Appendix I

Rate of Movement of Charge in a CCD Due to Diffusion and Space Charge

A simple diffusion calculation of the rate of charge transfer through the CCD registers is made in this appendix. Three mechanisms contribute to the rate at which charge is transferred. These are: diffusion, due to the charge gradient present; self-induced drift, due to the fields associated with the carrier space charge; and drift associated with the fringing field between the electrodes. Numerical solutions of the relevant equations show that self-induced drift tends to dominate during short times, with diffusion becoming more prominent as the charge density drops; this is because the former mechanism depends on the carrier density. However, the fringe field at the gate is extremely important. In a well-designed, fast device, the fringe field effect is usually dominant. If surface state recombination effects are negligible, devices with 10-μm gates are operable at 10 MHz and devices with 4-μm gates are operable at frequencies as high as 100 MHz, with both operable down to extremely low frequencies of the order of a few hundred hertz.

Here we will calculate only the diffusion effects, and give some discussion of space-charge effects. For simplicity, we will make the calculations for a p-type carrier. The diffusion current density J_p for p-type carriers moving in the y direction is

$$J_p = -qD\frac{\partial p}{\partial y} \tag{I.1}$$

where p is the hole density per unit volume, q is the charge of a hole, and D is the diffusion coefficient of the holes. Using the continuity equation, we find that

$$\frac{\partial J_p}{\partial y} + q\frac{\partial p}{\partial t} = 0 \tag{I.2}$$

Hence the differential equation for the motion is

$$D\frac{\partial^2 p}{\partial y^2} = \frac{\partial p}{\partial t} \tag{I.3}$$

We suppose that the gate is of length L, and that the boundary conditions, referring to Fig. I.1, are zero carrier gradient, or current, at $y = 0$, and zero carrier concentration at $y = L$. We solve this equation by the method of separation of variables, that is, we use a product solution, writing

$$p = T(t)Y(y) \tag{I.4}$$

Inserting Eq. (I.4) into Eq. (I.3) gives the result

$$D\frac{T''}{T} = \frac{Y'}{Y} \tag{I.5}$$

This leads to a solution satisfying the boundary conditions of the form

$$p_k(y, t) = A_k \cos \lambda_k y e^{-\lambda_k^2 Dt} \tag{I.6}$$

where

$$\lambda_k = \frac{\pi}{2L}(2k + 1) \tag{I.7}$$

If, in addition, we now require the initial carrier profile $p(y, 0)$ to be uniform, we can use a square-wave expansion to find the coefficients A_k in the series for $p_0 = p(y, 0)$, and write

$$p(y, t) = \sum_{k=0}^{\infty} A_k \cos \lambda_k y e^{-\lambda_k^2 Dt} \tag{I.8}$$

with

$$A_k = \frac{4p_0}{(2k + 1)\pi} \tag{I.9}$$

Each of the terms in this sum decays exponentially, but the first term decays least rapidly, so it is the dominant term in most cases. Thus, after some time,

$$\frac{4p_0}{\pi} \qquad e^{-(\pi^2 D/4L^2)t} \tag{I.10}$$

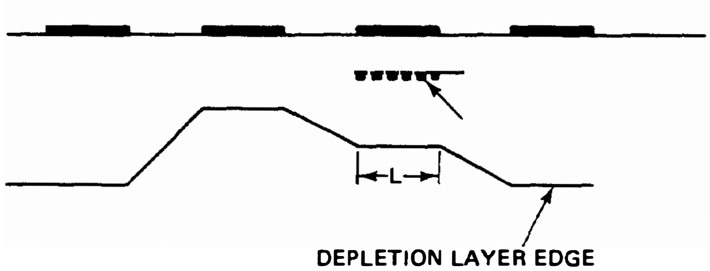

Figure I.1 Schematic of a CCD.

where the total charge $p_{tot} = \int_0^L p(y, t)\, dy$ remaining after time t is

$$p_{tot} = \frac{8}{\pi} p_{tot}(0) e^{-(\pi^2 D/4L^2)t} \tag{I.11}$$

Note that the decay of the total charge due to thermal diffusion can be considered to be exponential with a decay time constant of $\tau_0 = L^2/2.5D$. Thus it is highly advantageous to use as short a gate length as possible. It is also desirable to use as high a diffusion constant as possible, which implies that it is better to work with n-type carriers rather than with p-type carriers and, therefore, a p-type substrate.

As an example, with a gate length of 10 μm and $D = 20$ cm²/s, the decay time constant is $\tau_0 = 20$ ns. So with a clock rate of 5 MHz and a transfer time $t = 100$ ns, the remaining charge under the gate is 0.8%. Thus after 100 transfers, only 44% of the charge passes through the CCD. On the other hand, with $L = 5$ μm and $\tau_0 = 10$ ns, the remaining charge is 0.0045% and, after 100 transfers, 99.5% of the charge passes through the CCD.

The space-charge effects due to the carriers can be regarded as an enhanced diffusion mechanism. This will be shown in the following treatment. If we suppose that the charge per unit length is qp_s, the surface potential associated with this charge is

$$\phi_s = \frac{qp_s}{C_{0x}} + \text{constant} \tag{I.12}$$

where C_{0x} is the capacity per unit area of the gate oxide. The field associated with the surface potential is, in turn,

$$E_s = \frac{-\partial \phi_s}{\partial y} \tag{I.13}$$

It follows that the equation of motion of the carriers is

$$J_s = \frac{-q^2 \mu p_s (\partial p_s/\partial y)}{C_{0x}} \tag{I.14}$$

where μ is the mobility of the carriers. Thus the carrier density obeys the differential equation

$$\frac{q\mu}{2C_{0x}} \frac{\partial^2}{\partial y^2} (p_s^2) = \frac{\partial p_s}{\partial t} \tag{I.15}$$

It is as if the carriers have an enhanced diffusion coefficient D_{enh}, where

$$\begin{aligned} D_{enh} &= \frac{q\mu p_s}{C_{0x}} \\ &= D \frac{qp_s}{C_{0x}(kT/q)} \end{aligned} \tag{I.16}$$

where from the Einstein relation $D = \mu kT/q$, T is the temperature and k is Boltzmann's constant.

When the carrier density is high, the voltage corresponding to the space charge $\phi_s = qp_s/C_{0x}$ can be of the order of 1 to 2 V, and much larger than kT/q. In this case, the space-charge effect is the strongest effect present. Thus, initially, when charge is released from a register, it moves relatively rapidly due to space-charge effect, but as the density drops, diffusion becomes the dominant effect.

Computed results of this kind are shown in Fig. I.2. At first the charge drops rapidly; it then drops less rapidly as normal diffusion effects become dominant.

It is advantageous to trap the carriers in a "buried layer" (i.e., in a potential well around a layer of negatively charged acceptors); then they will be some distance from the gates. Due to this effect, the carrier velocity is increased, which means that higher-speed devices can be made.

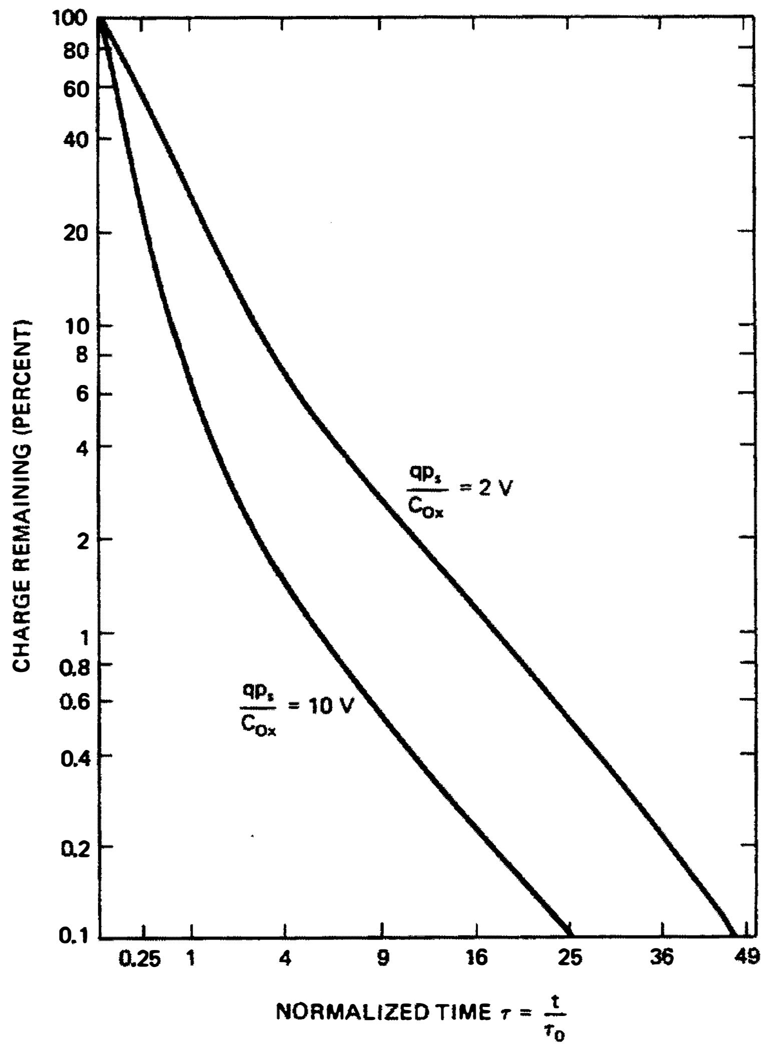

Figure I.2 Fraction of charge remaining under a plate as a function of the square root of the normalized time $\tau = t/\tau_0$. The space charge effect is strong in the region where $\tau < 1$. The curves are straight parallel lines over the diffusion portion of the characteristic. (From Séquin and Tompsett [1].)

REFERENCE

1. C. H. Séquin and M. F. Tompsett, *Charge Transfer Devices*, Suppplement 8 for *Advances in Electronics and Electron Physics*, L. Marton, series ed. New York: Academic Press, Inc., 1975, Chapter VI, pp. 201–35.