part of the cycle that a considerable error has very little effect on the overall behavior. Making the indicated substitution yields the equation

$$V_{b'} = \frac{KT}{q} \ln \left[1 + \frac{I_e(1 - \alpha_I \alpha_N)}{I_{\text{ed}}} \right]. \tag{G-3}$$

Solving for I_e gives

$$I_e = \frac{I_{\text{ed}}}{(1 - \alpha_I \alpha_N)} [e^{qV_{b'}/KT} - 1]. \tag{G-4}$$

Now let

$$I_R = \frac{I_{\text{ed}}}{(1 - \alpha_I \alpha_N)}. \tag{G-5}$$

Substituting this into equation (G-4) gives

$$I_e = I_R \,(e^{qV_{b'}/KT} - 1), \tag{G-6}$$

which is in the form of the well known Shockley equation for an ideal p-n junction. I_R is generally in the order of three times the value of I_{ed} and thus is a very small current. Therefore, for practical purposes, equation (G-6) is equivalent to equation (G-7) if the transistor is active during at least a part of each cycle:

$$I_e = I_R \, e^{qV_{b'}/KT} \tag{G-7}$$

We now assume that a sinusoidal signal is applied between the base and emitter. In general, some dc bias is also applied and the base-to-emitter voltage has the form

$$V_{b'} = E \cos \omega t + E_0. \tag{G-8}$$

The emitter current is then given by

$$I_e = I_R \exp\left[\frac{q}{KT}(E\cos\omega t + E_0)\right]. \tag{G-9}$$

This can be rewritten in the form:

$$I_e = I_R \exp\left(\frac{qE_0}{KT}\right)\exp\left(\frac{q}{KT}E\cos\omega t\right). \tag{G-10}$$

Let us now examine the term

$$\exp\left(\frac{q}{KT}E\cos\omega t\right). \tag{G-11}$$

To expand this, we use the Bessel function expansion of the form*

$$\exp z\cos\theta = I_0(z) + 2\sum_{n=1}^{\infty} I_n(z)\cos n\theta, \tag{G-12}$$

where $I_n(z)$ represents a modified Bessel function of the first kind and of order n. These modified Bessel functions are also referred to as the hyperbolic Bessel functions and are related to the familiar $J_n(z)$ Bessel functions much as the trigonometric functions are related to the hyperbolic functions. Thus,

$$I_n(z) = (i^{-n})J_n(iz), \tag{G-13}$$

where

$$i = \sqrt{-1}.$$

In series form,

$$I_n(z) = \sum_{j=0}^{\infty}\frac{(z/2)^{n+2j}}{j!(n+j)!}. \tag{G-14}$$

Substituting $z = qE/KT$ into equation (G-12) and substituting equation (G-12) into equation (G-10), we have

$$I_e = I_R \exp\left(\frac{qE_0}{KT}\right)\left[I_0\left(\frac{Eq}{KT}\right) + 2\sum_{n=1}^{\infty} I_n\left(\frac{Eq}{KT}\right)\cos n\omega t\right]. \tag{G-15}$$

For convenience in notation, let $V = Eq/KT$. Substituting this into equaion (G-15) and writing out a few terms, we have

$$I_e = I_R(\exp qE_0/KT)[I_0(V) + 2I_1(V)\cos\omega t + 2I_2(V)\cos 2\omega t + \cdots]. \tag{G-16}$$

*See page 106 of reference 33.

The d c component is given by

$$I_e(\text{mean}) = I_R \exp qE_0/KT\, I_0(V). \tag{G-17}$$

Substituting this into equation (G-16) to eliminate E_0, we have

$$I_e = I_e(\text{mean})\left[1 + \frac{2I_1(V)}{I_0(V)}\cos \omega t + \frac{2I_2(V)}{I_0(V)}\cos 2\omega t + \cdots\right]. \tag{G-18}$$

As stated earlier, $I_c = \alpha_N I_e$, but α_N is nearly unity; hence, $I_c \doteq I_e$. To determine the transconductance, we form the ratio,

$$g_m = \frac{i_c(\text{fund})}{E \cos \omega t}, \tag{G-19}$$

which is given by

$$g_m = \frac{2I_e(\text{mean})\, I_1(V) \cos \omega t}{I_0(V)\, E \cos \omega t} = \frac{2I_e(\text{mean})\, I_1(V)}{EI_0(V)}. \tag{G-20}$$

In the small-signal case, $V = 0$ and $I_0(V) = 1$, while

$$I_1(V) = \frac{V}{2} = \frac{Eq}{2KT}. \tag{G-21}$$

Substituting these in equation (G-20) gives

$$g_{m0} = \frac{2I_e(\text{mean})\, Eq}{2KTE} = \frac{I_e(\text{mean})\, q}{KT}, \tag{G-22}$$

which is the well known small-signal value. Substituting this in equation (G-20) for I_e(mean) gives

$$\frac{g_m}{g_{m0}} = \frac{2I_1(V)}{VI_0(V)}. \tag{G-23}$$

It is also possible to determine the input impedance of the transistor from equation (G-18) making use of the fact that the base current is related to the emitter current by the factor $\beta + 1$; hence,

$$I_b = \frac{I_e}{\beta + 1}. \tag{G-24}$$

The equivalent input resistance is given by

$$R_{\text{in}} = \frac{E \cos \omega t}{I_b(\text{fund})}, \tag{G-25}$$

which is

$$R_{\text{in}} = \frac{E \cos \omega t\, (\beta + 1) I_0(V)}{2I_e(\text{mean})\, I_1(V) \cos \omega t} = \frac{E(\beta + 1) I_0(V)}{2I_e(\text{mean})\, I_1(V)}. \qquad \text{(G-26)}$$

In the small-signal case $V = 0$ and, using the limit values of equation (G-21), we have

$$R_{\text{in}0} = \frac{E(\beta + 1)}{2I_e(\text{mean})\, (Eq/2KT)} = \frac{(\beta + 1) KT}{qI_e}, \qquad \text{(G-27)}$$

which is the well known small-signal value. Substituting this into equation (G-26) gives

$$\frac{R_{\text{in}}}{R_{\text{in}0}} = \frac{VI_0(V)}{2I_1(V)}. \qquad \text{(G-28A)}$$

For purposes of computer analysis it is possible to approximate equation (G-28A) by the simplified equation

$$\frac{r_e}{r_{e0}} = \left[1 + \left(\frac{Eq}{2KT\lambda}\right)^2\right]^{1/2} = \left[1 + \left(\frac{5.78 \times 10^{-3} E}{T\lambda}\right)^2\right]^{1/2}. \qquad \text{(G-28B)}$$

The diffusion capacitance for large-signal voltages may also be computed. The charge stored in the base region is proportional to the base current; hence

$$Q = MI_b, \qquad \text{(G-29)}$$

where M is a constant of proportionality related to the carrier lifetime. Substituting equation (G-24) into (G-7) and the resultant equation into (G-29) gives

$$Q = \frac{MI_R}{(\beta + 1)} e^{qV_{b'}/KT}. \qquad \text{(G-30)}$$

The reactive component of the base current is given by

$$i = \frac{dQ}{dt}. \qquad \text{(G-31)}$$

Differentiating equation (G-30) and substituting in to (G-31) gives

$$i = \frac{MI_R}{(\beta + 1)} \frac{q}{KT} e^{qV_{b'}/KT} \frac{dV_{b'}}{dt}. \qquad \text{(G-32)}$$

Substituting $V_{b'} = E \cos \omega t + E_0$ gives

$$i = \frac{-MI_R qE\omega}{(\beta + 1) KT} \exp\left(\frac{qE_0}{KT}\right) \left[\exp\left(\frac{qE}{KT} \cos \omega t\right)\right] \sin \omega t. \qquad \text{(G-33)}$$

For convenience in notation, let

$$P = -\frac{MI_R qE\omega}{(\beta + 1)KT} \exp\left(\frac{qE_0}{KT}\right); \qquad \text{(G-34)}$$

then

$$i = P\left(\exp \frac{qE \cos \omega t}{KT}\right) \sin \omega t. \qquad \text{(G-35)}$$

The fundamental component of this current may be found by Fourier analysis using the formulas given in equations (H-5), (H-6), and (H-7) of Appendix H. Thus the coefficient of the sin ωt term is given by

$$b_1 = \frac{1}{\pi} \int_0^{2\pi} i \sin \theta \, d\theta, \qquad \text{(G-36)}$$

where $\theta = \omega t$. Substituting for i from equation (G-35) gives

$$b_1 = \frac{1}{\pi} \int_0^{2\pi} P\left(\exp \frac{qE \cos \theta}{KT}\right) \sin \theta \sin \theta \, d\theta. \qquad \text{(G-37)}$$

Using the identity, $\sin^2 \theta = \frac{1}{2} - \frac{1}{2} \cos 2\theta$,

$$b_1 = \frac{1}{2\pi} \int_0^{2\pi} P\left(\exp \frac{qE \cos \theta}{KT}\right) d\theta$$

$$-\frac{1}{2\pi} \int_0^{2\pi} P\left(\exp \frac{qE \cos \theta}{KT}\right) \cos 2\theta \, d\theta. \qquad \text{(G-38)}$$

The first integral may be evaluated using the form*

$$\frac{1}{2\pi} \int_0^{2\pi} (\exp z \cos \theta) \, d\theta = I_0(Z), \qquad \text{(G-39)}$$

and the second, using the form**

$$\frac{1}{2\pi} \int_0^{2\pi} (\exp z \cos \theta) \cos n\theta \, d\theta = (-1)^n I_n(z).$$

*See page 162 of reference 33.
**See page 51 of reference 33.

Using these forms, we have

$$b_1 = P\left[I_0\left(\frac{qE}{KT}\right) - I_2\left(\frac{qE}{KT}\right)\right]. \qquad \text{(G-40)}$$

It can be shown that***

$$I_0(z) - I_2(z) = \frac{2I_1(z)}{z}; \qquad \text{(G-41)}$$

therefore,

$$b_1 = \frac{2PI_1(qE/KT)}{qE/KT} = \frac{2PI_1(V)}{V}. \qquad \text{(G-42)}$$

The coefficient of the cos ωt term is given by

$$a_1 = \frac{1}{\pi}\int_0^{2\pi} P\left(\exp\frac{qE\cos\theta}{KT}\right)\sin\theta\cos\theta\, d\theta. \qquad \text{(G-43)}$$

Using the identity

$$\sin\theta\cos\theta = \tfrac{1}{2}\sin 2\theta, \qquad \text{(G-44)}$$

we have

$$a_1 = \frac{1}{2\pi}\int_0^{2\pi} P\left(\exp\frac{qE\cos\theta}{KT}\right)\sin 2\theta\, d\theta. \qquad \text{(G-45)}$$

The value of this integral can be shown to be zero.* Thus the fundamental component of capacitive current is given by

$$i = \frac{2PI_1(V)}{V}\sin\omega t. \qquad \text{(G-46)}$$

The equivalent capacitance is given by the relationship,

$$\omega C = \frac{i(\text{fund})\,(90^\circ\text{ leading})}{e(\text{fund})}. \qquad \text{(G-47)}$$

Now $-\sin\omega t$ leads $\cos\omega t$ by 90 degrees; hence:

$$\omega C_{de} = \frac{-2PI_1(V)}{EV}, \qquad \text{(G-48)}$$

***See page 163 of reference 33.

*See page 51 of reference 33.

since $e = E \cos \omega t$. Substituting for P from equation (G-34),

$$C_{\text{de}} = \frac{Mq}{KT(\beta+1)} \frac{2I_1(V)}{V} I_R \, e^{qE_0/KT}. \tag{G-49}$$

Substituting equation (G-17) for $I_R \exp qE_0/KT$ gives

$$C_{\text{de}} = \frac{M}{\beta+1} \frac{qI_e(\text{mean})}{KT} \frac{2I_1(V)}{VI_0(V)}. \tag{G-50}$$

For the small-signal case, we may use the limit values given in (G-21) and

$$C_{\text{deo}} = \frac{M}{\beta+1} \frac{qI_e(\text{mean})}{KT}. \tag{G-51}$$

Therefore, from the circuit of Figure 6-5 and equation (6-16),

$$\frac{M}{\beta+1} = \frac{1}{2\pi f_t}. \tag{G-52}$$

Also dividing equation (G-50) by equation (G-51), we have

$$\frac{C_{\text{de}}}{C_{\text{deo}}} = \frac{2I_1(V)}{VI_0(V)}. \tag{G-53}$$

The bias shift resulting from the signal voltage can be determined from equation (G-17). With signal present,

$$I_e(\text{mean}) = I_R(\exp qE_0/KT)\, I_0(V). \tag{G-54}$$

With no signal, $V = 0$ and $I_0(V) = 1$; hence,

$$I_e(\text{no signal}) = I_R \exp qV_{be}/KT. \tag{G-55}$$

We wish to determine E_0 so that

$$I_e(\text{mean}) = I_e(\text{no signal}).$$

Equating gives

$$(\exp qE_0/KT)\, I_0(V) = \exp qV_{be}/KT. \tag{G-56}$$

Solving for qE_0/KT gives

$$\frac{qE_0}{KT} = \frac{qV_{be}}{KT} - \ln I_0(V), \tag{G-57}$$

or

$$E_0 - V_{be} = \frac{KT}{q} \ln I_0(V). \tag{G-58}$$

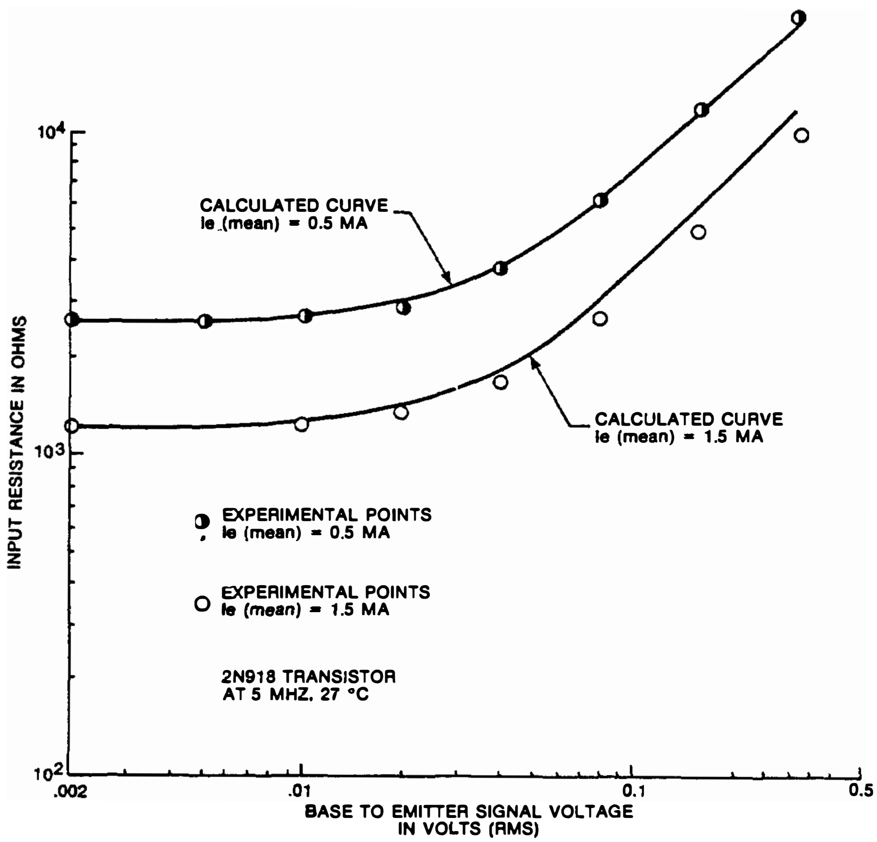

Figure G-2. Input resistance versus signal voltage.

The bias shift is given by

$$V_{\text{bias}} = E_0 - V_{be} = \frac{KT}{q} \ln I_0(V) \tag{G-59}$$

Noting that $V = qE/KT$, we have

$$V_{\text{bias}} = \frac{KT}{q} \ln I_0\left(\frac{qE}{KT}\right). \tag{G-60}$$

In order to verify the results of this analysis, a series of measurements was made on a type 2N918 transistor. Tests were run at emitter currents of both 0.5 mA and at 1.5 mA. The ac sinusoidal input voltage was varied from 0.002 to

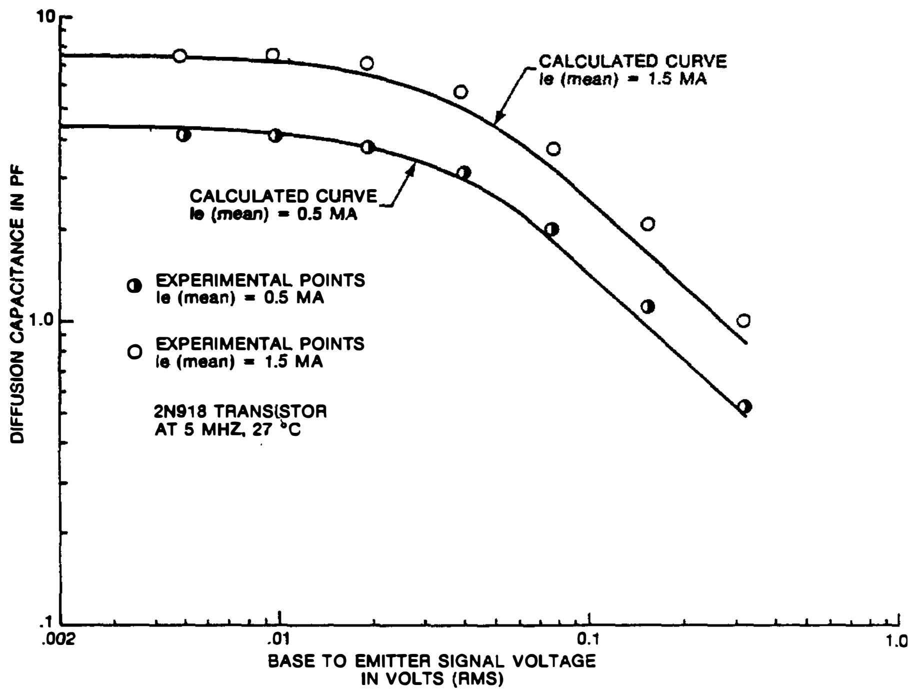

Figure G-3. Diffusion capacitance versus signal voltage.

0.3 V. As the signal voltage was increased, the bias circuit was readjusted to maintain the emitter current at the indicated value. The resistance and capacitance measurements were made, using a Boonton *RX* meter, by adjusting the oscillator to obtain the desired ac voltage. The graph of Figure G-2 shows the input resistance variation as a function of signal level, while Figure G-3 shows how the input capacitance varies with drive. The curves are plotted from equation (G-28B), and the circles correspond to measured points. As can be seen, the agreement is excellent.