Appendix A

Derivation of the Complex Equation for Oscillation

Using the block diagram of Figure A-1, in which the active element is represented by its Y-parameters and the feedback network by its Z-parameters, the complex equation for oscillation can be derived. By the definition of Y-parameters, the currents and voltages of the active device can be described by the following equations:

$$I = y_{11} V + y_{12} V' \tag{A-1}$$

$$I' = y_{21} V + y_{22} V'. \tag{A-2}$$

Also by definition, the currents and voltages of the feedback network can be described by the equations:

$$V' = -Z_{11} I' - Z_{12} I \tag{A-3}$$

$$V = -Z_{21} I' - Z_{22} I. \tag{A-4}$$

Arranging the equations symmetrically,

$$V y_{11} + V' y_{12} - I + 0I' = 0 \tag{A-5}$$

$$V y_{21} + V' y_{22} + 0I - I' = 0 \tag{A-6}$$

$$0V + V' + I Z_{12} + I' Z_{11} = 0 \tag{A-7}$$

$$V + 0V' + I Z_{22} + I' Z_{21} = 0. \tag{A-8}$$

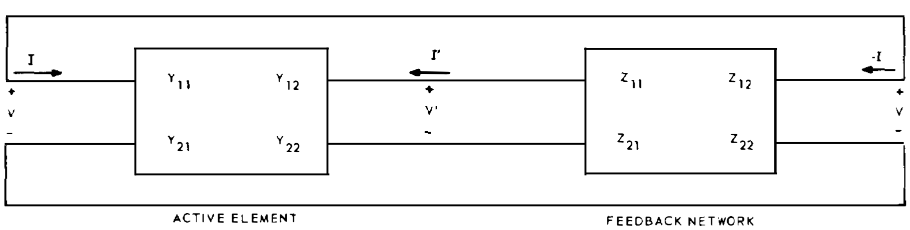

Figure A-1. General oscillator: block diagram.

Solving these equations for V using determinants,

$$V = \frac{\begin{vmatrix} 0 & y_{12} & -1 & 0 \\ 0 & y_{22} & 0 & -1 \\ 0 & 1 & Z_{12} & Z_{11} \\ 0 & 0 & Z_{22} & Z_{21} \end{vmatrix}}{\begin{vmatrix} y_{11} & y_{12} & -1 & 0 \\ y_{21} & y_{22} & 0 & -1 \\ 0 & 1 & Z_{12} & Z_{11} \\ 1 & 0 & Z_{22} & Z_{21} \end{vmatrix}}. \qquad \text{(A-9)}$$

It will be seen that the numerator of this expression is zero, making $V = 0$ for every case except that for which the denominator also is zero; in that case, V is indeterminate. We know, however, that if oscillation takes place, $V \neq 0$, and therefore it must be true that

$$\begin{vmatrix} y_{11} & y_{12} & -1 & 0 \\ y_{21} & y_{22} & 0 & -1 \\ 0 & 1 & Z_{12} & Z_{11} \\ 1 & 0 & Z_{22} & Z_{21} \end{vmatrix} = 0. \qquad \text{(A-10)}$$

Solving this determinant gives the equation:

$$y_{21}Z_{21} + y_{11}Z_{22} + y_{22}Z_{11} + y_{12}Z_{12} + \Delta y \Delta Z + 1 = 0 \qquad \text{(A-11)}$$

where

$$\Delta y = y_{11}y_{22} - y_{21}y_{12}$$

$$\Delta Z = Z_{11}Z_{22} - Z_{21}Z_{12}.$$

This equation is quite general and may be applied to almost any oscillator. If we choose, we may represent the active device by Y-parameters and the feedback network by Z-parameters or vice versa. It is usually more convenient to use the former, however.

It should be pointed out that the use of the two-port parameters implies that the circuits are linear. At large-signal amplitudes, the Y-parameters therefore must be defined as the ratios of fundamental components of current to fundamental components of voltage.

In working with transistors, it may be convenient to use the convention $y_{11} = y_i$, $y_{21} = y_f$, $y_{12} = y_r$, and $y_{22} = y_o$, which conforms to present usage on transistor data sheets. Making the same transition in the Z-parameter gives equation

(A-11) as

$$y_f Z_f + y_i Z_o + y_o Z_i + y_r Z_r + \Delta y \Delta Z + 1 = 0, \tag{A-12}$$

where

$$\Delta y = y_i y_o - y_f y_r$$
$$\Delta Z = Z_i Z_o - Z_f Z_r,$$

which is the form used throughout this book. [Equation (A-12) is presented and discussed at some length in reference 41.]