Appendix D

Determination of the Impedance Z_0 in Terms of $\Delta V/V$

We now determine the impedance Z_0 in Eq. (2.5.35) for a piezoelectric medium excited by a charge at its surface. We drop the subscript n where convenient and write Eq. (2.5.27) in the form

$$\frac{dA}{dz} + jkA = \frac{j\omega}{4}\int \rho_s \phi_0^* \, dl = \frac{j\omega}{4}\,\sigma\phi_0^* \tag{D.1}$$

where $\sigma = \rho_s w$, ρ_s is assumed to be uniform over the width w of the beam, and ϕ_0^* is the potential at the surface normalized to unit power.

We define a quantity Z_0 with the dimensions of impedance

$$Z_0 = \frac{\phi_0\phi_0^*}{2} \tag{D.2}$$

Then, by assuming ϕ_0 is real because of the normalization employed, Eq. (D.1) can be written in the form

$$\frac{dA}{dz} + jkA = \frac{j\omega(2Z_0)^{1/2}\sigma}{4} \tag{D.3}$$

This coupling impedance is a measure of the strength of the electrical potential at the surface of the substrate and, hence, the coupling to an interdigital transducer placed on the substrate. It can be calculated directly by carrying out the field theory for Rayleigh wave propagation along the substrate. Alternatively, it can be related to K^2 or to $\Delta V/V$.

We shall now show how to relate the impedance Z_0 to $\Delta V/V$, the relative change in the velocity of the wave when a perfect conductor is laid down on the substrate.

When a perfect conductor is laid down on the substrate, the propagation constant of the wave changes from k to k'. We assume that $|k' - k| << |k|$. In this case, A now varies as $\exp(-jk'z)$, as does the surface charge on the metal $\sigma(z)$. We can therefore write Eq. (D.3) in the form

$$k - k' = \frac{\omega(2Z_0)^{1/2}\sigma}{4A} \tag{D.4}$$

We now need a further relation between $\sigma(z)$ and $A(z)$ to determine the change in k. This can be obtained by considering the potential at the metal conductor, which is made up of two parts: (1) the potential due to the surface wave ϕ_a, where

$$\phi_a = A\phi_0 \tag{D.5}$$

and (2) the electrostatic potential ϕ_s due to the charge σ itself. The total potential at the surface is

$$\phi = \phi_s + \phi_a \tag{D.6}$$

At a perfect conductor, we have the simple condition $\phi = 0$.

The solution of Poisson's or Laplace's equation creates a potential ϕ_s, the *electrostatic potential*, due to a charge distribution $\sigma(z)$, which varies as $\exp(-jk'z)$. Using the notation of Eq. (D.6), the solution of Laplace's equation in the region $y \geqslant 0$ is

$$\begin{aligned} \phi_s &= Ce^{-k'y}e^{-jk'z} \\ E_{ys} &= -\frac{\partial\phi_s}{\partial y} = Ck'e^{-k'y}e^{-jk'z} \\ E_{zs} &= -\frac{\partial\phi_s}{\partial z} = jk'Ce^{-k'y}e^{-jk'z} \end{aligned} \tag{D.7}$$

For the region $y < 0$ within the substrate, if the substrate is semi-infinite so that $\phi_s \rightarrow 0$ as $y \rightarrow -\infty$, then

$$\begin{aligned} \phi_s &= De^{k'y}e^{jk'z} \\ E_{ys} &= -Dk'e^{k'y}e^{-jk'z} \\ E_{zs} &= jk'De^{k'y}e^{-jk'z} \end{aligned} \tag{D.8}$$

where C and D are constants.

The electrostatic potential is continuous at $y = 0$, so that $C = D$. Furthermore, the value of D_y must obey the boundary condition

$$\varepsilon_0 E_{ys}^{+} - \varepsilon E_{ys}^{-} = D_{ys}^{+} - D_{ys}^{-} = \frac{\sigma}{w} \tag{D.9}$$

where the substrate is assumed to be isotropic, with a permittivity ε, and the medium above the substrate has a permittivity ε_0.

It then follows that

$$\phi_s(0) = \frac{\sigma}{k'w(\varepsilon_0 + \varepsilon)} \tag{D.10}$$

This is the electrostatic potential at the surface of the substrate due to the surface charge in the conductor. We can now use Eqs. (D.4)–(D.6) and Eq. (D.10), with the condition that $\phi(0) = 0$, to obtain

$$\frac{\Delta V}{V} = \frac{k - k'}{k'} = \frac{-\omega w \phi_0^2}{4}(\varepsilon_0 + \varepsilon) \tag{D.11}$$

It follows that

$$Z_0 = \frac{2}{\omega w(\varepsilon_0 + \varepsilon)}\left|\frac{\Delta V}{V}\right| \tag{D.12}$$

Thus the impedance Z_0 is directly proportional to the quantity $\Delta V/V$, the relative change in acoustic velocity when an infinitesimally thin metal conductor is laid down on the piezoelectric substrate.