6
Discussion of Transistors

The selection of a transistor type for a crystal oscillator is largely based on engineering judgment. The following factors that should be considered are discussed in this chapter:

a. Temperature requirements.
b. Maximum frequency requirements.
c. Output power requirements.
d. Input and output impedance.
e. Available power gain.
f. Interchangeability requirements.
g. Cost, availability, etc.

In addition to these, several other characteristics affect the oscillator performance to a lesser degree. A number of these are also discussed.

The transistor chosen must obviously be operable over the required temperature range. In addition, the variation in transistor characteristics with temperature must be compatible with the oscillator circuit. Roughly, the β of a transistor decreases by about 50 percent from room temperature to $-55°C$ and increases by about 50 percent from room temperature to the maximum permissible operating temperature. However, some transistors are better than others in this respect. The saturation resistance increases with temperature. At VHF frequencies, the characteristics of transistors are less dependent on temperature, since the low-frequency parameters have little influence on VHF performance.

Generally, bipolar transistors are used for crystal oscillators because of their larger transconductance at low power levels. The use of field effect devices as crystal oscillators is increasing, however, in connection with oscillators implemented with logic gates and other integrated circuits.

The cutoff frequency f_t of a transistor is some measure of the phase shift through it at the operating frequency. This phase shift is lagging and causes the oscillator frequency to decrease and to become more dependent on the transistor. Therefore, it is desirable to use a transistor with a cutoff frequency at least an order of magnitude higher than the operating frequency.

Larger output powers obviously require higher transistor dissipation. It must be remembered, however, that the allowable quartz crystal dissipation often limits the maximum power output of a stable oscillator.

For high-stability oscillators, it is desirable to minimize the effects of the transistor on the frequency. For this reason the input and output capacitances of the transistor are often swamped out by the addition of external input and output capacitors. (This is particularly convenient in the Pierce, Colpitts, and Clapp oscillator circuits.) If the input and output capacitances of the transistor are small, they can be swamped out effectively without the external capacitors becoming large enough to prevent oscillation. Therefore, it is desirable to use transistors with low input and output capacitances.

6.1. TRANSISTOR EQUIVALENT CIRCUITS

A large number of equivalent circuit representations have been used for transistors in various applications. Obviously, different representations work better or are more practical in certain applications than in others. It has been found that for crystal oscillator design, the Y-parameter representation and the hybrid π equivalent circuit are very useful. Consequently, these circuits will be reviewed briefly prior to incorporating them in the derivation of oscillator equations. The Y-parameter representation is quite versatile in that any linear device can be characterized using the approach, whether it be a single transistor or a combination of devices such as a gate or an integrated amplifier. It is often more convenient to perform measurements on a device, particularly at VHF frequencies, using S-parameters; therefore, the equations required to convert from S-parameters to Y-parameters are also included.

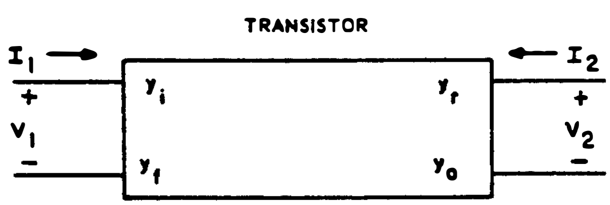
It is possible to perform a complete oscillator analysis based on S-parameters. Such an analysis would be most useful in the design of microwave oscillators. Since quartz crystals are generally not used beyond 150 MHz, the Y-parameter approach is more appropriate for the present work.

The hybrid π equivalent circuit of a transistor is, of course, much more closely related to the physical properties of the device. It is useful in the design of crystal oscillators because it can easily be adapted and used in a nonlinear model and thus is used in connection with analyses to predict the amplitude of oscillation. The Y-parameters can also be derived from the hybrid π equivalent circuit, and these are given in section 6.3.

6.2. Y-PARAMETER MODEL

The Y-parameter representation of a transistor or device is based on the assumption that the device is linear. This is a valid assumption during the initial buildup of oscillation and can therefore be used to predict the starting conditions for oscillation. The starting conditions are obviously an important part of the design and are studied in great detail. After the signal becomes large, the Y-parameters can still be useful if we define them as the ratios of the fundamental components of current to the fundamental components of voltage.

The Y-parameter representation of a device is shown in Figure 6-1 along with an equivalent circuit which can be used in Figure 6-2.



$$I_1 = y_i V_1 + y_r V_2$$

$$I_2 = y_f V_1 + y_o V_2$$

Figure 6-1. Y-Parameter representation of a transistor.

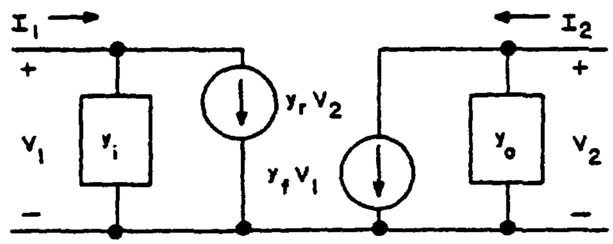


Figure 6-2. Transistor equivalent circuit using Y-parameters.

The term y_i is the input admittance with the output short-circuited:

$$y_i = \left.\frac{I_1}{V_1}\right|_{V_2=0} .$$

In like manner, y_o is the short-circuit output admittance:

$$y_o = \left.\frac{I_2}{V_2}\right|_{V_1=0} ;$$

y_f is the forward transfer admittance (also referred to as the trans-conductance):

$$y_f = \left.\frac{I_1}{V_2}\right|_{V_1=0} .$$

and y_r is the reverse transfer admittance:

$$y_r = \left.\frac{I_1}{V_2}\right|_{V_1=0} .$$

A subscript e, b, or c is added to the Y-parameters to indicate the circuit configuration; thus, y_{ib} is the common-base input admittance while y_{ie} is the common-emitter input admittance. If the common-emitter parameters are given on the transistor data sheet, the common-base parameters can be calculated using the following relationships:

$$y_{ib} = y_{ie} + y_{oe} + y_{fe} + y_{re} \tag{6-1}$$

$$y_{fb} = -(y_{fe} + y_{oe}) \tag{6-2}$$

$$y_{rb} = -(y_{re} + y_{oe}) \tag{6-3}$$

$$y_{ob} = y_{oe} \tag{6-4}$$

Also Y-parameters can be calculated from S-parameters as follows.

If we assume a common-emitter configuration so that $y_{ie} = y_{11}$, $y_{fe} = y_{21}$, $y_{re} = y_{12}$, and $y_{oe} = y_{22}$,

$$y_{11} = \frac{(1 - S_{11})(1 + S_{22}) + S_{12}S_{21}}{Z_o[(1 + S_{22})(1 + S_{11}) - S_{12}S_{21}]} \tag{6-5}$$

$$y_{21} = \frac{-2S_{21}/Z_o}{(1 + S_{11})(1 + S_{22}) - S_{12}S_{21}} \tag{6-6}$$

$$y_{12} = \frac{-2S_{12}/Z_o}{(1 + S_{11})(1 + S_{22}) - S_{21}S_{12}} \tag{6-7}$$

$$y_{22} = \frac{(1 + S_{11})(1 - S_{22}) + S_{21}S_{12}}{Z_o[(1 + S_{11})(1 + S_{22}) - S_{12}S_{21}]} \tag{6-8}$$

For purposes of oscillator analysis, it is often convenient to break the Y-parameters into their real and imaginary parts (as listed on transistor data sheets). Therefore, the following standard designations will be used

$$y_i = g_i + jb_i$$

$$y_o = g_o + jb_o$$

$$y_f = g_f + jb_f$$

$$y_r = g_r + jb_r$$

Physically,

$$g_i = \frac{1}{R_{\text{in}}}, \quad b_i = \omega C_{\text{in}}, \quad g_o = \frac{1}{R_{\text{out}}}, \quad \text{and} \quad b_o = \omega C_{\text{out}}. \tag{6-9}$$

6.3. HYBRID π EQUIVALENT CIRCUIT

The hybrid π equivalent circuit of a transistor is shown in Figure 6-3. Unfortunately, at all but low frequency, where capacitances can be neglected, the application of this circuit to oscillators leads

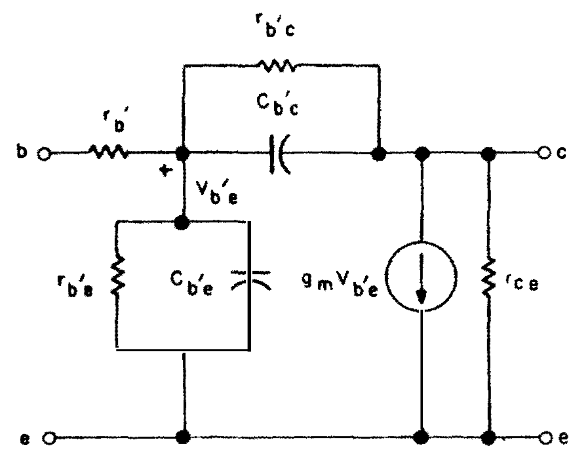


Figure 6-3. Hybrid π (common-emitter) equivalent circuit.

to complicated equations that are difficult to solve. Pritchard[39] has made certain simplifications to the equivalent circuit of Figure 6-3. Figure 6-4 shows this approximate high-frequency equivalent circuit for the common-emitter configuration.

The resulting Y-parameters are

$$y_{ie} = \frac{1}{r_{b'} - j(\omega_t/\omega)r_{e'}} \tag{6-10}$$

$$y_{re} = \frac{-\omega_t r_{c'} C_c}{r_{b'} - j(\omega_t/\omega)r_{e'}} \tag{6-11}$$

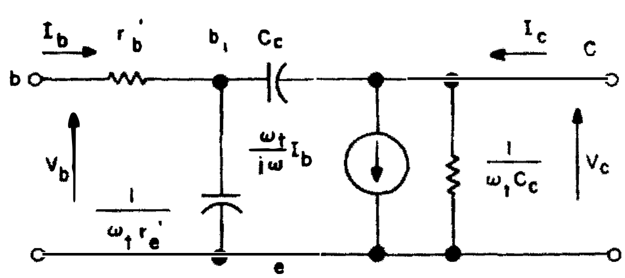


Figure 6-4. Approximate high-frequency equivalent circuit of a junction transistor.

$$y_{fe} = \frac{-j(\omega_t/\omega)}{r_{b'} - j(\omega_t/\omega)\, r_{e'}} \tag{6-12}$$

$$y_{oe} = \frac{\omega_t C_c(r_{b'} + r_{e'}) + j\omega C_c(r_{b'})}{r_{b'} - j(\omega_t/\omega)\, r_{e'}} \tag{6-13}$$

These equations establish the approximate correlation between internal parameters and terminal parameters. Since most of these expressions are rather involved, it is simpler to express circuit design equations in terms of the terminal parameters.[26]

6.4. NONLINEAR MODELS

In contrast to the case of small-signal properties of the transistor, very few results predicting the large-signal behavior are available in the literature. As a result, the engineer is generally unfamiliar with this aspect of transistor behavior.

As previously stated, the small-signal behavior holds only for base-to-emitter signal voltages up to about 10 mV. It is possible, however, to define equivalent linear properties at higher signal levels. This is done by forming the ratios of the fundamental components of voltage to the fundamental components of current. The ratios, of course, change with amplitude. It is the purpose of this section to present formulas which will enable the engineer to predict the large-signal properties of a transistor knowing the small-signal values and the signal amplitude. The predictions are accomplished using the hybrid π equivalent circuit. Two types of analysis are made; the first, in section 6.4.1, is valid for the intrinsic transistor, neglecting the base spreading resistance, and is useful for most crystal oscillator applications below 10–20 MHz. In some low-noise oscillator applications, however, it has been found desirable to use emitter degeneration to reduce $1/f$ noise. Therefore a second analysis, given in section 6.4.2, is presented to allow prediction of the amplitude of oscillation when degeneration is used. The results generally follow the same form, although the mathematics used in deriving them is considerably different.

6.4.1. Intrinsic Transistor Model*

The large-signal analysis of transistor parameters presented in this paragraph is based on the hybrid π equivalent circuit shown in Figure 6-5. The various elements of this circuit are the same as those presented in Figure 6-3, although they are represented differently in some cases. It should be understood that this circuit is only an approximate equivalent circuit of the transistor and represents some rather serious simplifications of the actual device.

The resistor $r_{bb'}$ is the base-spreading resistance and is neglected in the analysis.

The emitter resistance r_e is composed of intrinsic and extrinsic parts. The intrinsic part usually accounts for the largest portion of the resistance and, for small-signal conditions, is given by

$$r_{e0} = \frac{KT\lambda}{qI_e}; \tag{6-14}$$

at 27°C and $\lambda = 1$

$$r_{e0} = \frac{26}{I_e} \quad \text{with } I_e \text{ in mA.} \tag{6-15}$$

Here the subscript 0 refers to a small-signal value. As will be shown later, r_e varies considerably with signal level.

The base diffusion capacitance C_{De} is given by k/r_e and generally accounts for most of the transistor input capacitance in the active region.

The diffusion capacitance is related to the gain–bandwidth product, so that

$$k \doteq \frac{1}{2\pi f_t}. \tag{6-16}$$

C_{Te} is base-to-emitter transition capacitance (junction capacitance), which depends on the size of the base-to-emitter junction. It

*The application of these results to crystal oscillators is essentially parallel to the results discussed in connection with LC oscillators by Holford in Mullard Technical Communications, see reference 21.

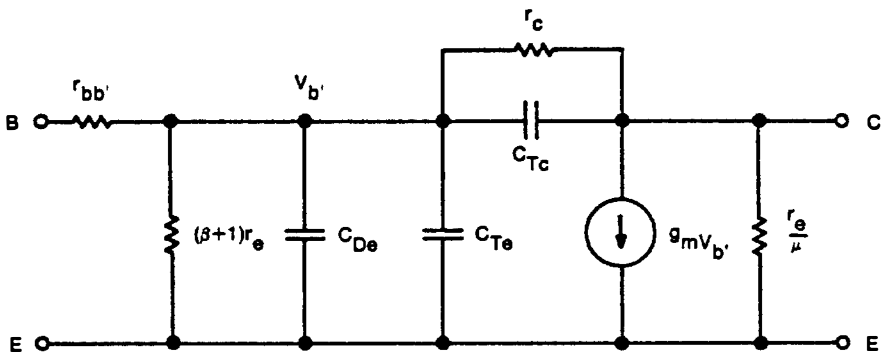


Figure 6-5. Hybrid π equivalent circuit of a transistor.

generally varies as the square root of the base-to-emitter voltage; thus:

$$C_{Te} = \frac{K_{Te}}{(V - V_{be})^{1/2}} \tag{6-17}$$

The base-to-collector transition (junction) capacitance C_{Tc} is dependent on the junction area and on the grading of the junction. It varies as some power of the applied voltage; thus:

$$C_{Tc} = \frac{K_{Tc}}{[V_{cb} + V]^{\alpha}} \tag{6-18}$$

where V is the contact potential and α is a function of the grading of the junction; α is usually between 0.5 and 0.1.

Transconductance is given approximately by $1/r_e$, and the feedback factor is given by μ which is considered to be a constant for this analysis. It is shown in Appendix G that r_e increases with signal level and is given by

$$\frac{r_e}{r_{e0}} = \frac{\left(\frac{Eq}{\lambda KT}\right) I_0 \left(\frac{Eq}{\lambda KT}\right)}{2I_1 \left(\frac{Eq}{\lambda KT}\right)} \tag{6-19}$$

where r_{e0} is the small-signal emitter resistance given by equation (6-14). E is the peak value of the base-to-emitter signal voltage assumed to be given by $E \cos \omega t$. For purposes of this analysis, it is convenient to define the voltages in terms of $q/\lambda KT$ units; thus, we let

$$V = \frac{qE}{\lambda KT}. \tag{6-20}$$

At 27°C and $\lambda = 1$,

$$V = \frac{E}{26} \tag{6-21}$$

when E is in millivolts. Thus, we have

$$\frac{r_e}{r_{e0}} = \frac{VI_0(V)}{2I_1(V)} \tag{6-22}$$

$I_0(V)$ and $I_1(V)$ are hyperbolic Bessel functions of the first kind of orders zero and one, respectively. Equation (6-22) is plotted in Figure 6-6 along with its reciprocal.

From the transistor equivalent circuit, we see that most of the parameters are either proportional to or inversely proportional to r_e. Thus knowing how r_e behaves with signal voltage, we know also how the input and output resistance, the transconductance, and the input capacitance behave.

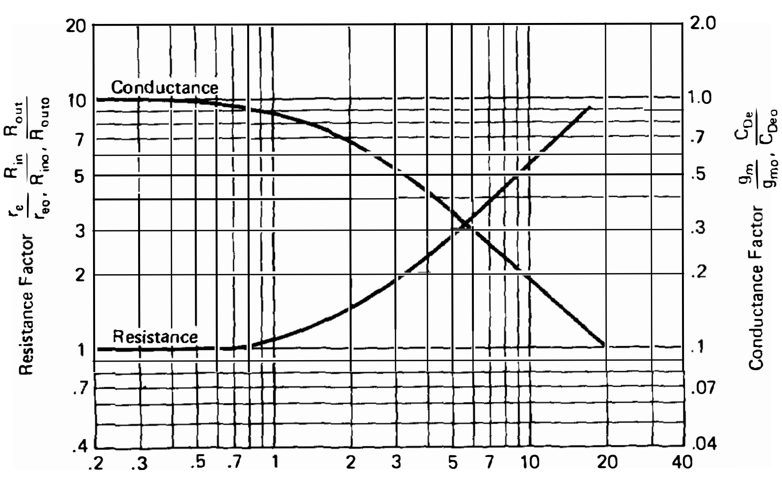


Figure 6-6. Transistor parameters versus signal voltage.

It should be pointed out that the analysis requires the small-signal parameters to be calculated at the final emitter current with signal applied. In general, this current is higher than the value with no signal. An equation is derived in appendix G which allows the final emitter current to be predicted. This is done by computing the change in the dc base-to-emitter voltage resulting from the signal. It is then only necessary to allow for this decrease in base-to-emitter voltage, V bias, when computing the values of the bias resistors. This equation is given by

$$\frac{qV_{\text{bias}}}{\lambda KT} = \ln I_0(V) \tag{6-23}$$

and is plotted in Figure 6-7.

It is possible to predict the amount of fundamental and harmonic

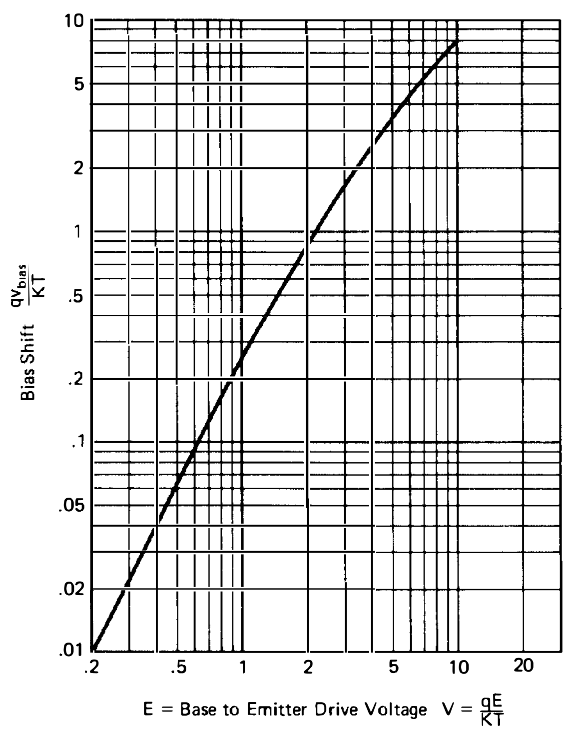


Figure 6-7. Base-to-emitter bias shift versus applied signal voltage.

signal present in the collector current when a sinusoidal signal is applied to the base. The equation predicting this behavior (see Appendix G), neglecting feedback, is given by:

$$i_c = 2I_e(\text{mean})\left[\frac{I_1(V)}{I_0(V)}\cos\omega t + \frac{I_2(V)}{I_0(V)}\right.$$

$$\left.\cos 2\omega t + \frac{I_3(V)}{I_0(V)}\cos 3\omega t + \cdots\right] \quad (6\text{-}24)$$

This equation is plotted in Figure 6-8 along with the equation giving the peak collector current, which is

$$i_c(\text{peak}) = I_e(\text{mean})\, e^v/I_0(V) \quad (6\text{-}25)$$

This graph may be used to determine the efficiency of transistors or oscillators used as frequency multipliers. It is interesting to note

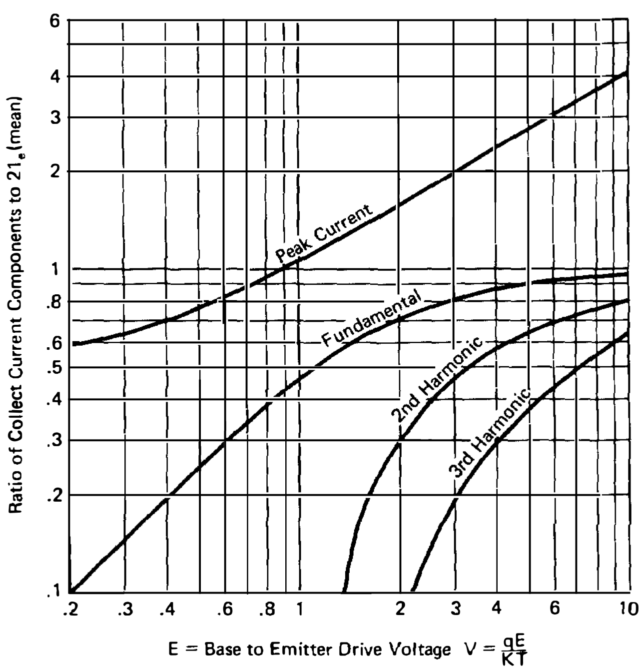


Figure 6-8. Collector current components versus applied signal voltage.

that the fundamental component of collector current approaches $2I_e$(mean) for large drive voltages.

The results presented in this paragraph are derived in Appendix G, as previously indicated. The appendix also contains a comparison of the predicted and measured results for a typical oscillator transistor using different values of emitter current.

Depending on the degree to which the assumptions made are valid, the analysis may on occasions yield a large error in the absolute value of the amplitude. In these cases the trend shown by the analysis may still be useful, however.

6.4.2. Nonlinear Model with Emitter Degeneration*

Basically the same types of curve have been derived for the case where the extrinsic emitter resistance is predominant. For this case, the equivalent circuit assumes the form shown in Figure 6-9, where R_f is the emitter degeneration resistor. Here the feedback term μ is assumed to be negligible. Also for this analysis $(r_{bb'} + R_f)_0$ is assumed to be negligible compared to $\beta(R_f + r_e)_0$. Under these conditions, it is shown in Appendix H that

$$\frac{(R_f + r_e)}{(R_f + r_e)_0} = \frac{\pi}{\theta - \frac{1}{2}\sin 2\theta} \tag{6-26}$$

Here again the subscript 0 refers to a small-signal value. θ is one-half the effective conduction angle in radians. The conduction angle can be computed in terms of the signal voltage and the mean emitter current by the equation:

$$\sin\theta - \theta\cos\theta = \frac{I_e(\text{mean})\,(r_e + R_f)_0\pi}{E} \tag{6-27}$$

where E is the peak value of the signal voltage applied between the base and ground, assumed to be of the form $E\sin\omega t$. Equations (6-26) and (6-27) can be plotted parametrically and are given in Figure 6-10 along with the reciprocal of equation (6-26). It should be noted that if R_f is zero, the term

$$\frac{E}{I_e(\text{mean})\,(r_e + R_f)_0} = \frac{E}{I_e(\text{mean})\,r_{e0}} = \frac{Eq}{KT\lambda}$$

*The application of these results to crystal oscillators is essentially parallel to the results discussed in connection with LC oscillators by Holford in Mullard Technical Communications, see reference 22.

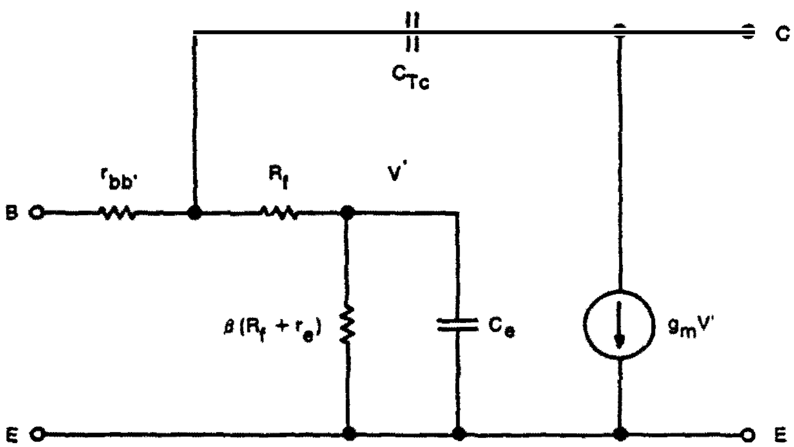
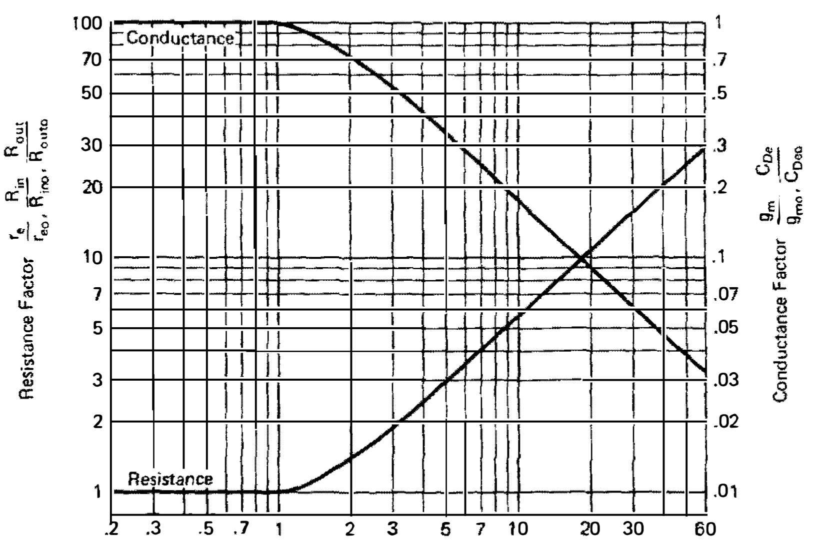


Figure 6-9. Approximate equivalent circuit with emitter degeneration.



E = Base to Emitter Signal Voltage

$V = \frac{E}{Ie(mean)\,(r_e + R_f)_o}$ in Volts/Amp./Ohm

Figure 6-10. Transistor parameters versus signal voltage.

as before. The curves are not identical, however, because the latter analysis acquires a considerable error if R_f is not larger than r_e.

As before, it is possible to compute the reduction in base-to-emitter voltage due to the presence of signal, which in this case is given by

$$\frac{V_{\text{bias}}}{I_e(\text{mean})\,(r_e + R_f)_0} = \frac{\pi}{\tan\theta - \theta} + 1. \qquad (6\text{-}28)$$

This equation is plotted in Figure 6-11.

It is also possible to predict the harmonic content of the collector current, the ac value of which is given by

$$i_c = I_e(\text{mean})\left\{\frac{[\theta - \frac{1}{2}\sin 2\theta]}{[\sin\theta - \theta\cos\theta]}\right. \sin\omega t$$

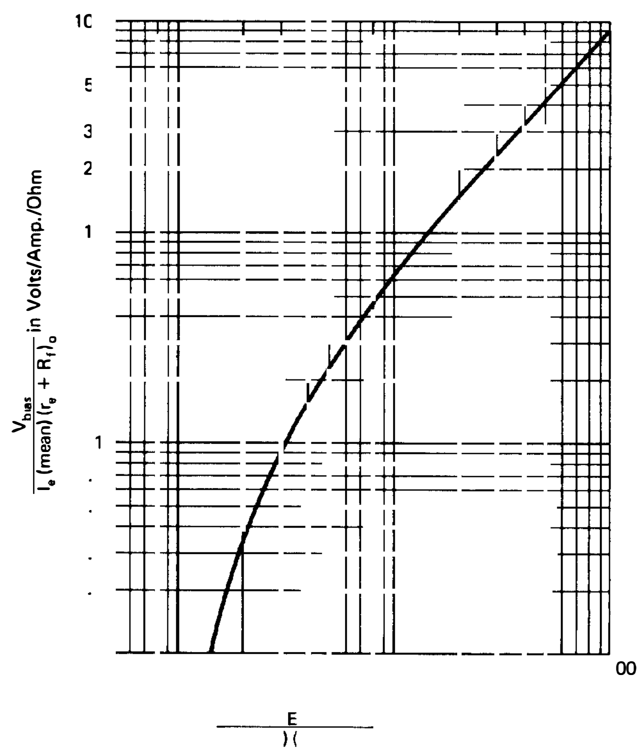


Figure 6-11. Base-to-emitter bias shift versus applied signal voltage.

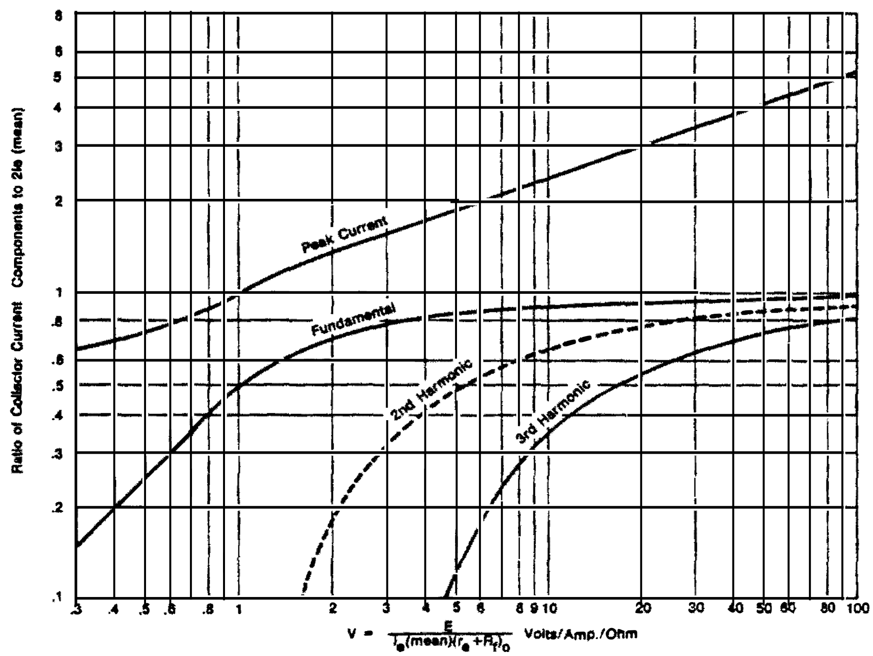


Figure 6-12. Collector current components versus applied signal voltage.

$$+\frac{[\cos\theta\sin 2\theta - \sin\theta - \frac{1}{3}\sin 3\theta]}{[\theta\sin\theta - \theta\cos\theta]}\cos 2\omega t$$

$$\left.+\frac{[\frac{2}{3}\cos\theta\sin 3\theta - \frac{1}{2}\sin 2\theta - \frac{1}{4}\sin 4\theta]}{[\sin\theta - \theta\cos\theta]}\sin 3\omega t + \cdots\right\} \quad (6\text{-}29)$$

This equation is plotted in Figure 6-12 along with the peak collector current, which is given by the equation

$$i_c(\text{peak}) = I_e(\text{mean})\,\frac{\pi(1-\cos\theta)}{\sin\theta - \theta\cos\theta} \quad (6\text{-}30)$$

The derivation of the results presented in this section is made in Appendix H.