Appendix C

Derivation of Y-Parameter Equations for the Grounded-Base Oscillator

The grounded-base crystal oscillator can be represented by the schematic diagram of Figure C-1.

Let the circuit be redrawn and the transistor replaced by a four-terminal network described by its Y-parameters. The diagram of Figure C-2 then results. If the transistor is represented by its Y-parameters, the feedback network must be represented by its Z-parameters if equation (A-12) is to be used.

For this analysis it is convenient to include the output resistance of the transistor in the load resistor R_T. This is accomplished by using a value of R_T which is equal to $R_L + (1/g_{ob})$. Often g_{ob} is negligible and the correction need not be used. The output capacity of the transistor will be lumped in parallel with L, thus making $y_{ob} = 0$ in the analysis. The reverse transfer admittance y_{rb} will be neglected to simplify the analysis.

The network which must be represented by its Z-parameters is given in Figure C-3.

From this circuit, the Z-parameters can be calculated using the following definitions:

Input impedance $$Z_i = \left.\frac{V_1}{I_1}\right|_{I_2=0} \tag{C-1}$$

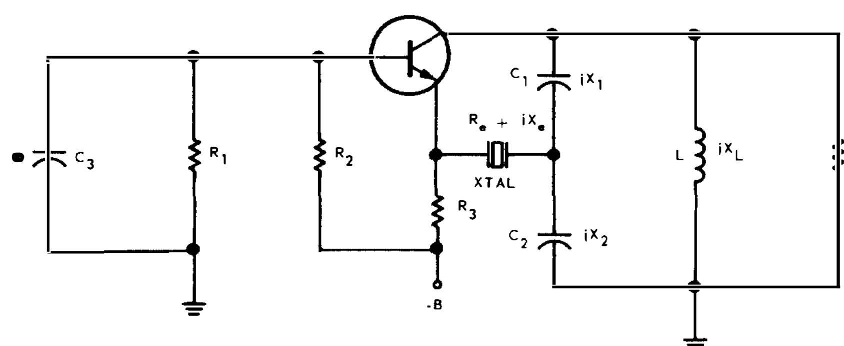

Figure C-1. Grounded-base oscillator: schematic diagram.

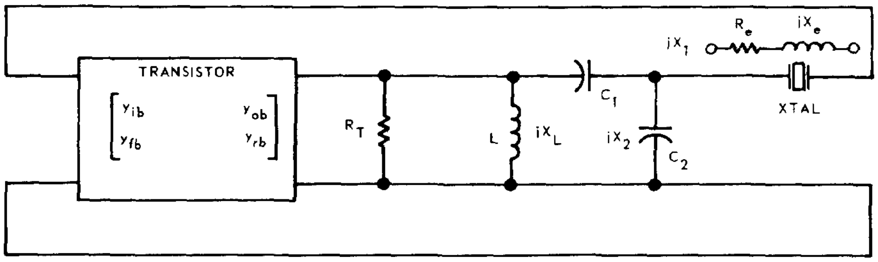

Figure C-2. Grounded-base oscillator: ac circuit diagram.

Output impedance $$Z_o = \left.\frac{V_2}{I_2}\right|_{I_1=0} \tag{C-2}$$

Forward transfer impedance $$Z_f = \left.\frac{V_2}{I_1}\right|_{I_2=0} \tag{C-3}$$

Reverse transfer impedance $$Z_r = \left.\frac{V_1}{I_2}\right|_{I_1=0} \tag{C-4}$$

To simplify the analysis, it will be assumed that $X_1 + X_2 + X_L = 0$. This is approximately resonance for the tank circuit.

From equation (C-1) it can be seen that Z_i is merely the input impedance of the network with the output open-circuited and can be written by inspection as $Z_i = R_T$.

The forward transfer impedance is given by the ratio of V_2 to I_1 with the output open-circuited. In order to calculate this, it is convenient to first deter-

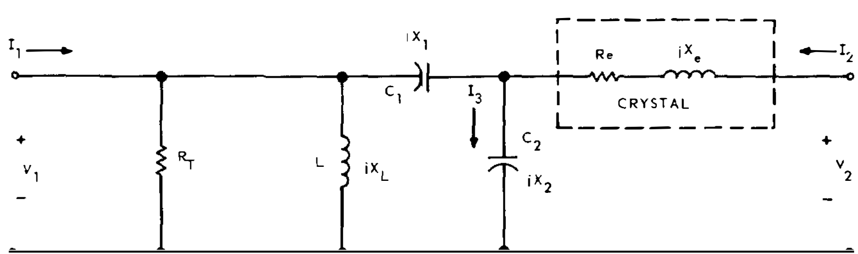

Figure C-3. Grounded-base oscillator feedback network: simplified diagram.

mine I_3 in terms of I_1, which is given by

$$I_3 = \frac{I_1\left(\frac{jR_TX_L}{R_T+jX_L}\right)}{\frac{jR_TX_L}{R_T+jX_L}+j(x_1+x_2)}. \tag{C-5}$$

Simplifying gives

$$I_3 = \frac{jI_1R_TX_L}{-X_L(X_1+X_2)+jR_T(X_1+X_2+X_L)}, \tag{C-6}$$

but

$$I_3 = \frac{V_2}{jX_2}.$$

Substituting this and making use of the assumption that

$$X_L + X_1 + X_2 = 0$$

gives

$$\frac{V_2}{I_1} = \frac{R_TX_2X_L}{X_L(X_1+X_2)} = \frac{R_TX_2}{X_1+X_2} = \frac{-R_TX_2}{X_L}; \tag{C-7}$$

therefore

$$Z_f = -\frac{R_TX_2}{X_L}.$$

Since the network is composed entirely of reciprocal elements, the forward and reverse transfer impedances are equal, so that

$$Z_f = Z_r = -\frac{R_TX_2}{X_L}. \tag{C-8}$$

From equation (C-2), it can be seen that Z_o is merely the output impedance of the network with the input open-circuited. It is given by

$$Z_o = R_e + jX_e + \frac{\left[\left(\frac{jR_TX_L}{R_T+jX_L}\right)+jX_1\right]jX_2}{\left[\left(\frac{jR_TX_L}{R_T+jX_L}\right)+jX_1\right]+jX_2}. \tag{C-9}$$

Simplifying this gives

$$Z_o = R_e + jX_e + \frac{jX_2\,[-X_1X_L + jR_T(X_1 + X_L)]}{jR_T(X_1 + X_2 + X_L) - X_L(X_1 + X_2)}. \qquad \text{(C-10)}$$

Making use of the assumption $X_1 + X_2 + X_L = 0$ and further simplifying gives

$$Z_o = R_e + jX_e + \frac{R_TX_2(X_1 + X_L) + jX_1X_2X_L}{X_L(X_1 + X_2)} \qquad \text{(C-11)}$$

and

$$\frac{\qquad\qquad}{X_L}. \qquad \text{(C-12)}$$

These results can be substituted in the general equation for oscillation as developed in Appendix A:

$$y_{fb}Z_f + y_{ib}Z_o + y_{ob}Z_i + y_{rb}Z_r + \Delta y_b \Delta Z + 1 = 0. \qquad \text{(C-13)}$$

Since y_{ob} is being accounted for the feedback network, and since y_{rb} is assumed to be zero, $\Delta y = y_{ib}y_{ob} - y_{fb}y_{ob} = 0$. This simplifies equation (C-13) to

$$y_{fb}Z_f + y_{ib}Z_o + 1 = 0. \qquad \text{(C-14)}$$

Substituting for Z_f and Z_o gives

$$\frac{-y_{fb}R_TX_2}{\quad} \quad \Big[\quad \frac{R_TX_2^2 - jX_1X_2X_L}{\quad}\Big] \qquad \text{(C-15)}$$

Substituting $y_{fb} = g_{fb} + jb_{fb}$ and $y_{ib} = g_{ib} + jb_{ib}$ gives

$$\frac{-(g_{fb} + jb_{fb})R_TX_2}{X_L} + (g_{ib} + jb_{ib})\left[R_e + jX_e + \frac{R_TX_2^2 - jX_1X_2X_L}{X_L^2}\right] + 1 = 0. \qquad \text{(C-16)}$$

Performing the indicated multiplications and collecting terms results in the following equation:

$$-g_{fb}R_TX_2 - jb_{fb}R_TX_2 + g_{ib}R_eX_L + jg_{ib}X_eX_L + jb_{ib}R_eX_L - b_{ib}X_eX_L + \frac{g_{ib}R_TX_2^2}{X_L} - jg_{ib}X_1X_2 + \frac{jR_TX_2^2b_{ib}}{X_L} + b_{ib}X_1X_2 + X_L = 0. \qquad \text{(C-17)}$$

This equation can be separated into real and imaginary components. The real parts are

$$-g_{fb}R_TX_2 + g_{ib}R_eX_L - b_{ib}X_eX_L + \frac{g_{ib}R_TX_2^2}{X_L} + b_{ib}X_1X_2 + X_L = 0. \tag{C-18}$$

Substituting $g_{ib} = 1/R_{\text{in}}$ and simplifying gives

$$g_{fb} = \frac{1}{R_T}\left(\frac{X_L}{X_2}\right)\left[\frac{R_e + R_{\text{in}}}{R_{\text{in}}}\right] + \frac{1}{R_{\text{in}}}\left(\frac{X_2}{X_L}\right) + \frac{b_{ib}X_1}{R_T} - b_{ib}\left(\frac{X_e}{R_T}\right)\left(\frac{X_L}{X_2}\right) = 0. \tag{C-19}$$

The imaginary parts are

$$-b_{fb}R_TX_2 + g_{ib}X_eX_L + b_{ib}R_eX_L - g_{ib}X_1X_2 + \frac{R_TX_2^2b_{ib}}{X_L} = 0. \tag{C-20}$$

Again substituting $g_{ib} = 1/R_{\text{in}}$ and simplifying gives

$$X_e = b_{fb}R_TR_{\text{in}}\left(\frac{X_2}{X_L}\right) + X_1\left(\frac{X_2}{X_L}\right) - b_{ib}R_{\text{in}}\left[R_e + R_T\left(\frac{X_2}{X_L}\right)^2\right]. \tag{C-21}$$

The optimum value of X_L/X_2 with respect to transistor gain can be found by differentiating equation (C-19):

$$\frac{d(g_{fb})}{d(X_L/X_2)} = \frac{1}{R_T}\left[\frac{R_e + R_{\text{in}}}{R_{\text{in}}}\right] - \frac{1}{R_{\text{in}}}\left(\frac{X_2}{X_L}\right)^2 - b_{ib}\left(\frac{X_e}{R_T}\right). \tag{C-22}$$

If it is required that the crystal operate at series resonance $X_e = 0$ or if b_{ib} is negligible, then equation (C-22) simplifies to

$$\frac{d(g_{fb})}{d(X_L/X_2)} = \frac{1}{R_T}\left[\frac{R_e + R_{\text{in}}}{R_{\text{in}}}\right] - \frac{1}{R_{\text{in}}}\left(\frac{X_2}{X_L}\right)^2. \tag{C-23}$$

Solving this for X_L/X_2 gives

$$X_L/X_2 = -\sqrt{R_T/(R_e + R_{\text{in}})} \tag{C-24}$$

The minimum g_{fb} then is given by

$$|g_{fb}|_{\text{min}} = \left|-\frac{2}{R_{\text{in}}}\sqrt{(R_e + R_{\text{in}})/R_T} + b_{ib}(X_1/R_T)\right|. \tag{C-25}$$

NOTE. This assumes that the crystal is at series resonance, $X_e = 0$.