Appendix A

ISOTROPIC SYSTEM

$$\mathbf{c} = \begin{bmatrix} c_{11} & c_{12} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{12} & c_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{44} \end{bmatrix} \qquad c_{44} = \frac{c_{11} - c_{12}}{2}$$

(isotropy condition with two independent components)

$$\mathbf{p} = \begin{bmatrix} p_{11} & p_{12} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{11} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{12} & p_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{44} \end{bmatrix} \qquad p_{44} = \frac{c_{11} - p_{12}}{2}$$

(two independent components)

$$e_{Ij} = r_{Ij} = 0$$

All units are MKS.

density $\rho \times 10^3$ kg/m^3;
stiffness components $c \times 10^9$ N/m^2;
electro-optic components $r \times 10^{-12}$ m/V (unless otherwise noted, r is at .633 μm).

The piezoelectric, photoelastic, and permittivity components are dimensionless.

	c_{11}	c_{12}	p_{11}	p_{12}	ρ	ϵ
Fused quartz	78.5	31.2	.121	.27	2.2	3.8
Lucite	8.5	1.43	.3	.28	1.2	

CUBIC SYSTEM

classes: 23, $\bar{4}3m$, $m3m$

$$\mathbf{c} = \begin{bmatrix} c_{11} & c_{12} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{12} & c_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{44} \end{bmatrix}$$

(three independent components)

All classes have three independent components:

$$\mathbf{p} = \begin{bmatrix} p_{11} & p_{12} & p_{12} & 0 & 0 & 0 \\ p_{21} & p_{11} & p_{12} & 0 & 0 & 0 \\ p_{21} & p_{21} & p_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{44} \end{bmatrix}$$

(either three or four independent components)

$$\mathbf{e} = \begin{bmatrix} 0 & 0 & 0 & e & 0 & 0 \\ 0 & 0 & 0 & 0 & e & 0 \\ 0 & 0 & 0 & 0 & 0 & e \end{bmatrix}$$

(one independent component)

	Class	ρ	e	r	ϵ
Bismuth germanium oxide	23	9.2	.99		38
Gallium arsenide	$\bar{4}3m$	5.3	.154	1.4	12.5
Gallium phosphide	$\bar{4}3m$	4.13	−.1	−1	11.1
Indium antimony	$\bar{4}3m$	5.77	.7		17.7
Germanium	$m3m$	5.3	—	—	
Silicon	$m3m$	2.33	—	—	

	c_{11}	c_{12}	c_{44}	p_{11}	p_{12}	p_{14}
Bismuth germanium arsenide	128	31	25.5			
Gallium arsenide	119	53.8	59.4	−.165	−.14	−.072
Gallium phosphide	141	62.5	70.5	−.151	−.082	−.074
Germanium	129	48.3	67.1	.27	.235	.125
Silicon	166	64	80	−.1	.009	−.11
Silver iodide	26.2	23	8.2			
Indium antimony	67.2	36.7	30.2			

(photoelastic constants for Ge at 10.6 μm, for Si at 3.39 μm, and for GaAs at 1.15 μm)

TETRAGONAL CLASSES: 4, $\bar{4}$, 4/*m*, 422, 4*mm*, $\bar{4}$2*m*, 4/*mmm*

$$\mathbf{c} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & c_{16} \\ c_{12} & c_{11} & c_{13} & 0 & 0 & c_{26} \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ c_{61} & c_{62} & 0 & 0 & 0 & c_{66} \end{bmatrix}$$

For classes 4, $\bar{4}$, and 4/*m*, $c_{26} = -c_{16}$ (seven independent components). For classes 422, 4*mm*, $\bar{4}$2*m*, and 4/*mm*, $c_{26} = c_{16} = 0$ (six independent components).

$$\mathbf{p} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & 0 & 0 & p_{16} \\ p_{12} & p_{11} & p_{13} & 0 & 0 & p_{26} \\ p_{31} & p_{31} & p_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & p_{45} & 0 \\ 0 & 0 & 0 & p_{54} & p_{44} & 0 \\ p_{61} & p_{62} & 0 & 0 & 0 & p_{66} \end{bmatrix}$$

For classes 4, $\bar{4}$, and 4/*m*, $p_{16} = -p_{26}$, $p_{45} = -p_{54}$, and $p_{26} = -p_{61}$; for these classes, there are 11 independent components. For classes 422, 4*mm*, $\bar{4}$2*m*, and 4/*mm*, $p_{16} = p_{26} = p_{45} = p_{54} = p_{62} = p_{61} = 0$, and there are seven independent components.

$$\mathbf{e} = \begin{bmatrix} 0 & 0 & 0 & e_{14} & e_{15} & 0 \\ 0 & 0 & 0 & e_{24} & e_{25} & 0 \\ e_{31} & e_{32} & e_{33} & 0 & 0 & 0 \end{bmatrix}$$

For class 4, $e_{25} = -e_{14}$, $e_{24} = e_{15}$, $e_{31} = e_{32}$. Class $\bar{4}$ is the same as class 4 with $e_{33} = 0$. Class 422 is the same as $\bar{4}$ with $e_{15} = e_{31} = 0$. For class 4*mm*, $e_{14} = e_{25} = 0$ (five independent components). Class $\bar{4}2m$ is the same as 4*mm* with $e_{31} = e_{33} = 0$. For classes 4/*m* and 4/*mmm*, $e_{Ij} = 0$.

$$\boldsymbol{\epsilon} = \begin{bmatrix} \epsilon_{11} & 0 & 0 \\ 0 & \epsilon_{11} & 0 \\ 0 & 0 & \epsilon_{33} \end{bmatrix}$$ (all clases have two independent components

	Class	ρ	ϵ_{11}	ϵ_{33}	e_{14}
Lead molybdate	4/*m*	6.95	34	41	
Rutile	4/*mmm*	4.26	89	173	
Tellurium dioxide	422	6.0	23	25	.22

	c_{11}	c_{12}	c_{13}	c_{33}	c_{44}	c_{66}
Lead molybdate	109	68	53	92	27	34
Rutile	266	173	136	470	124	189
Tellurium dioxide	56	51	22	106	26.5	66

	p_{11}	p_{12}	p_{13}	p_{31}	p_{33}	p_{44}	p_{66}
Rutile	−.001	.17	−.17	−.10	−.06	.01	−.07
Tellurium dioxide	.007	.19	.34	.1	.24	−.17	−.046
Lead molybdate	.24	.24	.026	.18	.3	.067	.05
	$p_{61} = .013$		$p_{45} = .01$		$p_{16} = .017$		

HEXAGONAL SYSTEM

Classes: 6, $\bar{6}$, 6/*m*, $\bar{6}m2$, 6*mm*, 622, and 6/*mmm*; we consider only the latter three classes.

$$\mathbf{c} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}$$

$$c_{66} = \frac{c_{11} - c_{12}}{2}$$

(five independent components)

The photoelastic matrix has the same form as the stiffness matrix for the 6/*mmm*, 622, and 6*mm* classes, and there are six independent components (because $p_{31} \neq p_{13}$).

The form of the piezoelectric matrix is

$$\mathbf{e} = \begin{bmatrix} 0 & 0 & 0 & e_{14} & e_{15} & 0 \\ 0 & 0 & 0 & e_{24} & e_{25} & 0 \\ e_{31} & e_{32} & e_{33} & 0 & 0 & 0 \end{bmatrix}$$

For class 622, $e_{25} = -e_{14}$, $e_{31} = e_{32} = e_{33} = 0$ (one component). For class 6*mm*, $e_{15} = e_{24}$, $e_{31} = e_{32}$, and $e_{14} = e_{25} = 0$ (three components). For class 6/*mmm*, $e_{Ij} = 0$.

The form of the permittivity matrix is (for all classes)

$$\boldsymbol{\epsilon} = \begin{bmatrix} \epsilon_{11} & 0 & 0 \\ 0 & \epsilon_{11} & 0 \\ 0 & 0 & \epsilon_{33} \end{bmatrix} \qquad \text{(two independent components)}$$

	Class	ρ	ϵ_{11}	ϵ_{33}		
Cadmium sulfide	6*mm*	4.8	9	9.5		
Zinc oxide	6*mm*	5.7	8.6	10		
	c_{11}	c_{12}	c_{13}	c_{33}	c_{44}	
Cadmium sulfide	91	58	51	94	15	
Zinc oxide	210	121	105	211	43	
	p_{11}	p_{12}	p_{13}	p_{31}	p_{33}	p_{44}
Cadmium sulfide	−.14	−.066	−.057	−.041	−.2	.054
	e_{15}	e_{31}	e_{33}	r_{15}	r_{31}	r_{33}
Cadmium sulfide	−.21	−.24	.44	1.7	2.45	2.9
Zinc oxide	−.48	−.57	1.32		1.4	2.6

(r_{Ij} for cadmium sulfide at 10.6 μm)

TRIGONAL SYSTEM

Classes: 3, $\bar{3}$, $\bar{3}m$, 32, and 3*m*; we consider only the latter three classes.

$$\mathbf{c} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & 0 & 0 \\ c_{12} & c_{11} & c_{13} & c_{24} & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ c_{14} & c_{24} & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & c_{56} \\ 0 & 0 & 0 & 0 & c_{56} & c_{66} \end{bmatrix} \qquad c_{66} = \frac{c_{11} - c_{12}}{2}$$

For classes $\bar{3}m$, 32, and $3m$, $c_{24} = -c_{14}$ and $c_{56} = 2c_{14}$ (six independent components). The form of the photoelastic matrix is identical to that of the stiffness matrix for classes $\bar{3}m$, 32, and $3m$ (eight independent components, because $p_{14} \neq p_{41}$ and $p_{31} \neq p_{13}$). The form of the piezoelectric matrix for class $3m$ (lithium niobate) is

$$\mathbf{e} = \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 2e_{21} \\ e_{21} & -e_{21} & 0 & e_{15} & 0 & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \end{bmatrix} \quad \text{(four independent components)}$$

The form of the piezoelectric matrix for class 32 (crystal quartz) is

$$\mathbf{e} = \begin{bmatrix} e_{11} & -e_{11} & 0 & e_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & -e_{14} & 2e_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{(two independent components)}$$

For class $\bar{3}m$, $e_{Ij} = 0$.

The form of the permittivity matrix is (for all classes)

$$\boldsymbol{\epsilon} = \begin{bmatrix} \epsilon_{11} & 0 & 0 \\ 0 & \epsilon_{11} & 0 \\ 0 & 0 & \epsilon_{33} \end{bmatrix} \quad \text{(two independent components)}$$

	Class	ρ	ϵ_{11}	ϵ_{33}
Lithium niobate	$3m$	4.64	44	29
Lithium tantalate	$3m$	7.4	41	43
Sapphire	$\bar{3}m$	4.0	10	10
Quartz	32	2.7	4.5	4.6

	c_{11}	c_{12}	c_{13}	c_{33}	c_{14}	c_{44}
Lithium niobate	203	53	75	245	9	60
Lithium tantalate	233	47	80	275	−11	94
Sapphire	494	158	114	496	−23	145
Quartz	87	7	12	107	−18	58

	p_{11}	p_{12}	p_{13}	p_{14}	p_{31}	p_{33}	p_{41}	p_{44}
Lithium niobate	−.026	.09	.13	−.075	.18	.07	−.15	.146
Lithium tantalate	−.08	.08	.09	−.026	.09	−.04	−.085	.03
Sapphire	−.23	−.03	.02	0	−.04	−.2	.01	−.1
Quartz	.16	.27	.27	−.03	.3	.1	−.05	−.08

	e_{15}	e_{22}	e_{31}	e_{33}	r_{15}	r_{22}	r_{31}	r_{33}
Lithium niobate	3.7	2.5	.2	1.3	28	3.4	8.6	31
Lithium tantalate	2.6	1.6	0	1.9	20	1	7.5	33
Sapphire		nonpiezoelectric				nonelectro-optic		
		e_{11}	e_{14}					
Quartz		.171	−.044					

ORTHORHOMBIC SYSTEM

Classes: 222, *mmm*, and *mm*2. The form of the stiffness matrix for both classes is

$$\mathbf{c} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}$$

All nine elements are independent. The form of the photoelastic matrix is

$$\mathbf{p} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & 0 & 0 & 0 \\ p_{21} & p_{22} & p_{23} & 0 & 0 & 0 \\ p_{32} & p_{32} & p_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{66} \end{bmatrix}$$

All 12 elements are generally independent.

The form of the piezoelectric matrix for class 222 is

$$\mathbf{e} = \begin{bmatrix} 0 & 0 & 0 & e_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & e_{25} & 0 \\ 0 & 0 & 0 & 0 & 0 & e_{36} \end{bmatrix} \qquad \text{(three independent components)}$$

The form of the piezoelectric matrix for class 2*mm* is

$$\mathbf{e} = \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{24} & 0 & 0 \\ e_{31} & e_{32} & e_{33} & 0 & 0 & 0 \end{bmatrix} \quad \text{(five independent components)}$$

For class *mmm*, $e_{Ij} = 0$.

	Class	ρ
Barium sodium niobate	2*mm*	5.3
Rochelle salt	222	1.8

	c_{11}	c_{12}	c_{13}	c_{22}	c_{23}	c_{33}	c_{44}	c_{55}	c_{66}
Barium sodium niobate	240	104	50	247	52	135	65	66	76
Rochelle salt	28	17	15	41	20	39	6.7	2.9	10

	e_{15}	e_{24}	e_{31}	e_{32}	e_{33}
Barium sodium niobate	2.8	3.4	−.4	−.3	4.3

	e_{14}	e_{25}	e_{36}
Rochelle salt	2.2	.154	.12