Appendix E

Derivation of Approximate Equations for the Pierce Oscillator Analysis

The input impedance Z_L may be written as

$$Z_L = \frac{jX_2(R_e + jX_e + jX_1)}{R_e + jX_2 + jX_1 + jX_e}.$$

If $X_1 + X_2 + X_e = 0$, then the expression simplifies to

$$Z_L = \frac{-X_2(X_e + X_1) + jR_eX_2}{R_e}.$$

Again applying the preceding assumption,

$$Z_L = \frac{X_2}{R_e}(X_2 + jR_e).$$

If we now assume that $X_2 >> R_e$, the input impedance simplifies to

$$Z_L = \frac{X_2^2}{R_e}.$$

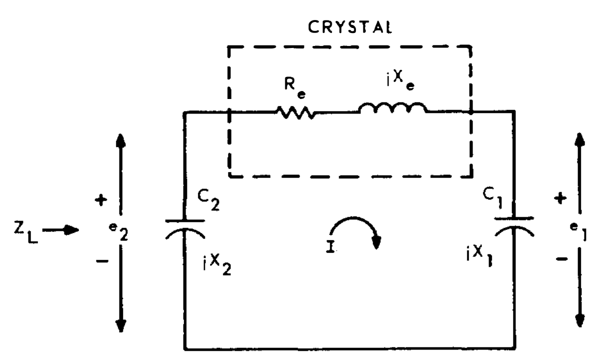

Figure E-1. Pierce oscillator π-network: simplified diagram.

The voltage e_1 may be written as $e_1 = jX_1 I$, where

$$I = \frac{e_2}{R_e + j(X_e + X_1)}.$$

Combining these gives

$$\frac{e_1}{e_2} = \frac{jX_1}{R_e + j(X_1 + X_e)}.$$

If $X_1 + X_2 + X_e = 0$, then this expression simplifies to

$$\frac{e_1}{e_2} = \frac{jX_1}{R_e - jX_2}.$$

If now we assume that $X_2 >> R_e$, then

$$\frac{e_1}{e_2} = -\frac{X_1}{X_2}.$$

Also, since

$$X_1 = -\frac{1}{\omega C_1}$$

and

$$X_2 = -\frac{1}{\omega C_2},$$

then

$$\frac{e_1}{e_2} = -\frac{C_2}{C_1}.$$