Phase Noise Measurements

Noise - A stench to the ear,
Undomesticated music,
The chief product and authenticating sign of civilization

Ambrose Bierce - "The Devils Dictionary" 1907

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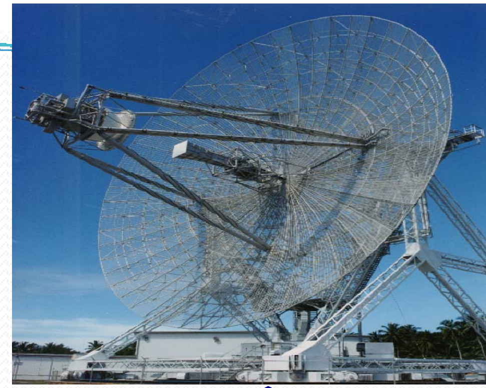
Topics

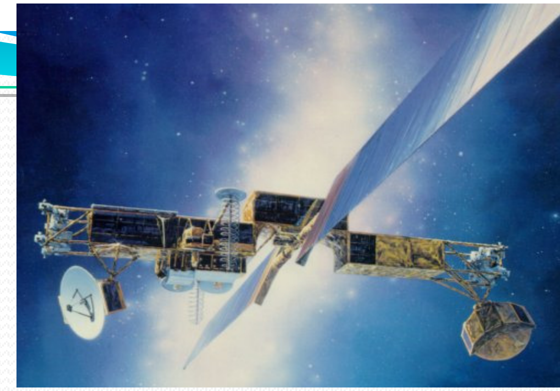
- Introduction and Review
- Noise Types
- Measurement Methods
- Measurement Calibration
- Common Measurement Problems
- Conclusions

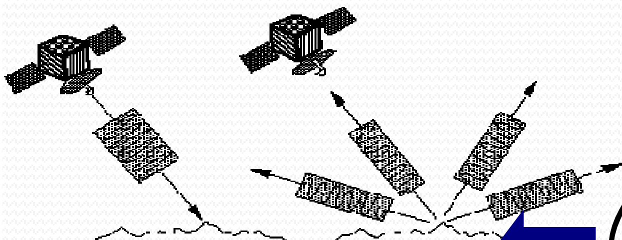
Weather Monitoring

RADAR

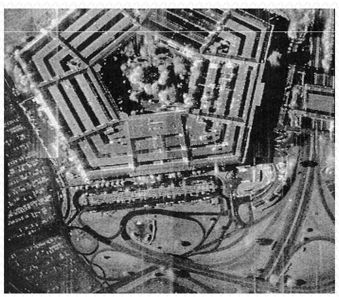


Satellite Communication



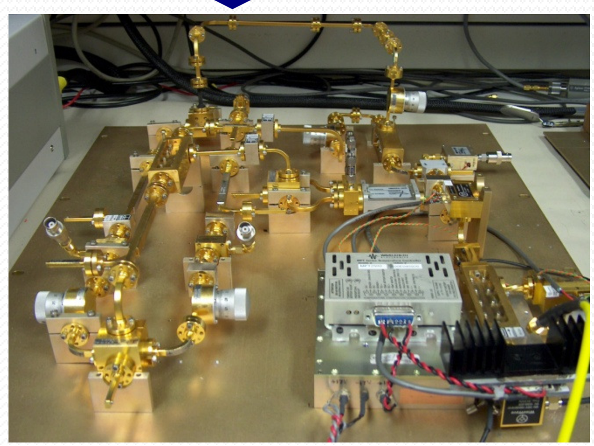


INDUSTRY and MILITARY NEEDS for SPECTRAL PURITY

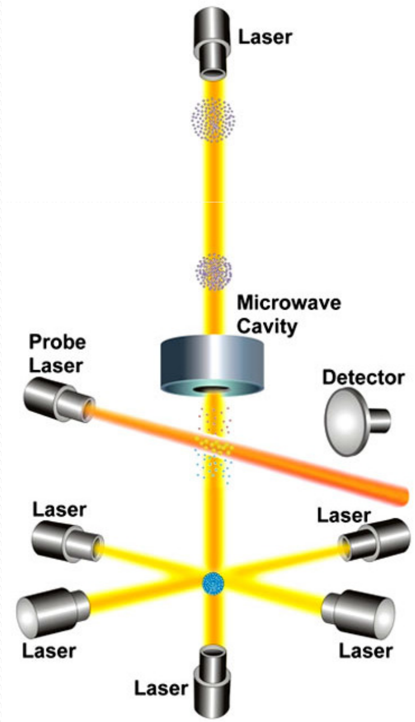


Imaging RADAR

- Navigation
- Defense & Homeland Security
- Secure Communication
- Astronomy & Geodesy



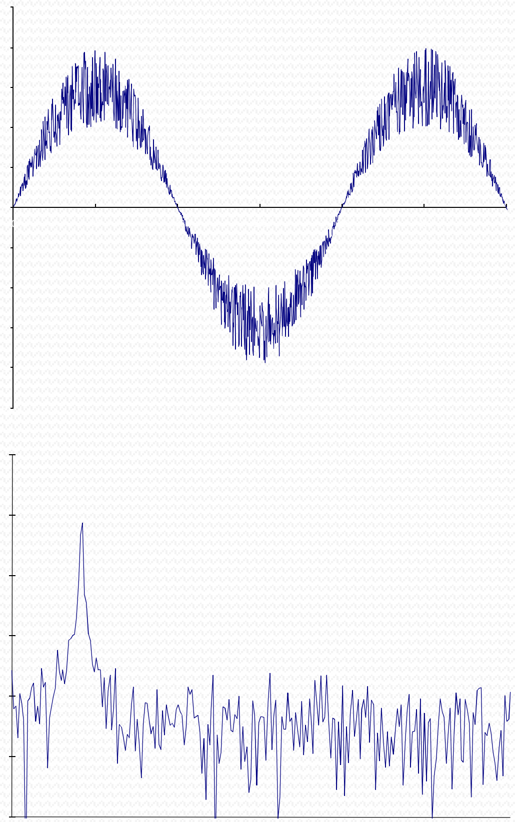
Noise Metrology



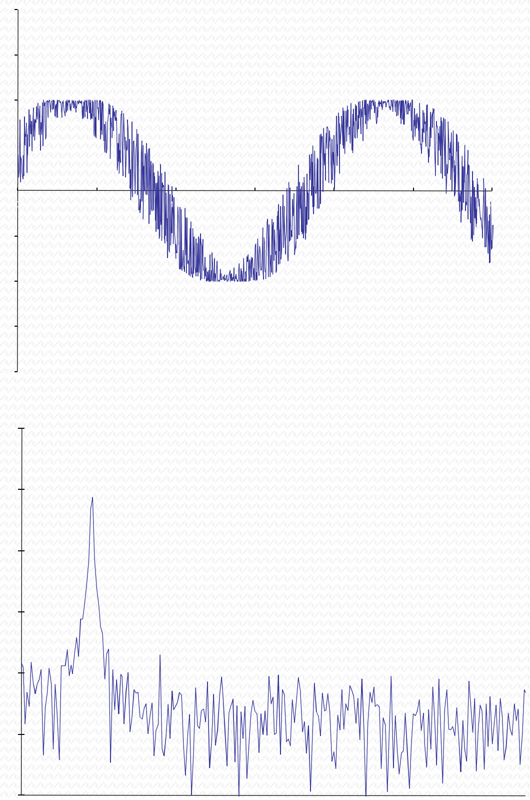
Atomic Frequency Standards & Spectroscopy

NOISE

AM Noise



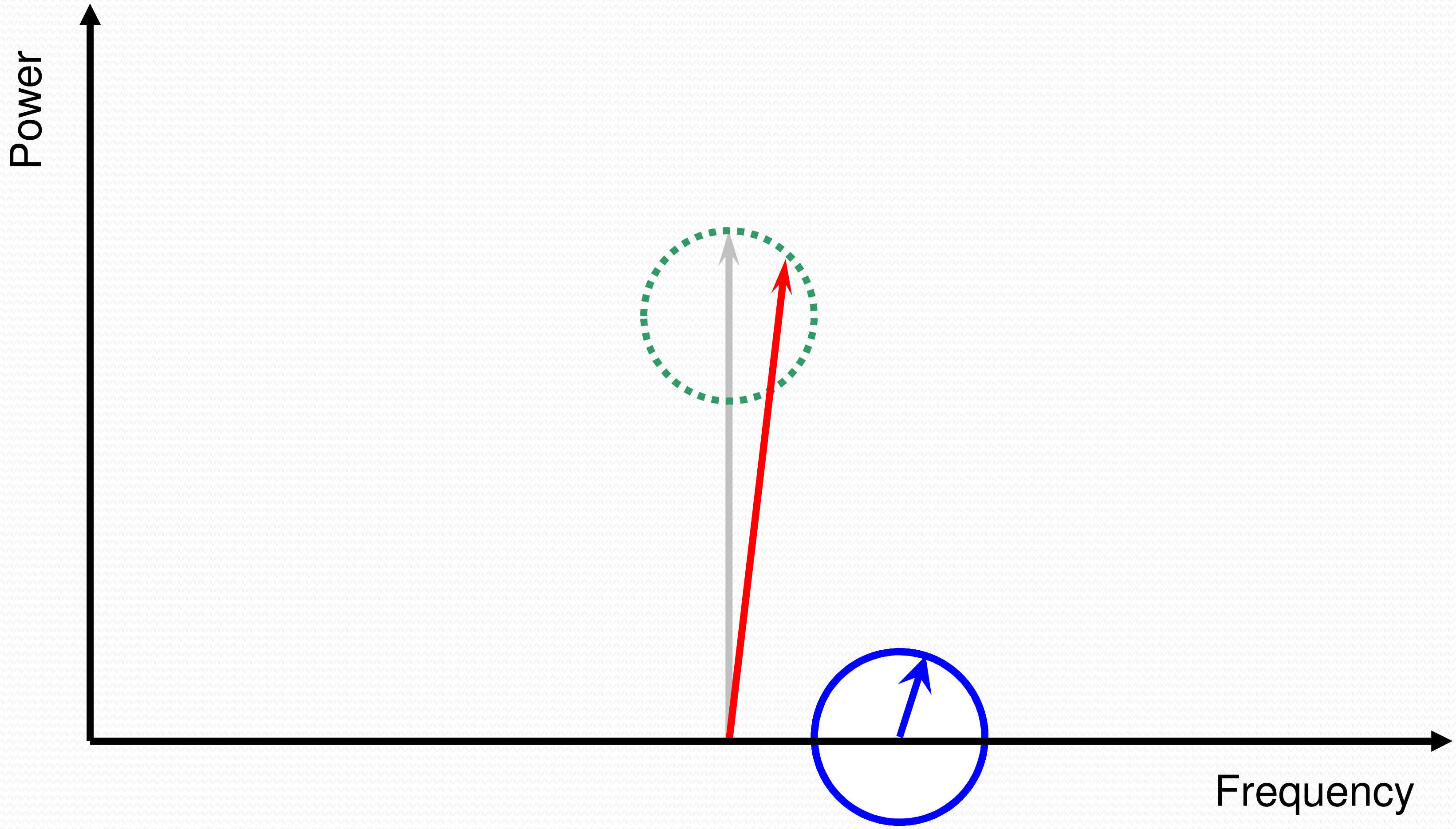
PM Noise



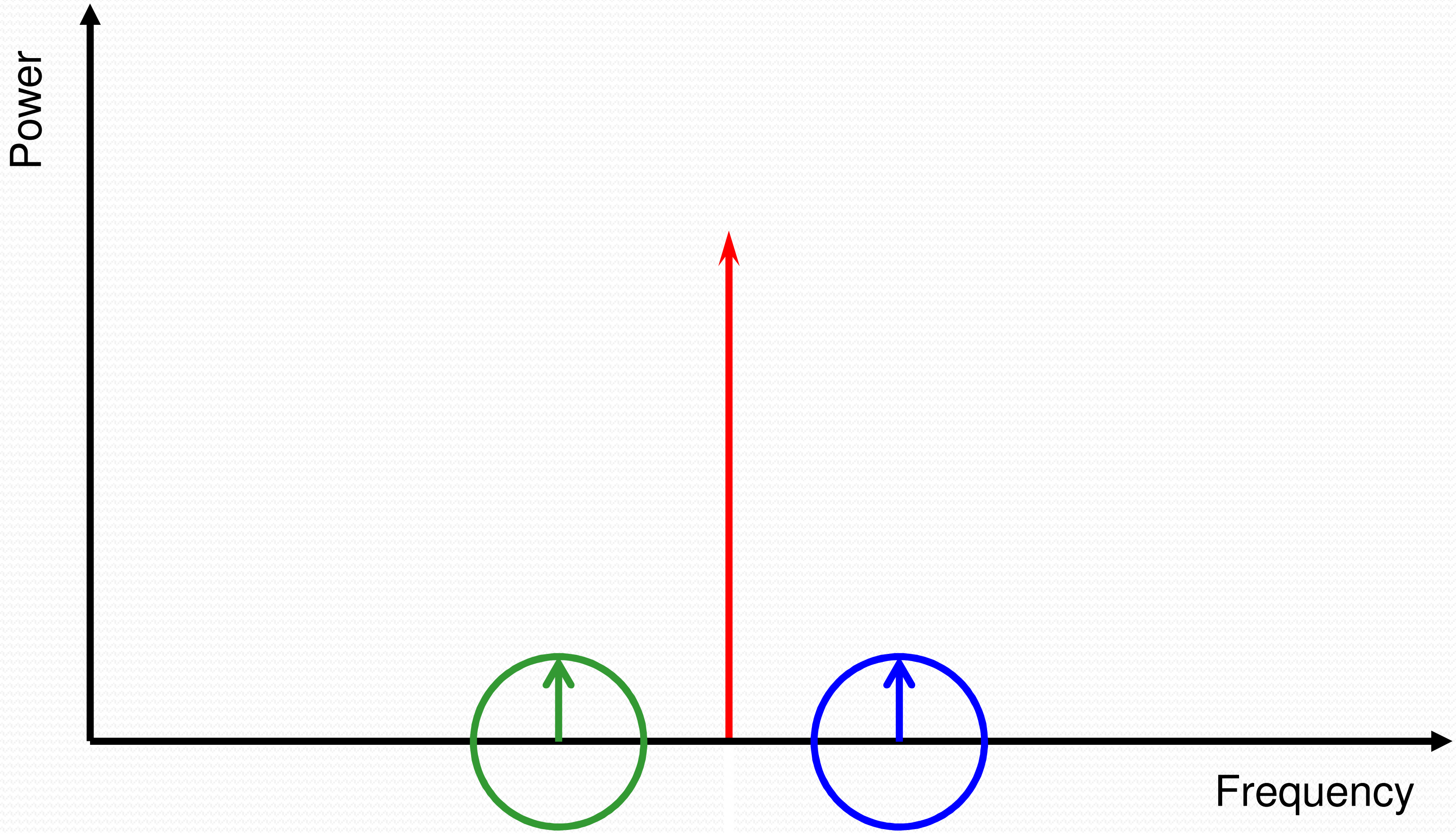
Difficulty of using a Spectrum Analyzer

- IF bandwidth too wide
- SA internal reference too noisy
- Not enough dynamic range
  - Phase noise is often below -170 dBc
- Cannot distinguish between AM and PM noise

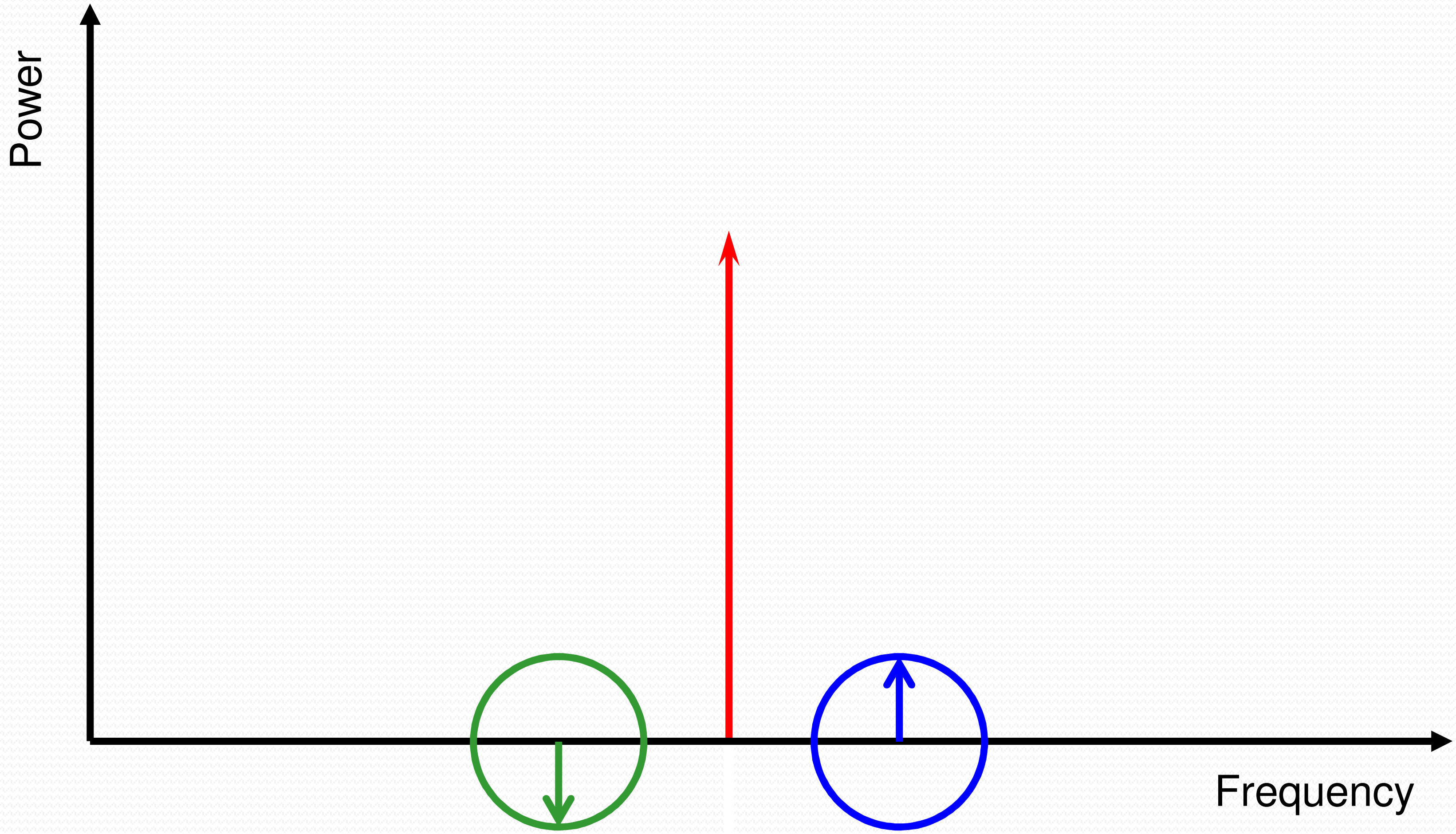
Single Sideband Modulation



Amplitude Modulation



Phase Modulation



Theory Review

Basic Model for Noisy Signal

$$V(t) = A(1+\alpha(t))\cos(2\pi\upsilon_0 t + \phi(t))$$

where:

$$A \quad = average\ amplitude$$

$$\alpha(t) = fractional\ amplitude\ fluctuations$$

$$\upsilon_0 \quad = average\ frequency$$

$$\phi(t) = phase\ flucuations$$

Basic Definitions

$$V(t) = A(1+\alpha(t))\cos(2\pi\upsilon_0 t + \phi(t))$$

$$phase = 2\pi\upsilon_0 t + \phi(t)$$

$$\omega(t) = \frac{d}{dt}[phase]$$

$$\upsilon(t) = \frac{1}{2\pi}\frac{d}{dt}[2\pi\upsilon_0 t + \phi(t)] = \upsilon_0 + \frac{1}{2\pi}\frac{d}{dt}\phi(t)$$

Fractional frequency deviation

$$y(t) = \frac{\upsilon(t) - \upsilon_0}{\upsilon_0} = \frac{1}{2\pi\upsilon_0}\frac{d}{dt}\phi(t)$$

$$S_y(f) = PSD^1[y(t)] = \frac{2}{T}\left|Y_T(f)\right|^2 \qquad 0 < f < \infty \qquad [\frac{1}{Hz}]$$

Definition of Phase Noise

$$S_\phi(f) = PSD^1(\phi(t)) = \frac{2}{T}\left|\Phi_T(f)\right|^2 \qquad 0 < f < \infty \qquad [\frac{rad^2}{Hz}]$$

Single sideband phase noise

$$\mathcal{L}(f) \equiv \tfrac{1}{2} S_\phi(f) \quad [dBc/Hz]$$

$$S_\phi(f) = \left(\frac{\nu_0}{f}\right)^2 S_y(f)$$

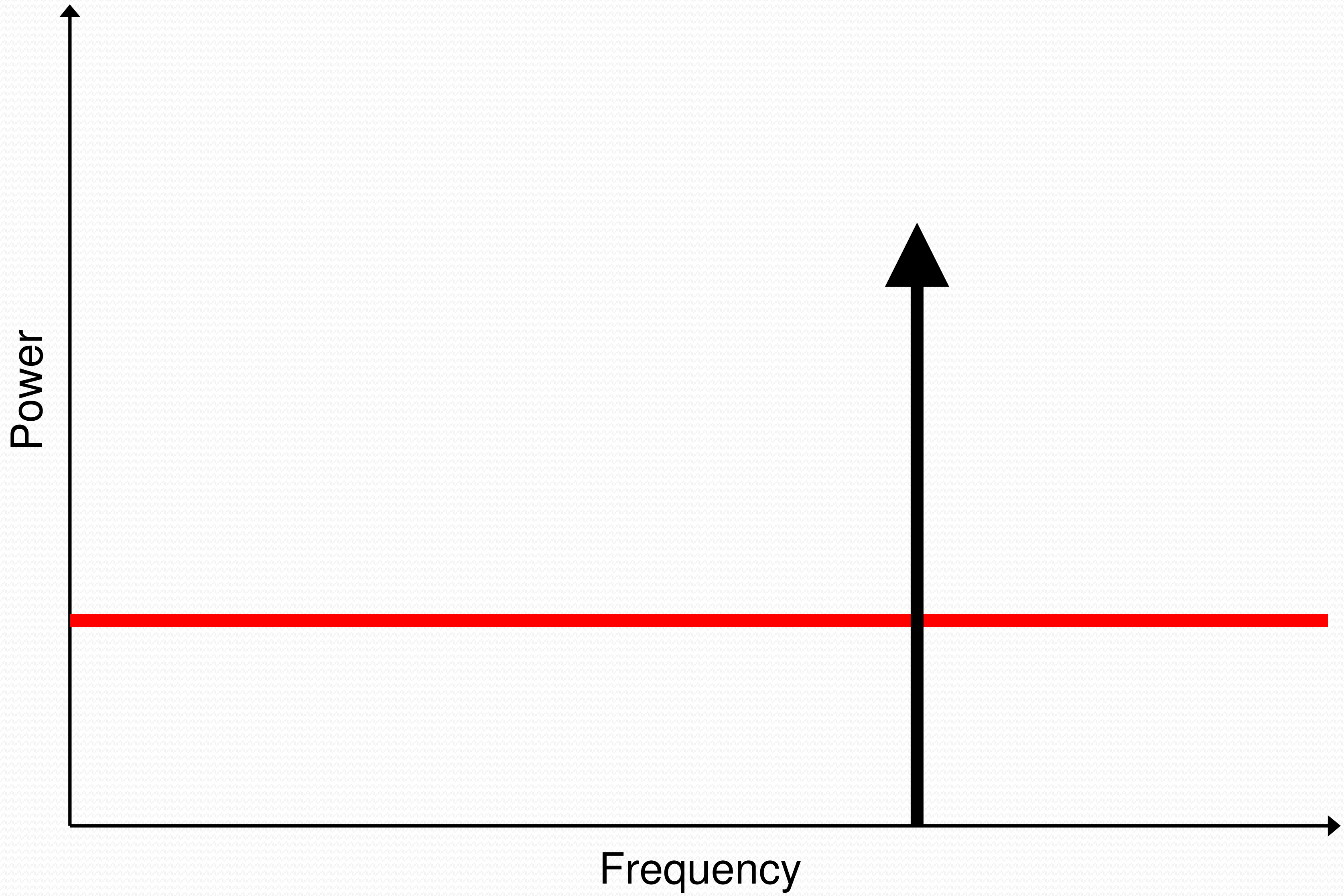
Definition of Amplitude Noise

$$S_\alpha(f) = PSD^1(\alpha(t)) = \frac{2}{T}\left|\mathrm{A}_T(f)\right|^2 \qquad 0 < f < \infty \qquad [\frac{1}{Hz}]$$

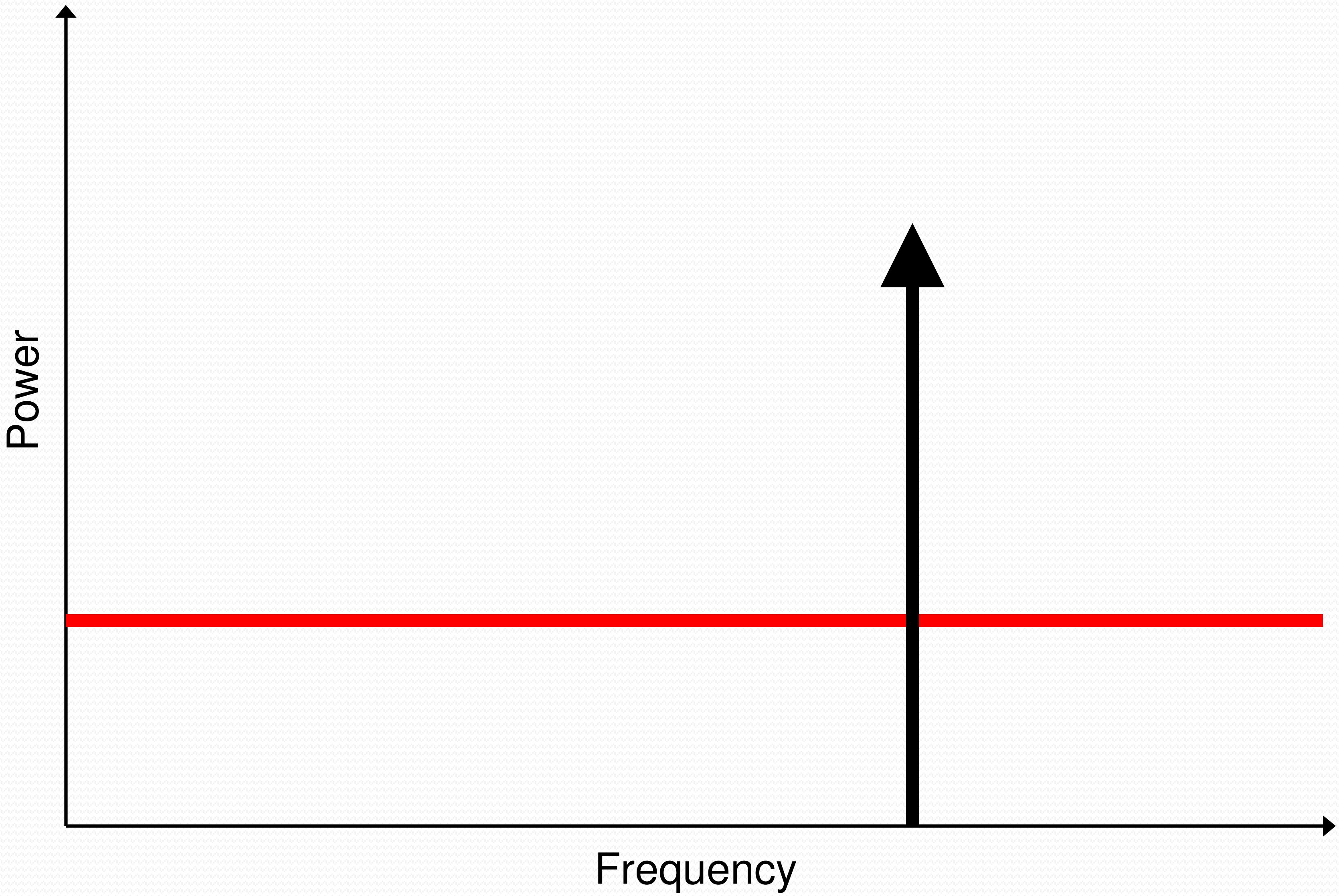
Noise Types

- Additive Noise
  - f^{0} Thermal
  - Shot noise
- Multiplicative Noise
  - Flicker f^{-1}
  - Higher order colored noise types $f^{-2}, f^{-3}, f^{-4} \ldots$

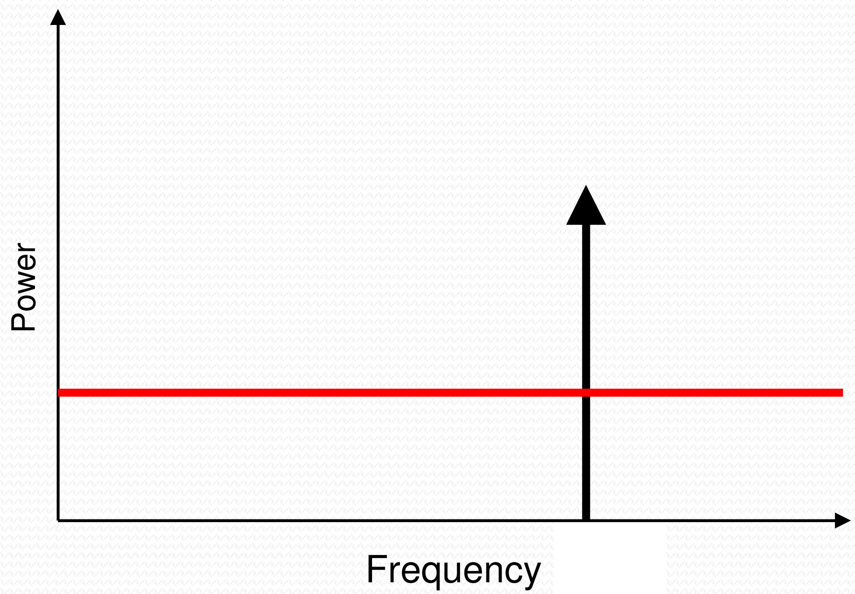
Additive Noise

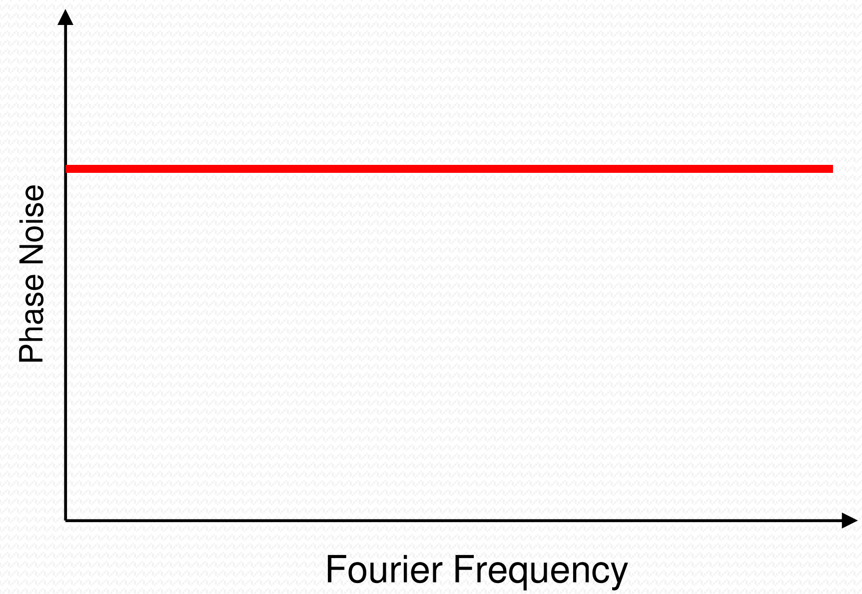


Additive Noise



Additive Noise



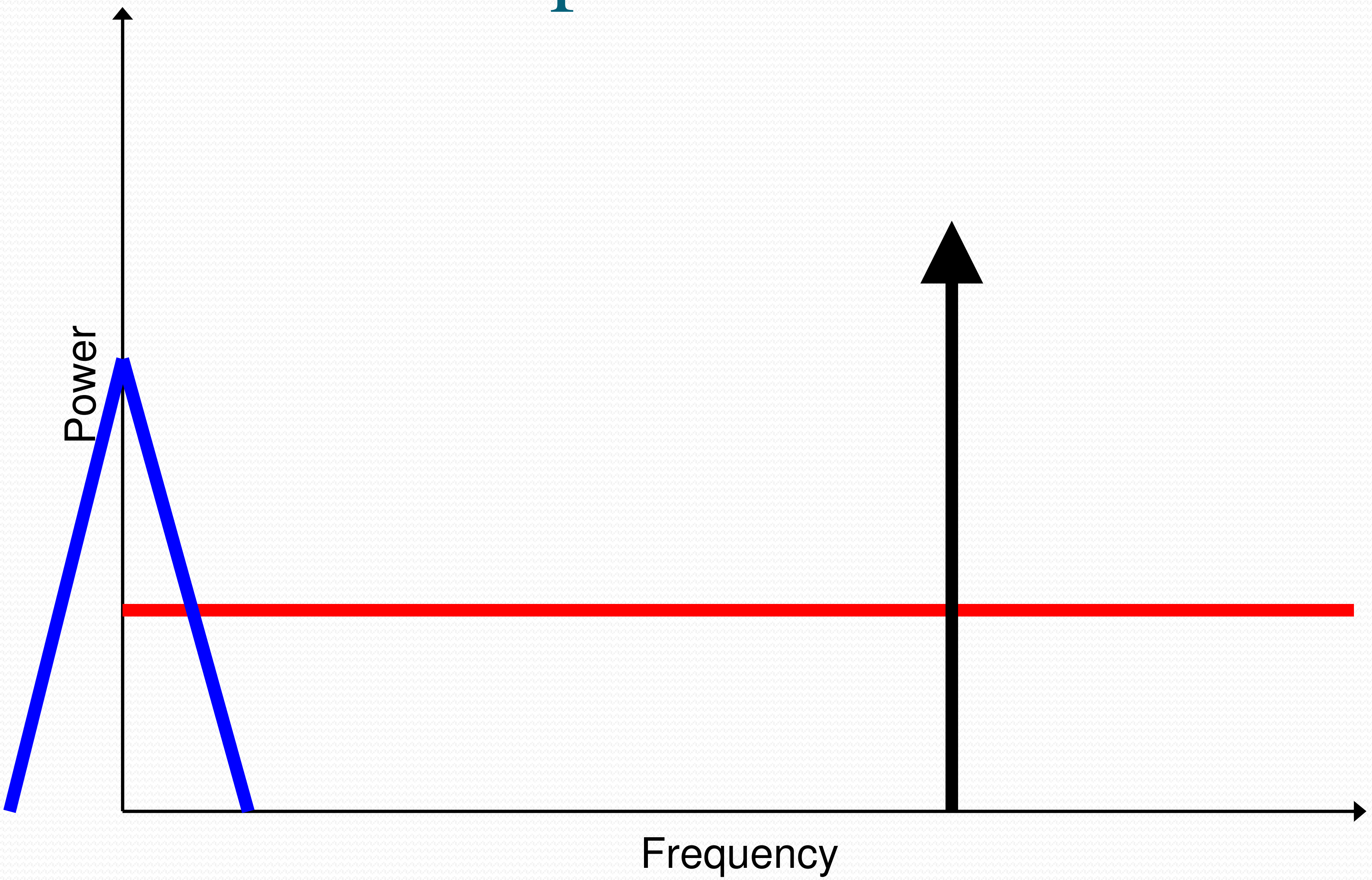


Since it is uncorrelated to the carrier, additive noise always appears as equal amount of AM and PM Noise.

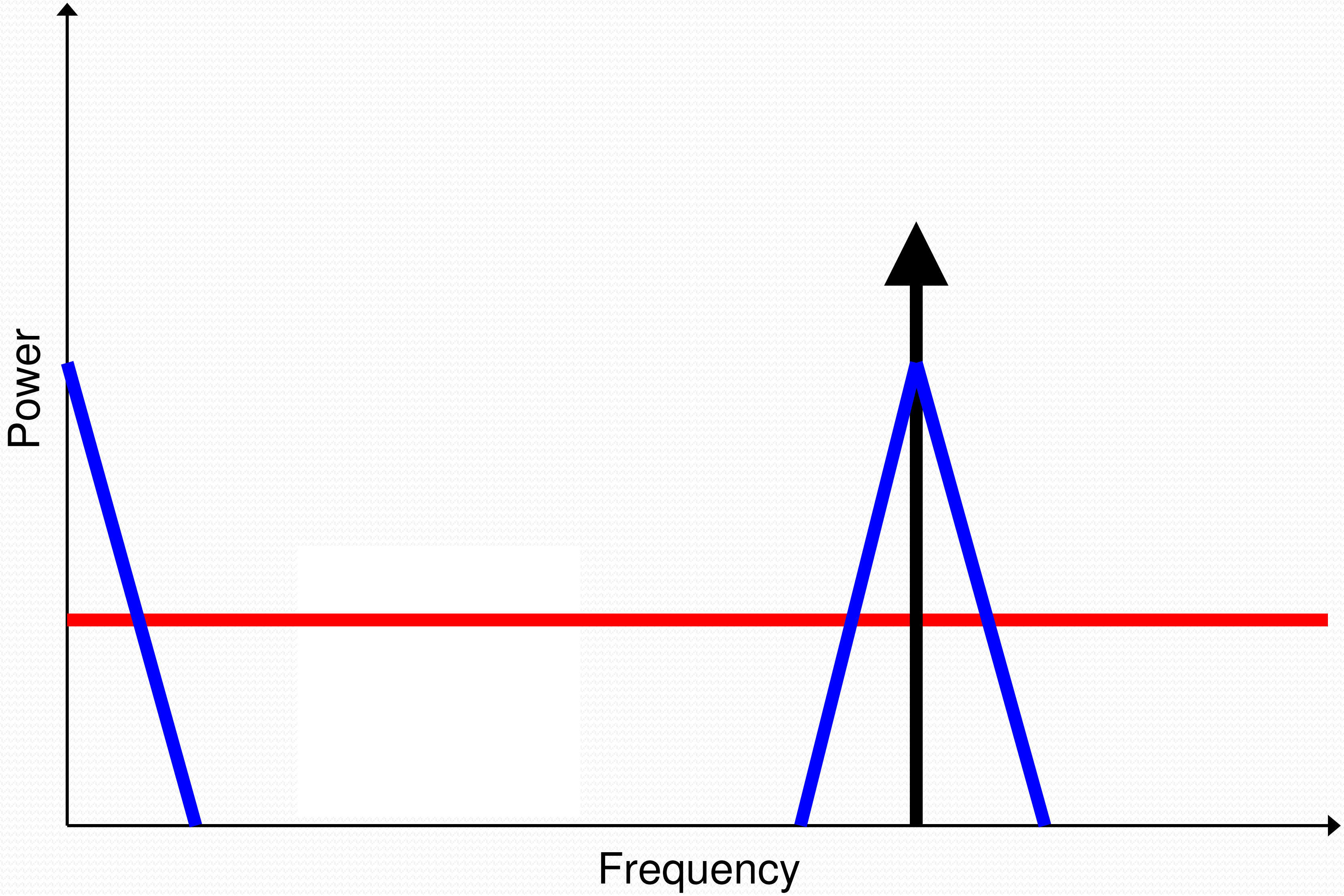
For Amplifiers:

$$S_\varphi(f) = S_a(f) = \frac{kTB}{P_I} NF = -174 + NF - P_I \quad @\, 300K$$

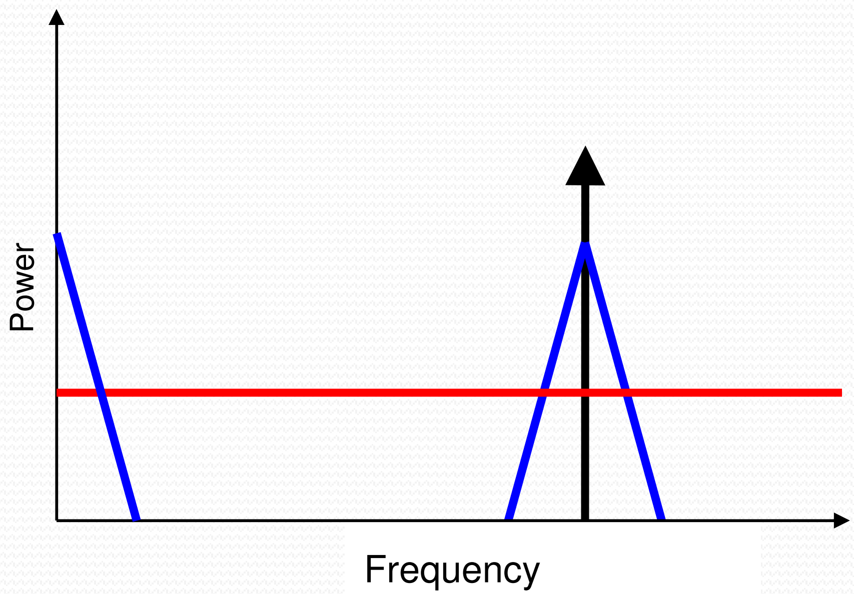
Multiplicative Noise

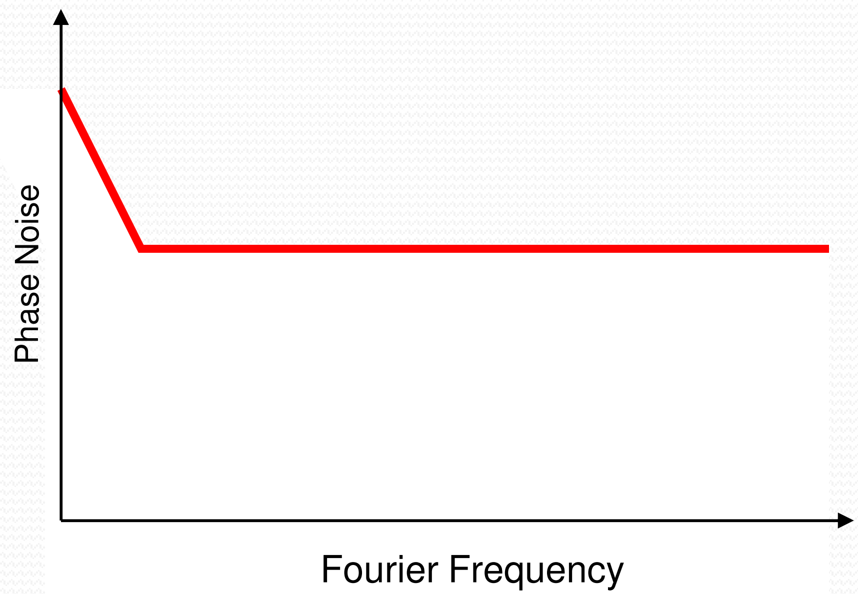


Multiplicative Noise



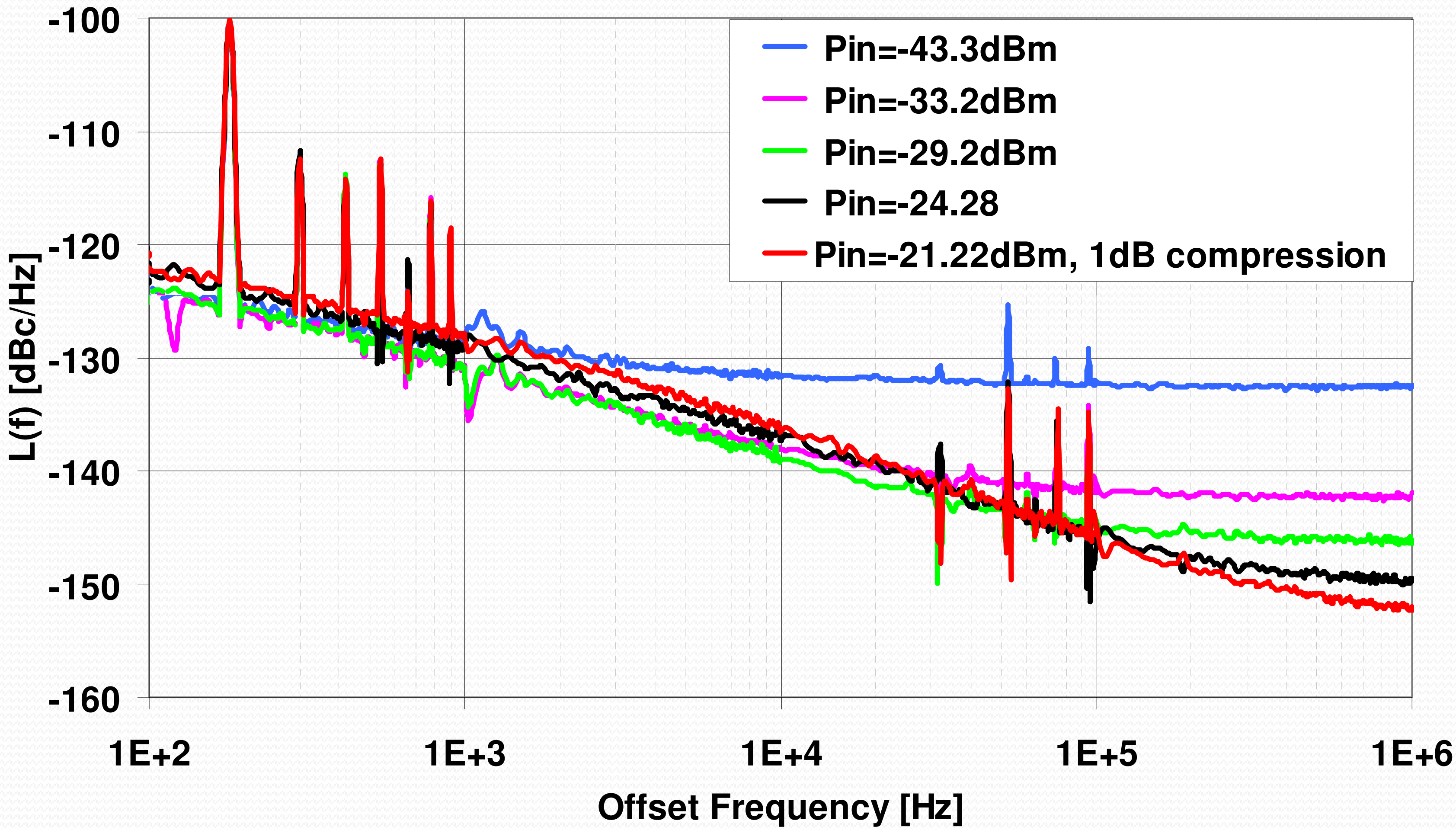
Additive and Multiplicative Noise





Additive and Multiplicative Noise

10 GHz, Gain=32.5dB, NF=1



Noise Figure

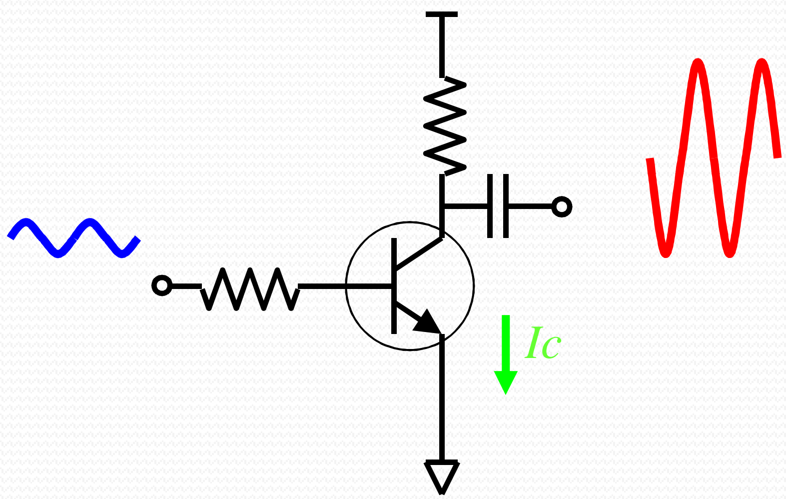
- Noise figure is only a figure of merit of thermal additive noise.
- It has ZERO correlation to flicker or any other type of multiplicative noise
- In the linear region of operation. The phase noise floor for a device can be determined from input power and noise figure as follows:

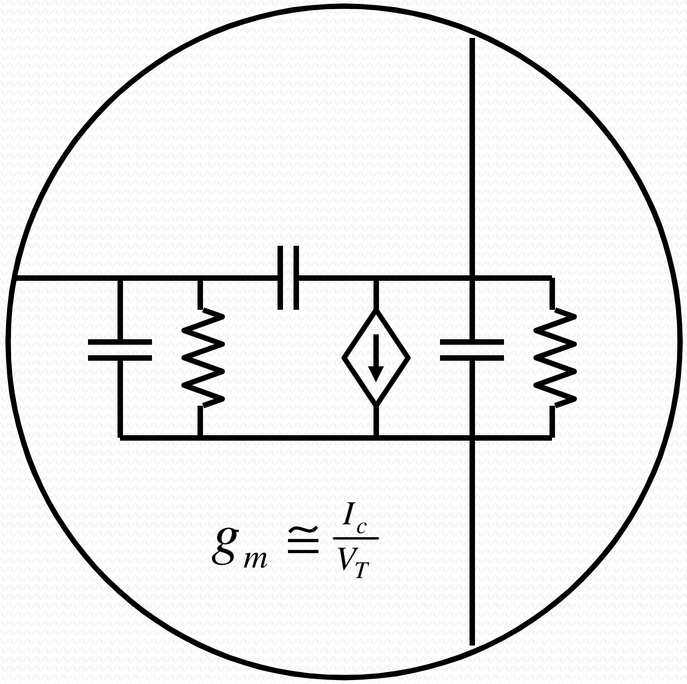
$$S_\varphi(floor) = S_\alpha(floor) = \frac{kT_0BNF}{P_{in}}$$

$$= -174 + NF - P_{in} \quad [dBrad^2/Hz] \;\; or \;\; [dB/Hz]$$

$$\mathscr{L}(floor) = -177 + NF - P_{in} \quad [dBc/Hz]$$

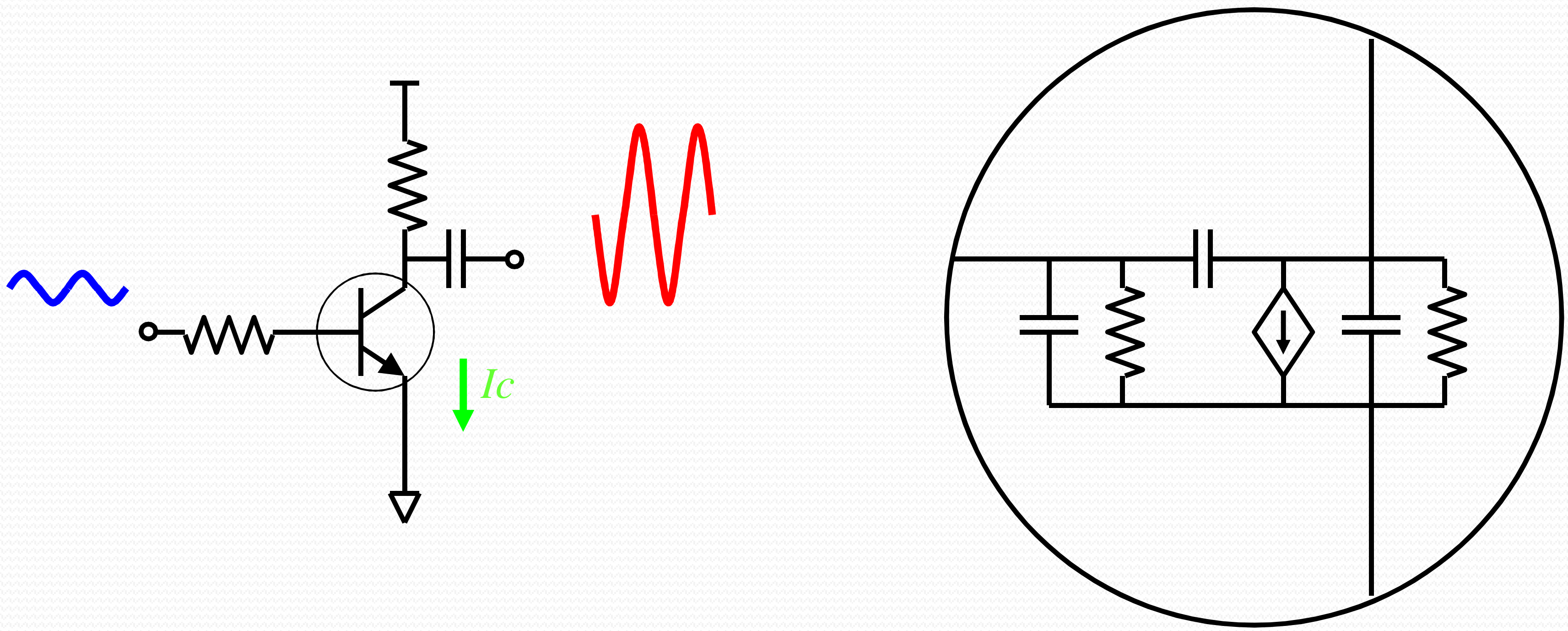
Multiplicative AM Noise





Unlike additive noise, multiplicative AM and PM are not fundamentally correlated because they up convert by different mechanisms.

Multiplicative PM Noise

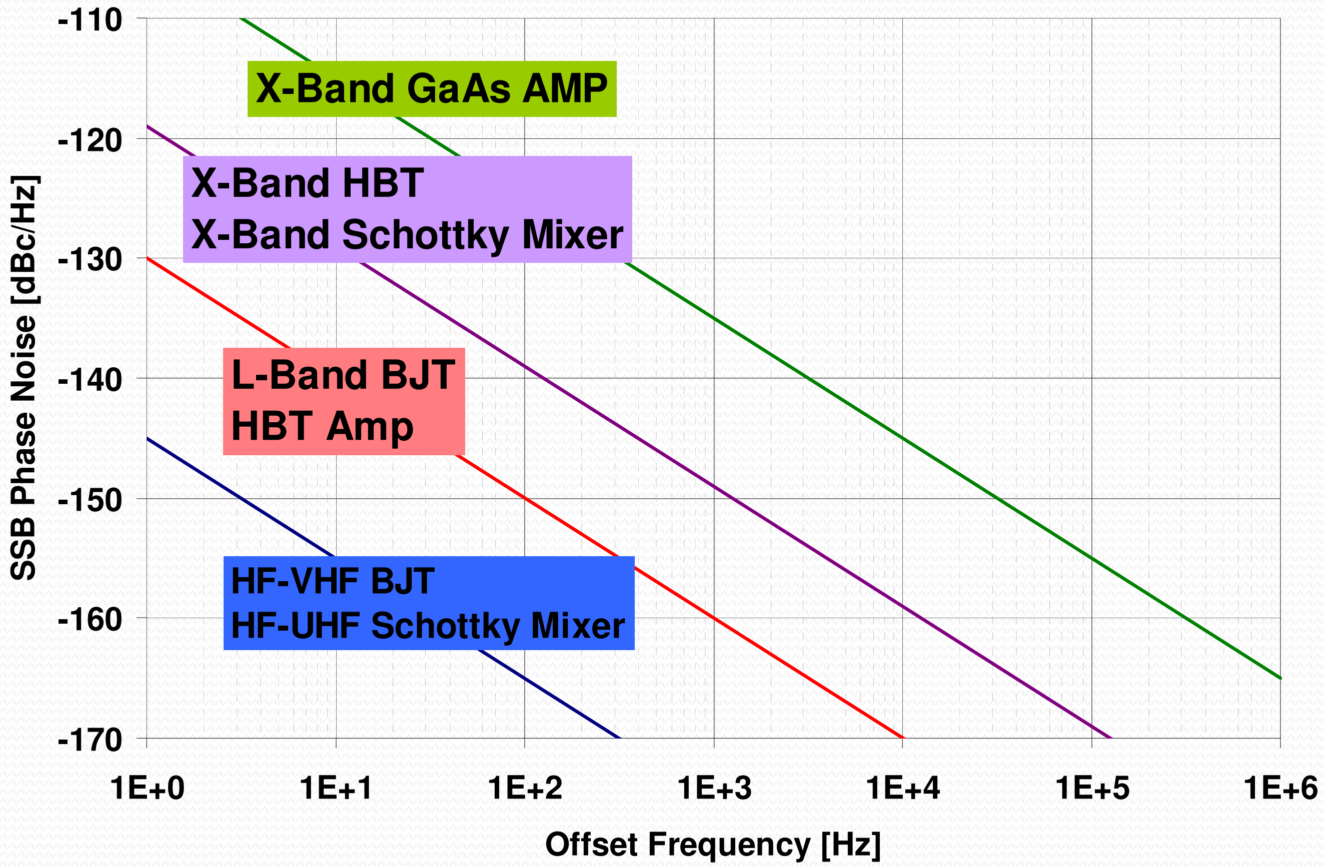


High Linearity => Low Multiplicative PM Noise (Flicker)

Separation between P1dB and IP3 is a simple indicator

> 15 dB has potential for low flicker

Typical PM Flicker Noise for Different Semiconductor Types



Courtesy of M. Driscoll

Noise Summary

Additive Noise Summary

- Noise power is uncorrelated to signal power
- AM = PM
- AM and PM noise levels vary inversely with carrier power

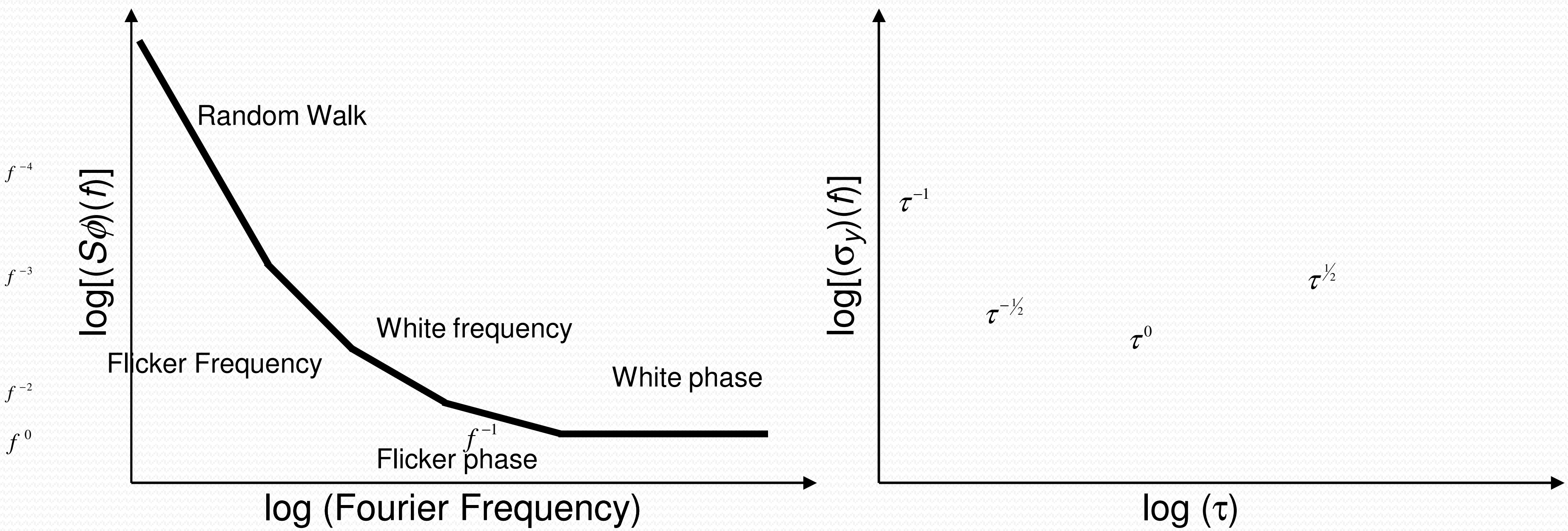
Multiplicative Noise Summary

- Noise power is correlated to signal power
- AM ≠ PM
- AM and PM noise levels are independent of carrier power

Typical Noise Types

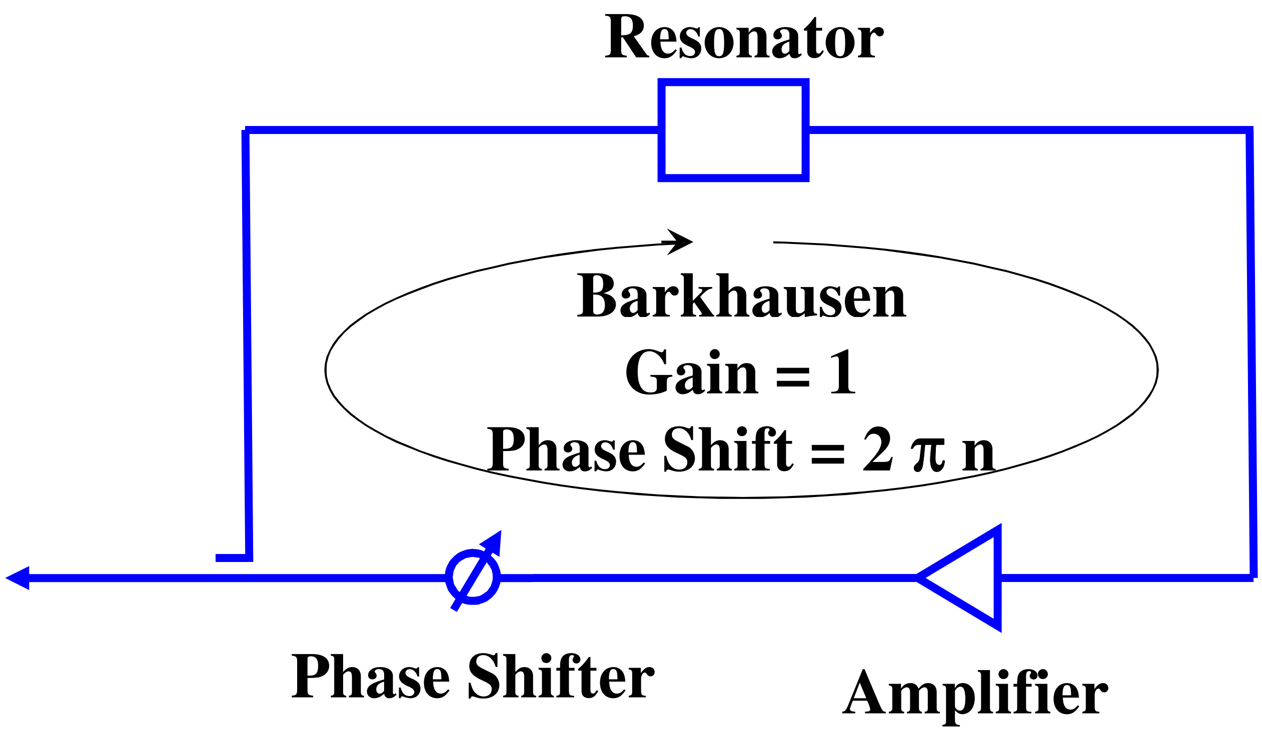
- Passive Devices
  - Thermal f^{0}
  - Some have flicker (magnetics, carbon resistors) f^{-1}
  - Higher order noise may come from temperature effects f^{-4}
- Active Devices
  - Almost all have thermal and flicker f^{0} and f^{-1}
  - Possible temperature effects f^{-4}
- Sources (May some or all of the higher order types)
  - White PM or Thermal f^{0}
  - Flicker PM f^{-1}
  - White FM f^{-2}
  - Flicker FM f^{-3}
  - Random Walk f^{-4}

Noise Types



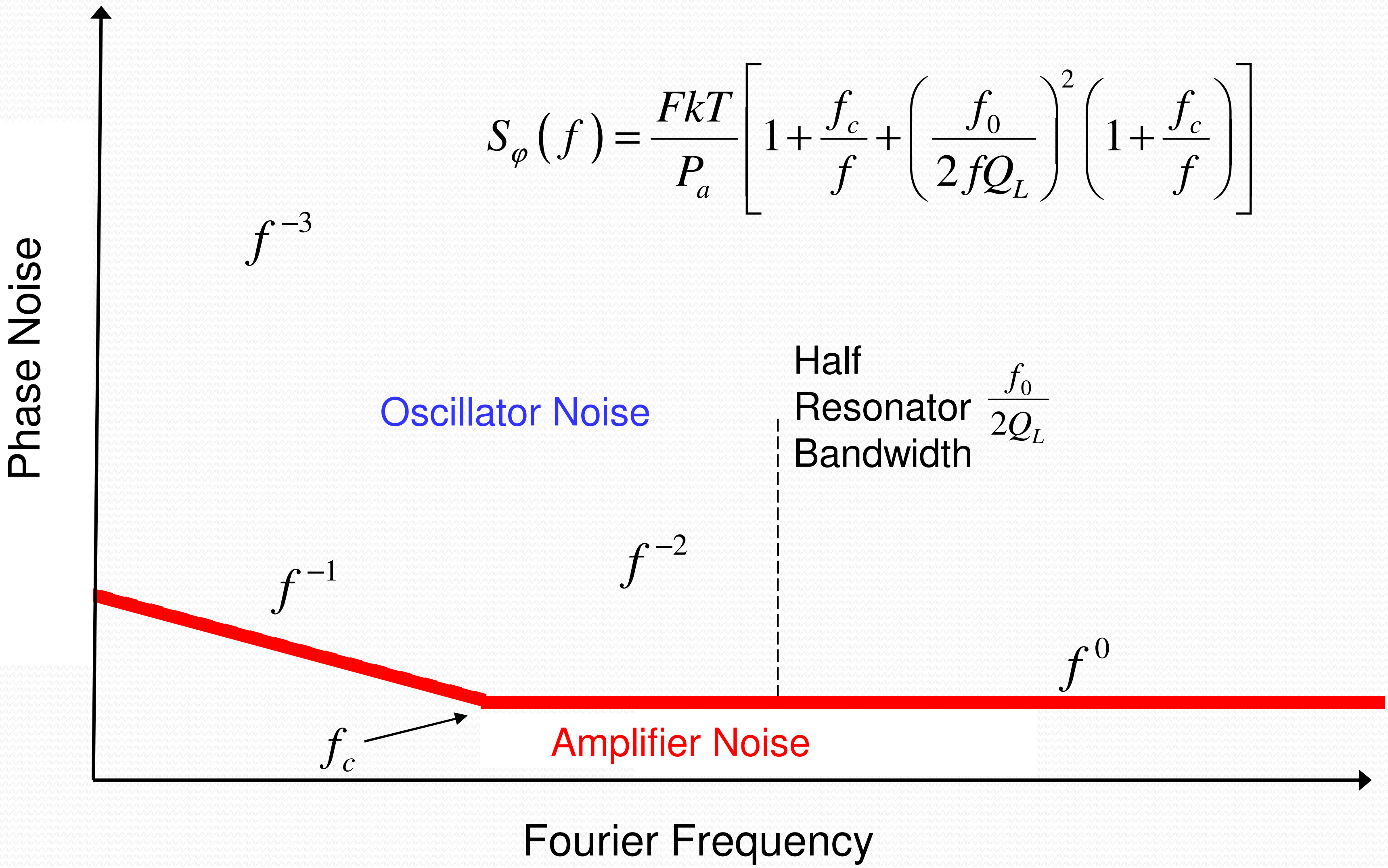
White phase	Thermal Noise, Shot Noise
Flicker phase	Electronics, recombination-generation, traps
White frequency	Resonator, integrated white phase
Flicker Frequency	Resonator, integrated flicker phase
Random Walk	Temperature, Shock, Vibration, Resonator

Leeson's Effect

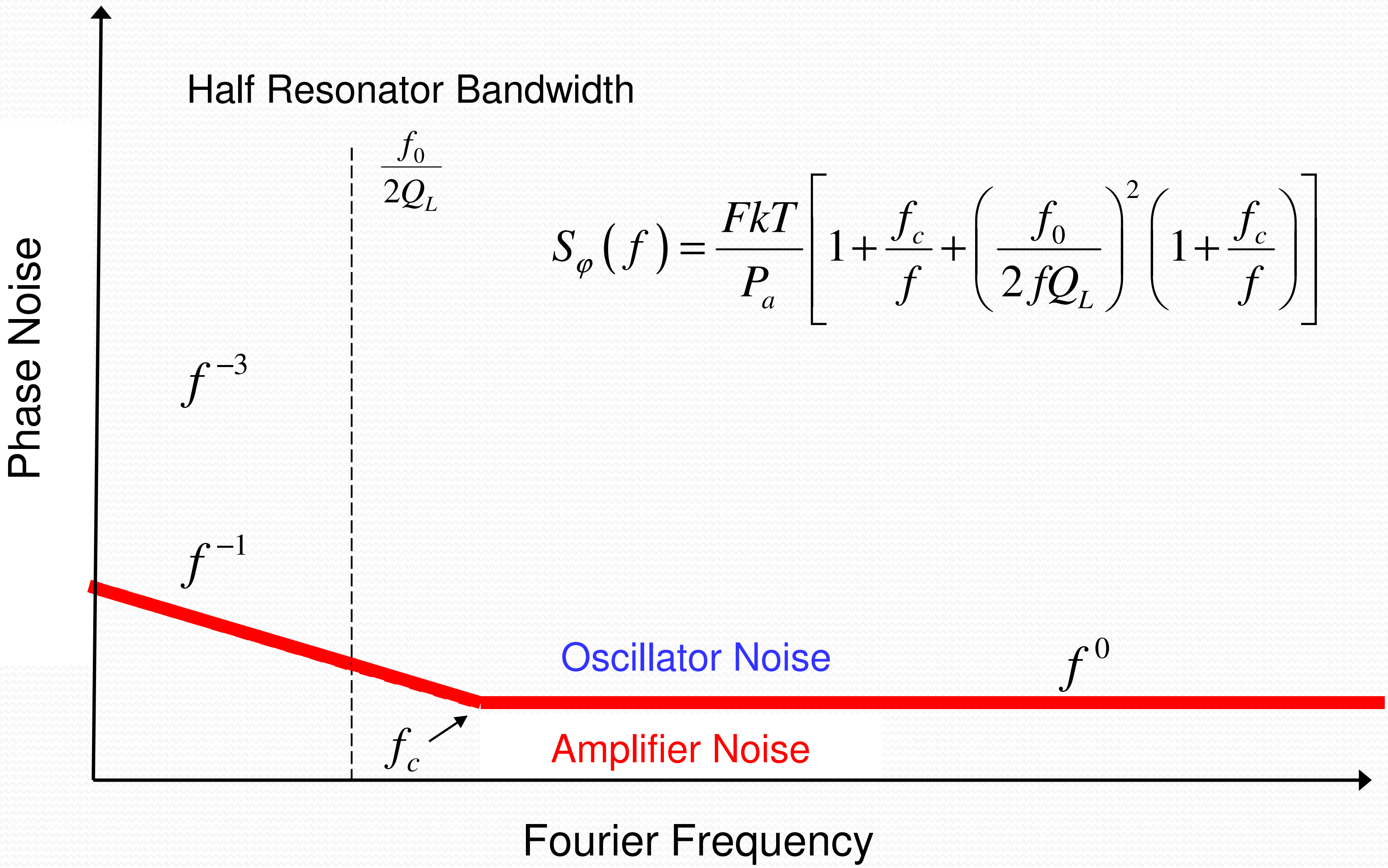


Leeson's Effect - Low Q

$$S_\varphi(f) = \frac{FkT}{P_a}\left[1+\frac{f_c}{f}+\left(\frac{f_0}{2fQ_L}\right)^2\left(1+\frac{f_c}{f}\right)\right]$$

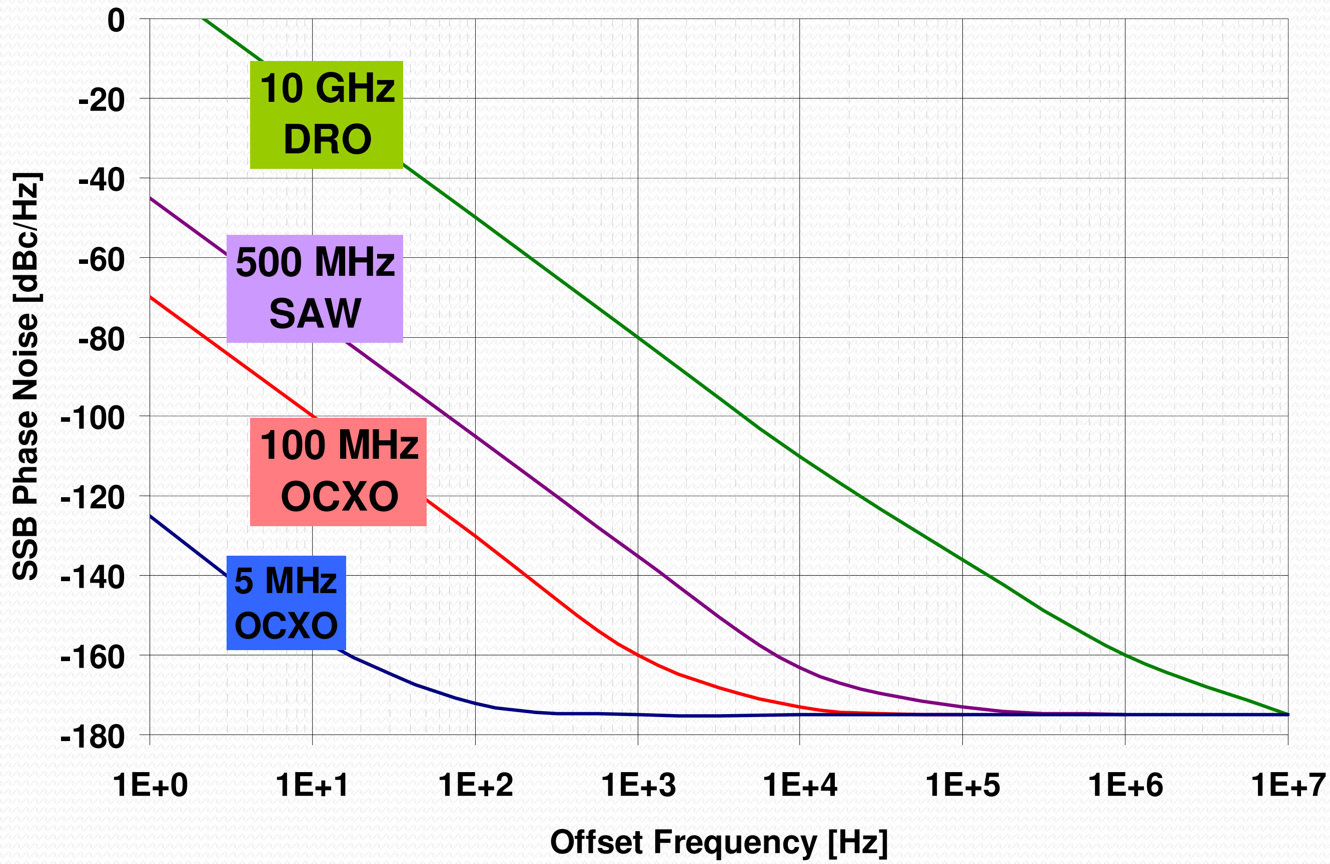


Leeson's Effect - High Q

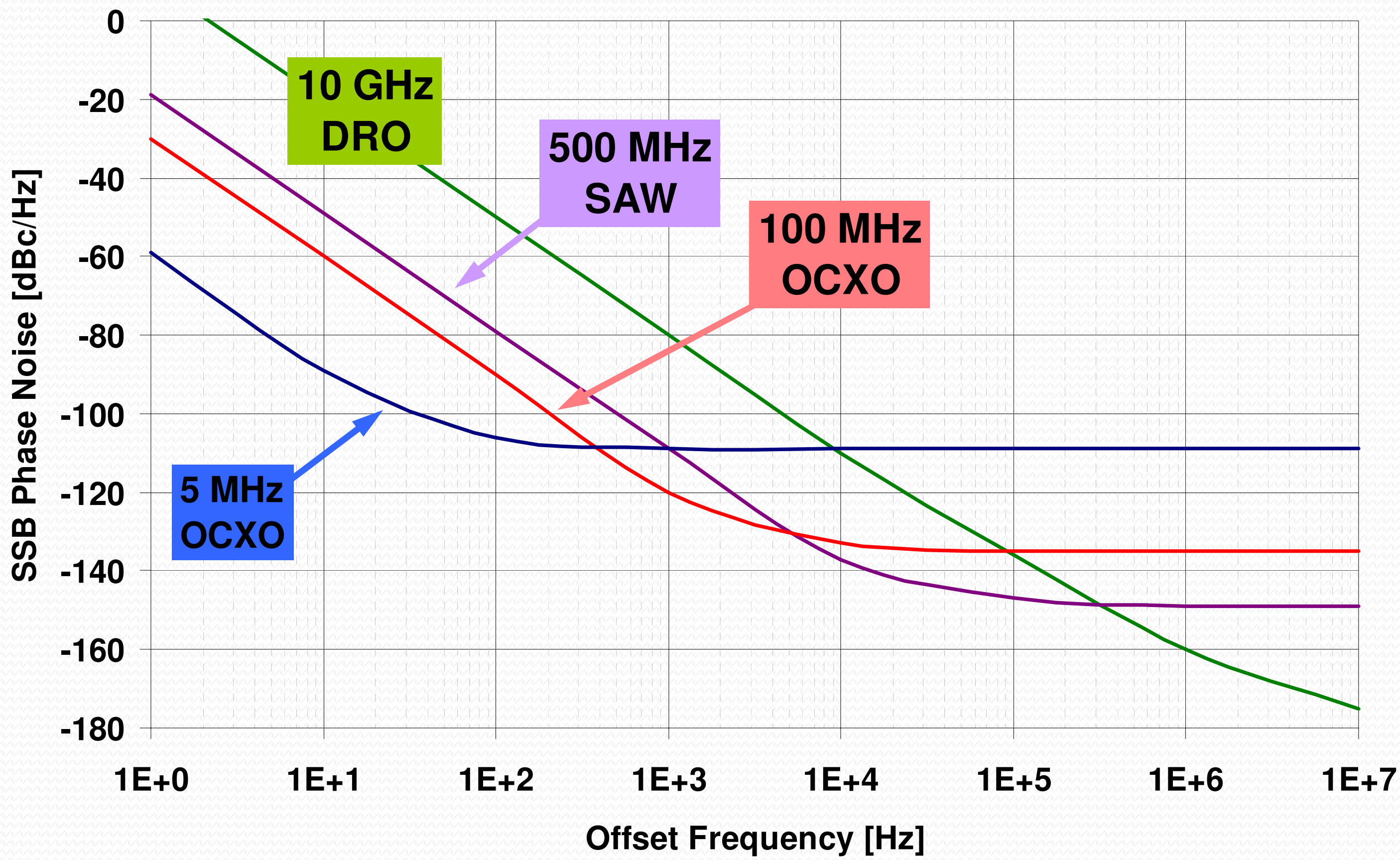


$$S_\varphi(f) = \frac{FkT}{P_a}\left[1 + \frac{f_c}{f} + \left(\frac{f_0}{2fQ_L}\right)^2\left(1 + \frac{f_c}{f}\right)\right]$$

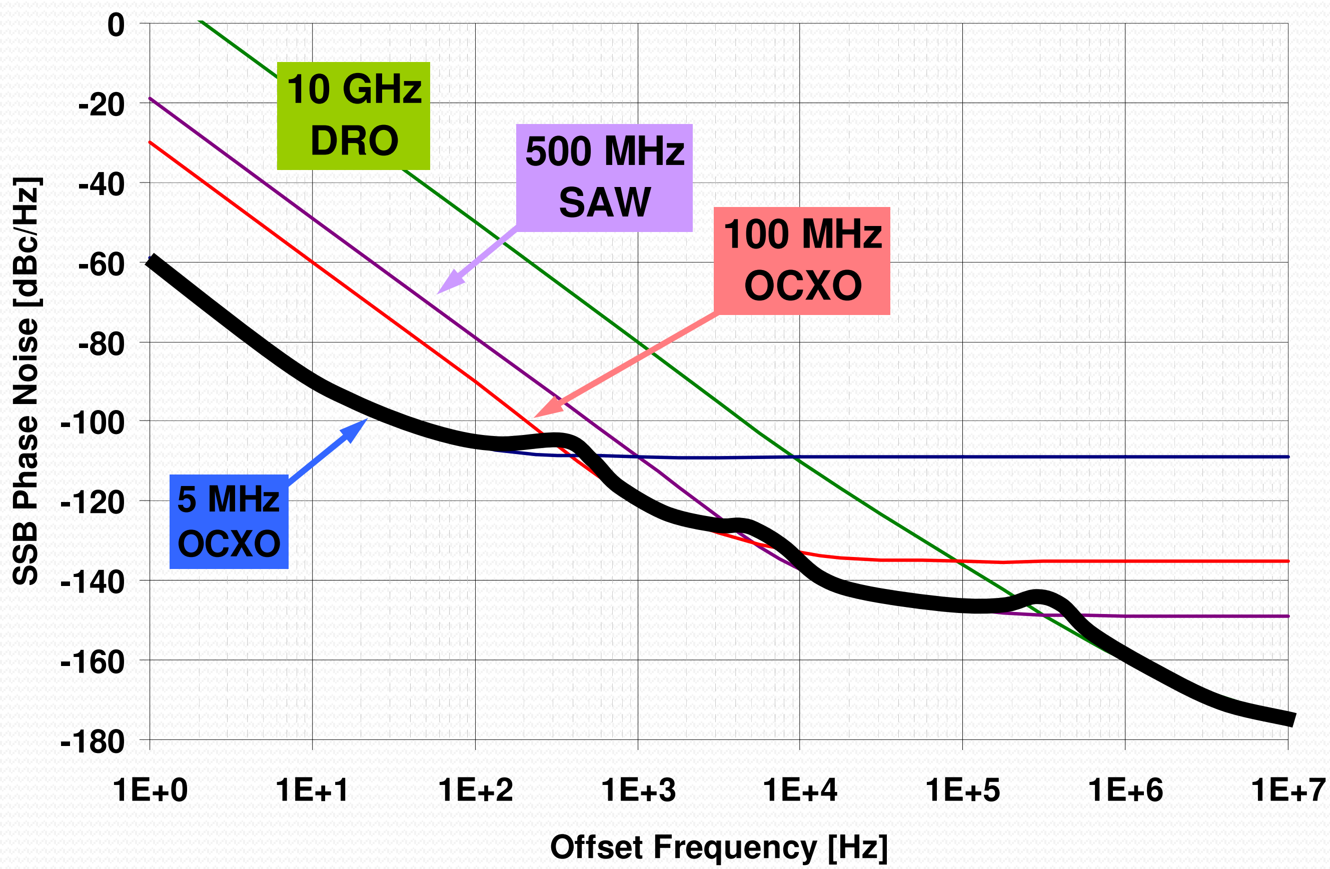
Four Sources at Different Frequencies and Similar Power



All Sources Normalized to 10 GHz

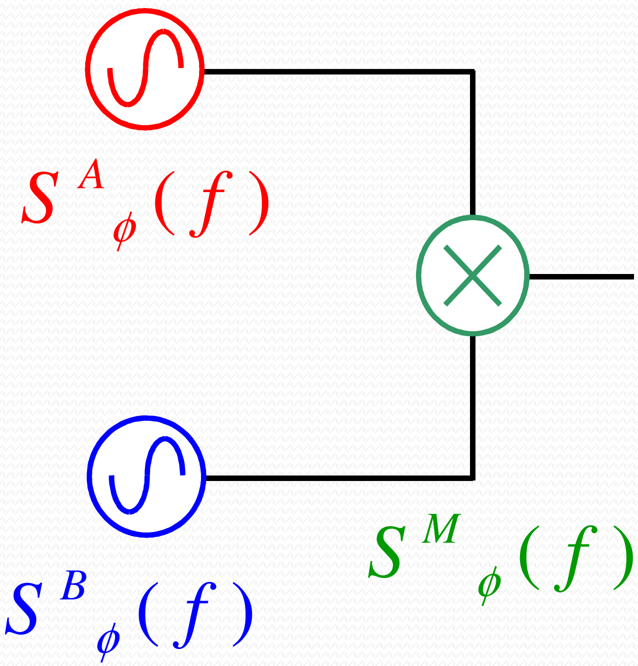


Composite Phase Noise Synthesis



Effects of Frequency Manipulation on Phase Noise

Translation or Mixing



$$S^A_\phi(f) + S^B_\phi(f) + S^M_\phi(f)$$

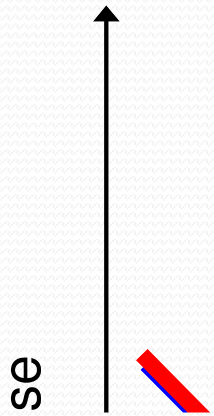
- Residual Noise
- AM to PM Conversion
- Noise Folding

Frequency Multiplication

$$v_2(t) = \cos[N(\omega t + \varphi(t)]$$

$$\omega_2 = N\omega$$

$$S_{\phi 2}(f) = \frac{2}{T}\left|N\Phi_{1,T}(f)\right|^2 = N^2 S_\phi(f)$$



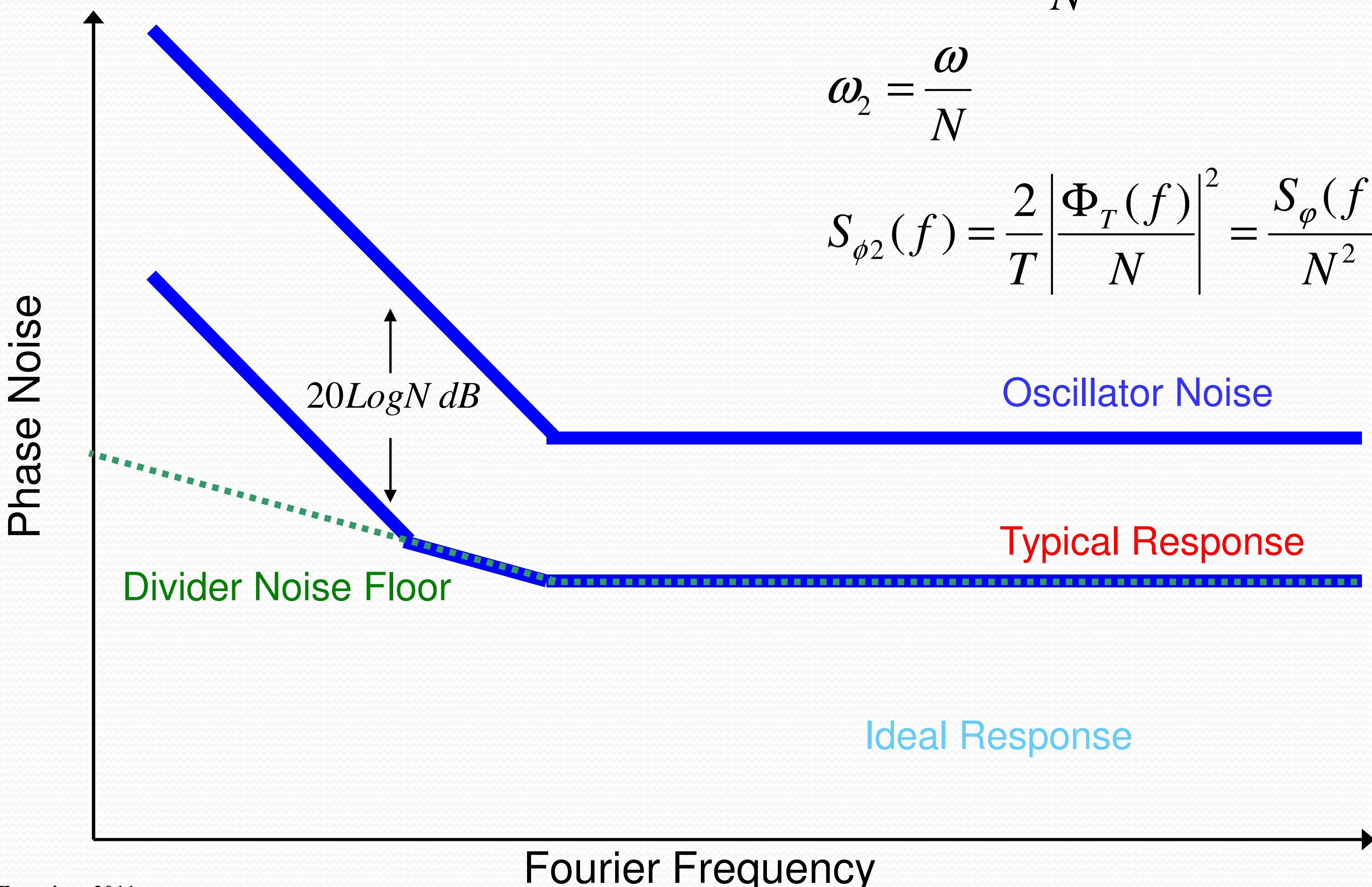
Fourier Frequency

Frequency Division

$$v_2(t) = \cos[\frac{1}{N}(\omega t + \varphi(t))]$$

$$\omega_2 = \frac{\omega}{N}$$

$$S_{\phi 2}(f) = \frac{2}{T}\left|\frac{\Phi_T(f)}{N}\right|^2 = \frac{S_\varphi(f)}{N^2}$$



Extracting Noise

Two noisy signals

$$V_1(t) = A_1(1+\alpha_1(t))\cos(\omega_1 t + \varphi_1(t))$$

$$V_2(t) = A_2(1+\alpha_2(t))\cos(\omega_2 t + \varphi_2(t))$$

Multiply them together

$$V_1(t) \bullet V_2(t) = \left[\frac{A_1 A_2 (1 + \alpha_1(t) + \alpha_2(t) + \alpha_1\alpha_2(t))}{2} \right] \bullet$$

$$\left\{ \cos\left[(\omega_1 + \omega_2)t + \varphi_1(t) + \varphi_2(t) \right] + \cos\left[(\omega_1 - \omega_2)t + \varphi_1(t) - \varphi_2(t) \right] \right\}$$

Phase Noise

Set ω = ω$_1$ = ω$_2$

$$V_1(t)\bullet V_2(t)=\left[\frac{A_1A_2(1+\alpha_1(t)+\alpha_2(t)+\alpha_1\alpha_2(t))}{2}\right]\bullet\left\{\cos\left[\varphi_1(t)-\varphi_2(t)+\phi_3\right]+\cos\left[(\qquad+\varphi_1(t)-\varphi_2(t)\right]\right\}$$

$$\left[\frac{A_1A_2(1+\alpha_1(t)+\alpha_2(t)+\alpha_1\alpha_2(t))}{2}\right]$$

When $\phi_3 = 90°$ $\qquad \sin x \cong x, \quad \cos(x+90)\cong x$

$$\left[\frac{A_1A_2(1+\alpha_1(t)+\alpha_2(t)+\alpha_1\alpha_2(t))}{2}\right]\cos\left[\phi_1(t)-\phi_2(t)+90\right]$$

$$\left[\frac{A_1A_2}{2}\right]\left[\phi_1(t)-\phi_2(t)\right]$$

$$h_{LPF} * \left[V_1(t)\bullet V_2(t)\right]\cong k_d\left[\varphi_1(t)-\varphi_2(t)\right]\quad when\quad \omega_1=\omega_2\ \wedge\ \phi_3=90°$$

Amplitude Noise

$$\omega_1 = \omega_2$$

$$V_1(t) = V_2(t) = A_1[1+\alpha_1(t)]\cos[\omega_1 t + \varphi_1(t)] = A_2[1+\alpha_2(t)]\cos[\omega_2 t + \varphi_2(t)]$$

$$\left[\frac{A^2(1+2\alpha(t)+\alpha^2(t))}{2}\right]\cos[\phi_3]$$

When $\phi_3 = 0°$ **and cos(0) = 1**

Remove DC term and neglect higher order term

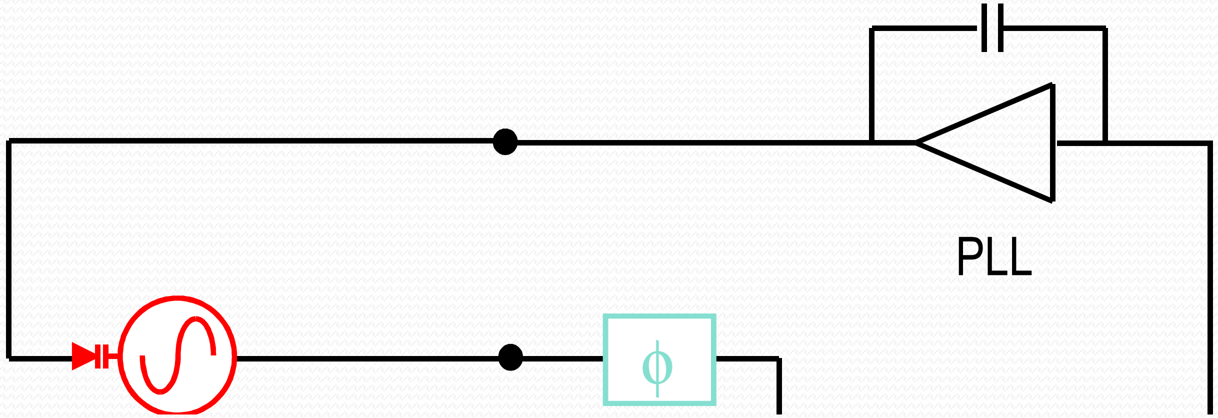
$$\left[\cancel{\frac{A^2}{2}} + A^2\alpha(t) + \cancel{\frac{A^2\alpha^2}{2}}(t)\right]$$

$$h_{LPF} * \left[V^2(t)\right] \cong \frac{A^2}{2} + k_a\alpha(t) \quad when \quad \phi_3 = 0°$$

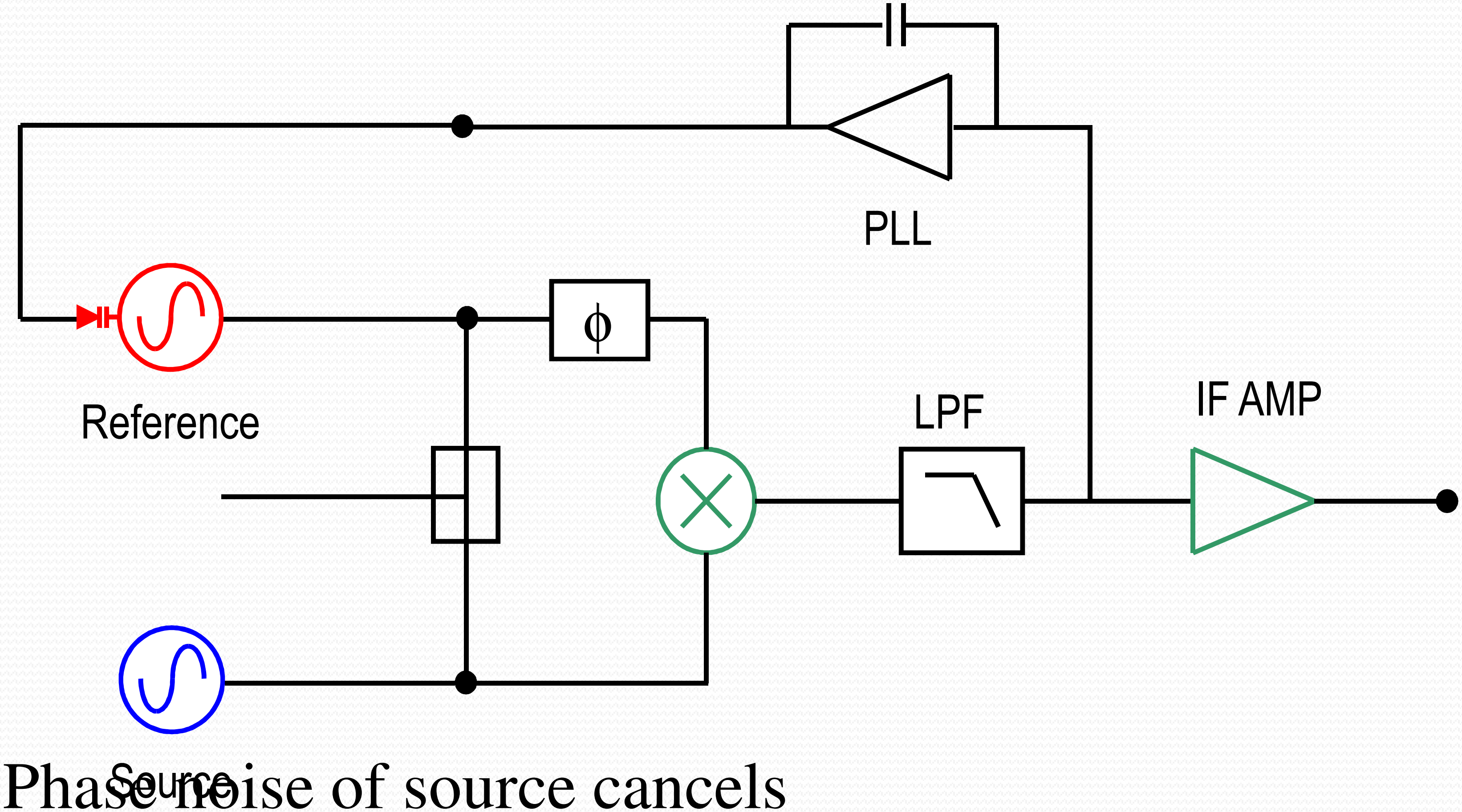
Phase Noise Measurement Types

- Homedyne or Residual – Single source
  - Noise Floor
  - Any two port device
- Heterodyne - Two source measurement
- Frequency Discriminator
  - Delay Line
  - Cavity Resonator
- Digital Measurement Systems

Two Oscillator Measurement



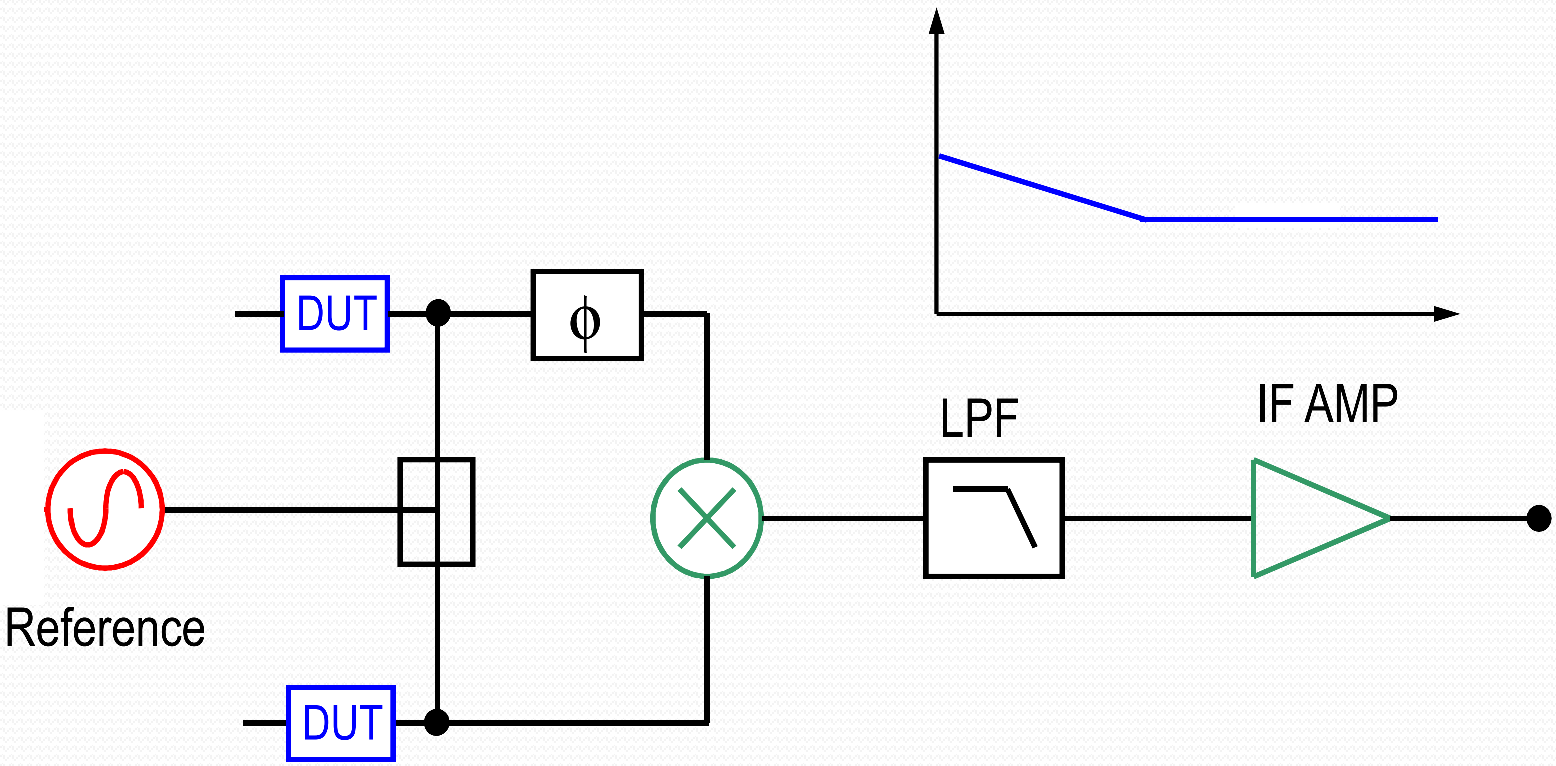
Noise Floor Measurement



- Phase noise of source cancels

$$S_{\phi_{NoiseFloor}}(f) = \cancelto{0}{\frac{[\Delta\phi_R]^2(f) - [\Delta\phi_R]^2(f)}{K_d^{\,2}(f)BW}} + \frac{PSD[V_{rms_{system}}(f)]}{K_d^{\,2}(f)}$$

Two Port Device Measurements



- Phase noise of source cancels
- Two devices are needed if they have a long delay
- Two devices are needed if they change the frequency

System Calibration

- Static Phase shift (PM)
- Kd or Beat Frequency Method (PM)
- Modulation
  - Single Sideband (AM/PM)
  - Phase Modulation (PM)
  - Frequency Modulation (PM)
  - Amplitude Modulation (AM)
- Noise Standards (AM/PM)

Static Phase Shift

- **A known phase shift is introduced, and the corresponding voltage change measured**

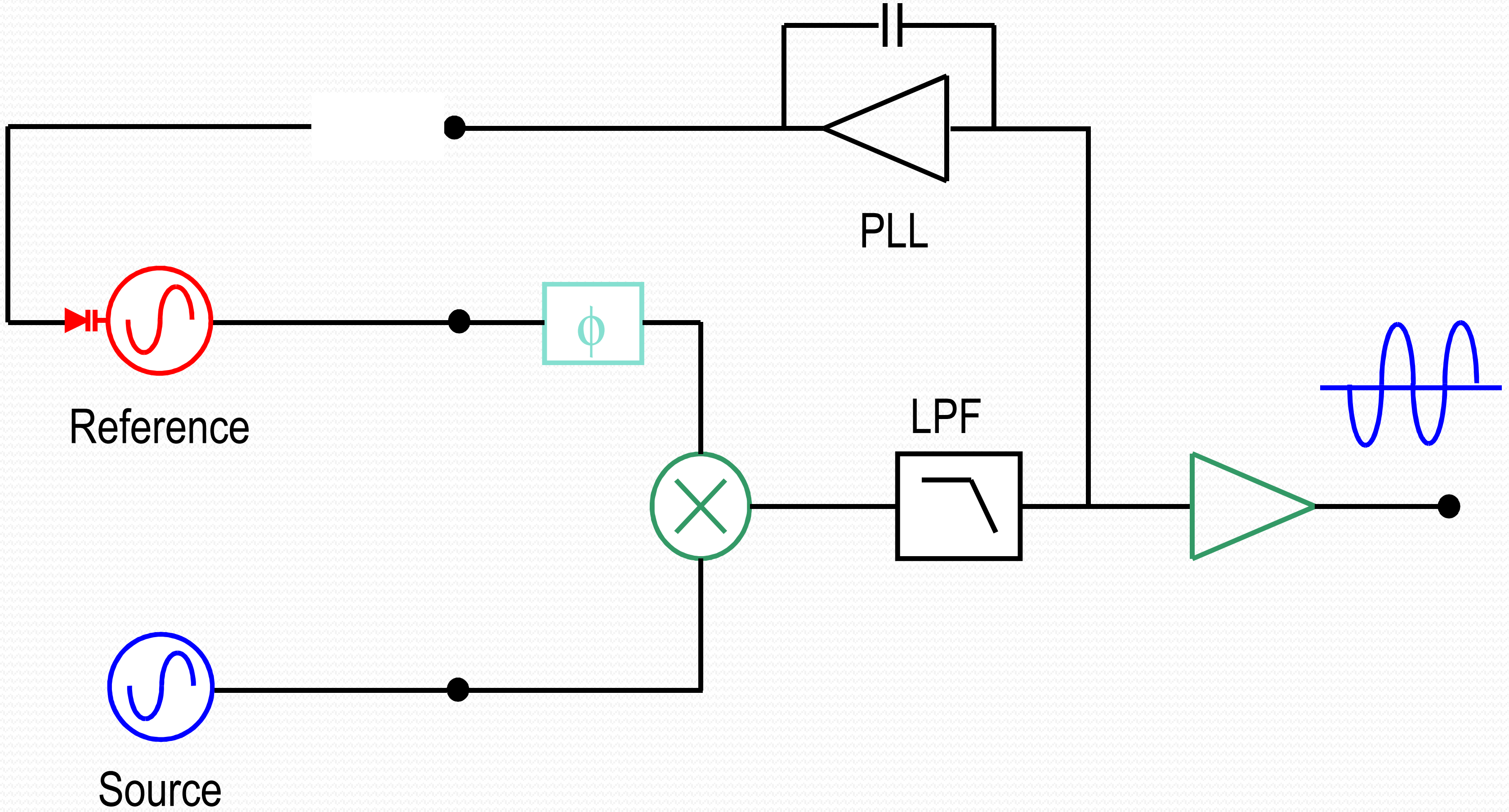
- **Adjustable phase shifter (mechanical or electrical)**
  - **Switched delay lines**
  - **Programmable phase shift in a synthesizer**

$$K_d = \frac{Voltage\ Change}{Phase\ Shift}\left[\frac{V}{rad}\right] \qquad S_\phi(f) = \frac{PSD[V_{rms}(f)]}{K_d^{\,2}}$$

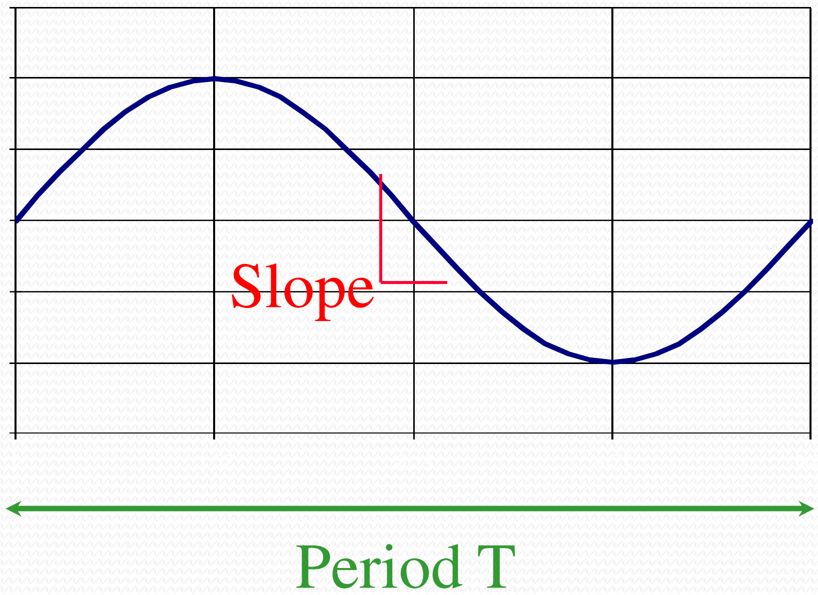
Remember:

$$\mathcal{L}(f) = \tfrac{1}{2} S_\phi(f) \quad [dBc/Hz]$$

Two Oscillator Beat Frequency Calibration



Calculating Mixer Sensitivity K_d

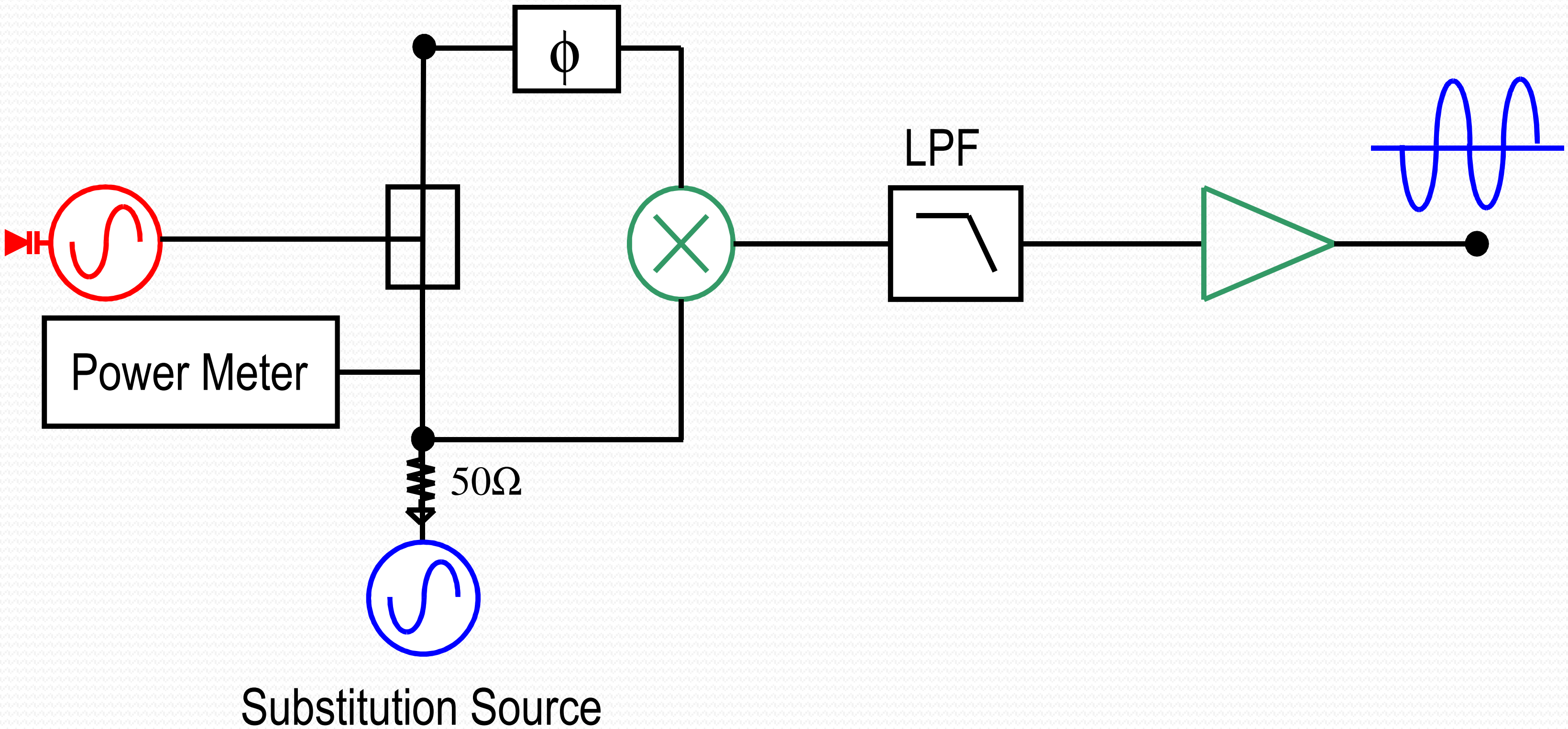


$$K_d(v/rad) = \frac{Slope(v/s) \cdot T(s)}{2\pi(rad)}$$

Typically 0.3 to 0.4 V/rad for a saturated double balanced Schottky diode mixer

- Make sure both positive and negative slopes are equal in magnitude and symmetric

Single Source Kd Calibration



Mixer Sensitivity

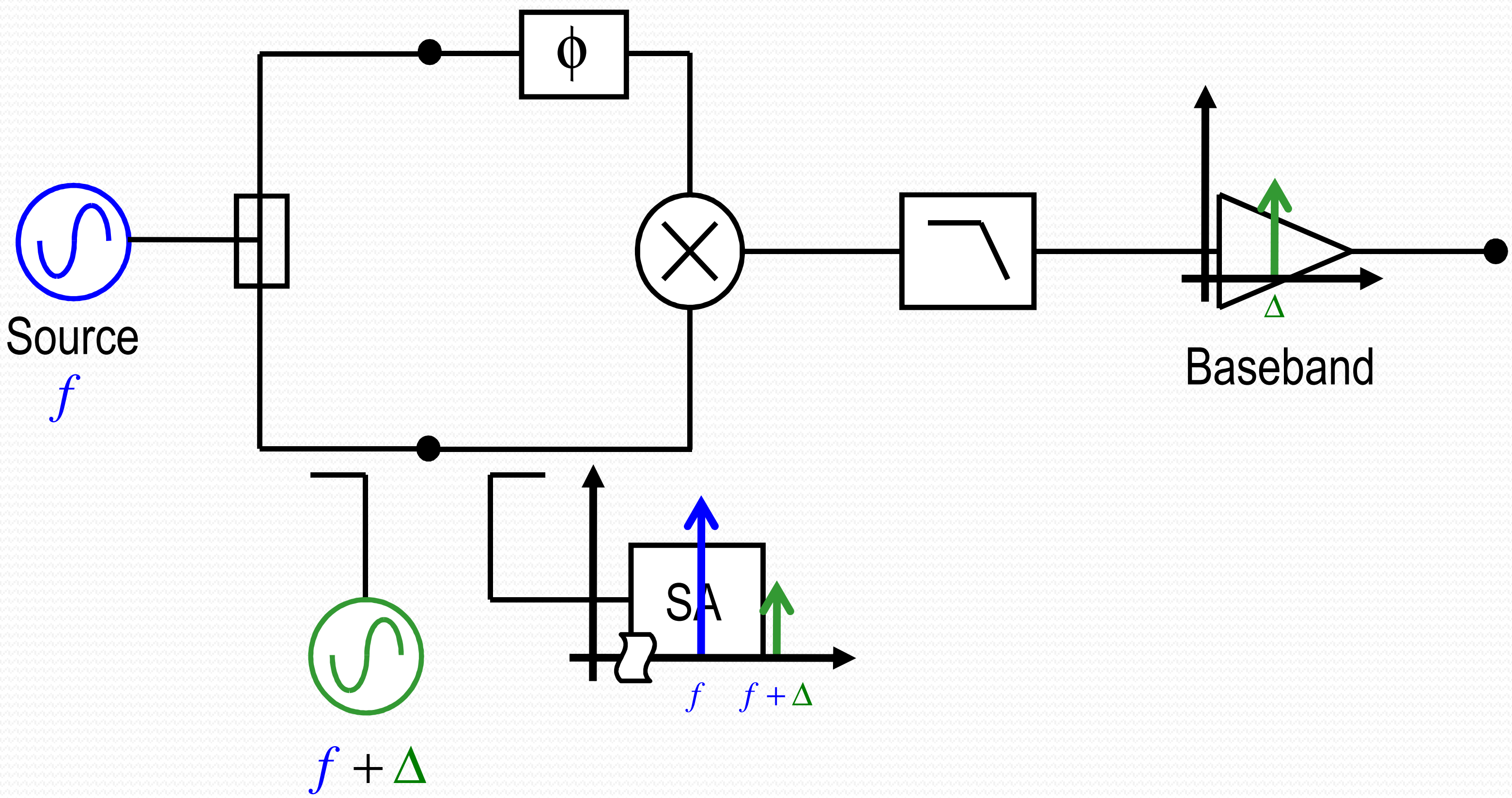
- Frequency
- RF and LO power
- Mixer termination at all three ports
- Cable lengths

Calibration conditions must replicate the measurement conditions as closely as possible

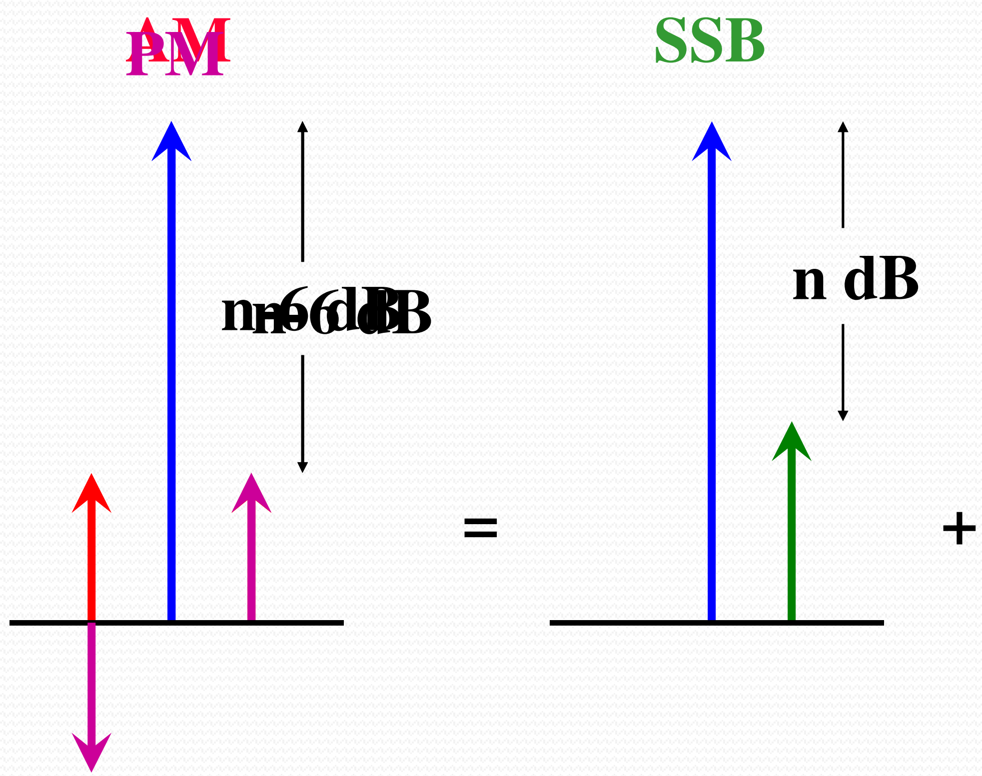
Possible Errors Using K_d Calibration

- Determines gain only at a single frequency
- Noise is suppressed inside PLL bandwidth
- Requires open loop PLL or substitution source
- Beat configuration does NOT match actual measurement configuration
- Injection Locking

SSB Calibration Method



Single Sideband Modulation



Generates equal amounts of AM and PM Noise
Phase and amplitude modulation is
detected at ½ the SSB ratio

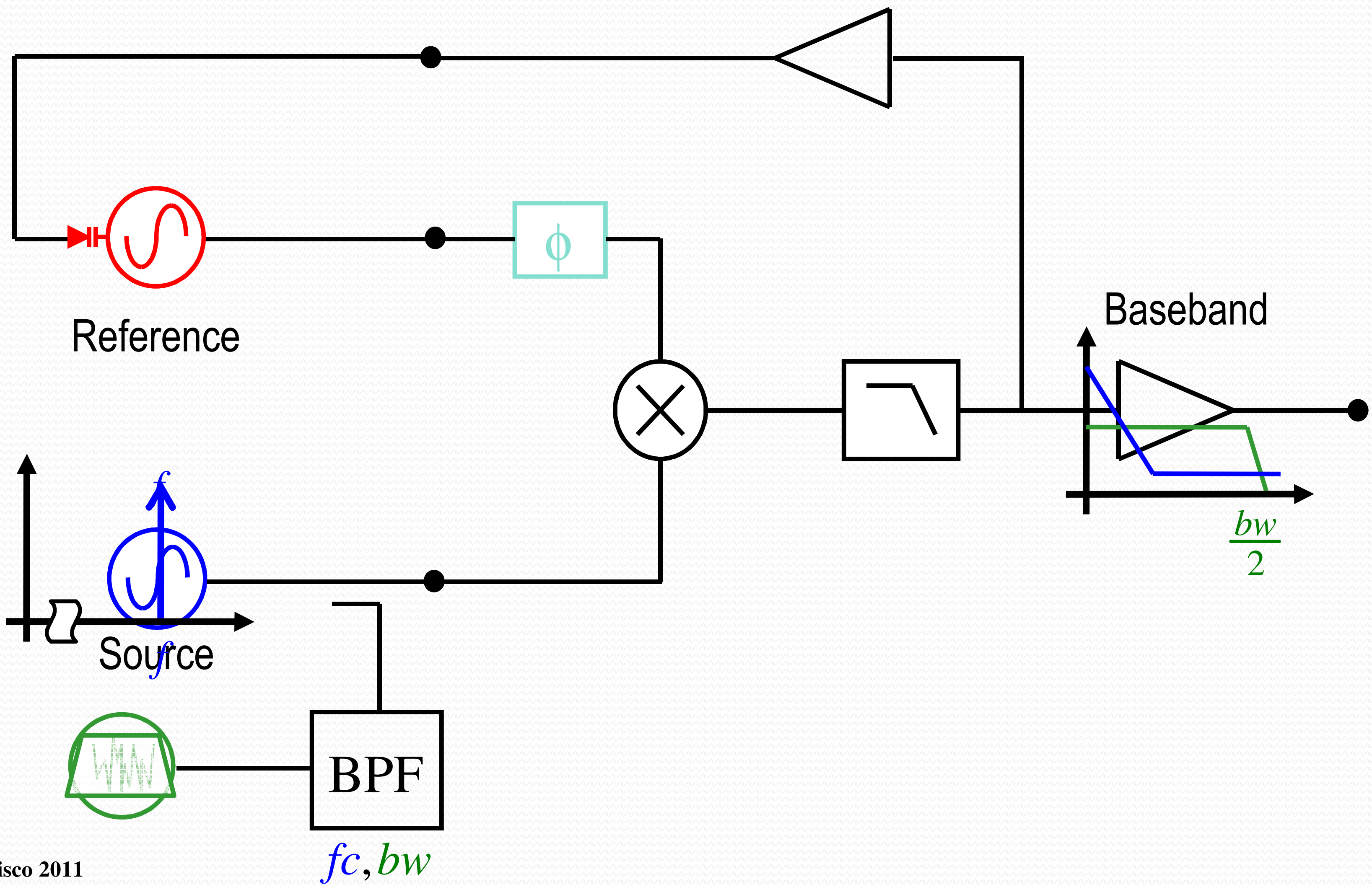
$$S_\varphi(f_c) \cong S_\alpha(f_c) \cong \frac{P_{\nu_0+f_c}}{2P_{\nu_0}} \cong \frac{P_{\nu_0-f_c}}{2P_{\nu_0}}$$

A SSB tone at n dB below the carrier
creates phase and amplitude noise with

$$S_\varphi(f_{cal}) = n - 3dB \quad or \quad \mathcal{L}(f_{cal}) = n - 6dB$$

$$S_\alpha(f_{cal}) = n - 3dB$$

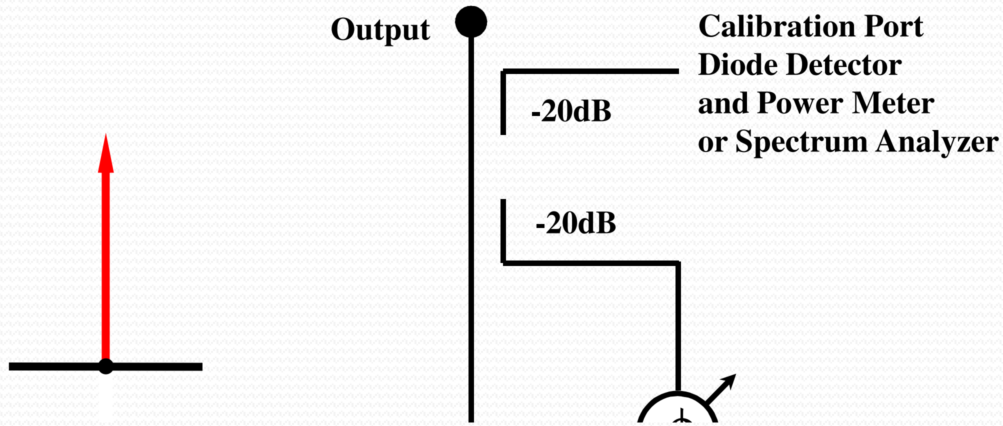
Additive Noise Calibration Method



AM/PM Modulator

- Can be adjusted for pure PM or AM modulation
- Extremely flat frequency response
- Calibrates $K_d(f)$ with system locked
- Can be used to find true quadrature for minimizing AM leakage

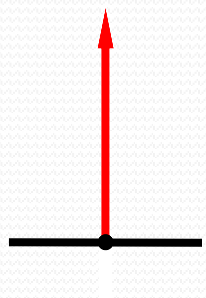
AM Modulator

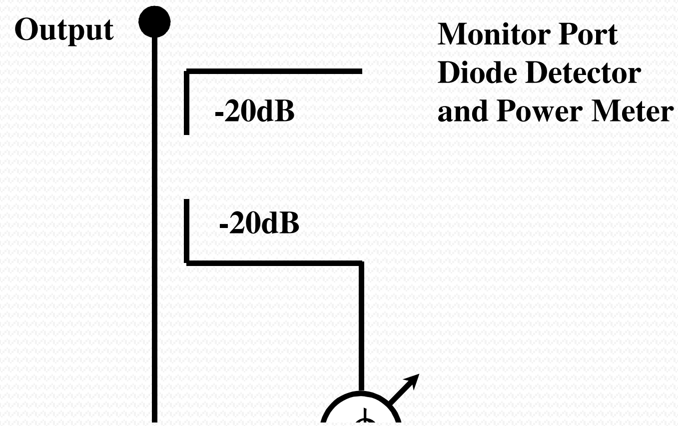


MAXIMIZE detected AM signal
By adjusting phase shifter

$$S_\alpha(f_c) \cong \frac{P_{\nu_0 - f_c} + P_{\nu_0 + f_c}}{P_{\nu_0}}$$

PM Modulator





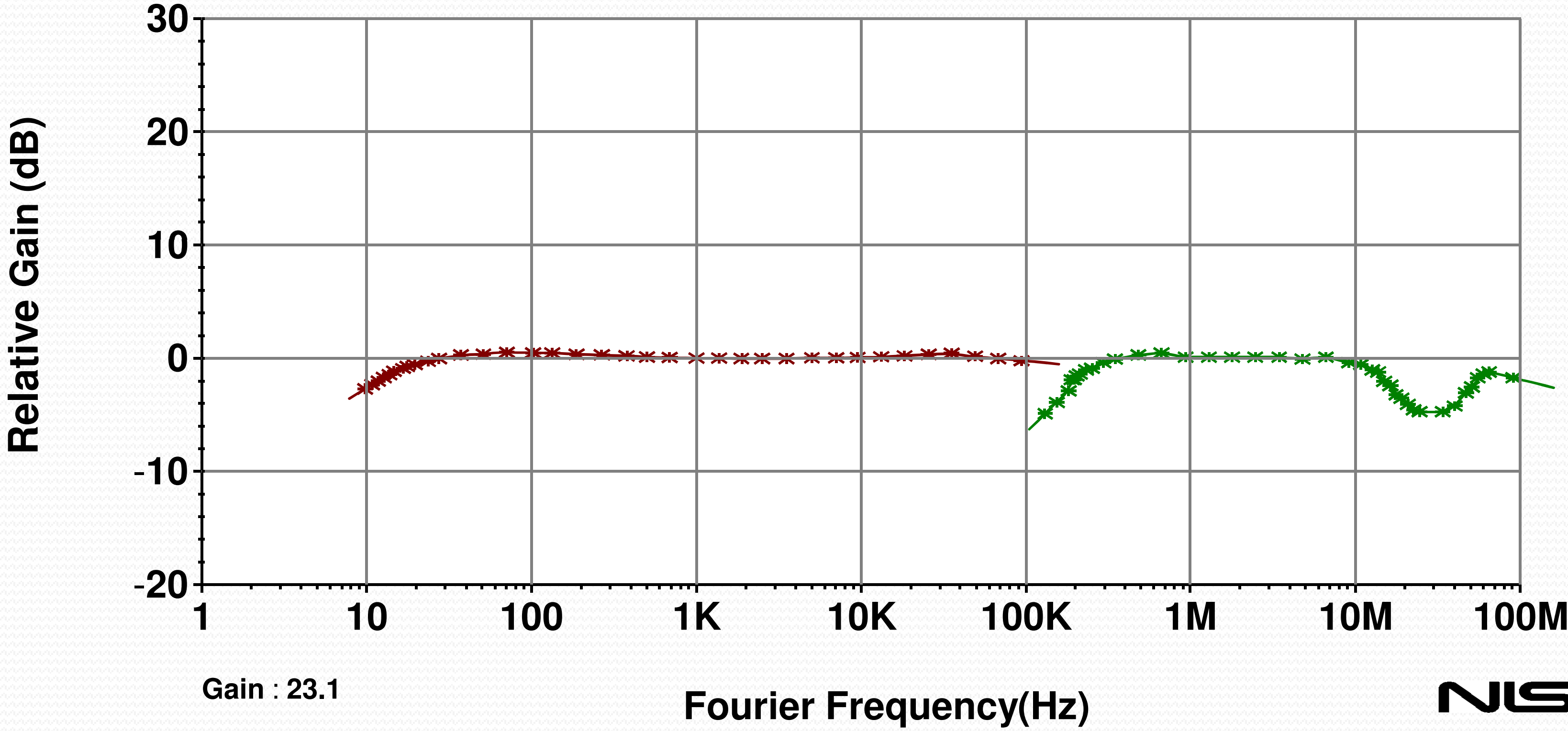
MINIMIZE detected AM signal
By adjusting phase shifter

$$S_\varphi(f_c) \cong \frac{P_{\nu_0 - f_c} + P_{\nu_0 + f_c}}{P_{\nu_0}}$$

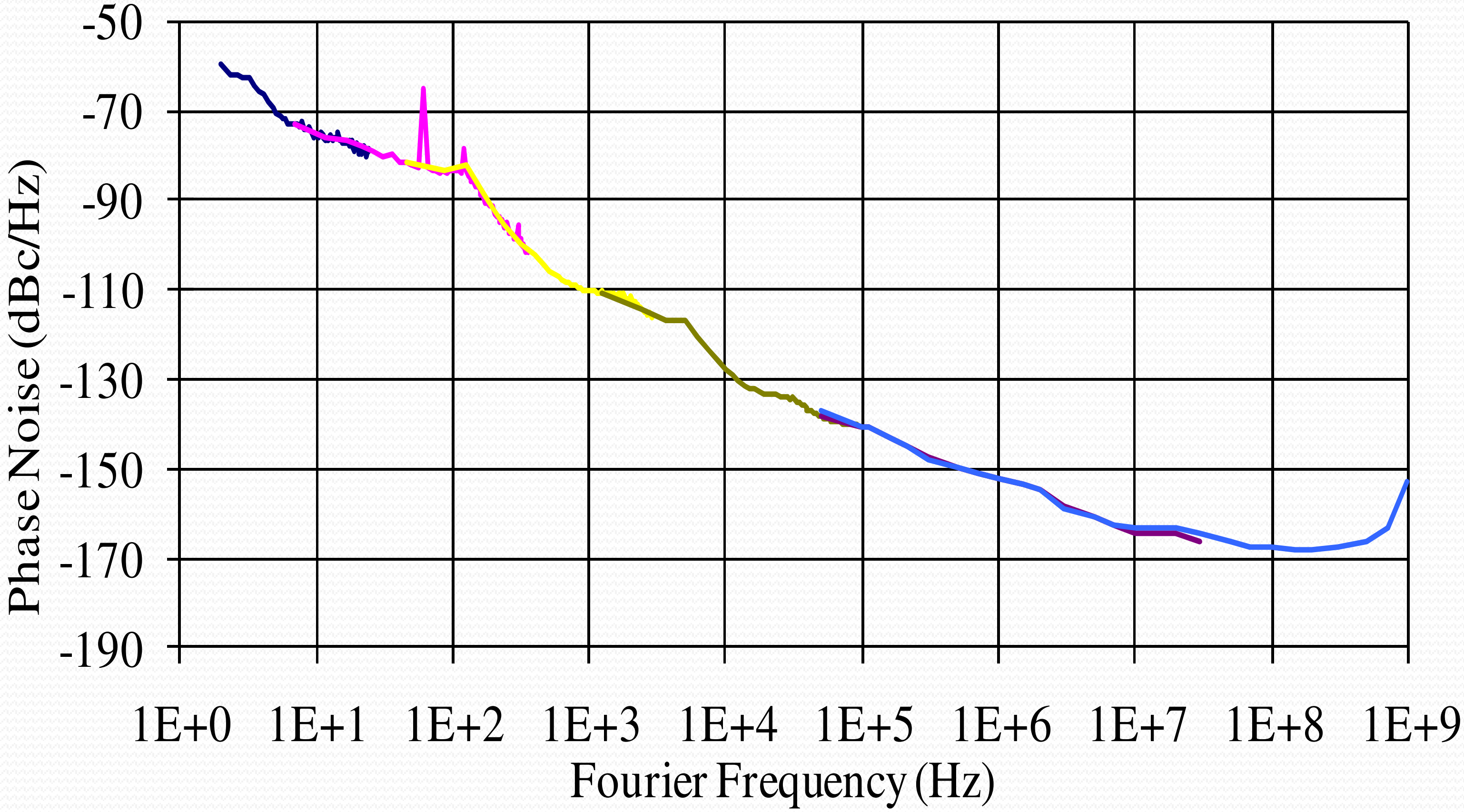
Tips for Measuring Gain vs. Fourier Frequency using Swept Modulation

- Measure power spectrum not PSD
- Use flattop windows for FFT
- Only small number of averages required
- 3-5 points per decade
- Create gain curve with cubic spline or linear curve
- Make sure tone does not saturate IF amplifiers

Calibration Curve at X-band



Phase Noise of X-band Synthesizer



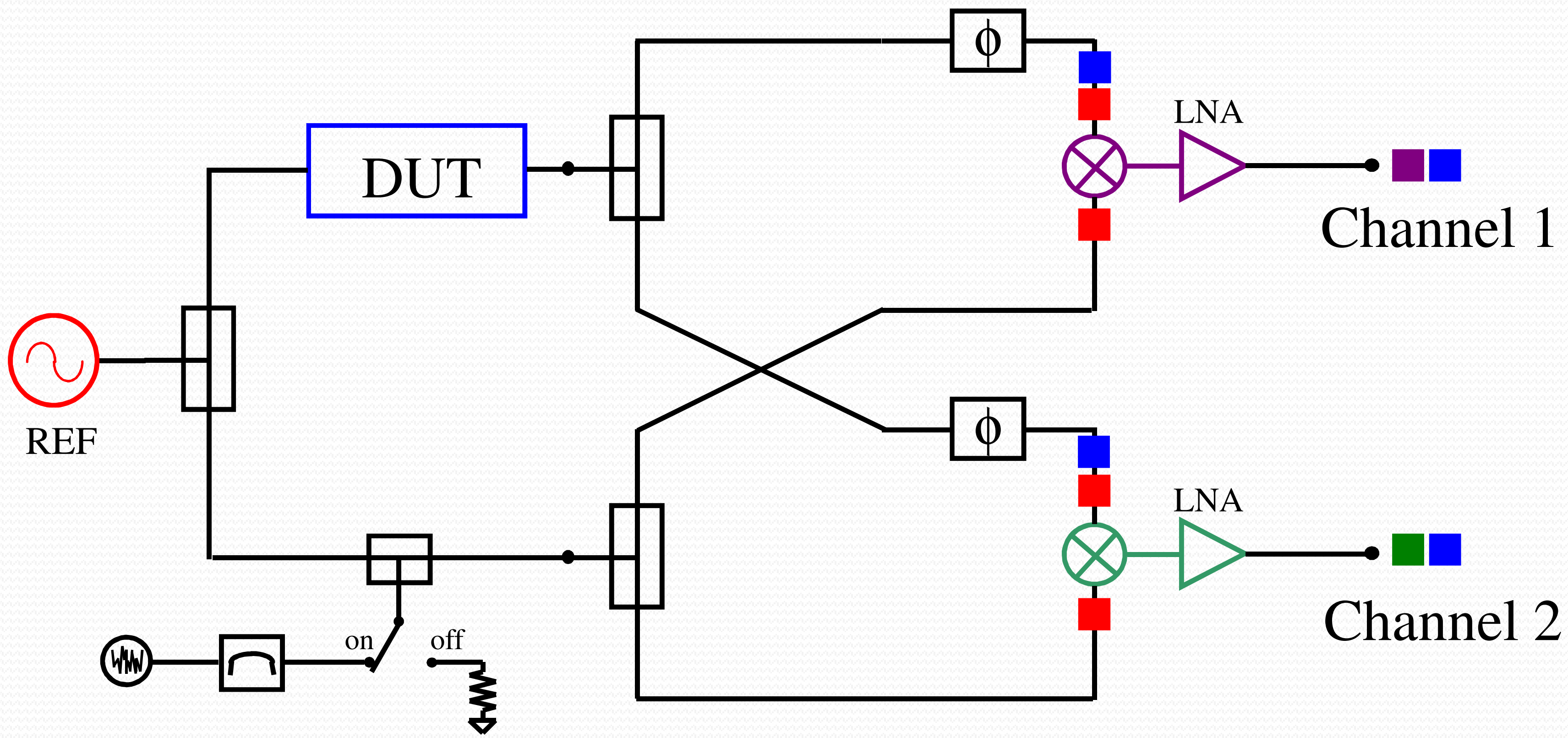
Tips for Measuring Noise

- Measure Power Spectral Density in Vrms/√Hz
- Use Hanning window
- Confidence interval depends on number of averages
- Confidence interval depends also on resolution and video bandwidth for swept analyzers
- Measure system noise floor.

Noise Floor Reduction Techniques

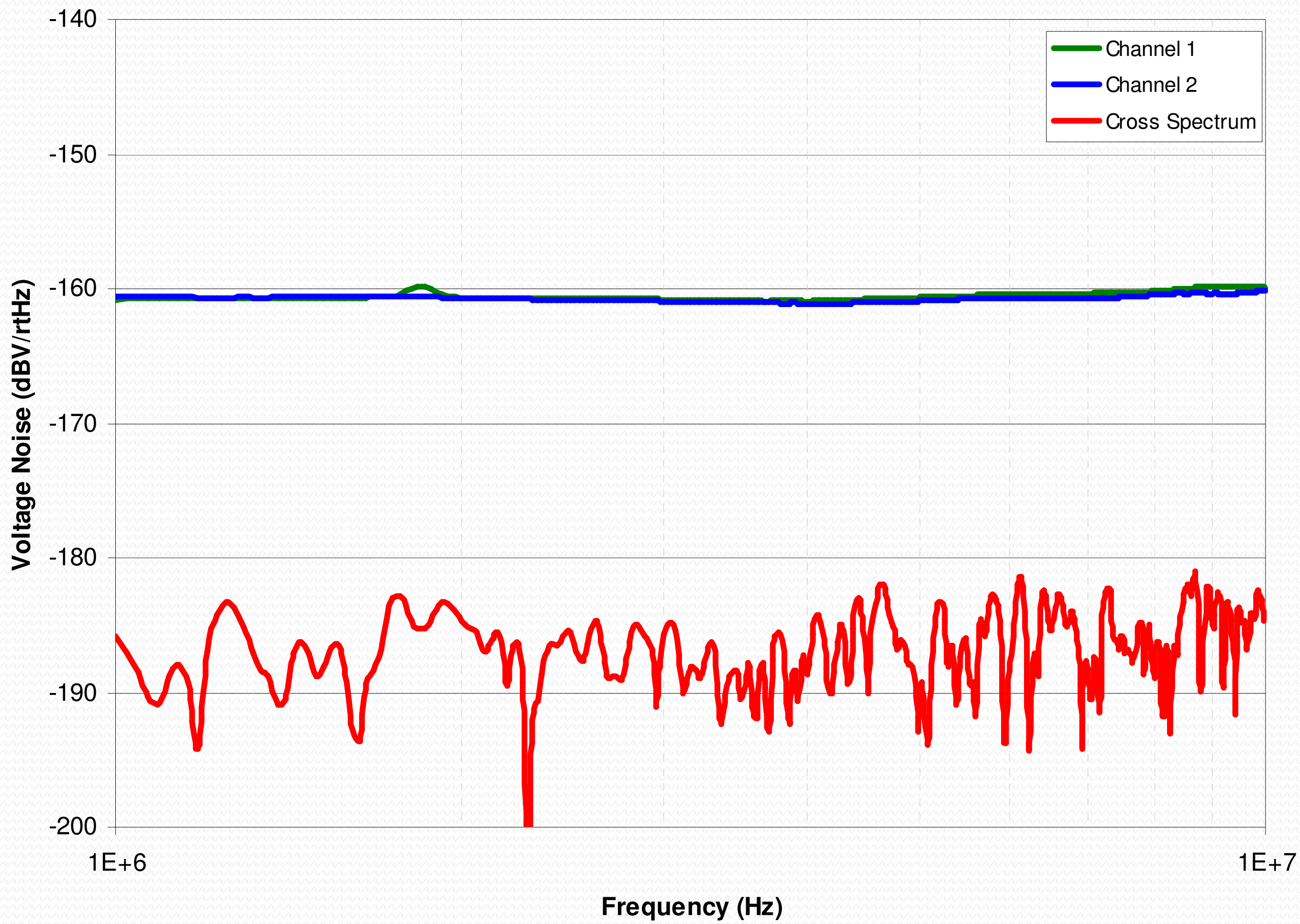
- Cross-Correlation
- Carrier Suppression

Cross-correlation PM Noise System for Two Port Measurements



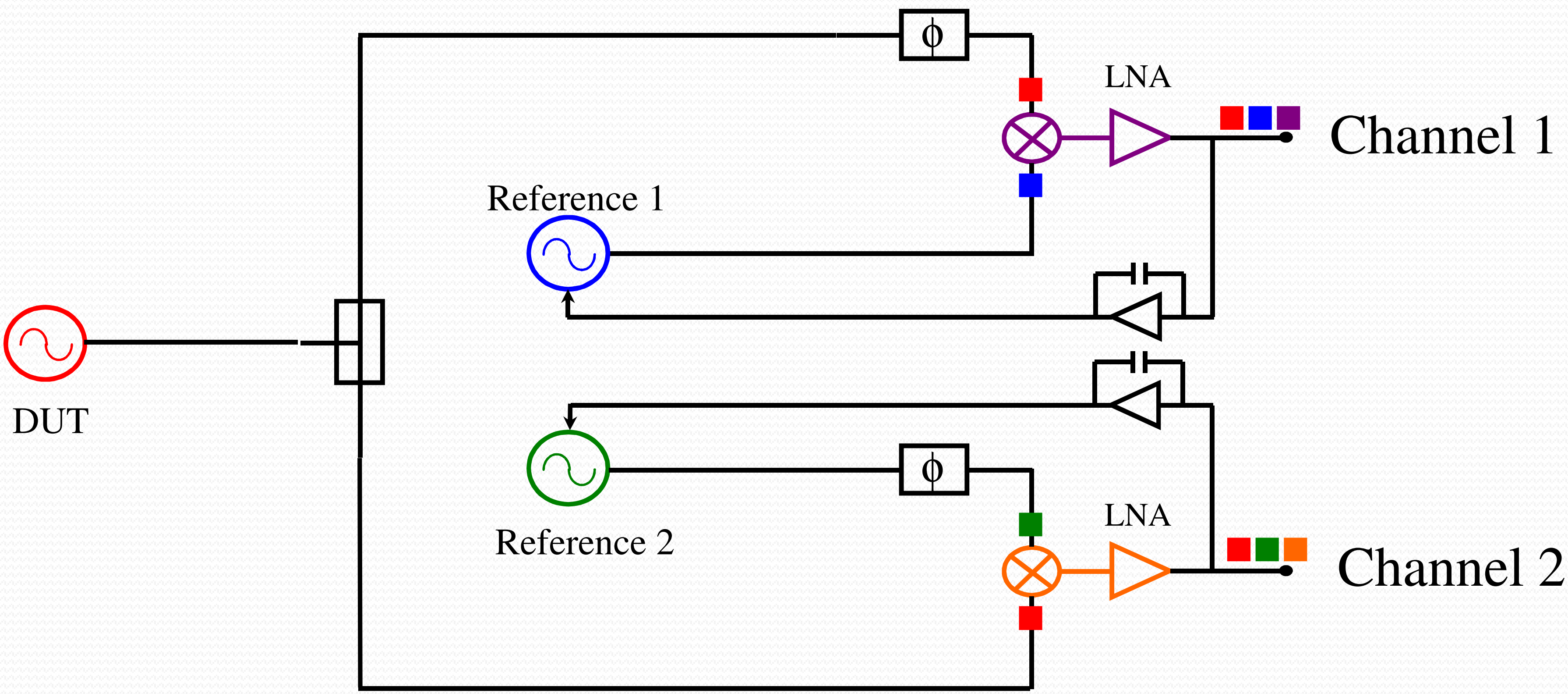
$$Noise_{Crosscorrelated} = Noise_{Correlated_{1,2}} + \frac{Noise_{Uncorrelated_1} + Noise_{Uncorrelated_2}}{\sqrt{N_{Averages}}}$$

Correlated Noise Measurements



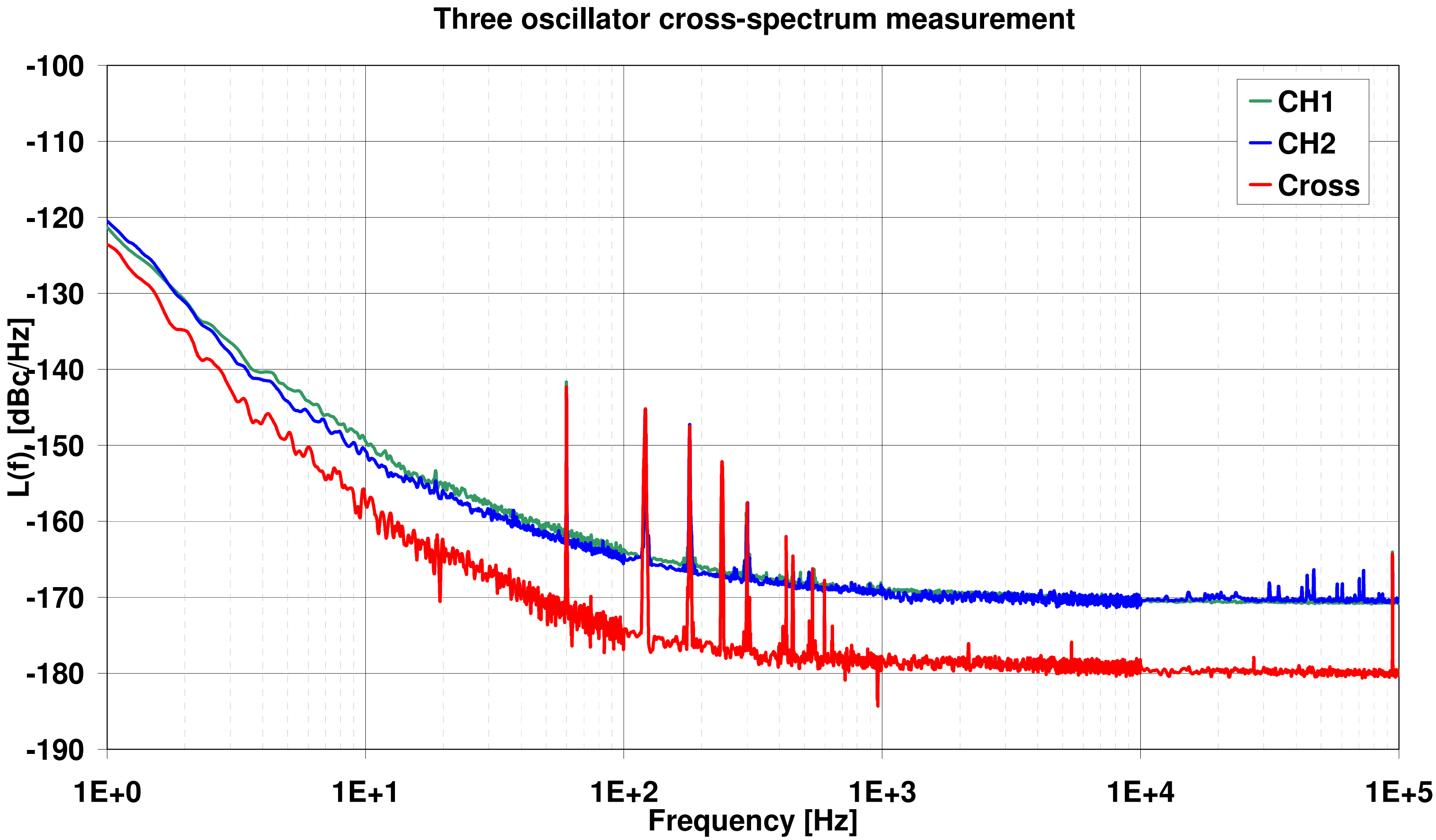
100,000 Ave : 35 min

Cross-correlation Oscillator PM Measurements

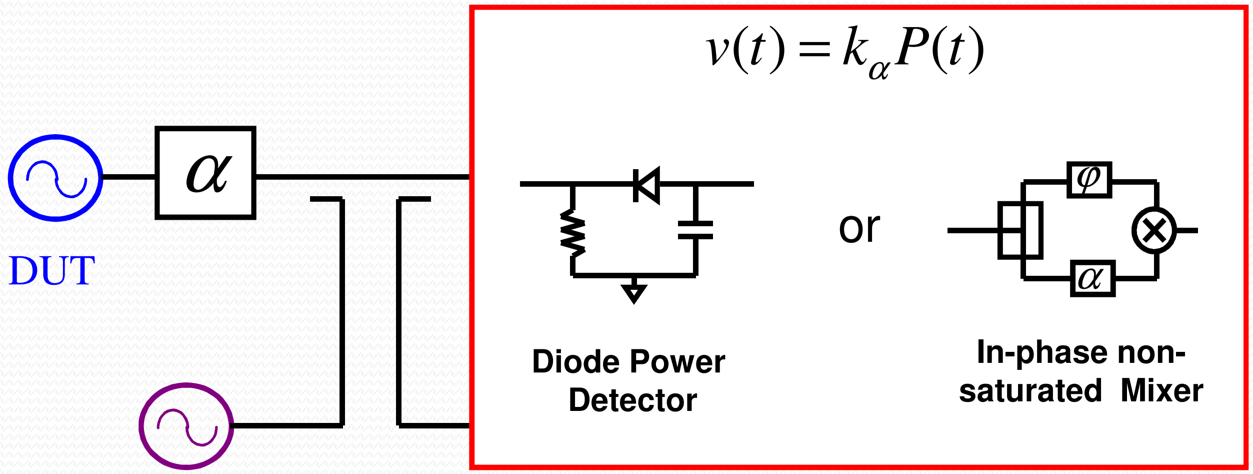


$$S_\phi(f)_{\mathrm{Cross}_{1,2}} = S_\phi(f)_{\mathrm{DUT}} + \frac{S_\phi(f)_{\mathrm{Ref}_1} + S_\phi(f)_{\mathrm{Ref}_2} + S_\phi(f)_{\mathrm{System}_1} + S_\phi(f)_{\mathrm{System}_2}}{\sqrt{N_{\mathrm{Averages}}}}$$

Cross-correlation Oscillator PM Measurements



AM Measurements*



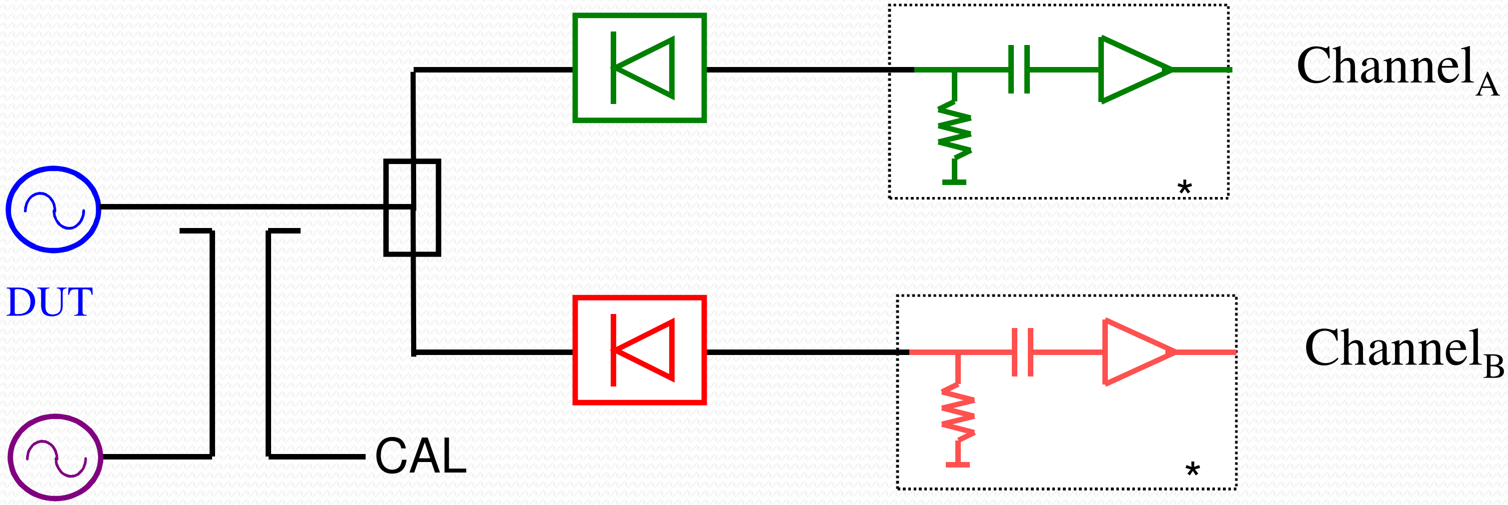
$$k_\alpha P_0 = \frac{\Delta v}{\Delta P / P_0}$$

$$v(t) = k_\alpha P_0 (1 + \alpha(t))^2 \cong k_\alpha P_0 (1 + 2\alpha(t) + \cancel{\alpha^2(t)})$$

$$S_\alpha(f) = \frac{S_{Vrms}(f)}{4 k_\alpha^2 P_0^2}$$

**The Measurement of AM noise of Oscillators – Rubiola2005*

Cross-correlation Source AM Measurements



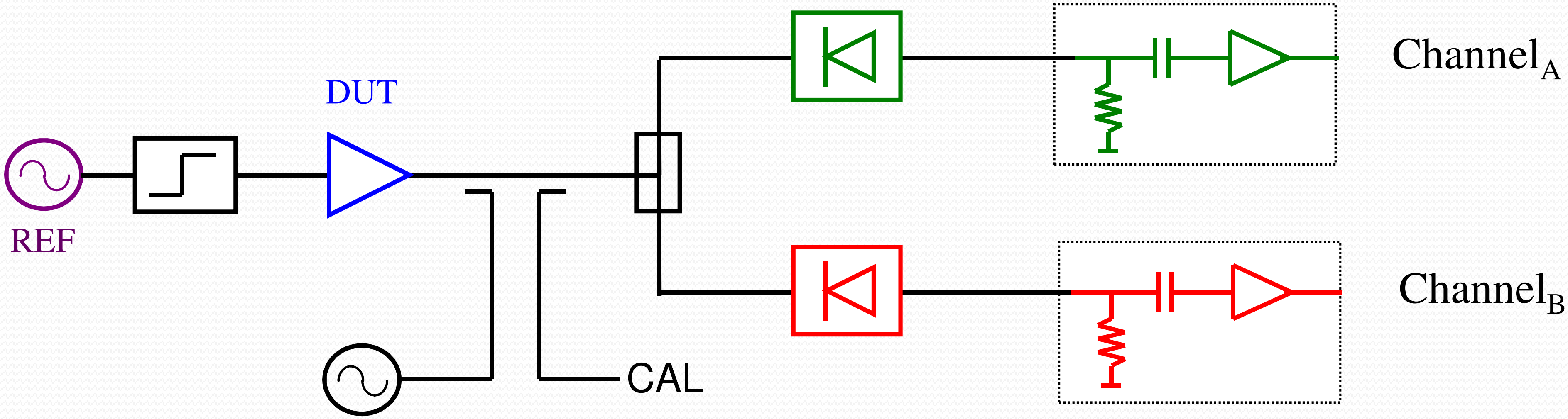
$$S_\alpha(f)_{CrossAB} = S_\alpha(f)_{\mathrm{DUT}} + \frac{S_{System_1}(f) + S_{System_2}(f)}{\sqrt{2N_{Ave}}}$$

$$S_\alpha(f)_{CrossAB} = \frac{S_{v_{AB}}(f)}{4k_{\alpha_A}^2 P_A^2 k_{\alpha_B}^2 P_B^2}$$

$$S_\alpha(f)_{CrossAB} = \frac{S_{v_{AB}}(f)}{K_a} \quad , \quad K_a = 4k_{\alpha_A}^2 P_A^2 k_{\alpha_B}^2 P_B^2$$

The Measurement of AM noise of Oscillators – Rubiola2005

Two-port AM Measurements



$$S_a(f)_{Cross_{1,2}} = S_\alpha(f)_{DUT} + S_\alpha(f)_{REF} + \frac{S_\alpha(f)_{System_A} + S_\alpha(f)_{System_B}}{\sqrt{2N_{Averages}}}$$

Reference AM noise must be less then DUT noise

Saturation can help in reducing AM Source noise

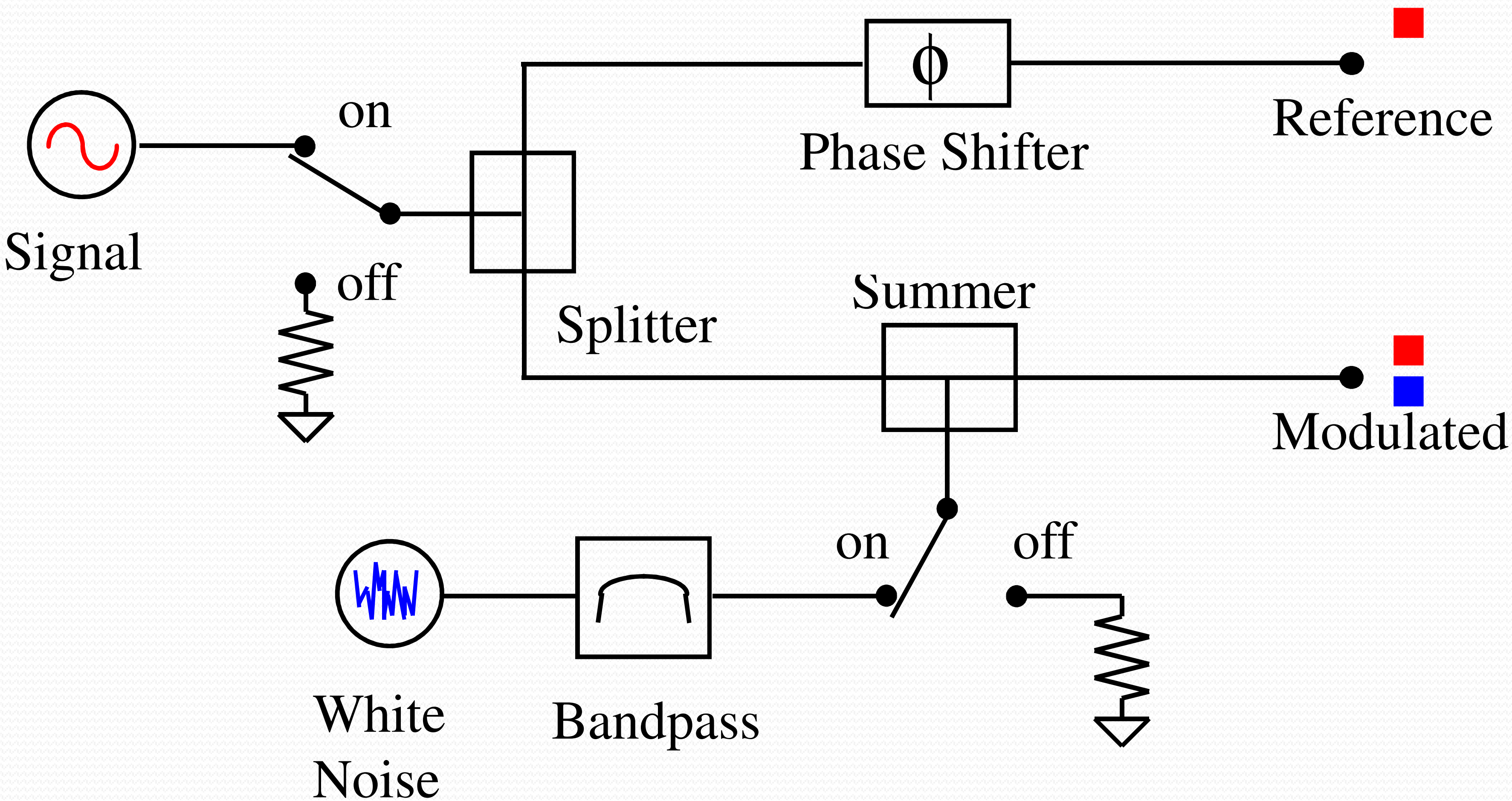
Digital Measurement Systems

- Carrier frequencies are sampled directly with A/D converters and phase information is extracted mathematically.
- No phase lock required
- Signals can be of different frequencies.
- Limited frequency range 1 - 400 MHz.
- Fourier Range of 1 mHz to 1 MHz.
- Limited sensitivity

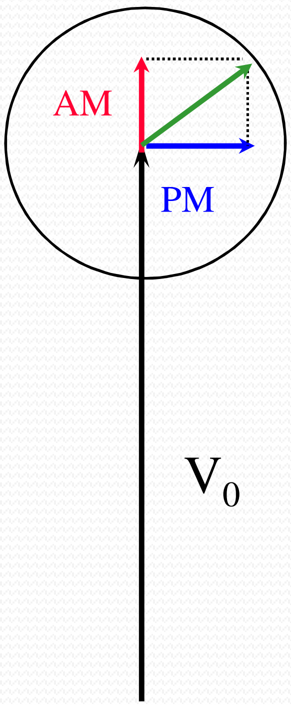
Integral PM and AM Noise Standards

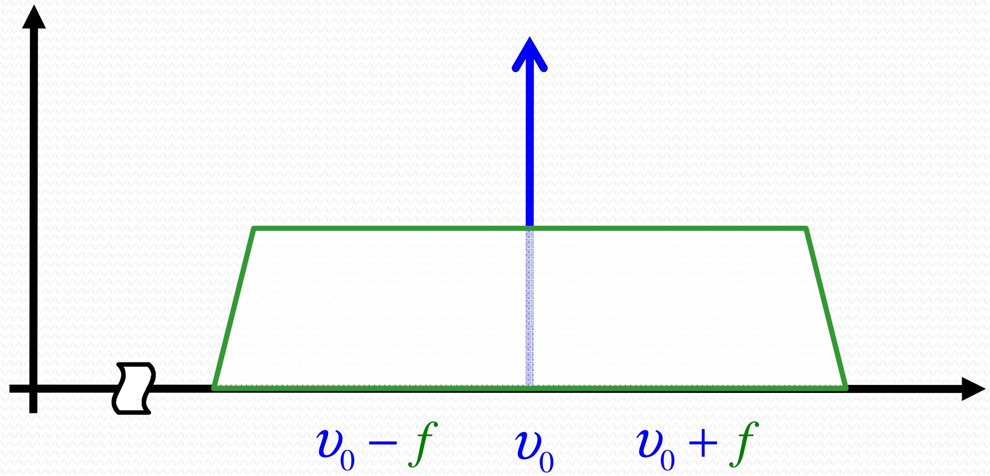
- Low noise signal source
- Two outputs with extremely low differential AM and PM noise
- Calibrated noise source
- Greatly simplifies AM and PM measurements
- Generates a calibrated level of equal AM and PM noise

Block Diagram of NIST PM/AM Noise Standard



Added Noise Appears as Equal Amounts of AM and PM



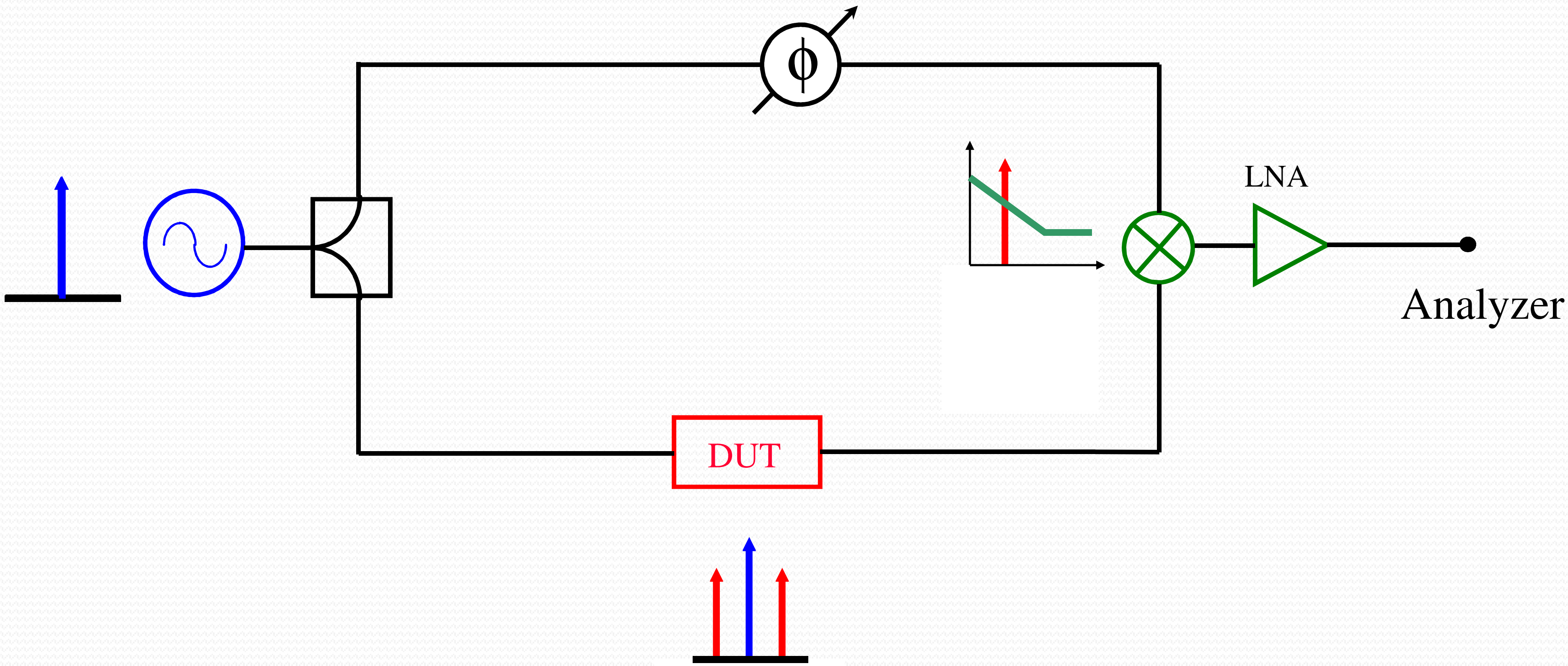


$$S_a(f) = S_\phi(f) = \frac{PSDV_n(\upsilon_0 - f) + PSDV_n(\upsilon_0 + f)}{2V_0^2}$$

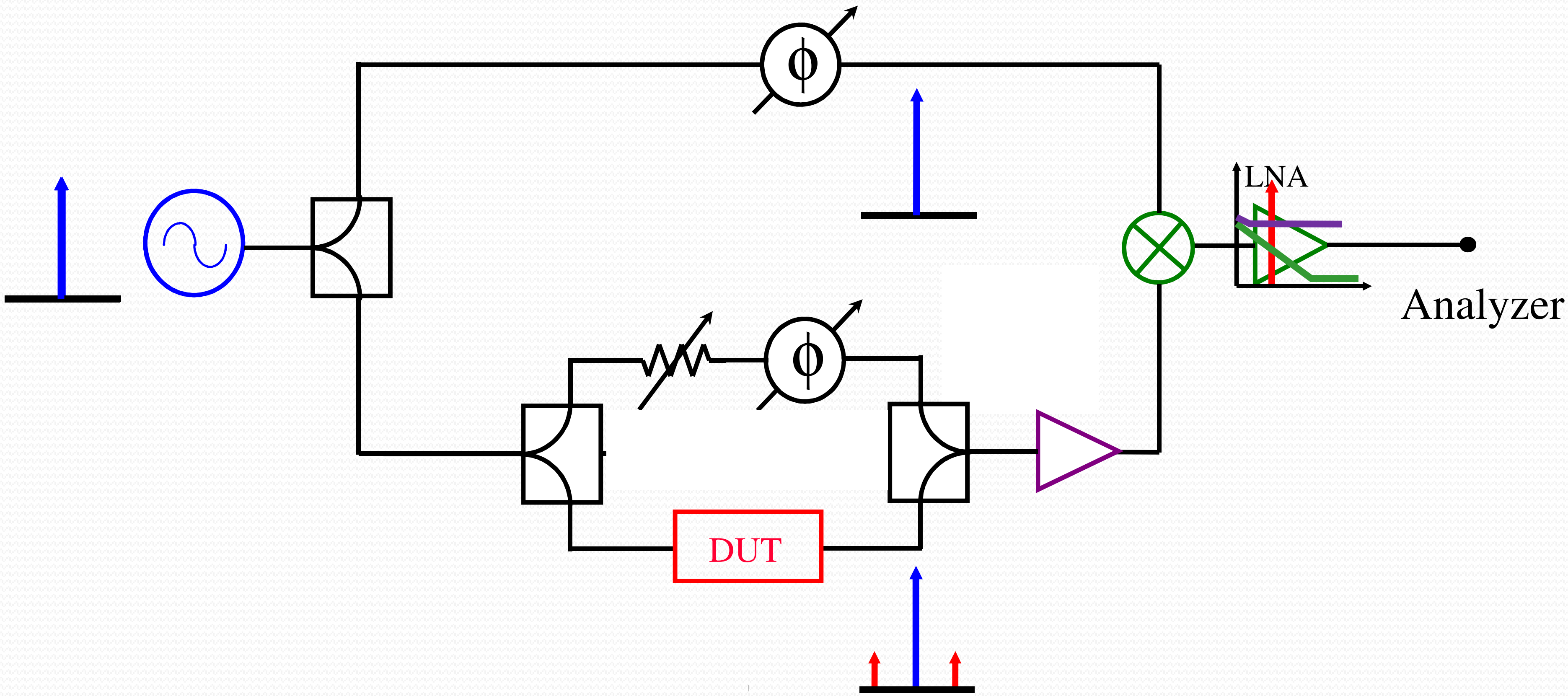
While

$$\int_0^\infty S_\phi(f) << 0.1$$

Basic Carrier Suppression Technique (Interferometric)

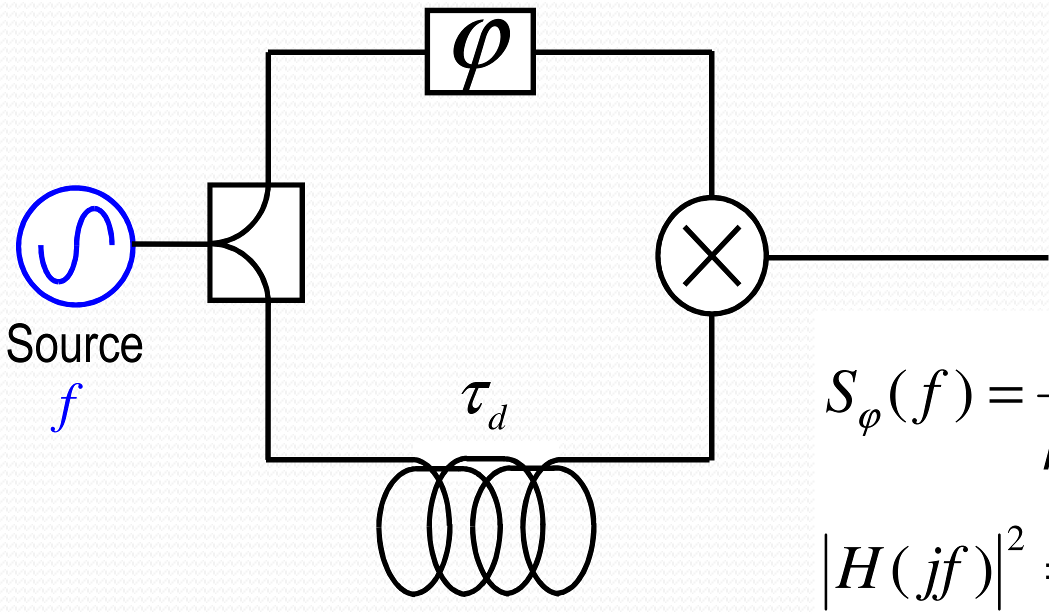


Basic Carrier Suppression Technique (Interferometric)



Delay Line Measurement Systems

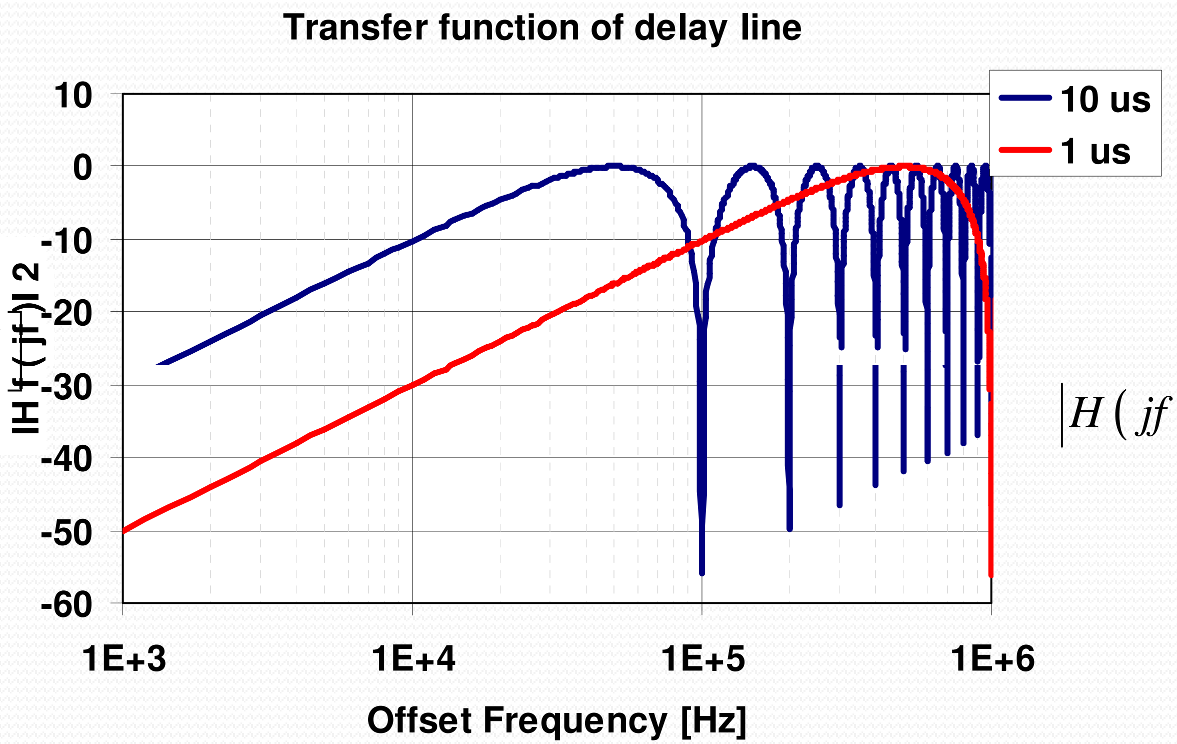
Delay line Measurement System



$$S_\varphi(f) = \frac{S_v(f)}{k_d^2 \left| H(jf) \right|^2}$$

$$\left| H(jf) \right|^2 = 4\sin^2(\pi f \tau_d)$$

Delay Line Transfer Function



$$10 Log\left[\left|H(jf)\right|^2\right]$$

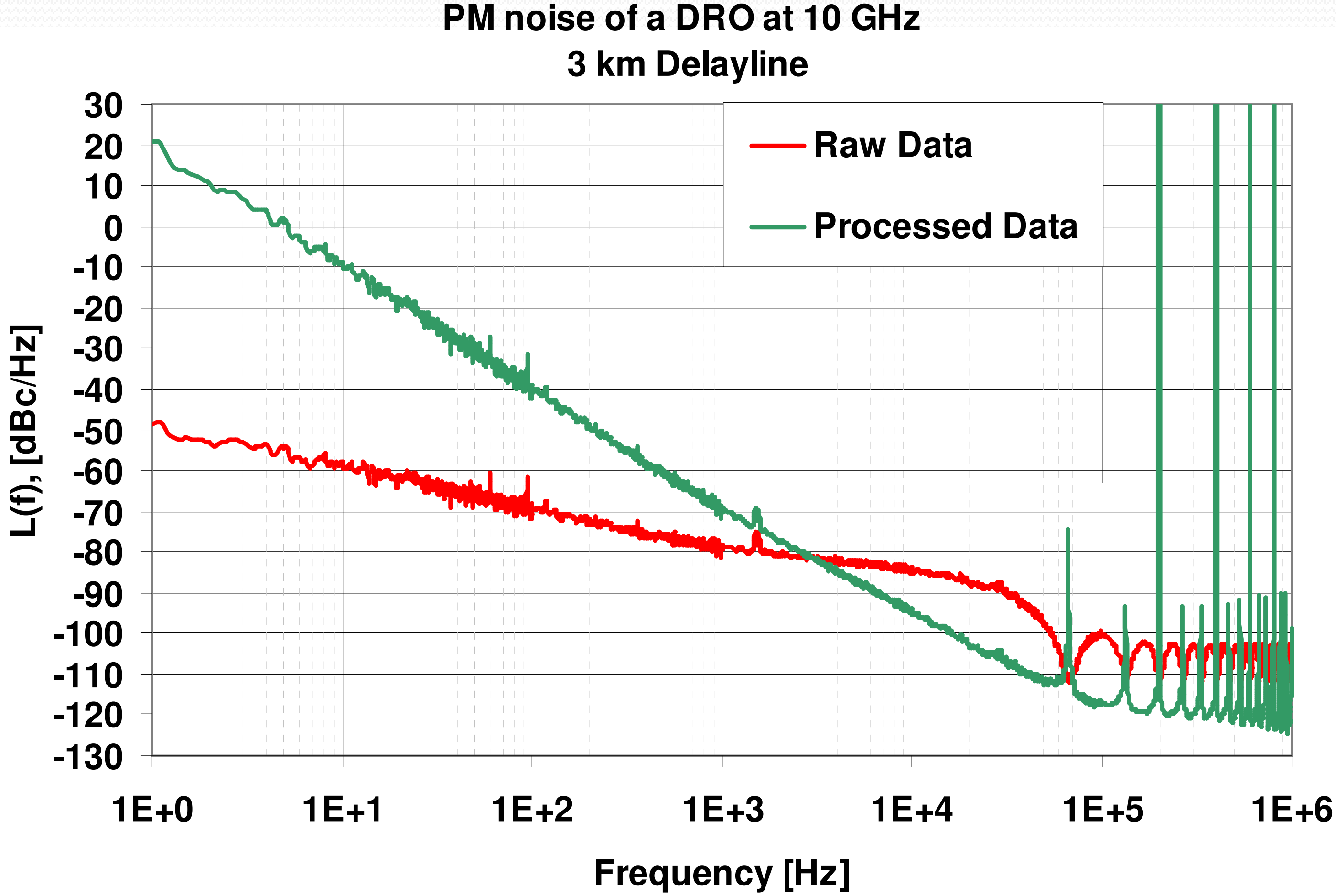
$$\left|H(jf)\right|^2 = 4\sin^2(\pi f \tau)$$

System sensitivity is proportional to τ_d

Long delays reduce sensitivity due to insertion loss

Transfer Function has nulls at n/τ_d

Delay Line Transfer Function



System sensitivity is proportional to τ_d

Long delays reduce sensitivity due to insertion loss

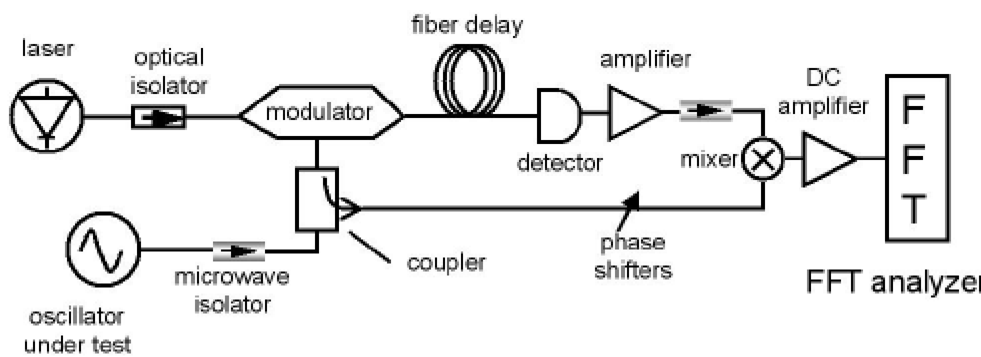
Transfer Function has nulls at n/τ_d

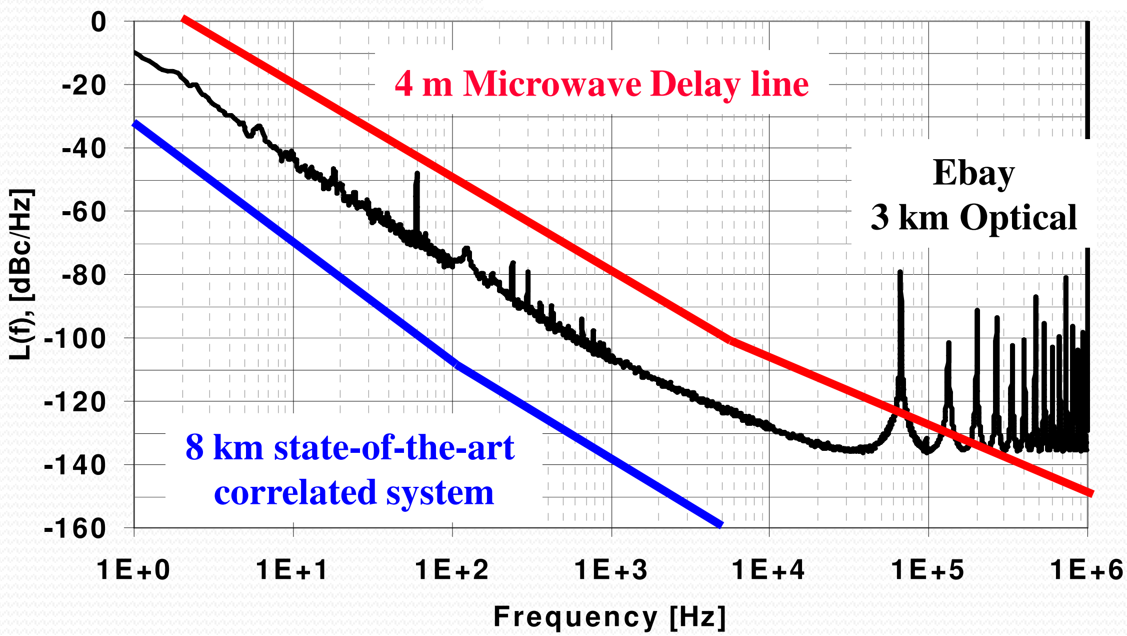
Single and Dual Source Measurements

Direct Frequency Comparison (Delay Line)	Direct Phase Comparison
Advantages:	
Doesn't require reference source No PLL effects Simple basic calibration	Lowest noise floor Noise scales as f^{-1} near carrier Noise floor easy to determine Very wide band performance Can be used for residual measurement
Disadvantages:	
Noise floor scales as f^{-3} near carrier Noise floor harder to determine Multiple delay lines required to cover different measurement conditions Not wideband – Frequency response has nulls Not useful for residual measurements Long delays have higher insertion loss	Requires reference of comparable quality Requires PLL to maintain phase lock between sources Calibration needed for measuring inside PLL loop bandwidth

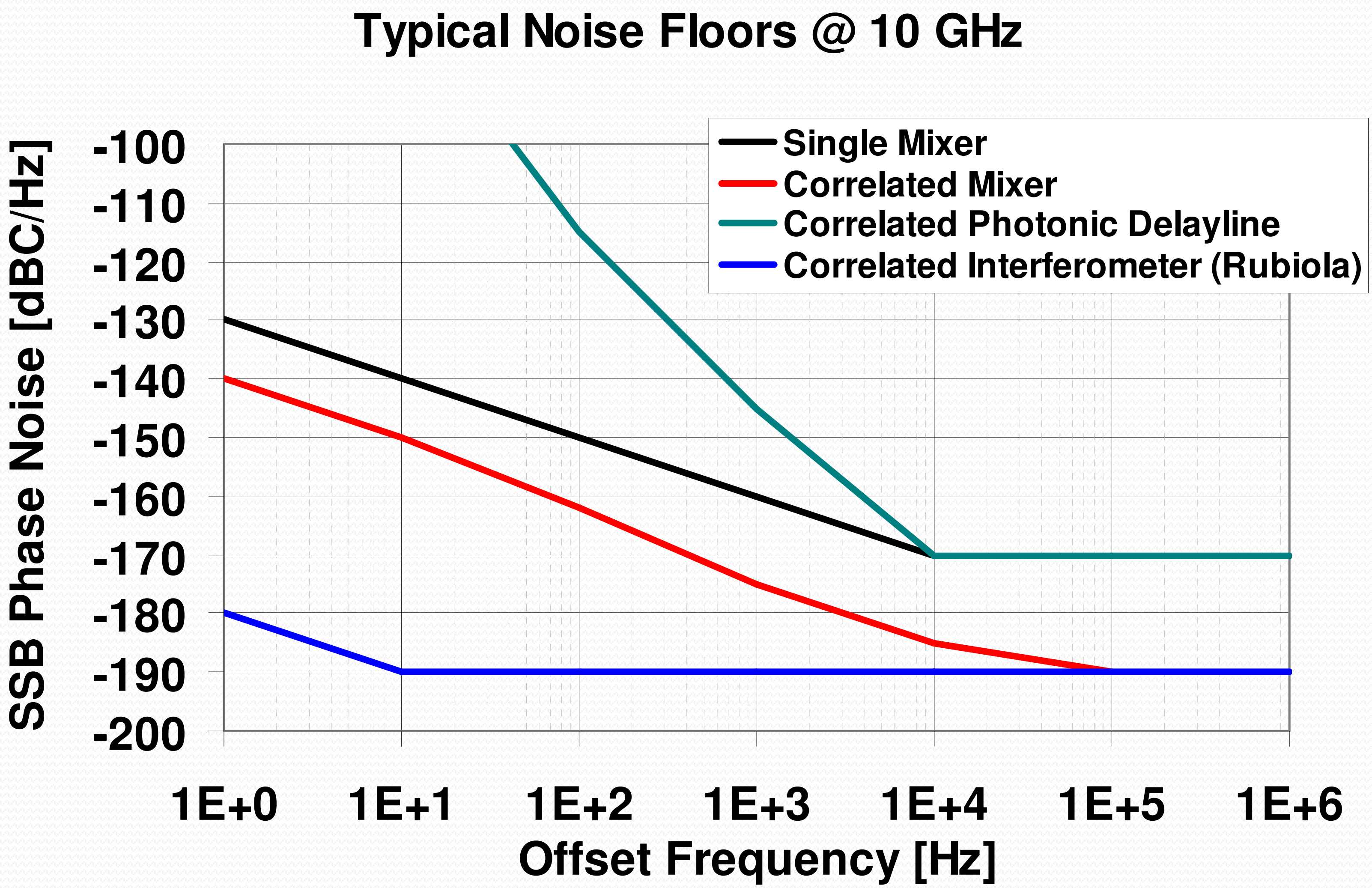
ptical Encoded Delay Line Measurement

~0.2 dB/km insertion loss vs 1 dB/m





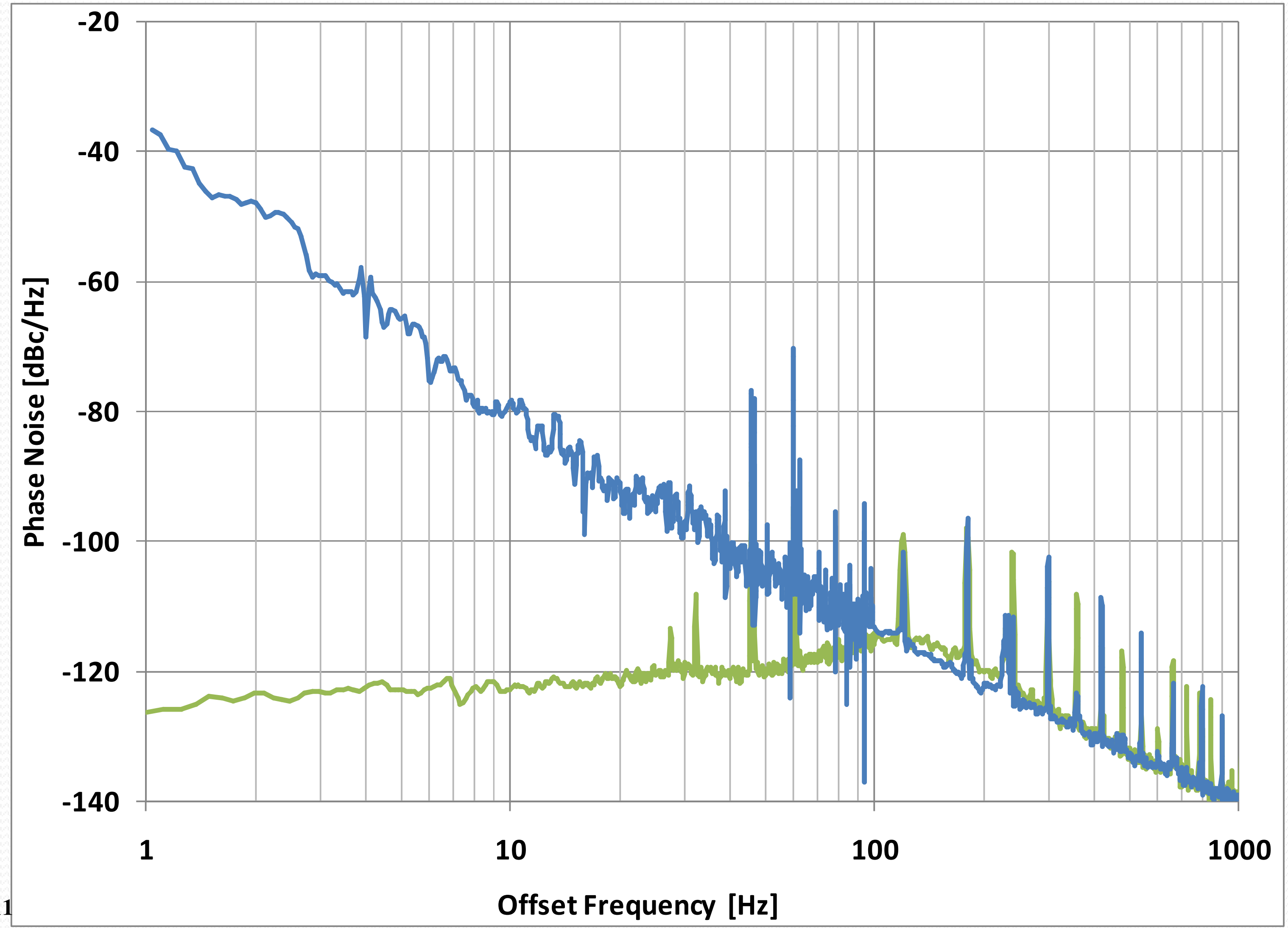
Comparison of Noise Floors



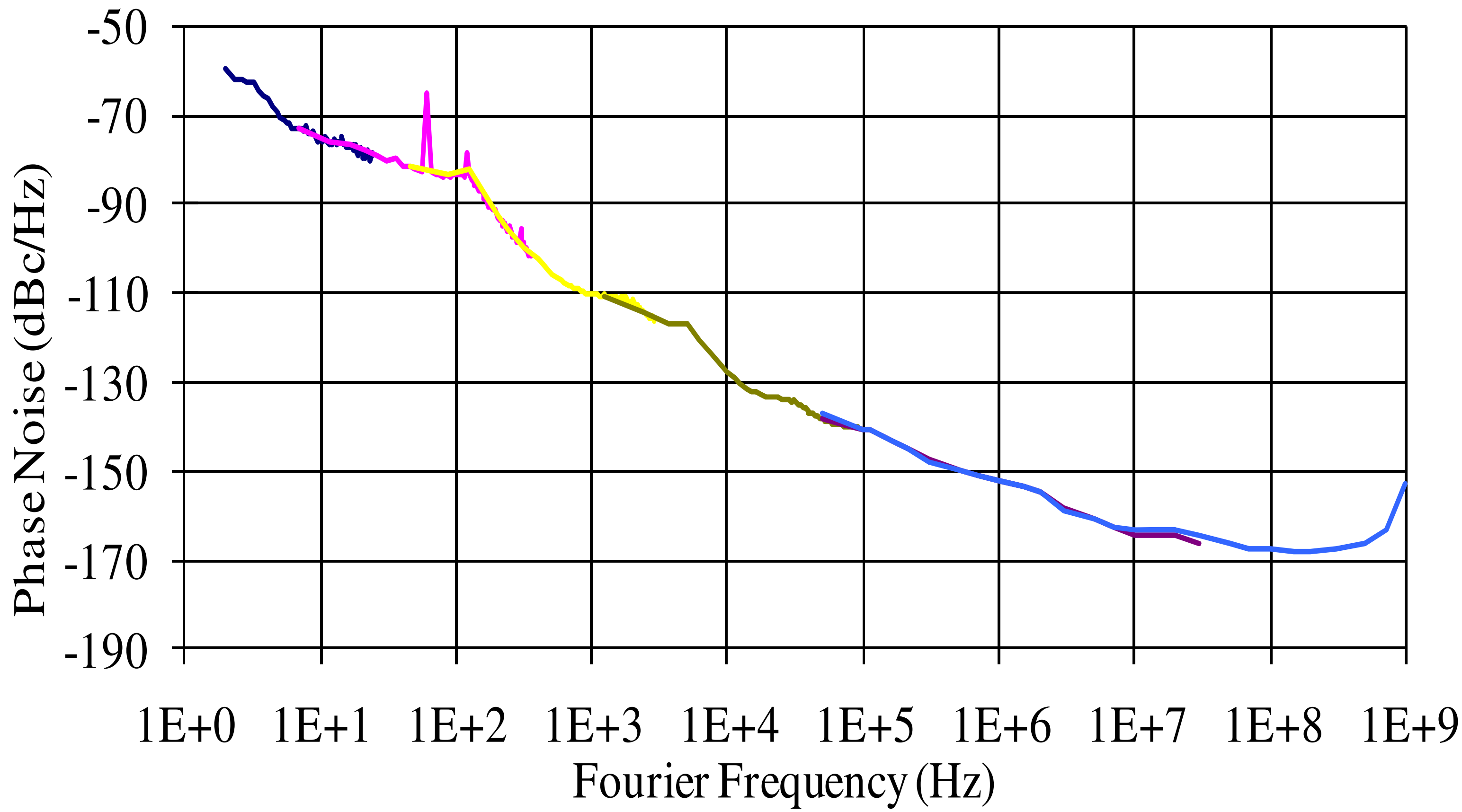
Measurement Problems

- Mechanical Instabilities
- Phase locked loop effects
- IF Gain Flatness
- Delay effects
- RBW effects
- AM Leakage
- FM Port noise
- Ground Loops

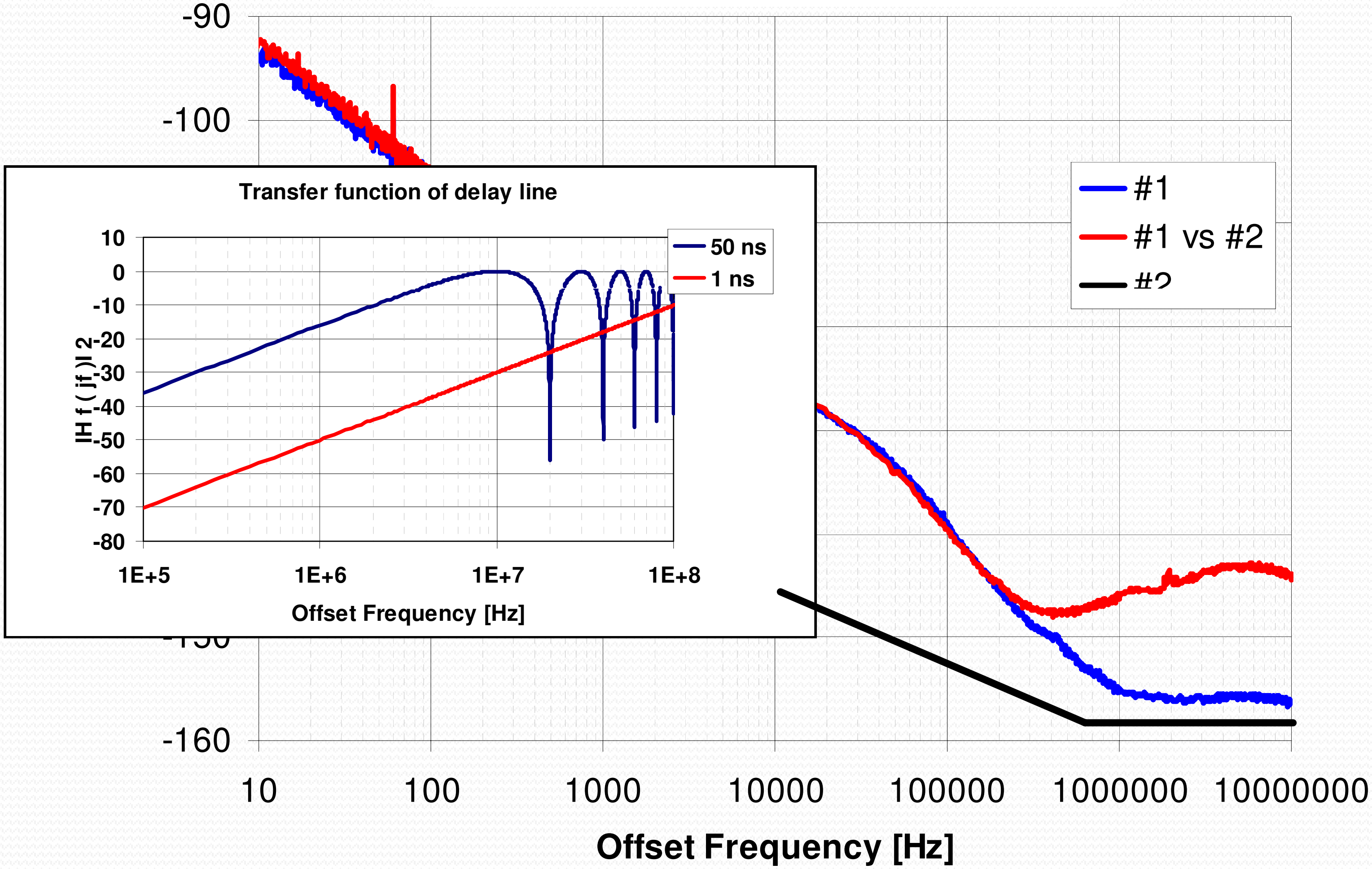
Phase Locked Loop Effects



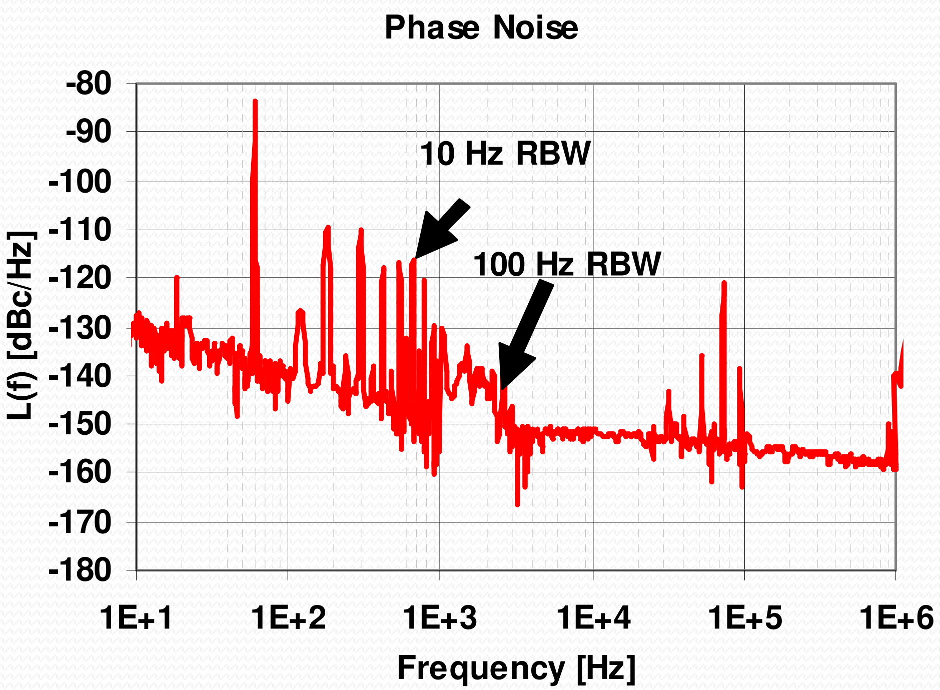
IF Gain Reduction in Presence of Noise Floor

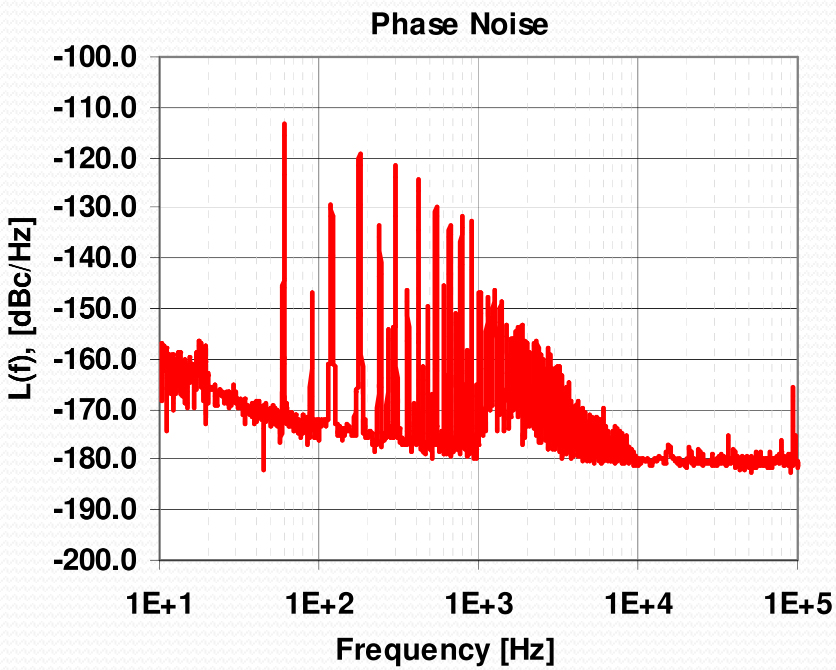


Delay Mismatch in Residual Measurements

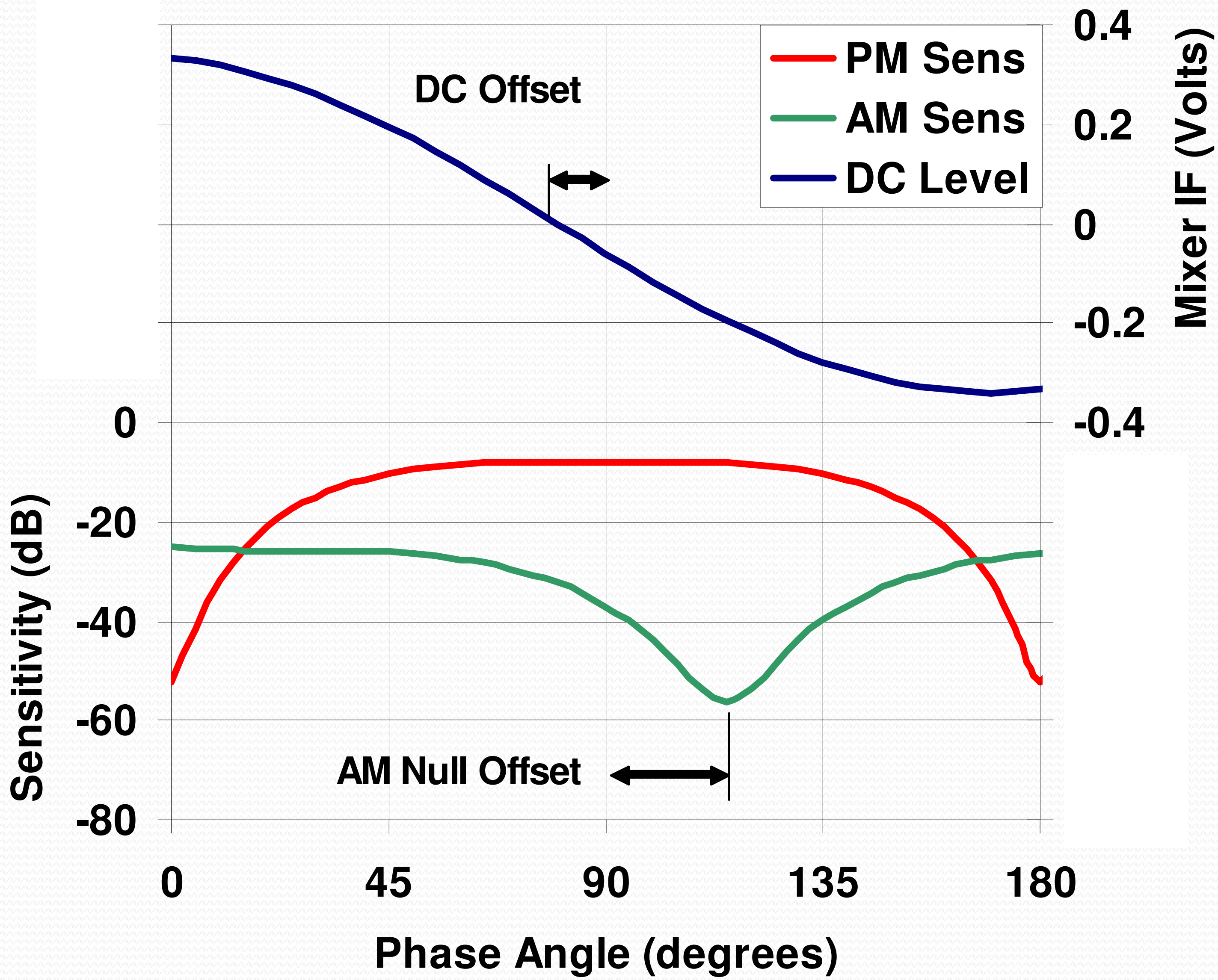


Insufficient Resolution Bandwidth

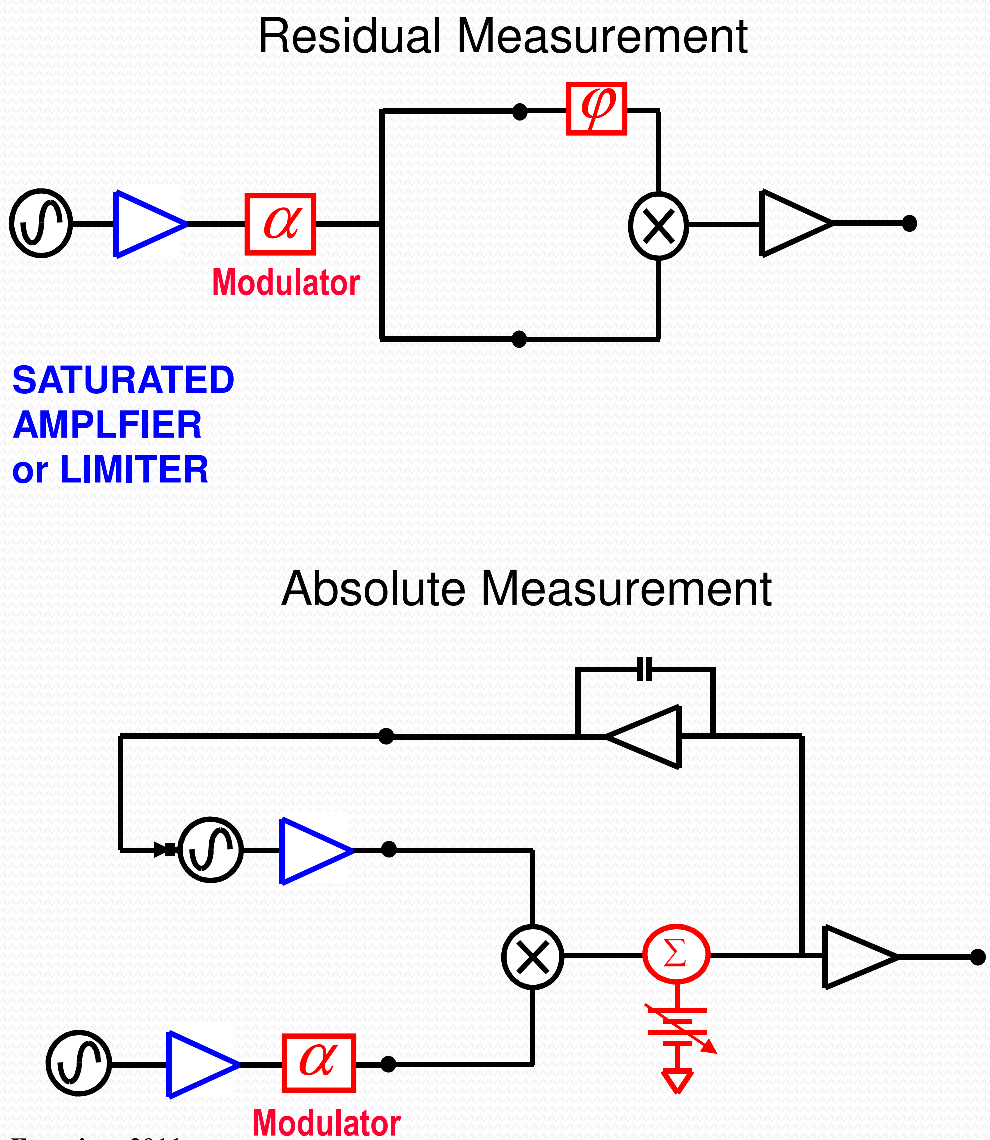


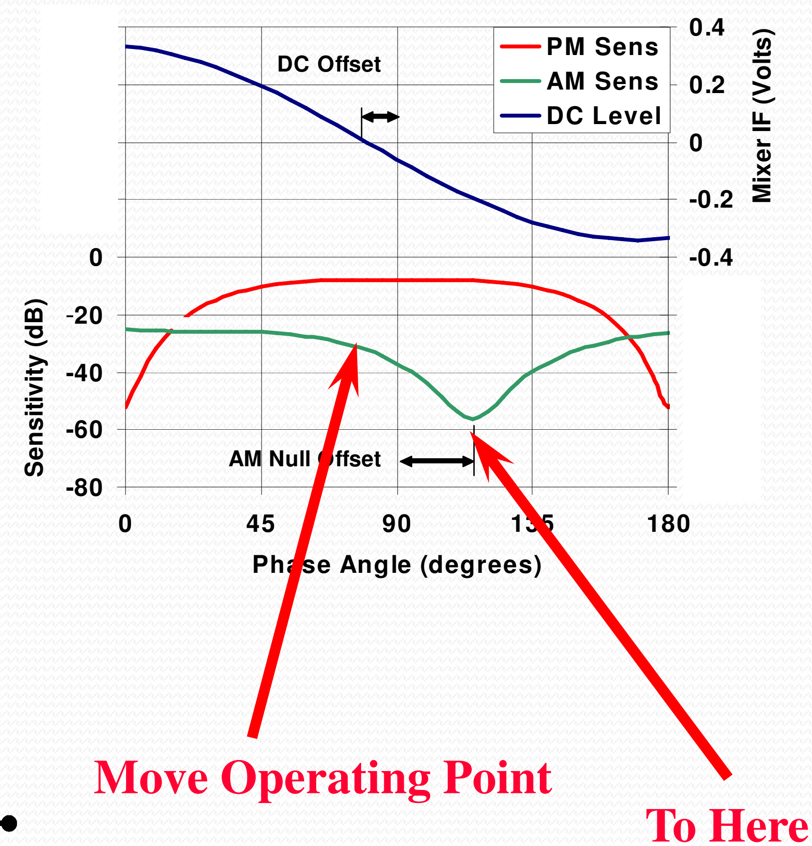


AM to PM Leakage in Mixer

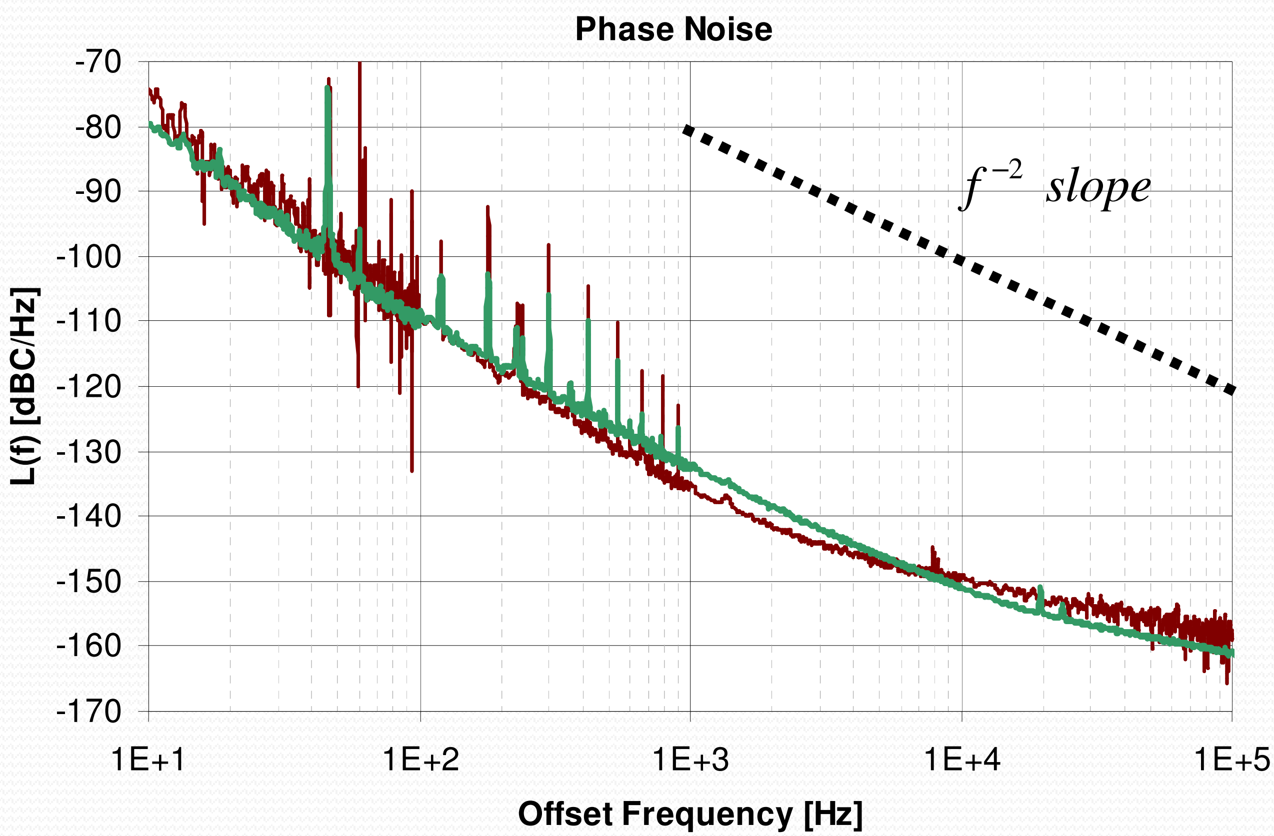


AM to PM Leakage in Mixer





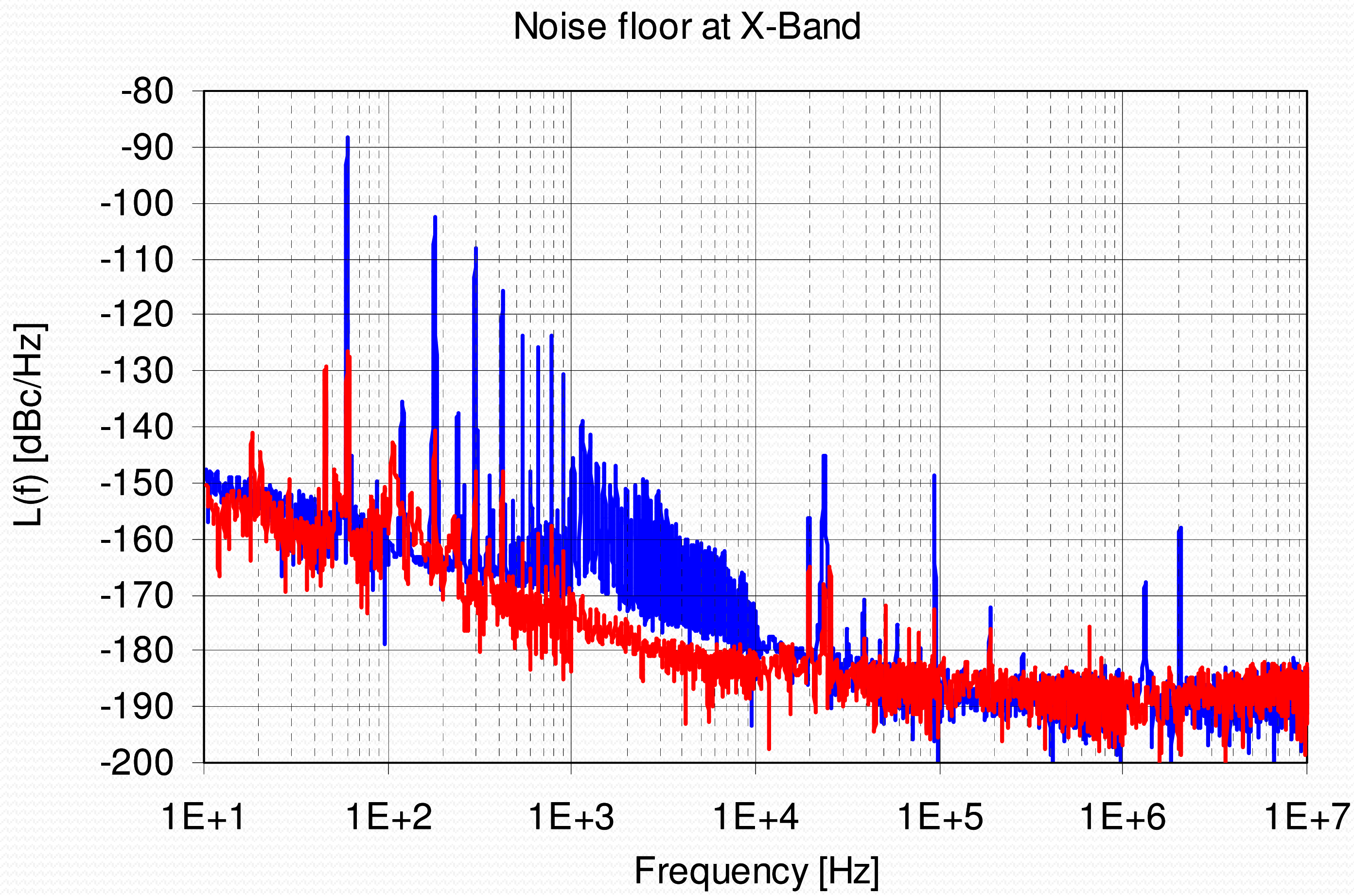
FM Port (PLL) Noise



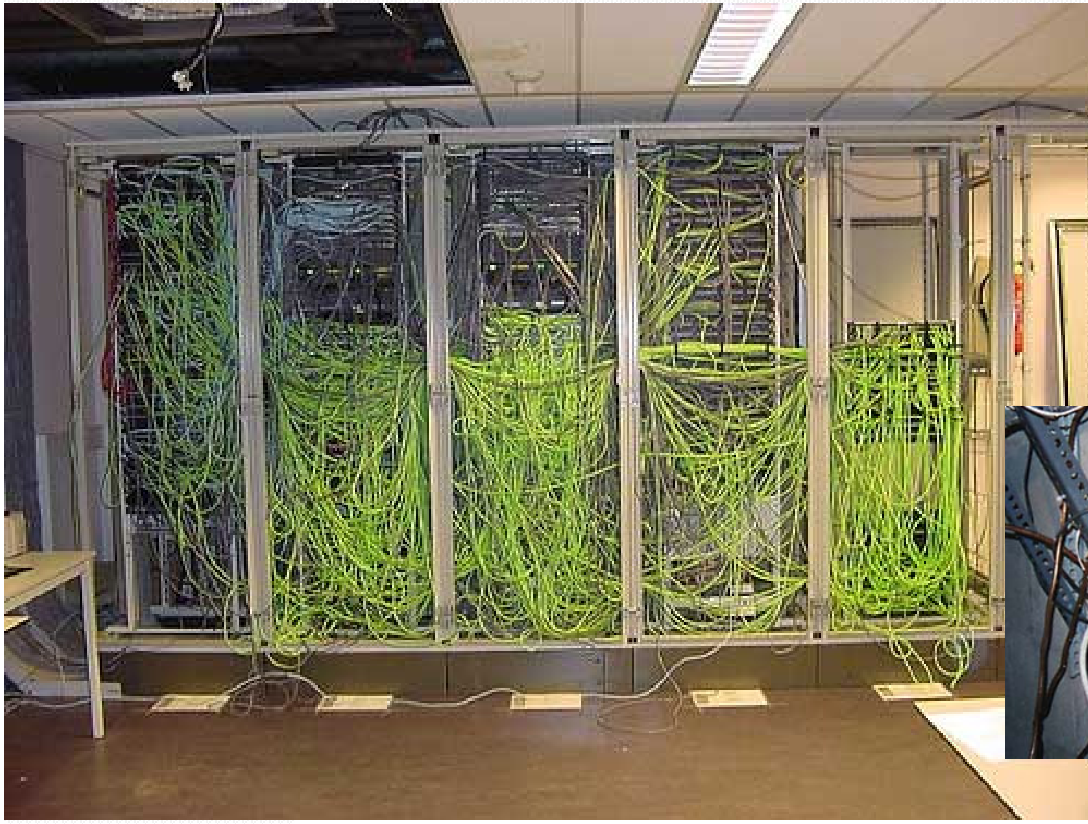
$$S_\phi(f) = \left(\frac{\nu_0}{f}\right)^2 S_y(f) = \left(\frac{K_\nu}{f}\right)^2 PSD(V_{rms})$$

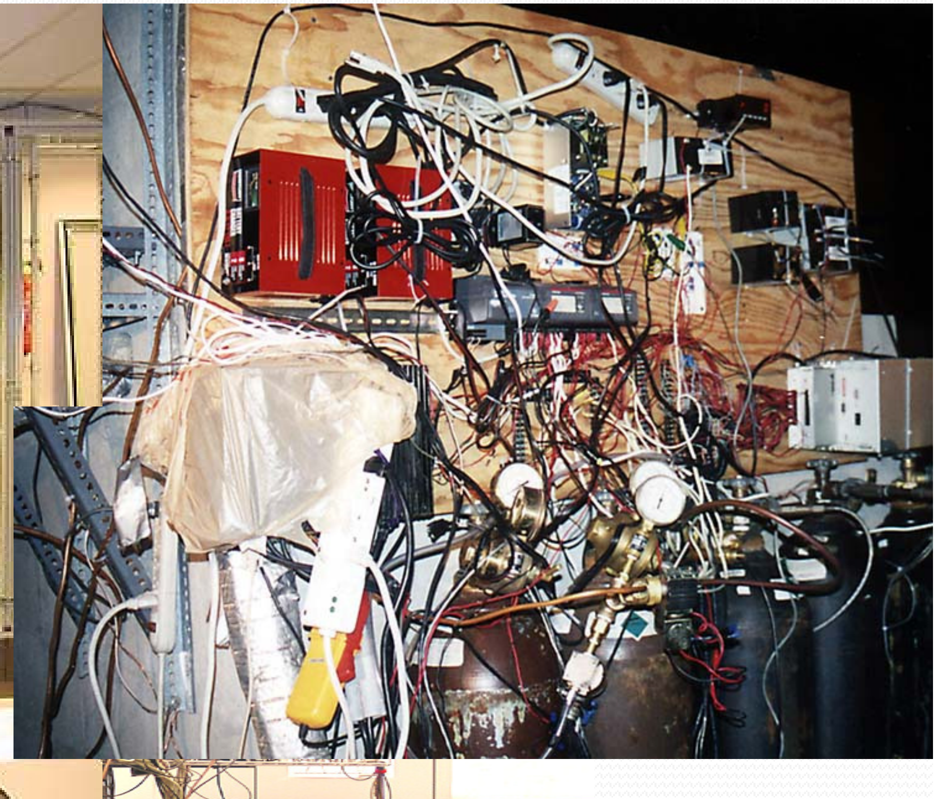
K_v is VCO sensitivity in Hz/V

Ground Loops and EMI



Ground Loops and EMI





Ground Loops and EMI

- Power devices from batteries
- Shielding – Faraday and magnetic
- DC Blocks
- Replace IF Gain with RF gain (Interferometric)
- Isolate or remove computer connections
- Plug in AC power all at same place
- EMI filtered power strips/ Ferrite Cores
- Ground lifting plugs (dangerous)
- Isolation transformers (dangerous)

Conclusions

- Great care must be taken when calibrating the mixer sensitivity.
- Calibrating Kd versus Fourier can be achieved by utilizing a modulation technique.
- For ultra-low noise floors cross-correlation or carrier-suppression techniques can be used.

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2. D.B. Sullivan, D.W. Allan, D.A. Howe, and F.L. Walls, *Characterization of Clocks and Oscillators*, NIST Tech. Note 1337, 1-342, 1990
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8. W. F. Walls, *Cross-correlation phase noise measurements*, Proc.Freq. Control Symp., IEEE, New York, 1992, May 27-29 1992, pp. 257-261.