Appendix A

Stress, Strain, and the Reduced Notation

STRESS AND STRAIN VECTORS

In this appendix we derive the three-dimensional forms of the stress and strain tensors somewhat more fully and rigorously than in Secs. 1.1 and 1.5 of the text. We also describe the commonly used reduced notation based on the symmetry of the S and T tensors [1].

Strain Tensor

A point $\mathbf{r}$ in the material is displaced by stress to a point $\mathbf{r} + \mathbf{u}$, where $\mathbf{u}$ is the displacement vector. Suppose that we consider length l in the material between the point $\mathbf{r}$ and $\mathbf{r} + \delta\mathbf{r}$. After displacement, l changes to l' and, as illustrated in Fig. A.1, we can write

$$l^2 = (\delta r)^2 = (\delta x_1)^2 + (\delta x_2)^2 + (\delta x_3)^2 \tag{A.1}$$

and

$$l'^2 = (\delta\mathbf{r} + \delta\mathbf{u})^2 = l^2 + 2\delta\mathbf{u} \cdot \delta\mathbf{r} + (\delta\mathbf{u})^2 \tag{A.2}$$

We shall express δu_x in the form

$$\delta u_x = \frac{\partial u_x}{\partial x}\delta x + \frac{\partial u_x}{\partial y}\delta y + \frac{\partial u_x}{\partial z}\delta z \tag{A.3}$$

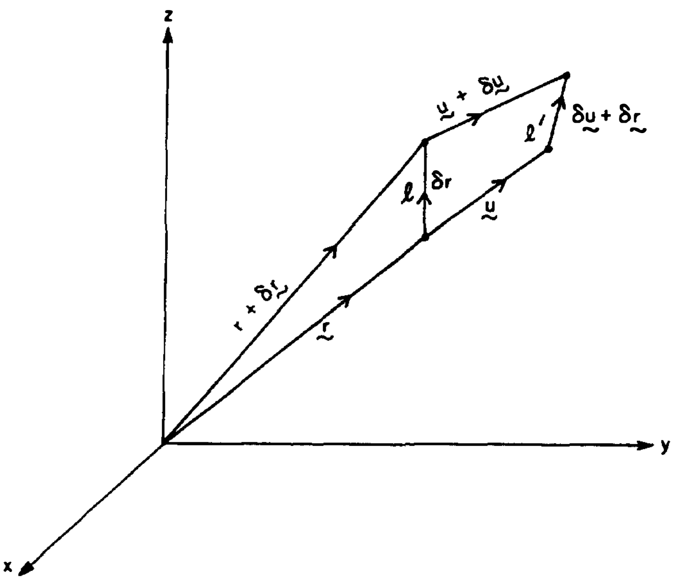

Figure A.1 Notation used in Eq. (A.1).

with similar notations for δu_y and δu_z. These relations can be summarized conveniently using tensor notation (see Sec. 1.5) and written in the form

$$\delta u_i = \frac{\partial u_i}{\partial x_j}\,\delta x_j \tag{A.4}$$

This is a shorthand notation for

$$\delta u_i = \sum_j \frac{\partial u_i}{\partial x_j}\,\delta x_j \tag{A.5}$$

where i can be x, y, or z, and where for a given i, the summation over the subscript j is understood.

We may write a Taylor expansion for Eq. (A.2) and keep terms to second order in δx_i. This procedure yields the result

$$l'^2 = l^2 + 2\frac{\partial u_i}{\partial x_j}\,\delta x_i\,\delta x_j + \frac{\partial u_i}{\partial x_j}\frac{\partial u_i}{\partial x_k}\,\delta x_j\,\delta x_k \tag{A.6}$$

where, as discussed in Sec. 1.5, the tensor notation now implies double summations over the three independent suffixes i, j, and k on the right-hand side of Eq. (A.6). Note that we have replaced the vector $\delta\mathbf{r}$ by δx_i in tensor notation. Similarly, a scalar product $\mathbf{A}\cdot\mathbf{B}$ is A_iB_i.

The change in l^2 is a true measure of the deformation of the material. If, instead, we were to use the change in $\delta\mathbf{r}$ as a criterion, this vector could be changed by a pure rotation of a rigid material without changing the length l; thus the change in $\delta\mathbf{r}$ would not be a measure of the deformation in this case.

We can now interchange the suffixes i and k in the third term of Eq. (A.6), and write the second term in the form

$$\frac{\partial u_i}{\partial x_j}\,\delta x_i\,\delta x_j = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\delta x_i\,\delta x_j \tag{A.7}$$

In this case, Eq. (A.6) can be written as

$$l'^2 = l^2 + 2S_{ij}\,\delta x_i\,\delta x_j \tag{A.8}$$

where S_{ij} is known as the strain tensor and is defined as

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j}\right) \tag{A.9}$$

We see from this definition that S_{ij} is a symmetric tensor. For small displacements, we can neglect the last term in Eq. (A.9) as being of second order. From now on, we shall write

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{A.10}$$

Alternatively, we can use a symbolic notation **S**, much like that for a vector, and define the strain S_{ij} in the form

$$\mathbf{S} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} = \begin{bmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{bmatrix} \tag{A.11}$$

where now the subscripts 1, 2, and 3 are equivalent to x, y, and z, respectively, and are used interchangeably with them in the literature. It follows that

$$\frac{\partial S_{ij}}{\partial t} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \tag{A.12}$$

where $\mathbf{v} = \partial\mathbf{u}/\partial t$ is the velocity of a particle in the material. This is equivalent to the one-dimensional equation of conservation of mass, given as Eq. (1.1.8), but it yields more information than just conservation of mass. If we take only the diagonal terms, we see that $\nabla \cdot \mathbf{v} = \partial v_i/\partial x_i$ and that the equation of conservation of mass is

$$\rho_{m0}\,\nabla \cdot \mathbf{v} + \frac{\partial \rho_{m1}}{\partial t} = 0 \tag{A.13}$$

We use, from Eq. (A.12), the relation

$$\nabla \cdot \mathbf{v} = \frac{\partial}{\partial t}\left(S_{11} + S_{22} + S_{33}\right) \tag{A.14}$$

This is directly equivalent to Eq. (1.1.8) and will be derived in another way below. The diagonal terms are associated with longitudinal strain; the off-diagonal terms are associated with shear strain.

Change in Volume

It will be noted that the volume of a small portion δV of the material is $\delta x_1\ \delta x_2\ \delta x_3$. After deformation, it becomes $\delta V'$, where

$$\begin{aligned} \delta V' &= (\delta x_1 + \delta u_1)(\delta x_2 + \delta u_2)(\delta x_3 + \delta u_3) \\ &= \delta V\left(1 + \frac{\partial u_1}{\partial x_1}\right)\left(1 + \frac{\partial u_2}{\partial x_2}\right)\left(1 + \frac{\partial u_3}{\partial x_3}\right) \\ &\approx \delta V\left(1 + \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}\right) \end{aligned} \tag{A.15}$$

It follows that

$$\delta V' - \delta V = \delta V(S_{11} + S_{22} + S_{33}) \tag{A.16}$$

We see that the sum of the diagonal components of the strain tensor is the relative volume change $(\delta V' - \delta V)/\delta V$. *The shear terms do not contribute to a change in volume.* We can also see this result from substituting Eq. (A.14) in Eq. (A.13). This yields the result

$$\rho_{m1} = -\rho_{m0}(S_{11} + S_{22} + S_{33}) \tag{A.17}$$

which is identical to Eq. (A.15) for

$$\rho_{m1}/\rho_{m0} = -(\delta V' - \delta V)/\delta V \tag{A.18}$$

Stress Tensor

Here we shall give a different and more detailed derivation for stress than that given in Sec. 1.5. The force in the x direction on a body of volume V is $\int F_x\, dV$, where the force F_x is a scalar quantity. We can always write a scalar quantity as the divergence of a vector. Thus we put

$$\begin{aligned} F_x &= \nabla \cdot \mathbf{A} \\ F_y &= \nabla \cdot \mathbf{B} \\ F_z &= \nabla \cdot \mathbf{C} \end{aligned} \tag{A.19}$$

Then, from Gauss's theorem, we can write

$$\int_V F_x\, dV = \int_V \nabla \cdot \mathbf{A}\, dV = \int_S \mathbf{A} \cdot d\mathbf{s}, \quad \text{etc.} \tag{A.20}$$

where the surface integral is taken around the enclosing volume V.

It is apparent that we need nine components, A_x, A_y, A_z, B_x, B_y, B_z, C_x, C_y, and C_z, to express $\int F_x\, dV$, $\int F_y\, dV$, and $\int F_z\, dV$. In tensor notation, we write

$$F_i = \frac{\partial T_{ij}}{\partial x_j} \tag{A.21}$$

which is shorthand for

$$F_i = \sum_j \frac{\partial T_{ij}}{\partial x_j} \tag{A.22}$$

or

$$F_x = \frac{\partial T_{xx}}{\partial x} + \frac{\partial T_{xy}}{\partial y} + \frac{\partial T_{xz}}{\partial z} \tag{A.23}$$

and so on. The quantity T_{ij} is called the *stress tensor*. In our previous notation, we see that $A_x = T_{xx} = T_{11}$, $A_y = T_{xy} = T_{12}$, $A_z = T_{xz} = T_{13}$, and so on.

It follows that the average force on an element of volume dV is

$$\frac{1}{dV}\int \nabla \cdot \mathbf{T}\, dV = \frac{1}{dV}\int \frac{\partial T_{ij}}{\partial x_j}\, dV = \frac{1}{dV}\int T_{ij}\, ds_j = \frac{1}{dV}\oint \mathbf{T} \cdot \mathbf{n}\, ds \tag{A.24}$$

where we define $\nabla \cdot \mathbf{T}$ as $\partial T_{ij}/\partial x_j$, and where ds_j is the surface element vector directed along the outward normal. The force on a surface in the z direction therefore has three components normal to the surface that comprise the vector $\mathbf{C}$ in Eq. (A.19); these are T_{xz}, T_{yz}, and T_{zz}. The first two terms are shear stresses that tend to distort the surface of an isotropic material, as shown in Fig. A.2(b).

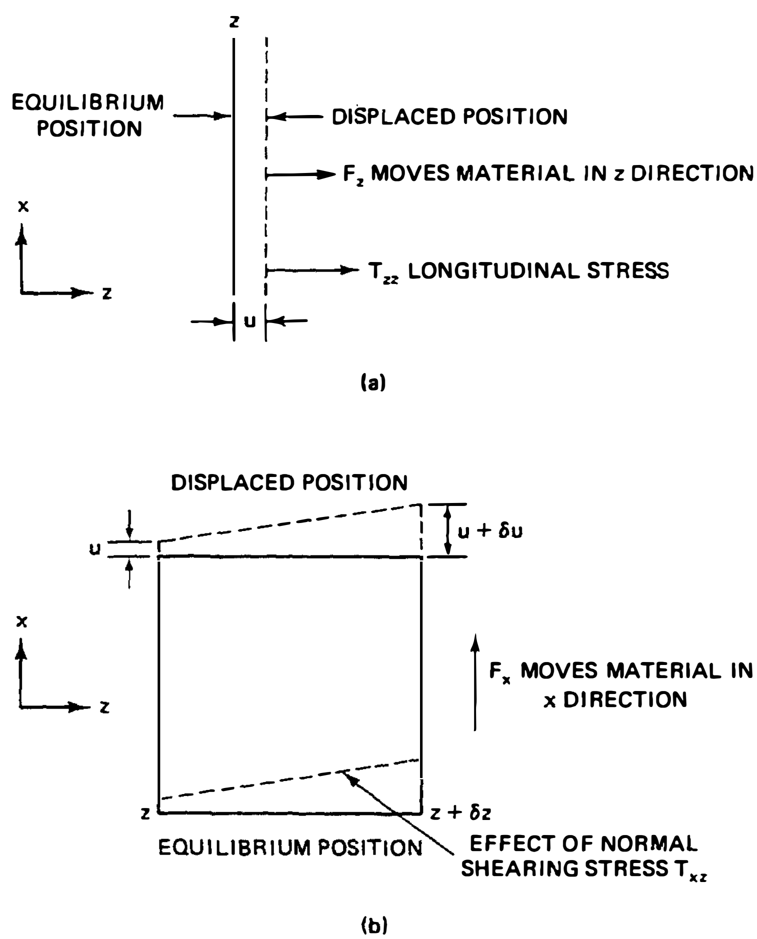

Figure A.2 Effect of normal longitudinal and shear stresses at a surface: (a) longitudinal stress; (b) shear stress.

The last term is a longitudinal stress, which acts as shown in Fig. A.2(a). All the stress components are applied to a cube, illustrated in Fig. 1.5.1. Because $\int T_{ij}\, ds_j = \int T_{ji}\, ds_i$, it can be shown that $T_{ij} = T_{ji}$ (i.e., T is a symmetric tensor).

Equation of Motion

The force on an element dV is $\int \mathbf{T} \cdot \mathbf{n}\, ds$ due to internal stresses. Thus, if only internal stresses are applied, we can write the equation of motion for first-order displacements as

$$\rho_{m0} \frac{\partial^2 \mathbf{u}}{\partial t^2} = \lim_{dV \to 0} \left[\frac{\int \mathbf{T} \cdot \mathbf{n}\, ds}{dV} \right] \tag{A.25}$$

or

$$\rho_{m0} \frac{\partial^2 \mathbf{u}}{\partial t^2} = \nabla \cdot \mathbf{T} \tag{A.26}$$

which is equivalent to

$$\rho_{m0} \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial T_{ij}}{\partial x_j} \tag{A.27}$$

where ρ_{m0} is the mass density of the material.

SYMBOLIC NOTATION AND ABBREVIATED SUBSCRIPTS

Strain Tensor

To reduce the complexity of the stress and strain tensors, it is helpful to use symmetry and to work with an abbreviated subscript notation. Here we shall describe this abbreviated subscript notation and show how it is used.

We first consider the strain tensor S_{ij}, defined as

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \tag{A.28}$$

Because the strain is a symmetric tensor, we can replace S_{yx} with S_{xy}, and so on. Thus we can use a reduced notation with fewer subscripts. The standard reduced notation can be expressed in matrix form:

$$\mathbf{S} = \begin{bmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{bmatrix} = \begin{bmatrix} S_1 & \frac{S_6}{2} & \frac{S_5}{2} \\ \frac{S_6}{2} & S_2 & \frac{S_4}{2} \\ \frac{S_5}{2} & \frac{S_4}{2} & S_3 \end{bmatrix} \tag{A.29}$$

Note that the notation follows a cyclic order, with the longitudinal strain terms corresponding to the subscripts 1, 2, and 3, respectively, and the shear strain terms corresponding to the subscripts 4, 5, and 6, respectively, as shown in the following table:

Normal tensor notation	Reduced notation	Corresponding strain
xx	1	Longitudinal in x direction
yy	2	Longitudinal in y direction
zz	3	Longitudinal in z direction
$yz = zy$	4	Shear $y - z$
$zx = xz$	5	Shear $z - x$
$xy = yx$	6	Shear $x - y$

Note that the off-diagonal terms are multiplied by $\frac{1}{2}$ so that we can write the strain in the form of a column matrix:

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} \tag{A.30}$$

This can be done only because the $\frac{1}{2}$ was used in our definitions in Eq. (A.29). Following Auld [1], it is convenient to define the matrix in Eq. (A.30) with a symbolic notation, writing

$$\mathbf{S} = \nabla_s \mathbf{u} \tag{A.31}$$

where $\nabla_s u$ is defined as the symmetric part of $\nabla \mathbf{u}$. The symetric operator $\nabla_s u$ is defined by the first matrix on the right-hand side of Eq. (A.30). In the unreduced tensor notation, the symmetry of ∇_s is apparent because

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{A.32}$$

A simple example of longitudinal motion in the x direction with propagation in the x direction is defined by the relation $S_1 = \partial u_x/\partial x$; this follows from Eq. (A.30). On the other hand, a plane shear wave, in which propagation is in the z direction but particle displacement is only in the y direction, is defined by the relations $u_x = u_z = 0$ and $S_4 = \partial u_y/\partial_z$. In this case, all other components of strain are on zero. The first case corresponds to a longitudinal wave passing through a flat plate; the second case corresponds to the flexural motion of a thin strip.

Stress Tensor

The stress tensor may be stated in terms of reduced subscripts just as the strain tensor was. Thus we write

$$\mathbf{T} = \begin{bmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{bmatrix} = \begin{bmatrix} T_1 & T_6 & T_5 \\ T_6 & T_2 & T_4 \\ T_5 & T_4 & T_3 \end{bmatrix} \tag{A.33}$$

Note that the $\frac{1}{2}$ terms are not required here. The equation of motion for symbolic notation is

$$\nabla \cdot \mathbf{T} = \rho_{m0}\frac{\partial \mathbf{v}}{\partial t} \tag{A.34}$$

This can be put in reduced tensor form, by writing

$$\rho_{m0}\frac{\partial}{\partial t}\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \nabla \cdot \mathbf{T} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ 0 & 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix}\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} \tag{A.35}$$

For example, if the stress field has only one component, a shear stress $T_5 = T_{xz}$ propagating in the z direction, then $\nabla \cdot T$ becomes $\partial T_5/\partial z$ and corresponds to an acceleration in the x direction.

Elasticity

Similarly, the elasticity tensor c_{ijkl} can be expressed in reduced notation. Because $S_{ij} = S_{ji}$ and $T_{ij} = T_{ji}$, it follows that $c_{ijkl} = c_{jikl} = c_{ijlk} = c_{jilk}$, which reduces the number of independent constants required from 81 to 36. Furthermore, because of symmetry, $c_{ijkl} = c_{klij}$. This further reduces the required number of independent constants in an arbitrary medium to 21. Thus we write

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{bmatrix}\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix} \tag{A.36}$$

or

$$\mathbf{T} = \mathbf{c} : \mathbf{S} \tag{A.37}$$

where the general term is c_{IJ}; we use capital subscripts to denote the reduced notation and take $c_{IJ} = c_{JI}$.

Example: Cubic Crystal

Most crystals have certain symmetries that reduce the required number of constants. For instance, a cubic crystal looks the same in the x, $-x$, y, $-y$, z, and $-z$ directions. This implies that $c_{11} = c_{22} = c_{33}$, $c_{44} = c_{55} = c_{66}$, and $c_{12} = c_{13} = c_{23}$. All other diagonal terms are zero because of the mirror symmetry. Thus we find that for a cubic crystal,

$$\mathbf{c} = \begin{bmatrix} c_{11} & c_{12} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{12} & c_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{44} \end{bmatrix} \tag{A.38}$$

When there is shear wave propagation along the z axis, with motion in the x direction, it follows from Eq. (A.30) that

$$S_5 = \frac{\partial u_x}{\partial z} \tag{A.39}$$

and from Eqs. (A.36) and (A.38) ı

$$T_5 = c_{44} S_5 \tag{A.40}$$

Assuming that the RF components t), then $v_x = j\omega u_x$. It follows from Eq. (A.34) or Eq. (A.35) that

$$\frac{\partial T_5}{\partial z} = j\omega \rho_{m0} v_x \tag{A.41}$$

However, from Eqs. (A.39) and (A.40), we see that

$$c_{44} \frac{\partial v_x}{\partial z} = j\omega T_5 \tag{A.42}$$

Equations (A.41) and (A.42) are the transmission-line equations for shear wave propagation. Assuming that the waves propagate as exp $(\pm j\beta_s z)$, we see that for shear waves in a cubic crystal,

$$\beta_s^2 = \omega^2 \left(\frac{\rho_{m0}}{c_{44}} \right) \tag{A.43}$$

If, on the other hand, we consider longitudinal motion in the z direction with only u_z or v_z finite, we find that the propagation constant β_l is given by the relation

$$\beta_l^2 = \omega^2 \left(\frac{\rho_{m0}}{c_{11}} \right) \tag{A.44}$$

Example: Isotropic Material

In this case, which is very much like that of the cubic crystal, the **c** tensor is of the same form as that of Eq. (A.38), with the additional condition that $c_{12} = c_{11} - 2c_{44}$. Note that the c_{12} term corresponds to the ratio of the longitudinal stress in the x direction to the longitudinal strain in the y direction. Such terms occur because when a material is compressed in one direction, it tends to expand in a perpendicular direction. The relation given follows from the requirement that the tensor **c** keeps the same form; however, the axes are rotated from their original position.

It follows that an isotropic medium has only two independent elastic constants. These are usually called the Lamé constants, defined as

$$\lambda = c_{12} \qquad \mu = c_{44} \tag{A.45}$$

with

$$c_{11} = c_{12} + 2c_{44} = \lambda + 2\mu \tag{A.46}$$

The **c** matrices for different types of crystals are tabulated in Appendix A.2 of B. A. Auld's *Acoustic Fields and Waves in Solids* [1]. The similar **s** matrices, for which $\mathbf{s} = \mathbf{c}^{-1}$ or

$$\mathbf{S} = \mathbf{s} : \mathbf{T} \tag{A.47}$$

are also tabulated by Auld.

PIEZOELECTRIC TENSORS

The piezoelectric tensor e_{ijk} can also be expressed in reduced notation in the forms

$$T_I = c_{IJ}^E S_J - e_{Ij} E_j \tag{A.48}$$

and

$$D_i = \varepsilon_{ij}^S E_j + e_{iJ} S_J \tag{A.49}$$

These relations can also be written in the equivalent forms

$$\mathbf{T} = \mathbf{c}^S : \mathbf{S} - \mathbf{e} \cdot \mathbf{E} \tag{A.50}$$

and

$$\mathbf{D} = \boldsymbol{\varepsilon}^S \cdot \mathbf{E} + \mathbf{e} : \mathbf{S} \tag{A.51}$$

where, as before, we have used capital subscripts to express the reduced notation, and the notation i, j, k to represent the x, y, and z axes.† We define the general e_{iJ} tensor as follows:

$$e_{iJ} = \begin{bmatrix} e_{x1} & e_{x2} & e_{x3} & e_{x4} & e_{x5} & e_{x6} \\ e_{y1} & e_{y2} & e_{y3} & e_{y4} & e_{y5} & e_{y6} \\ e_{z1} & e_{z2} & e_{z3} & e_{z4} & e_{z5} & e_{z6} \end{bmatrix} \tag{A.52}$$

†Often, in the literature, the commonly used notation drops distinctions between uppercase and lowercase subscripts: $e_{x1} = e_{11}$, $e_{z6} = e_{36}$, $e_{z3} = e_{33}$, and so on.

The e_{Ij} tensor is defined similarly as

$$e_{Ij} = \begin{bmatrix} e_{1x} & e_{1y} & e_{1z} \\ e_{2x} & e_{2y} & e_{2z} \\ e_{3x} & e_{3y} & e_{3z} \\ e_{4x} & e_{4y} & e_{4z} \\ e_{5x} & e_{5y} & e_{5z} \\ e_{6x} & e_{6y} & e_{6z} \end{bmatrix} \tag{A.53}$$

Note that $e_{1x} = e_{x1}$, and so on.

Symmetry once more reduces the number of independent quantities required from a possible 27. Thus, for a 43-m class of cubic crystal (e.g., gallium arsenide or indium antimonide) only one constant, e_{x4}, is required, where

$$e_{iJ} = \begin{bmatrix} 0 & 0 & 0 & e_{x4} & 0 & 0 \\ 0 & 0 & 0 & 0 & e_{x4} & 0 \\ 0 & 0 & 0 & 0 & 0 & e_{x4} \end{bmatrix} \tag{A.54}$$

For shear wave propagation in the z direction, with motion in the x direction, e_{x4} is the constant required. The longitudinal wave piezoelectric coupling constant in this direction is zero. On the other hand, for propagation along a $\langle 111 \rangle$ axis, we need the form of e_{iJ} in a rotated system, with axes in the $\langle 111 \rangle$ direction. In this case, there is a finite longitudinal piezoelectric coupling constant in the $\langle 111 \rangle$ direction, which can be determined from e_{x4}.

An isotropic material, or a cubic crystal with a center of symmetry such as silicon, is not piezoelectric. Thus $e_{x4} = 0$.

REFERENCE

1. The notation and physics in this appendix are dealt with in more detail in B. A. Auld, *Acoustic Fields and Waves in Solids*, Vol. 1 (New York: John Wiley & Sons, Inc., 1973).