Appendix J

Acoustooptic Effect in Anisotropic Crystals

We have only considered, in the main body of this text, the effect of acoustooptic diffraction in an isotropic crystal. We review here an important technique with major advantages, the use of shear or longitudinal acoustic waves to diffract an optical wave in an anisotropic crystal. We will demonstrate the important effects that occur in this case by using as an example shear wave diffraction in isotropic materials and an anisotropic material tellurium dioxide (TeO_2).

One advantage is the high value of the piezooptic coupling coefficient p and the acoustooptic factor of merit M, which for TeO_2 can be 515 times the value in fused quartz. A second reason for the use of TeO_2 is associated with the very slow acoustic shear wave velocity 0.617 km/sec in the [110] direction; this makes it possible to obtain relatively large Bragg angles at frequencies in the 50 to 100 MHz range. A third reason is associated with the fact that it is possible to obtain a diffracted wave with a constant amplitude over a broad acoustic input frequency range *without changing the input angle* θ_i.

We shall show, here, how this latter effect comes about when a polarized optical wave interacts with an acoustic shear wave. Tellurium dioxide is a tetragonal uniaxial crystal which is optically active. In an optically active material, a right hand circularly polarized light wave propagating along the optical axis, i.e., in the z direction or [001] direction has a refractive index n_0 different from the refractive index n_1 of a left hand circularly polarized wave propagating along the optical axis.

The general relation for the change in the dielectric impermeability due to interaction with an acoustic wave is

$$\Delta B_{ij} = p_{ijkl} S_{kl} \tag{J.1}$$

Suppose we consider, first of all, an isotropic material through which a plane linearly polarized optical wave, polarized in the x direction, propagates in the z direction. If this wave interacts with an acoustic shear wave propagating in the x direction with its particle velocity in the y direction, it follows from Eq. (J.1) that there is a perturbation in the dielectric impermeability of the form

$$\Delta B_{xy} = P_{xyxy} S_{xy} \tag{J.2}$$

This expression, in turn, implies that there is a perturbation in the dielectric constant of the form $\Delta\varepsilon_{xy}$. So, it follows that the field E_x gives rise to a displacement density $D_y = \Delta\varepsilon_{xy} E_x$ or to a plane polarized wave in the medium with its polarization at right angles to the original incident wave. Therefore, one advantage of this type of interaction is that it is easy to use a polarizer to separate the diffracted wave from the incident wave.

For an anisotropic material like TeO_2, one possible mode of operation is to use an incident circularly polarized wave propagating along the optical axis, the [001] direction, which interacts with a pure acoustic shear wave propagating in the [100] direction with the particle velocity in the [001] direction. It may be shown by a similar argument to that given above that the diffracted wave will be one with the opposite circular polarization, at least when the diffraction angle is small.

We now consider how the Bragg diffraction equations change from those for an isotropic material. We follow the analysis of Dixon[1,2].

The Bragg diffraction equations are

$$\omega_0 \pm \omega_a = \omega_1 \tag{J.3}$$

and

$$\mathbf{k}_0 \pm \mathbf{k}_a = \mathbf{k}_1 \tag{J.4}$$

Here, the subscript 0 refers to the undiffracted wave with a refractive index n_0, and the subscript 1 to the diffracted wave with a refractive index n_1 and the opposite circular polarization.

Referring to Fig. 4.9.6, we can write

$$\omega_1 = \omega_0 - \omega_a \tag{J.5}$$

$$-k_1 \sin\theta_R = k_0 \sin\theta_i - k_a \tag{J.6}$$

and

$$k_1 \cos\theta_R = k_0 \cos\theta_i \tag{J.7}$$

when $k_1 \neq k_0$, it follows by squaring and adding Eqs. (J.6) and (J.7), that

$$\sin\theta_i = (k_a/2k_0) + (k_0^2 - k_1^2)/2k_a k_0 \tag{J.8}$$

with

$$\sin\theta_R = (k_a/2k_0) - (k_0^2 - k_1^2)/2k_a k_0 \tag{J.9}$$

Suppose that we assume that $k_0 > k_1$, and that for small deflection angles the variation of k_0 and k_1 with angle in an anisotropic material can be neglected compared to other effects. Then, as a function of frequency or k_a, it will be seen that the first term in Eq. (J.8) increases with frequency and the second term decreases with frequency. Here, we have assumed that k_0 and k_1 do not vary much for small diffraction angles. The second terms in Eqs. (J.8) and (J.9) would not exist in an isotropic medium. At a particular frequency ω_{a0}, the value of θ_i is minimum. For frequencies around this minimum value, θ_i hardly changes. In an isotropic medium, on the other hand, the optimum value of θ_i varies linearly with frequency. Therefore, operation of a TeO_2 acoustooptic Bragg cell can occur over a much larger bandwidth than in a simple isotropic medium. Because of the low phase velocity of the shear wave in the TeO_2 Bragg cell, and its highly anisotropic characteristics, the optimum frequency of operation is approximately 50 MHz.

By adding Eqs. (J.8) and (J.9), it will be seen that

$$\sin \theta_i + \sin \theta_R = k_a/k_0 \tag{J.10}$$

It follows that when the incident angle is kept fixed, the angle of diffraction varies linearly with frequency *in just the same manner as in an isotropic material.*

It has been shown by Yano et al.[3] that the operation of this type of Bragg cell can have certain problems due to nonlinear coupling at higher input powers from the diffracted wave to a second wave, a condition which occurs near 50 MHz, at the center of the band. This difficulty can be avoided by operation of the Bragg cell at angles slightly off axis (only a few degrees). Interestingly enough, in this case, the input beam can be linearly polarized; but the polarization direction is quite critical. With this type of Bragg cell, the diffraction efficiency can be as much as 95%, and operation with as much as 80% efficiency can be obtained over an octave bandwidth centered at 75 MHz.

REFERENCES

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