The Fundamental Theory of Low Noise Oscillators with Special Reference to Some Detailed Designs
IEEE Frequency Control Symposium Tutorial
6th June, 2000, Kansas City

Jeremy K A Everard
Department of Electronics
University of York

Presented at the 2000 IEEE Int'l Frequency Control Symposium Tutorials
June 6, 2000, Kansas City, Missouri, USA

©2000 JKA Everard, University of York
Low Noise Oscillators

• Oscillator models
• Noise theories for thermal (additive noise)
• Optimisation for minimum sideband noise
• Flicker noise measurement and reduction
• Oscillator designs
  – LC oscillators
  – SAW oscillators
  – Transmission line oscillators
• Tuning - varactor limitations
• Non-linear CAD
• Detailed designs

©2000 JKA Everard, University of York
Spectrum of Oscillator

Amplitude

1 Hz

Δf

©2000 JKA Everard, University of York
Oscillator Models
Block model of oscillator

\[ P_{AVI} \quad FkT \quad F, G \quad P_{AVO} \]

Resonator

©2000 JKA Everard, University of York
Model by Splitting original input into 2 identical inputs

One for noise injection

One for feedback

Model like Op-Amp with two inputs added and $\beta_0 G = 1$

$$\frac{V_{OUT}}{V_{IN2}} = \frac{G}{1 - (\beta G)}$$
Model of Resonator
Resonator Response

\[ \beta = \left( \frac{R_{IN}}{R_{IN} + R_{OUT}} \right) \left( 1 - \frac{Q_L}{Q_0} \right) \frac{1}{\left( 1 \pm 2 jQ_L \frac{df}{f_o} \right)} \]

If \( R_{OUT} = R_{IN} \) then the insertion loss of the resonator is \( S_{21} = 2\beta \), therefore:

\[ S_{21} = \left( 1 - \frac{Q_L}{Q_0} \right) \frac{1}{\left( 1 \pm 2 jQ_L \frac{df}{f_o} \right)} \]

Insertion loss increases as \( Q_L \) tends to \( Q_0 \)

- \( S_{21} = 6\text{dB} \) when \( Q_L/Q_0 = 1/2 \)
- \( S_{21} = 9\text{dB} \) when \( Q_L/Q_0 = 2/3 \)

©2000 JKA Everard, University of York
Resonator response versus $Q_L/Q_0 = 0.1, 0.5, 2/3, 0.9$
Insertion loss vs $Q_L$
At resonance $\Delta f$ is zero and $V_{OUT}/V_{IN2}$ is very large

\[
\frac{V_{OUT}}{V_{IN2}} = \frac{G}{1 - \frac{1}{1 + \left(2 j Q_L \frac{df}{f_0}\right)}} = \frac{1}{\left(1 - Q_L/Q_0\right) \left(\frac{R_{IN}}{R_{OUT} + R_{IN}}\right) \left(1 - \frac{1}{1 + \left(2 j Q_L \frac{df}{f_0}\right)}\right)}
\]

Interested in noise ‘skirts’

simplifies to:

\[
\frac{V_{OUT}}{V_{IN2}} = \frac{G}{\pm 2 j Q_L \frac{\Delta f}{f_0} \left(1 - Q_L/Q_0\right) \left(\frac{R_{IN}}{R_{OUT} + R_{IN}}\right) \left(\pm 2 j Q_L \frac{\Delta f}{f_0}\right)}
\]

©2000 JKA Everard, University of York
Noise in terms of power in 1Hz BW

• Calculate input noise power in 1Hz BW
  – initially calculate square of I/P voltage
• Assume O/P power limited
  – or at least always calculate in terms of O/P *
• Equation starts to break down very close to carrier at offsets typically << 1Hz
  – this is not usually a problem
\[ V_{IN} = \sqrt{FkTR_{IN}} \quad \text{Noise input} \]

\[
(V_{\text{OUT}} \Delta f)^2 = \frac{FkTR_{IN}}{4(Q_L)^2 \left( \frac{R_{IN}}{(R_{\text{OUT}} + R_{IN})} \right)^2 \left( 1 - \frac{Q_L}{Q_0} \right)^2 \left( \frac{f_o}{\Delta f} \right)^2}
\]

Separate constants and variables

\[
(V_{\text{OUT}} \Delta f)^2 = \frac{FkTR_{IN}}{4(Q_0)^2 \left( \frac{Q_L}{Q_0} \right)^2 \left( \frac{R_{IN}}{(R_{\text{OUT}} + R_{IN})} \right)^2 \left( 1 - \frac{Q_L}{Q_0} \right)^2 \left( \frac{f_o}{\Delta f} \right)^2}
\]

©2000 JKA Everard, University of York
AM and PM noise model

\[
(V_{\text{OUT}} \Delta f)^2 = \frac{F k T R_{\text{IN}}}{8 (Q_0)^2 (Q_L/Q_0)^2 \left(\frac{R_{\text{IN}}}{R_{\text{OUT}} + R_{\text{IN}}}\right)^2 (1 - Q_L/Q_0)^2} \left(\frac{f_o}{\Delta f}\right)^2
\]

©2000 JKA Everard, University of York
\[ L_{FM} = \frac{(V_{OUT} \Delta f)^2}{(V_{OUT\,MAX\,RMS})^2} \]

\[ L_{FM} = \frac{FkTR_{IN}}{8(Q_0)^2 \left(\frac{Q_L}{Q_0}\right)^2 \left(1 - \frac{Q_L}{Q_0}\right)^2 \left(R_{IN} / (R_{OUT} + R_{IN})\right)^2 \left(V_{OUT\,MAX\,RMS}\right)^2} \left(\frac{f_o}{\Delta f}\right)^2 \]

Define power \( P_{AVO} \) or \( P_{RF} \)
Noise spectrum of oscillator

- O/P power
- Noise floor at O/P: $= GFkT/2$
- Sideband noise remains constant for a given loaded Q and amplifier noise figure
- 3dB BW of resonator

©2000 JKA Everard, University of York
\[ P_{RF} = \frac{(V_{OUT MAX RMS})^2}{R_{OUT} + R_{LOSS} + R_{IN}} \]

\[ L_{FM} = \frac{FkT(R_{OUT} + R_{IN})^2}{8(Q_0)^2 (Q_L/Q_0)^2 R_{IN} (1 - Q_L/Q_0)^2 P_{RF} (R_{OUT} + R_{LOSS} + R_{IN})} \left( \frac{f_o}{\Delta f} \right)^2 \]

The ratio of sideband noise in a 1Hz BW at offset \( \Delta f \) to the total power is therefore:

\[ L_{FM} = \frac{FkT}{8(Q_0)^2 (Q_L/Q_0)^2 (1 - Q_L/Q_0) P_{RF}} \left( \frac{R_{OUT} + R_{IN}}{R_{IN}} \right) \left( \frac{f_o}{\Delta f} \right)^2 \]
If $R_{\text{OUT}}$ is zero as in a high efficiency oscillator

$$L_{FM} = \frac{FkT}{8(Q_0)^2 \left( \frac{Q_L}{Q_0} \right)^2 (1 - Q_L/Q_0)P_{RF}} \left( \frac{f_o}{\Delta f} \right)^2$$

If $R_{\text{OUT}} = R_{\text{IN}}$

Most amplifiers have similar I/P and O/P impedance

$$L_{FM} = \frac{FkT}{4(Q_0)^2 \left( \frac{Q_L}{Q_0} \right)^2 (1 - Q_L/Q_0)P_{RF}} \left( \frac{f_o}{\Delta f} \right)^2$$

©2000 JKA Everard, University of York
Power available at the output $P_{AVO}$ then:

$$P_{AVO} = \frac{\left(V_{OUT \ MAX \ RMS}^{MAX \ RMS}\right)^2}{4 R_{OUT}}$$

$$L_{FM} = \frac{FkT}{32(Q_0)^2 \left(Q_L/Q_0\right)^2 \left(1 - Q_L/Q_0\right)^2 P_{AVO}} \left(\frac{(R_{OUT} + R_{IN})^2}{R_{OUT} \cdot R_{IN}}\right) \left(\frac{f_o}{\Delta f}\right)^2$$

$$\left(\frac{(R_{OUT} + R_{IN})^2}{R_{OUT} \cdot R_{IN}}\right) = 4$$ minimum when $R_{OUT} = R_{IN}$

If $R_{OUT} = R_{IN}$

$$L_{FM} = \frac{FkT}{8(Q_0)^2 \left(Q_L/Q_0\right)^2 \left(1 - Q_L/Q_0\right)^2 P_{AVO}} \left(\frac{f_o}{\Delta f}\right)^2$$

©2000 JKA Everard, University of York
General equation which describes all three cases

\[ L_{FM} = A \cdot \frac{FkT}{8 (Q_0)^2 (Q_L/Q_0)^2 (1 - Q_L/Q_0)^N} P \left( \frac{f_0}{\Delta f} \right)^2 \]

1. \( N = 1 \) and \( A = 1 \) if \( P \) is defined as \( P_{RF} \) and \( R_{OUT} = 0 \)
2. \( N = 1 \) and \( A = 2 \) if \( P \) is defined as \( P_{RF} \) and \( R_{OUT} = R_{IN} \)
3. \( N = 2 \) and \( A = 1 \) if \( P \) is defined as \( P_{AVO} \) and \( R_{OUT} = R_{IN} \)
The effect of the load

- Load not included so far
- Incorporate as coupler/attenuator at O/P of amplifier which causes:
  - Reduction in open loop gain
  - Increase in amplifier noise figure
  - NB Closed loop gain does not change as this is set by the insertion loss of the resonator
- Effect of load reduced if amplifier has zero/low O/P impedance
Optimisation for minimum noise

• The amplifier gain and resonator loaded Q are directly linked:

\[ S_{21} = (1 - Q_L/Q_0) \]

• The noise factor is also dependent on loaded Q due to the change in source impedance
  – This is a second order effect and will be considered later
OPTIMISATION FOR MINIMUM NOISE

\[ L_{(fm)} = \frac{FKT}{8Q_o^2(Q_L/Q_o)^2(1-Q_L/Q_o)PFED} \times \left[ \frac{f_o}{\delta f} \right]^2 \]

For minimum noise if \( F \) is constant

\[ \frac{\delta (L_{(fm)})}{\delta (Q_L/Q_o)} = 0 \]

MINIMUM NOISE OCCURS WHEN:

\[ Q_L/Q_o = 2/3 \text{ and } G = 3 \]
Noise power (dB min.)

Theory

\( Q_L/Q_o \)

Experiment

Sideband noise v. \( Q_L/Q_o \)

0.67
What happens if the power that is limited is defined as Power available at the input of the amplifier.

The noise equation now becomes

$$L_{fm} = \frac{FkT}{8QL^2P_{avi}} (f_0/\Delta f)^2$$

but the gain has now disappeared.

One therefore expects that $Q_L$ should be high and made close to $Q_0$.

However if $Q_L$ tends to $Q_0$,

the amplifier gain and hence power have to be

Infinite.
High efficiency 1GHz oscillator
\[ V_{\text{out}} = G [V_{\text{in}}(1) + V_{\text{in}}(2)] \]

Oscillator model.
Basic Class E Amplifier

Class E Current Waveforms

$F_0 = 1.47 \text{ MHz}$
Figure 4  High Q Class E amplifier Load Network

Figure 5, PCB layout
Figure 14 Phase Noise Performance of Optimised Oscillator
Figure 11 Noise Figure Measurement System.
Double transmission line filter
Flicker noise: measurement and reduction
Transposed flicker noise

Low frequency flicker noise has \( \sim 1/f \) characteristic with flicker noise corner of \( F_C \)

This is modulated onto carrier causing transposed flicker noise with \( 1/\Delta f \) characteristic

The transposed flicker noise corner \( \Delta F_C \) is not the same as \( F_C \)

In oscillator this causes \( (1/\Delta F)^3 \) characteristic at offsets below \( \Delta F_C \)

©2000 JKA Everard, University of York
Flicker noise produces noise degradation in oscillators.

- **Flicker noise**: Slope \(\approx 1/dF\)
- **Thermal Noise**
- **GaAs Oscillator**: Slope \(\approx (1/dF)^2\)
- **Silicon Oscillator**: Slope \(\approx (1/dF)^3\)
- **Flicker corner for Silicon**: 100 Hz - 5 KHz
- **Flicker corner for GaAs**: 500 KHz - 50 MHz
- **Transposed flicker noise**
FLICKER NOISE MEASUREMENT SYSTEM

REFERENCE LINE

0 or 0 - 90°

0 = 550°

L.O.

50Ω

f > 100MHz

DIPLEXER

I.F.

f ≤ 100MHz

3 CHANNEL LOW NOISE AMP

3 CHANNEL COMPUTER CONTROLLED LF BANDPASS FILTERS

COMPUTER + 4 CHANNEL CARD + FFT

FOUR CHANNEL INTERFACE

COMPUTER CONTROLLED INTERFACE

SOURCE AMPLIFIER

639MHz SIGNAL GENERATOR (H.P. 8662A)

3dB POWER SPLITTER

VARIABLE ATTENUATOR

AMPLIFIER UNDER TEST

G

D

R.F.

X

+27dBm

LOW NOISE AMP.
MARKER: CH1.
L: Left.
R: Right.
S: Stop.
Ctrl-L: Fast L.
Ctrl-R: Fast R.
E: Expand plot.
D: Reduce plot.
t = 0.5
CH1: -9.84μV
CH2: -17.5μV

TIMEBASE:
25.0Ksamples/s
2ms/div

CHANNEL 1:
70μV/div
Offset = 0V

CHANNEL 2:
14μV/div
Offset = 0V

X: 0 to 20.4ms.
Y: CH1: -140μV to 138μV. CH2: -28μV to 27.7μV.
CURRENT METHODS FOR TRANSPOSED FLICKER NOISE REDUCTION

1. Direct LF reduction
2. RF Detection and LF Cancellation
3. Transposed Gain Amplifiers
4. Feedforward Amplifiers
DIRECT LF REDUCTION

Noise reduction was discussed by Riddle and Trew, 1985, who designed the amplifier using a pair of FETS operated in push pull at the microwave frequency but operated in parallel at low frequencies via a low frequency connection between the two bias networks.

Pringent and Obregon, 1987, used a bias network with a low frequency negative feedback. This reduced the device gain at low frequencies and at the same time reduced the baseband and transposed flicker noise. This assumed that the majority of the Flicker noise was generated by a gate noise source modulating the input non linear capacitor of the GaAs Fet.

An elegant implementation of the same idea was produced by Mizukami et al 1988 who developed a GaAs mmic in which the impedance presented to the source was arranged to rise at low frequencies. This method would be more difficult to implement with discrete FETs as the parasitics need to be very low indeed.

These methods have all reduced the low frequency flicker noise present at the device terminals, but this often does not necessarily correlate well with the oscillator flicker noise reduction. The transposed flicker noise depends on the nature of the internal noise sources, and the transposition mechanism. All of these vary greatly between device manufacturers.
Flicker Noise Reduction in GaAs Oscillators

Ultra low noise frequency discriminator for lowest noise X band oscillators - Ivanov, Tobar and Woode [32], [33]
Wide Bandwidth Flicker Noise Reduction in GaAs Amplifiers
M. Driscoll, FCS 1995
GAIN PRODUCED IN TRANSPOSED GAIN AMPLIFIER

- Gain
- Low Frequency Gain
- Frequency
- Transposed Gain
- Local Oscillator
- Loss of Mixers
TRANPOSED GAIN OSCILLATOR

O/P

Resonator

RF amp

Local Osc.

Delay Line
Amplifier Module
10kHz to 200MHz
21 ± 0.5dB Gain
$P_{1db} 10\text{dBm}$
Noise temp < 250K
delay 1.3nS (inverting)
15-24VDC @ 60mA
Phase Noise Performance

- $F_0 = 7.6\text{GHz}$
- $Q_0 = 44,000$
- $P_{AVO} = 8\text{dBm (6.3mW)}$
- Noise Figure $= 15\text{dB including image noise}$
- Flicker noise corner $\sim 1\text{kHz}$
- $L_{FM} = -136\text{dBc@10kHz (theory -139dBc)}$
Problems with TGO

O/P power max ~ 8dBm NF ~ 15dB

Therefore use FEEDFORWARD
Residual flicker noise reduction in 1 Watt GaAs Feedforward Amp
Broomfield and Everard FCS 2000

©2000 JKA Everard, University of York
Oscillator designs

- LC
- SAW
- Transmission Line
- Helical
- Tuning
- Detailed designs
  - LC
  - Transmission line
Carrier power level = 23.4 dBm
Sideband noise at 25 kHz offset = -106.40 dBm (1 Hz) - 6 dB for SSB = -135.8 dBc / Hz

VID AVG 30, RES BW 300 Hz, VBW 300 Hz, SWP 1.0s.

Oscillator phase noise against frequency.
Carrier power level = -5 dBm
Sideband noise at 250 Hz offset
= -92.50 dBm (1 Hz) = -87.5 dBc/Hz

VID AVG 10, RES BW 10 Hz,
VBW 10 Hz, SWP 10s.

Oscillator phase noise against frequency.
SURFACE ACOUSTIC WAVE RESONATOR

Interdigital Transducers

Reflector

Reflector
262MHz Surface Acoustic Wave Oscillator

[Diagram of oscillator circuit with components labeled, including Filtercon, 7812, Capacitors, Resistors, Inductors, and SAW Resonator.]
262 MHz SAW Oscillator

• Phase noise performance of -130dBc/Hz at 1kHz offset - limited by measurement

• Montress, Parker, Loboda and Greer [20] demonstrated high power 500MHz SAW oscillators with -140dBc/Hz at 1kHz offset
TRANSMISSION LINE OSCILLATOR

Delay line

\[ \alpha, \beta, \text{ ZoT, Veff} \]

Transmission line Resonator

Output coupler

Amplifier
Frequency Response of Resonator

Magnitude of S21 (dB)

Phase of S21 (degree)

normalised frequency (f/fo)
Close to the resonant peak and for small $\alpha L (\ll 0.05)$ and $\delta f/f_0 \ll 1$,

$$f_0 = \left(\frac{v_{\text{eff}}}{2L}\right)\{1 + (1/\pi)\tan^{-1}(2/X)\},$$

$$f_0 = \text{resonant frequency and } \delta f = f - f_0$$

$$S_{21}(\delta \varepsilon) = S_{21}(0)/\{1 + j2Q_L(\delta f/f_0)\}$$

$$S_{21}(0) = 1/\{1 + (\alpha L/2)X^2\}$$

$$Q_L = \pi S_{21}(0)X^2/4$$

From the last two equations it can be seen that the insertion loss and the loaded Q factor of the resonator are interrelated.

As the shunt capacitors (assumed to be lossless) are increased the insertion loss increases towards infinity and $Q_L$ increases to a limiting value of $\pi/2\alpha L$ which we will define as $Q_0$. 
Noise performance of 1.5GHz Osc.

- $Q_0 = 83$, $\alpha l = 0.019$, substrate $\varepsilon_r 10$
- O/P power = 3.1dBm
- Noise Figure = 3dB
- Noise performance = $-104$dBc/Hz @ 10kHz
- Within 2dB of the theory
HELICAL RESONATOR OSCILLATOR

Diagram showing the components of a helical resonator oscillator:
- Amplifier
- Output coupler
- Delay line
- Helical transmission line
- Inductor
Helical Resonator

©2000 JKA Everard, University of York
Helical resonator oscillators [22]

- 900 MHz, $Q_0 = 582$
- O/P power = 0dBm
- Noise Factor = 6dB
- Noise performance =
- -127dBc/Hz @ 25kHz
- Within 2dB of theory

- 1.6GHz, $Q_0 = 382$
- O/P power = 0dBm
- Noise Figure = 3dB
- Noise performance =
- -120dBc/Hz @ 25kHz
- Within 2dB of theory

$Z_0$ of helix = 340Ω measured using Time Domain Reflectometry

©2000 JKA Everard, University of York
$Q_0 > 500$ at 5GHz on low loss $\varepsilon_r$ 2.5 PCB

$Q_0 = 380$ at 4.8 GHz on $\varepsilon_r$10 [23]

results without screening, therefore near zero radiation loss

©2000 JKA Everard, University of York
QL versus S21 ( f_{r}=4.8 \text{ GHz } )
5 section 4.5GHz bandpass filter on $\varepsilon_r10$ [23]

low radiation loss

spurious out of band waves exist
Tuning: Varactor Limitations
OSCILLATOR TUNING - 2 Main types

1. Tunable Resonators
   - Offers broadband and narrowband tuning.
   - For low noise reduce loading caused by varactor to minimum

2. Phase Shift Method
   - Allows narrow band tuning
   - Causes $\cos^4\theta$ degradation

Diagram:
- Amplifier (Amp)
- Tunable Resonator
- Tunable phase shifter
- Adjust tap to ensure minimum loading by varactor
Noise degradation due to varactors

Power calculation by - Underhill [25]

- Power dissipated in varactor loss resistor, $r_s$, is: $P = \frac{(V_{rs})^2}{r_s}$

- The voltage across the capacitor in the resonator is: $V_C = QV_{rs}$

- Therefore the power dissipated in the varactor is: $P_v = \frac{V_C^2}{Q^2r_s}$

- The noise power in oscillators is proportional to $1/PQ_0^2$
• The figure of merit \((V_C^2/rs)\) should therefore be as high as possible

• Optimum performance obtained for large voltage handling characteristics and small series resistances in varactor
Apply this using new noise equations

- If $P$ is defined as $P_{AVO}$, $Q_L/Q_0 = 1/2$ and $R_{out} = R_{in}$, then $A = 1$ and $N = 2$ then

\[
L_{FM} = \frac{2 FkT}{Q_0^2 P_{AVO}} \left( \frac{f_0}{\Delta f} \right)^2
\]

- As $P_{AVO} = 2P_V$ then

\[
L_{FM} = \frac{FkTrs}{V_C^2} \left( \frac{f_0}{\Delta f} \right)^2
\]

Noise performance only dependant on $V_C$ and $rs$
Example

• A varactor with a series resistance of $1\Omega$ with an RF voltage of 0.25V rms at a frequency of 1GHz.

• The noise performance at 25kHz offset is -97dBc for an amplifier noise figure of 3dB
Improved by

• Reducing the tuning range by coupling varactor into resonator more lightly
• switching in tuning diodes using PIN diodes
• Increased voltage handling using back to back diodes
• improving the varactor
Varactor bias noise

- Flat noise spectral density on bias line causes $(1/\Delta f)^2$ noise in oscillator - same as thermal noise in oscillator -

- For low level modulation:

\[
L_{FM} = \frac{(K_F V_M)^2}{(2F_M)^2}
\]

$K_F$ = tuning sensitivity, Hz/Volt

$V_M$ = noise voltage, Volts/√Hz

$F_M$ = offset frequency, Hz
Bias resistor noise

\[
\frac{2}{\text{\textit{e}}_n} = 4kTB\text{\textit{r}}_b
\]

- Keep bias resistor value low
  - less than few hundred ohms eg 50Ω
- Use this to advantage in resonator design to suppress unwanted higher order resonances
3-6GHz resonator (5mm) on alumina
Two Alpha diodes CVE7900D
$C_{j0}=1.5\text{pf}$, $Q (-4\text{V, 50MHz}) = 7000$
$k$ (capacitance ratio) = 6
[26], [27]
Insertion loss and Q vs frequency, 3-6GHz resonator

©2000 JKA Everard, University of York
2 x 1 mm

8.4 - 9.8GHz GaAs MMIC resonator [27]

Variation of $Q_L/Q_0$ and $S_{21}$ vs frequency
NOISE DEGRADATION DUE TO OPEN LOOP PHASE ERROR
Effects of open loop phase error [24]

- Always oscillate at N*360°
- Resonator Q degradation as $Q \propto \frac{d\phi}{d\omega}$
- Insertion loss and hence gain increase
- Causes $\cos^4 \phi$ degradation in noise performance
- 45° causes 6dB noise degradation
- eg: At 10GHz with DRO Q=10,000, 1MHz offset causes 6dB degradation
Figure 4 Noise performance degradation with phase error

- 0 - Bipolar
- ● - GaAs
- - Theory
Non linear CAD for oscillators
Non Linear CAD

• Break Circuit at short circuit point
• Place current source and frequency dependent resistor at this point
• Make resistor:
  » Open circuit at fundamental
  » Short circuit at harmonics
• Adjust amplitude and fundamental frequency of current source to obtain zero volts.

©2000 JKA Everard, University of York
OPTIMISATION TECHNIQUE

$Z_\omega = O/C$ at fundamental and $S/C$ at harmonics

Adjust magnitude and frequency of $I$ to obtain zero $V$. 

Oscillating network

$Z_{in}$
TYPICAL OSCILLATOR CIRCUIT

Break point
Comparison of computed and measured data

<table>
<thead>
<tr>
<th></th>
<th>Predicted</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resonant Frequency</td>
<td>5.47 GHz</td>
<td>5.41 GHz</td>
</tr>
<tr>
<td>Ids</td>
<td>26.5 mA</td>
<td>24.0 mA</td>
</tr>
<tr>
<td>Vgs</td>
<td>-0.58 V</td>
<td>-0.53 V</td>
</tr>
<tr>
<td>Fundamental</td>
<td>9.3 dBm</td>
<td>8.6 dBm</td>
</tr>
<tr>
<td>1st harmonic</td>
<td>-14.7 dBm</td>
<td>-17 dBm</td>
</tr>
<tr>
<td>2nd harmonic</td>
<td>-20.1 dBm</td>
<td>-24 dBm</td>
</tr>
</tbody>
</table>
Measurement of coil $Q$

Adjust coil overlay to obtain low coupling

Place tuned circuit in between coils and measure response

raise to reduce coupling to obtain $Q_0$
Summary - low phase noise

- High unloaded Q and low noise figure
- Set resonator coupling to achieve $Q_L/Q_0 = 1/2 \rightarrow 2/3$
- Set the open loop phase error to be N.360
- Use a device and circuit configuration producing the lowest transposed flicker noise corner $\Delta F_C$
Summary - low noise tuning

- Incorporate varactor loss resistor into resonator and set $Q_L/Q_0$ as before.
- For narrow band tuning
  - loosely couple varactor into resonator and set $Q_L/Q_0$ as before or consider
  - low loss phase shifter in the feedback loop. Expect 6dB noise degradation if open loop phase error goes to 45 degrees.
- Arrange for low bias line noise eg $r_b=50\Omega$.
LC Design Example
Design Example

Design a 150 MHz oscillator using:

1. 235nH inductor with a $Q_0$ of 300.

2. An inverting amplifier with an input and output impedance of 50Ω
An LC resonator with losses can be represented as an LCR resonator as shown in Figure 24.

\[ Q_0 = \frac{\omega L}{R_{loss}} \]

The equivalent series resistance is 0.74Ω.
Assuming the amplifier has an O/P impedance $R_{out}$, The ratio $Q_L/Q_0$ is:

$$\frac{Q_L}{Q_o} = \frac{R_{loss}}{R_{loss} + R_{in} + R_{out}}$$

For $R_{in} = R_{out}$

$$\frac{Q_L}{Q_o} = \frac{R_{loss}}{R_{loss} + 2R_{in}}$$

Let $\frac{Q_L}{Q_o} = \frac{1}{2}$ as the $P_{AVO}$ definition can be used.

Then $R_{loss} = 2R_{in}$

then $R_{in} = \frac{R_{loss}}{2}$

©2000 JKA Everard, University of York
Therefore $R_{in} = R_{out} = 0.37 \Omega$ as shown in Figure 25.

Figure 25 LC resonator with scaled source and load impedances
As amplifier has $50\Omega$ input and output impedances:

Use LC transformer as shown in Figure 26.

**Figure 26 LC transformer to convert to 50Ω**

The equations for the series and shunt components are:

$$Q_s = Q_p = \sqrt{\left(\frac{R_p}{R_s} - 1\right)}$$
The Q of the series component is:

\[ Q_s = \frac{X_s}{R_s} \]

The Q of the shunt component is:

\[ Q_p = \frac{R_p}{X_p} \quad \text{Note:} \]

\[ R_p = \text{shunt resistance} \]

\[ R_s = \text{series resistance} \]

\[ X_s = \text{series reactance} = j\omega L \]

\[ X_p = \text{shunt reactance} = \frac{1}{j\omega c} \]

©2000 JKA Everard, University of York
\[ Q_s = Q_p = \sqrt{\left( \frac{R_p}{R_s} - 1 \right)} = \sqrt{\left( \frac{50}{0.37} - 1 \right)} = 11.58 \]

\[ X_s = 4.28 = j\omega L \quad L = 4.5 \text{ nH} \]

\[ X_p = 4.31 = \frac{1}{j\omega C} \quad C = 246 \text{ pf} \]
Incorporate 2 transforming circuits into resonator circuit as shown in Figure 27

Figure 27 LC resonator with impedance transformers
As the total inductance is 235nH, the part that resonates with the series capacitor is reduced by 9nH as shown in Figure 28.

![Resonator with total L = 235nH](image)

**Figure 28 Resonator with total L = 235nH**

It is now necessary to calculate the resonant frequency.

The part of the inductance which resonates with the series capacitance is reduced by the matching inductors to:

\[ 235 \text{nH} - (2 \times 4.5 \text{nH}) = 226 \text{nH} \]

\[
f = \frac{1}{2\pi\sqrt{LC}} \quad \quad \quad LC = \frac{1}{(2\pi f)^2}
\]

\[
C = \frac{1}{L(2\pi f)^2} = 5 \text{pF}
\]

©2000 JKA Everard, University of York
The circuit now becomes:

![Circuit Diagram]

Note the value of shunt capacitors: 246pf!

Figure 29 Final resonator circuit
Simulation of insertion loss, $S_{21}$, of resonator
Effect of parasitic components

• What is the effect of the parasitics in the shunt capacitors
• Investigate the effect of both a 1nH and 2nH parasitic inductance
• This increases the effective capacitance as close to resonance (reduces impedance)
Effect of parasitic inductance in shunt C

Yellow = 1nH
Green = 2nH - correct for this by reducing C from 246pf to 174pf
Note the phase shift at resonance is $180^\circ$.

So the amplifier should provide a further $180^\circ$.

If necessary a phase shifter should be included to ensure $N \times 360^\circ$ at the peak in the resonance as shown in Figure 30.

*Figure 30 Oscillator incorporating phase shifter*
1 GHz Transmission Line Osc.

- Design a 1GHz Transmission line Oscillator
  
  use 1.5mm FR4 PCB, $Z_0 = 50 \Omega$, $\varepsilon_{eff} = 3.3$

  \[
  Q_0 = \frac{\pi}{2\alpha L} \quad Q_L = \pi S_{21}(0) \left( \frac{X^2}{4} \right)
  \]

  \[
  \text{Length } L = \left( \frac{V_{eff}}{2f_0} \right) \left(1 + \left( \frac{1}{\pi} \right) \tan^{-1} \left( \frac{2}{X} \right) \right)
  \]

  \[
  X = 2\pi f_0 C Z_0 \text{ or } -Z_0/2\pi f_0 l
  \]
Find loss of line

• Measure loss of known length of line OR
• Build a number of resonators with varying $Q_L$. Extrapolate to $Q_0$ by drawing straight line as $S_{21} = (1 - Q_L/Q_0)$
• Simulate using field model
• Note that the loss of ‘low loss’ transmission lines can be deduced from resonator measurements
Calculate parameters

- From measurement $Q_0 = 39.3$
- $\alpha L = 0.04$
- For $\frac{L}{Q_0} = 1/2$ \quad $X = \sqrt{\frac{2}{\alpha L}}$
- $X = 7.07$ therefore as $X = 2\pi f_0 C Z_0 = -Z_0/2\pi f_0 l$
  - inductor $l = 1.125\text{nH}$
  - capacitor $C = 22.5\text{pf}$
- Line length=7.53cms for $l$ and 8.98cms for $C$
Transmission line resonator response using shunt inductor
Acknowledgements

• I would like to thank the UK Engineering and Physical Sciences Research Council for supporting this work.

• I would also like to thank Paul Moore at Philips research laboratories who, some 16 years ago, started me in the right direction and I also thank Jens Bitterling, Carl Broomfield, Michael Cheng, Frazer Curley, Paul Dallas and Mike Page-Jones for their help in generating new ideas and results.
REFERENCES