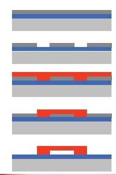
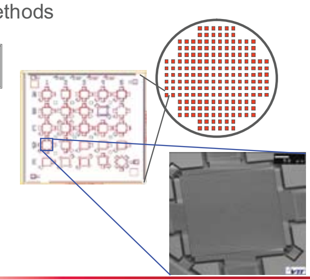
Theory and Analysis of
MEMS Resonators

V e Kaa akar
Chief Scientist, MEMS Resonators

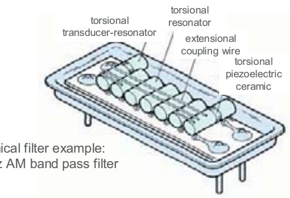
Why MEMS?
Batch fabrication of microresonators with IC-like processing and methods

one 200 mm wafer
> 50 000 chips!





Mechanical resonators and filters



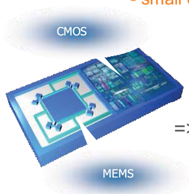
Mechanical filter example:
455 kHz AM band pass filter

- Low frequency mechanical filters ubiquitous between 40's and 80's. Bulky and costly by today's standards. Low frequency filter market dominated by ceramic and IC filters.
- High frequency mechanical filters still irreplaceable. Example: SAW band pass filters.
- As are quartz resonators used for oscillators.

Why micromechanical resonators?

- small size
- power consumption
- integrability

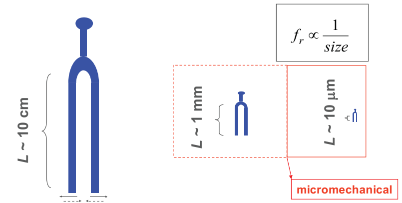
- what IC cannot do:
  - stability
  - small dissipations (high-Q)



=> *compact, high-performance devices!*

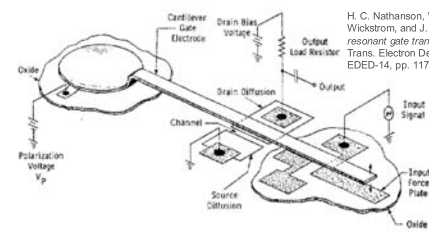
Resonance frequency of tuning fork

$$f_r \propto \frac{1}{size}$$



f_r ~ 1 kHz f_r ~ 100 kHz f_r ~ 10 MHz

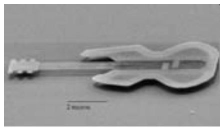
Examples of microresonators



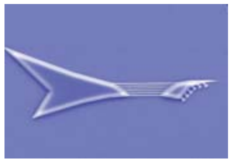
H. C. Nathanson, W. E. Newell, R. A. Wickstrom, and J. R. Davis, Jr., *"The resonant gate transistor"*, IEEE Trans. Electron Devices, vol. , vol. EDED-14, pp. 117–133, 1967.

First MEMS resonator: Resonating gold MOS gate in 60's!

Examples of microresonators



Microguitar (1997)



Improved microguitar (2003)
- you can actually play it
  (with a laser beam)

Examples of microresonators

Flexural polysilicon resonator

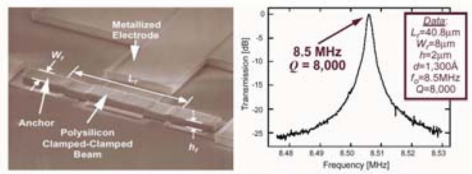


Fig. 2: SEM of an 8.5 MHz clamped-clamped beam μmechanical resonator with a typical measured spectrum [15].

C. T.-C. Nguyen et al., University of Michigan

Examples of microresonators

Contour mode polysilicon resonators

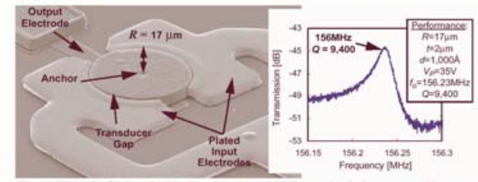
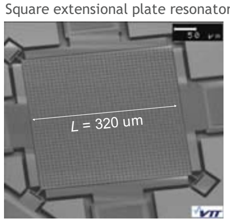


Fig. 4: SEM of a fabricated 156 MHz contour-mode disk μmechanical resonator with a measured frequency characteristic [20].

C. T.-C. Nguyen et al., University of Michigan

Examples of microresonators

Square extensional plate resonator

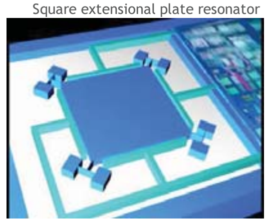


- First MEMS resonator to meet GSM specifications for noise (signal to noise ratio 150 dB!)
- Narrow electrode gaps needed to reduce the bias voltage (d = 200 nm and Vbias = 20 V gives R_m = 600 Ω)

P. Rantakari, et.al., "Low noise, low power micromechanical oscillator," in Transducers'05, Seoul, Korea, 5-9 Jun. 2005, pp. 2135- 2138.

Examples of microresonators

Square extensional plate resonator



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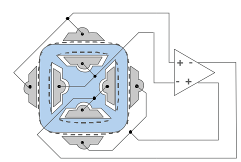
Examples of microresonators

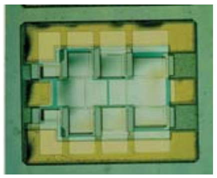


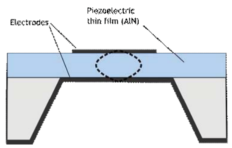
- First commercially available MEMS timing product from start-up SiTime
- Rm = 1 MΩ at V= 4.6. Differential measurement needed!
- The temperature drift and fabrication offsets are compensated with a phase locked loop (not shown).

Photo of MEMS die

Die bonded to an IC

Examples of microresonators

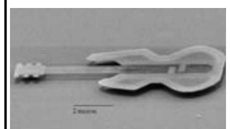




R. Ruby, *et al*, (Agilent Technologies), ISSCC 2001.

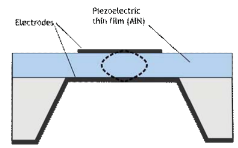
- Film Bulk Acoustic Resonator (FBAR)
- Used commercially in cell phones (filters, duplexers)

This course



Microguitar

VS.



FBAR

What separates commercial success from “mere” technology demonstrations?

Outline

- Part 1: modeling of microresonators
  - Lumped models for distributed resonator
  - Electrostatic and piezoelectric actuation
- Part 2: figures of merit
  - Quality factor
  - Electromechanical coupling coefficient
- Part 3: Oscillator applications
- Part 4: Filter applications
- LOTTERY!

Modeling challenge

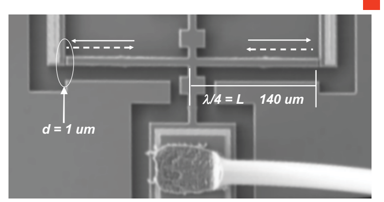
Vibrations of, for example, guitar string:

Which part is mass and which spring?

- Resonators usually have distributed mass and elasticity.
- Analyzing distributed resonators requires solving 3D stress fields either analytically or with finite element analysis (FEM).
- For system analysis, lumped mass-spring model or electrical equivalent model are preferred.

Resonator example:

Electrostatically actuated BAW resonator



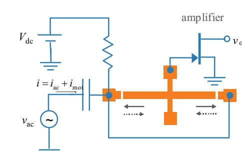
$f_r = 13$ MHz

$Q > 100\,000$

Ends extending and contracting (length extensional mode)
Mattila *et al*., 2002

Resonator example:

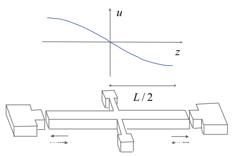
Electrostatically actuated BAW resonator



Longitudinal mode beam resonator [Mattila02]. A distributed system.

How to represent with a lumped mass spring model?

Distributed BAW resonator vibrations



Vibrations governed by the wave equation (longitudinal mode wave in beam):

$$\rho A\frac{\partial^2 U}{\partial t^2} - EA\frac{\partial^2 U}{\partial z^2} = 0$$

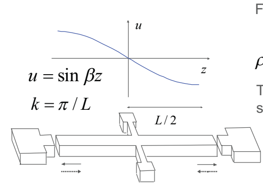
"Guess" the solution:

$$U = \sin(\beta z)e^{-j\omega_0 t} \Rightarrow \rho\omega_0^2 = E\beta^2$$

From boundary conditions:

$$\beta = \pi / L$$

Lumped model for forced vibrations



$u = \sin \beta z$

$k = \pi / L$

Forced beam vibrations:

$$\rho A\frac{\partial^2 U}{\partial t^2} + bA\frac{\partial U}{\partial t} - EA\frac{\partial^2 U}{\partial z^2} = 0$$

Try solution based on mode shape:

$$U(t,z) = x(t)\sin\frac{\pi z}{L}$$

$$\Rightarrow \rho A\frac{\partial^2 x}{\partial t^2}\sin\frac{\pi z}{L} + bA\frac{\partial x}{\partial t}\sin\frac{\pi z}{L} + EA\left(\frac{\pi z}{L}\right)^2 x\sin\frac{\pi z}{L} = 0$$

Lumped model for forced vibrations

(continued)

$$\rho A\frac{\partial^2 x}{\partial t^2}\sin\frac{\pi z}{L} + bA\frac{\partial x}{\partial t}\sin\frac{\pi z}{L} + EA\left(\frac{\pi z}{L}\right)^2 x\sin\frac{\pi z}{L} = 0$$

Multiply by mode shape $u = \sin\frac{\pi z}{L}$ and integrate over the beam length to give

$$\underbrace{\frac{\rho AL}{2}}_{m}\frac{\partial^2 x}{\partial t^2} + \underbrace{\frac{bAL}{2}}_{\gamma}\frac{\partial x}{\partial t} + \underbrace{\frac{EA\pi^2}{2L}}_{k}x = 0$$

$$m\frac{\partial^2 x}{\partial t^2} + \gamma\frac{\partial x}{\partial t} + kx = 0$$

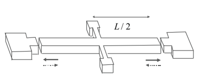
Compare to the geometrical mass and dc-spring:

$$m = \rho AL$$

$$k = E\frac{A}{L}$$

Lumped model for forced vibrations

(continued)



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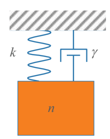


$$m = \frac{\rho AL}{2}$$

$$k = \frac{EA\pi^2}{2L}$$

- Lumped harmonic resonator can be used to model the vibration mode in distributed resonator.
- Distributed system has infinite number of resonances. Additional resonances can be modeled by having several harmonic resonators in parallel. Often one resonance is all that is needed.

Practical analysis

- Often mode shape and resonance frequency are known
  - FEM
  - Vibration encyclopedia (e.g. Roark's)
- Integrate mode shape to get the effective mass

$$\boxed{m = \int \rho U^2 \mathrm{d}V}$$

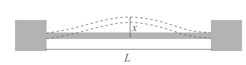
mode shape

- Calculate effective spring from

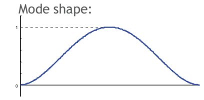
$$\boxed{k = \omega_0^2 m}$$

Example: clamped-clamped beam

Clamped-clamped resonator:



Mode shape:



Mode shape:

$$U_1(X) = C_1[\sinh\beta_1 X/L - \sin\beta_1 X/L + \alpha_1(\cosh\beta_1 X/L - \cos\beta_1 X/L)]$$

Effective mass:

$$m = \int \rho U^2 \mathrm{d}V = \rho A\int_0^L U_1^2(X)\mathrm{d}X = 0.40\rho AL \approx 8.8\ \mathrm{pkg}$$

Resonance frequency:

$$\omega_0 = \frac{\beta_1}{\sqrt{12}}\sqrt{\frac{E}{\rho}}\frac{w}{L^2}$$

Effective spring:

$$k = \omega_0^2 m \approx 819\ \mathrm{N/m}$$

Recap

- Resonators have distributed mass and spring
- The vibration mode can be modeled with effective mass and spring => lumped mechanical model
- Next step: electrical equivalent

Why electrical equivalents

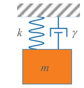
•Analytical work is simplified if all effects of physics, mechanics, and circuits are in the same domain.

•Circuit simulators are powerful and very robust

=>Filter and quartz oscillator design work is done with circuit simulators

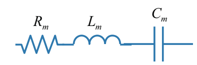
•Philosophical: it is good to start with a simple physics based model and add complexity if necessary. (Compare to starting with complex finite element simulation).

Step #1:
Electrical equivalent circuits



$$m\frac{\partial \dot{x}}{\partial t}+\gamma\dot{x}+k\int \dot{x}\mathrm{d}t=F(t)$$

$$\dot{x}=i$$
$$F=v$$
$$m=L_m$$
$$\gamma=R_m$$
$$k=1/C_m$$



velocity

force

$$L_m\frac{\partial i}{\partial t}+R_m i+\frac{1}{C_m}\int i\mathrm{d}t=v$$

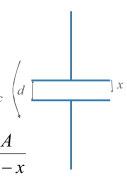
Step #2:
Electromechanical coupling

- Electromechanical coupling needed to:
  - convert excitation **voltage into force**
  - convert mechanical **displacement into signal current**
- Several transduction methods are possible:
  - Capacitive transduction.
    - Based on electrostatic force and conservation of charge.
    - Well suited for micromachining.
    - Widely used in MEMS gyroscopes and accelerometers.
  - Piezoelectric transduction.
    - Used in >99.9% of all commercial devices (quartz resonators, SAW and FBAR filters).
    - Material property. Unfortunately silicon is not piezoelectric.
  - Magnetic transduction.
  - Piezoresistive sensing.
  - Thermal actuation.

Not used commercial devices and therefore not covered here.

Capacitive transduction - force

$$v=V_{dc}+v_{ac}$$



$$C=\varepsilon\frac{A}{d-x}$$

$$C_0=\varepsilon\frac{A}{d}$$

$$F=\frac{\partial W}{\partial x}=\frac{\partial}{\partial x}\left(\frac{1}{2}Cv^2\right)$$

$$\approx\frac{1}{2}v^2\frac{C_0}{d}=\frac{C_0}{2d}\left(V_{dc}^2+2V_{dc}v_{ac}+v_{ac}^2\right)$$

dc-force

ac-force

assumed small

Force at excitation frequency:

$$F=\eta v_{ac} \quad \text{where} \quad \eta=V_{dc}\frac{C_0}{d}$$

Capacitive transduction -current

$$i=\frac{\partial Cv}{\partial t}=v\frac{\partial C}{\partial t}+C\frac{\partial v}{\partial t}$$

$$\approx V_{dc}\frac{\partial C}{\partial t}+C_0\frac{\partial v_{ac}}{\partial t}$$

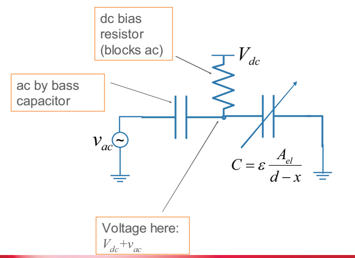
$$\boxed{V_{dc}\frac{\partial C}{\partial t}=V_{dc}\frac{\partial C}{\partial x}\frac{\partial x}{\partial t}\approx\eta\frac{\partial x}{\partial t}}$$

motional current

”normal” current

$$i\approx\eta\frac{\partial x}{\partial t}+C_0\frac{\partial v_{ac}}{\partial t}=\eta\frac{\partial x}{\partial t}+j\omega C_0 v_{ac}$$

Practical realization



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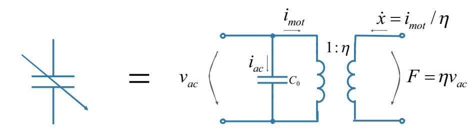
Transformer model for coupling



$F = \eta v_{ac}$

$$i = i_{mot} + i_{ac}$$
$$\approx \eta \dot{x} + j\omega C_0 v_{ac}$$

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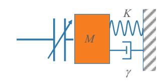
MEMS resonator model

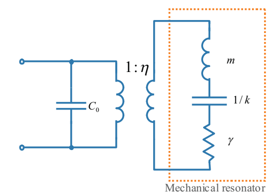
Lumped model



$F = \eta v_{ac}$

$v = \frac{i}{\eta}$

Electrical equivalent



$v = i \quad F = v$

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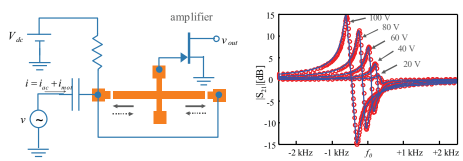
Comparison of simulation and experiment



Measurement and electrical equivalent model agree very well!

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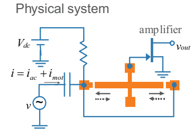
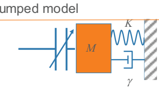
Putting it all together:

MEMS resonator model

Physical system

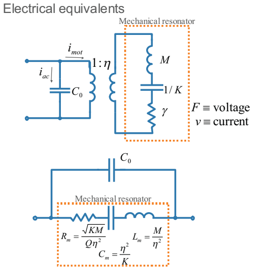


Lumped model



$F = \eta v_{ac} \quad i_{mot} = \eta v$

Electrical equivalents



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Typical values for the beam BAW

Mechanical parameters:

Spring constant	K	22	MN/m
Effective mass	M	340	pkg
Quality factor	Q	100 000	

Electrical parameters (U_{dc} = 100 V, d = 1 μm):

Motional capacitance	C_m	0.35	aF
Motional inductance	L_m	430	H
Motional resistance	R_m	350	kΩ

Electrical parameters (U_{dc} = 10 V, d = 0.1 μm):

Motional capacitance	C_m	35	aF
Motional inductance	L_m	4.3	H
Motional resistance	R_m	900	Ω

Motional impedance is a strong function of gap (and bias)

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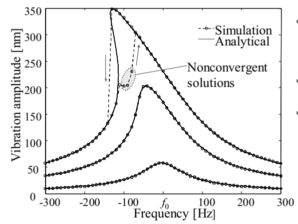
Recap

- Equations for mechanical resonator can be represented with a LRC-circuit
- Electrostatic coupling is based on variable capacitor
  - Electrostatic force and current through the capacitor depend on the electromechanical transduction factor η
- Next step: nonlinear effects

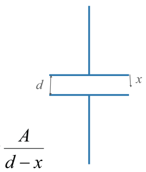
Nonlinear effects:
Amplitude limit due to nonlinear springs



- Can be of capacitive or mechanical origin (fundamentally limited by material nonlinearity)
- Bifurcation (hysteresis) at large vibration amplitudes.
- Sets limit to resonator power handling capacity!

V. Kaajakari et al, IEEE J. MEMS

Nonlinear effects:
Capacitive nonlinear springs



$$C = \varepsilon \frac{A}{d-x}$$

Taylor series expansion of the force:

$$F = \frac{\partial W}{\partial x} = \frac{\partial}{\partial x}\left(\frac{1}{2} C(x) V_{dc}^2\right)$$
$$= \frac{C_0}{2d} V_{dc}^2 \left(1 + \frac{2x}{d} + \frac{3x^2}{d^2} + \frac{4x^3}{d^3} + \ldots\right)$$

F ~ x, linear spring
Can be used to adjust frequency!

Nonlinear springs!

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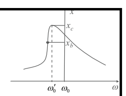
Nonlinear effects:
Power handling capacity



Nonlinear forced excitations:

$$m\frac{\partial^2 x}{\partial t^2} + \gamma \frac{\partial x}{\partial t} + k_0(1 + k_1 x + k_2 x^2)x = f(t)$$

Critical vibration amplitude: $x_c = \frac{2}{\sqrt{\sqrt{3}Q|\kappa|}}, \quad \kappa = \frac{3}{8}k_2 - \frac{5}{12}k_1^2$

Maximum energy stored in resonator: $E_{max} = \frac{1}{2} k_0 x_c^2$

Resonator power handling capacity: $P_{max} = \frac{E_{max}\omega_0}{Q}$

Signal to noise ratio ~ $\frac{E_{max}}{k_B T}$ (remember that thermal noise is $k_B T$)

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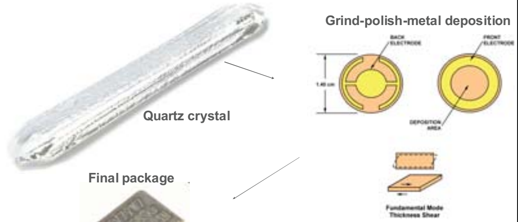
Nonlinearity: implications

- The signal is reduced with shrinking size
  - Bulk resonators typically have larger signal than microresonators
  - Nanomechanical is worse than micromechanical!
  - Signal-to-noise ratio (S/N) decreases with size
- Non-linear effects should be minimized for large S/N
  - Stiff structures (bulk, not flexural)
  - Linear coupling (piezoelectric, not electrostatic)
- Meeting phase noise specifications with microresonators is a challenge
- Microresonators may not be suitable for filters
  - Large interfering signal will mix and alias to signal band

Recap of nonlinearity

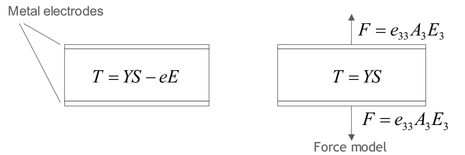
- The maximum signal level easy to evaluate if nonlinear spring constants are known
  - Trivial with capacitive coupling
  - Mechanical nonlinearity harder to model
- Fundamental challenge to miniaturization!
- Next: **piezoelectric transduction**

Piezoelectric resonators



- No bias voltage needed
- Strong electromechanical coupling
- Simple and cheap

Modeling piezoelectric coupling



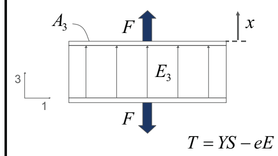
Constitutive model

- Stress is sum of mechanical and piezoelectric strain
- Coupled mechanical and electrical equations
- A lot of bookkeeping!

Lumped model

- Piezoelectric stress due to a lumped force at the location of electrodes
- Piezoelectric material modeled as a regular solid

Transduction factor for piezoelectric resonators

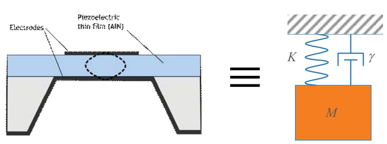


- Distributed vibrations can be modeled with a lumped mass and spring (as with capacitive MEMS)
- The electromechanical coupling can be modeled with effective electromechanical transduction factor (as with capacitive MEMS)

$$\boxed{\eta_{eff} = 2\frac{e_{33}A}{h}}$$

$$\boxed{\begin{aligned} F &= \eta_{eff} v_{ac} \\ i &= \eta_{eff}\dot{x} = j\omega_0\eta_{eff}x \end{aligned}}$$

Example: FBAR



$$m = \frac{\rho AL}{2}$$

$$k = \frac{EA\pi^2}{2L}$$

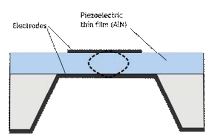
$$\eta_{eff} = 2\frac{e_{33}A}{h}$$

Simplest model:

- 1D vibrations in the thickness mode only
- Compare to 1D BAW we already analyzed

Caveat: real resonators have multiple "spurious modes"

Example: FBAR at 900 MHz



$$\rho = 3200\,\mathrm{kg/m^3}$$

$$A = (500\,\mu\mathrm{m})^2$$

$$h = 6\,\mu\mathrm{m}$$

$$e_{33} = 1.55\,\mathrm{C/m^2}$$

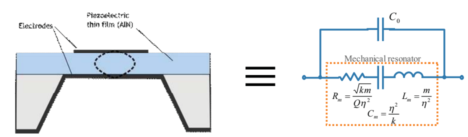
$$m = \frac{\rho AL}{2} = 2.4\,\mathrm{nkg}$$

$$k = \frac{EA\pi^2}{2h} = 82.2\,\mathrm{GN/m}$$

$$\eta_{eff} = 2\frac{e_{33}A}{h} = 0.1292\,\mathrm{N/V}$$

Example: FBAR at 900 MHz

continued



$$C_0 = \varepsilon_R\varepsilon_0\frac{A}{h} = 3.1\,\mathrm{pF}$$

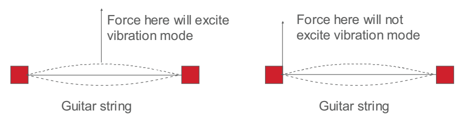
$$R_m = \frac{\sqrt{km}}{Q\eta^2} = 0.8\,\Omega$$

$$C_m = \frac{\eta^2}{k} = 200\,\mathrm{fF}$$

Recap of piezoelectric transduction

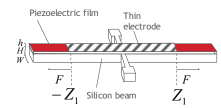
- Piezoelectric resonators can be modeled as lumped systems
  - Analysis identical to lumped models for electrostatic resonators
- The effective electromechanical transduction depends on the location of the electrodes and material constants
- Next: **Transducer location dependency**

Location of transducer matters



Lumped force

The effective force is also normalized by the mode shape.



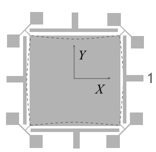
$$u = \sin \beta Z$$

Effective force counting the both forces and mode shape:

$$F_{eff} = F \sin \beta Z_1 - F \sin \beta(-Z_1) = 2F \sin \beta Z_1$$

Effective transduction factor: $\boxed{\eta_{eff} = \eta_0 2 \sin \beta Z_1}$

Lame resonator example



X-displacement:

$$U_X = \sin \frac{\pi X}{L} \cos \frac{\pi Y}{L}$$

Y-displacement:

$$U_Y = -\sin \frac{\pi Y}{L} \cos \frac{\pi X}{L}$$

Effective transduction factor for electrode 1:

$$U_X\left(X = \frac{L}{2}\right) = \cos \frac{\pi Y}{L}$$

$$\eta_{eff} = \eta_0 \int_{-\frac{L}{2}}^{\frac{L}{2}} \cos \frac{\pi Y}{L} \mathrm{d}Y = \frac{\eta_0}{2}$$

where

$$\eta_0 = \varepsilon_0 \frac{A}{d^2} V_{dc}$$

Recap of transducer location dependency

- The transduction factor is normalized by the resonator mode shape.
  - For maximum coupling, electrodes should be placed at the location of maximum vibration amplitude
- Next: **Part 2: figures of merit**

Outline

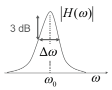
- Part 1: modeling of microresonators
  - Lumped models for distributed resonator
  - Electrostatic vs. piezoelectric actuation
- **Part 2: figures of merit**
  - Quality factor
  - Electromechanical coupling coefficient
- Part 3: Oscillator applications
- Part 4: Filter applications

Definitions for quality factor

(1) Physical definition

$$\boxed{Q = 2\pi \frac{\text{Energy stored}}{\text{Energy dissipated per cycle}}}$$

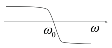
(2) Transmission peak width



$$\boxed{Q = \frac{\omega_0}{\Delta\omega}}$$

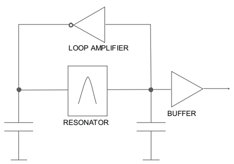
(3) Slope of transmission phase

$\Phi = \angle H(\omega)$



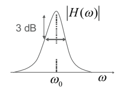
$$\boxed{Q = \frac{\omega_0}{2}\frac{d\Phi}{d\omega}}$$

Importance of quality factor in oscillators



- Low noise: resonator is effective narrow bandwidth filter
- Stability of the oscillator: small changes in circuit do not change oscillation frequency

$Q = \frac{\Delta\omega}{\omega_0}$

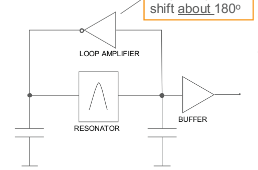


Leeson's formula for phase noise:

$$L \propto \frac{k_B T}{2P}\left[1 + \left(\frac{\omega_0}{2Q\Delta\omega}\right)^2\right]$$

Importance of quality factor in oscillators

Amplifier phase shift about 180°



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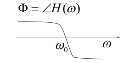
$\Phi = \angle H(\omega)$



$Q = \frac{\omega_0}{2}\frac{d\Phi}{d\omega}$

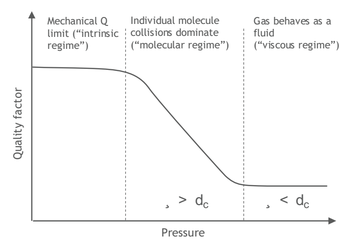
Resonator phase shift about 180° => frequency change due to finite Q

Example:
$Q = 100{,}000$
$d\Phi = 5^\circ = 0.087\,\text{rad}$
$\frac{d\omega}{\omega_0} = \frac{d\Phi}{2Q} = 0.4\,\text{ppm}$

Loss effects in microresonators

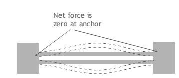
- Gas damping
  - Dominant loss mechanism unless vacuum packaging is used
  - Very high quality vacuum needed to minimize losses
- Anchor losses
  - Resonator vibrations couple to substrate/packaging
  - Significant especially for low frequency resonators
- Intrinsic losses
  - Material dependent viscous losses
  - Low for hard, crystalline materials (quartz, silicon…)

Gas damping

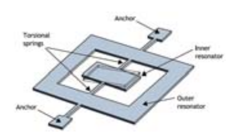


Minimizing anchor losses

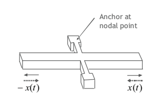
Simple clamped-clamped beam resonator exerts force on anchor and results in large losses



Anchor moments approximately cancel in tuning fork resonators

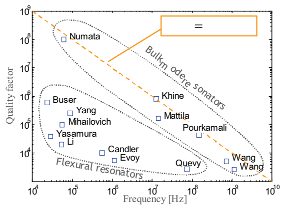


Vibration isolation with a double resonator structure



Bulk mode resonators (BAW) can be mounted at nodal point to minimize energy leak

Scaling law for intrinsic losses



- Anchor losses dominate in flexural resonators
- Intrinsic losses limit bulk mode resonators
- f·Q-product approximately constant for given material

Recap of quality factor

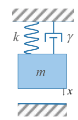
- Characterizes losses in resonator
  - Material losses
  - Anchor losses
  - Gas damping
- Important for oscillator signal quality and stability
- Important for filter performance
- Next: **Electromechanical coupling coefficient k_{eff}^2**

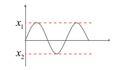
Electromechanical coupling coefficient

- Electromechanical coupling coefficient is defined as

$$k_{eff}^2 = \frac{\text{energy converted}}{\text{input energy}}$$

- For example, electrostatic resonator vibrating between two extremes x_1 and x_2 trades mechanical energy to electrical energy





Energy change:

$$\Delta W = \frac{1}{2}\frac{C_0^2}{d^2 k}V^4$$

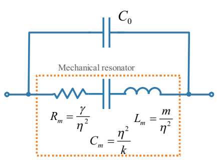
Stored electrical energy:

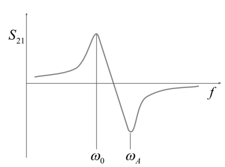
$$W = \frac{1}{2}C_0 V^2$$

$$\boxed{k_{eff}^2 = \frac{\Delta W}{W} = \frac{\eta^2}{kC_0} = \frac{C_m}{C_0}}$$

Electromechanical coupling coefficient

$$\boxed{k_{eff}^2 = \frac{\eta^2}{kC_0} = \frac{C_m}{C_0}}$$





Series resonance:

$$\omega_0 = \frac{1}{\sqrt{C_m L_m}}$$

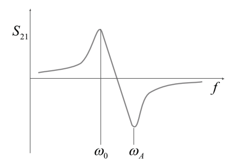
Parallel resonance (anti-resonance):

$$\omega_A = \omega_0\sqrt{1+\frac{C_m}{C_0}} = \omega_0\sqrt{1+k_{eff}^2}$$

Note:

$$k_{eff}^2 = \frac{\omega_A^2-\omega_0^2}{\omega_A^2} \approx 2\frac{\omega_A-\omega_0}{\omega_0}$$

Electromechanical coupling coefficient

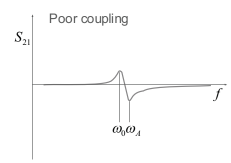


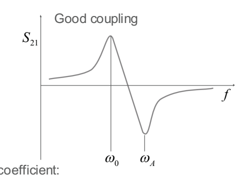
Coupling coefficient:
⇒Strength of coupling relative to mechanical stiffness
⇒Measures distance between resonance and anti-resonance.
⇒Tuning range for capacitive loaded oscillators
⇒Bandwidth for ladder filters

$$\boxed{k_{eff}^2 = \frac{\omega_A^2-\omega_0^2}{\omega_A^2} \approx 2\frac{\omega_A-\omega_0}{\omega_0}}$$

$$\boxed{k_{eff}^2 = \frac{\eta^2}{kC_0} = \frac{C_m}{C_0}}$$

Electromechanical coupling coefficient





$$\boxed{k_{eff}^2 = \frac{\omega_A^2-\omega_0^2}{\omega_A^2} \approx 2\frac{\omega_A-\omega_0}{\omega_0}}$$

$$\boxed{k_{eff}^2 = \frac{\eta^2}{kC_0} = \frac{C_m}{C_0}}$$

Coupling coefficient:
⇒ Strength of electromechanical coupling relative to mechanical stiffness
⇒ Measures distance between resonance and anti-resonance.
⇒ Tuning range for capacitive loaded oscillators
⇒ Bandwidth for ladder filters
⇒ Insertion loss for filters

Combining Q and k_{eff}^2

- Product of Q and k_{eff}^2 is a figure of merit for resonators:

$$\boxed{Qk_{eff}^2 = \frac{1}{\omega_0 C_m R_m}\frac{C_m}{C_0} = \frac{1}{\omega_0 C_0 R_m}}$$

- Importance easy to understand by noting that $Z_0 = \frac{1}{\omega_0 C_0}$ is the characteristic impedance in filter design:

$$\boxed{Qk_{eff}^2 = \frac{1}{\omega_0 C_0 R_m} = \frac{Z_0}{R_m}}$$

- In resonators, tells how much signal goes through motional resonators vs C0

Recap of electromechanical coupling coefficient

- Characterizes strength of coupling
- Depends on
  - stiffness (energy stored in spring)
  - capacitance (electrical energy stored)
  - transduction factor (energy converted)
- Important for oscillator signal magnitude
- Important for filter performance
- Next: **Part 3: Oscillator applications**

Outline

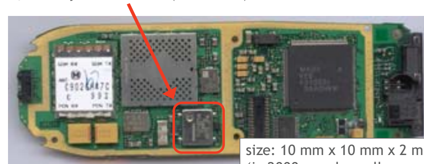
- Part 1: modeling of microresonators
  - Lumped models for distributed resonator
  - Electrostatic vs. piezoelectric actuation
- Part 2: figures of merit
  - Quality factor
  - Electromechanical coupling coefficient
- **Part 3: Oscillator applications**
- Part 4: Filter applications

Reference oscillators

- Radio communication, high speed data transfer
- High frequency, typically 26 MHz
- Low power consumption needed, typically 1 mA
- Moderate to high accuracy, typically +/- 20 ppm over consumer temperature band

Quartz resonator

Quartz crystal oscillator (10-40 MHz)

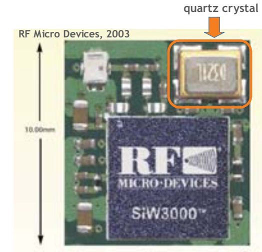


size: 10 mm x 10 mm x 2 mm
(in 2000, much smaller now!)

•advantages: well understood, low cost (<0.5 EUR), temperature stable to 1 ppm, long term stable to 0.1 ppm/year
•disadvantage: significant space consumer on circuit board!

Size disadvantage illustration
"Single-chip" Bluetooth radio

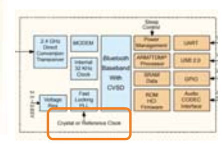
quartz crystal



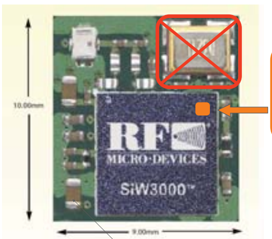
The IC can do almost everything

BUT

an analog, mechanical reference still needed for *frequency (or time)*!



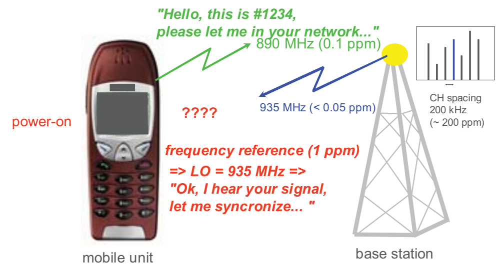
MEMS approach
Integrated reference and IC



Replace the bulky quartz crystal with an integrated MEMS-reference!

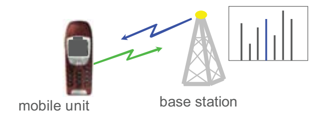
(the passives will disappear into the circuit board)

Why a frequency reference?



Why a frequency reference?

- To establish communication!
  - a fundamental need for wireless devices
  - especially important for portable devices
  - the more accurate the reference - the less scanning required (quick link start-up)
  - high-Q => low noise

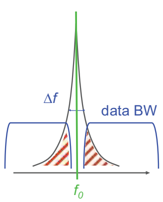


Silicon *micro*resonators - suitable for reference oscillator application?

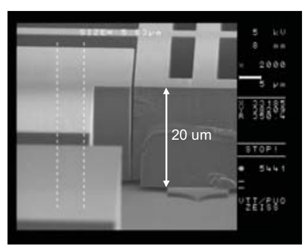
Critical issues to achieve in micro-size:

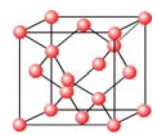
(1)Long-term stability
- material stability
- surface stability

(2) Noise performance
- spectral purification in transceivers (sensitivity and selectivity)



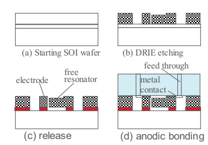
Long-term stability of silicon *micro*resonators (material stability)

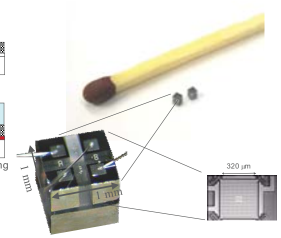




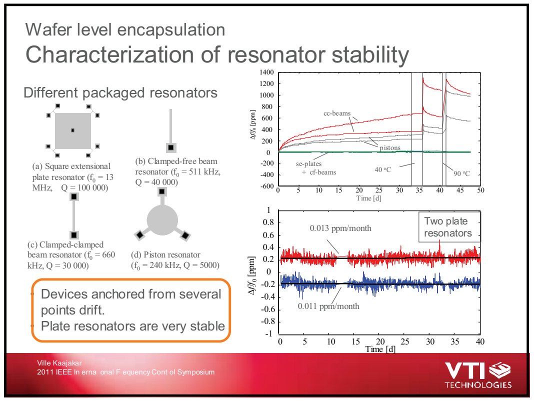
Diamond lattice => very stable material, suitable for reference application.

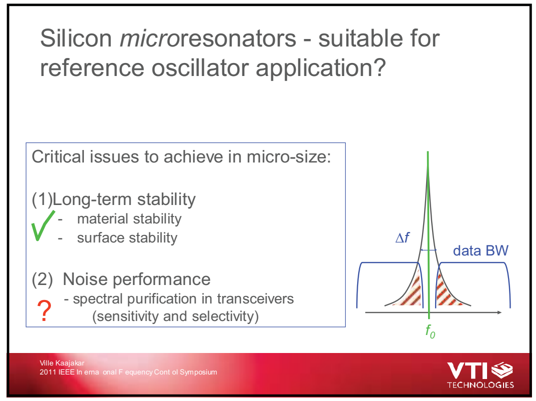
Wafer level vacuum encapsulation





Encapsulation process:
-feed throughs on glass cover (compact)
-hermetic sealing with anodic bonding (robust process)
-vacuum pressure 1 mPa


Wafer level encapsulation
Characterization of resonator stability
Different packaged resonators
(a) Square extensional plate resonator (f_0 = 13 MHz, Q = 100 000)
(b) Clamped-free beam resonator (f_0 = 511 kHz, Q = 40 000)
(c) Clamped-clamped beam resonator (f_0 = 660 kHz, Q = 30 000)
(d) Piston resonator (f_0 = 240 kHz, Q = 5000)
cc-beams
pistons
sq-plates
cf-beams
40 °C
90 °C
Time [d]
0.013 ppm/month
Two plate resonators
0.011 ppm/month
Time [d]
Devices anchored from several points drift.
Plate resonators are very stable
Ville Kaajakar
2011 IEEE In erna onal F equency Cont ol Symposium
VTI TECHNOLOGIES


Silicon *micro*resonators - suitable for reference oscillator application?
Critical issues to achieve in micro-size:
(1)Long-term stability
- material stability
- surface stability
(2) Noise performance
- spectral purification in transceivers (sensitivity and selectivity)
?
Δf
data BW
f_0
Ville Kaajakar
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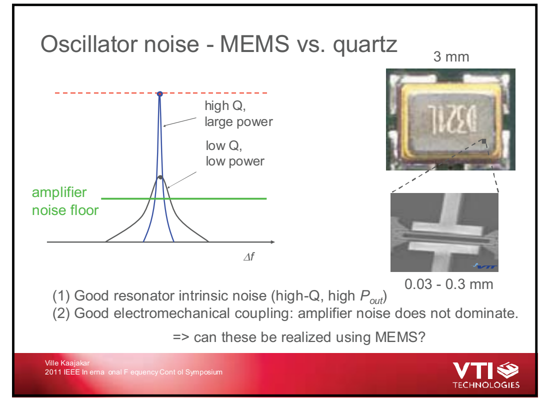
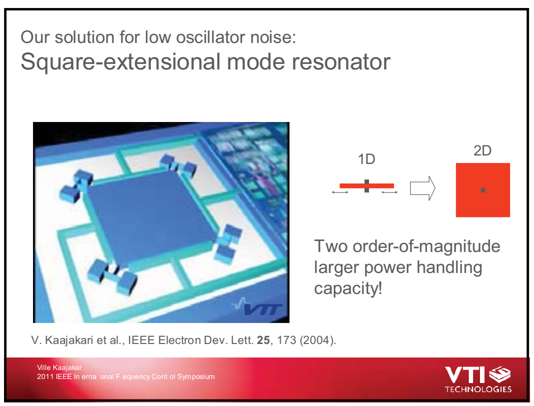
VTI TECHNOLOGIES


Oscillator noise - MEMS vs. quartz
3 mm
high Q, large power
low Q, low power
amplifier noise floor
Δf
0.03 - 0.3 mm
(1) Good resonator intrinsic noise (high-Q, high P_{out})
(2) Good electromechanical coupling: amplifier noise does not dominate.
=> can these be realized using MEMS?
Ville Kaajakar
2011 IEEE In erna onal F equency Cont ol Symposium
VTI TECHNOLOGIES


Our solution for low oscillator noise:
Square-extensional mode resonator
1D
2D
Two order-of-magnitude larger power handling capacity!
V. Kaajakari et al., IEEE Electron Dev. Lett. 25, 173 (2004).
Ville Kaajakar
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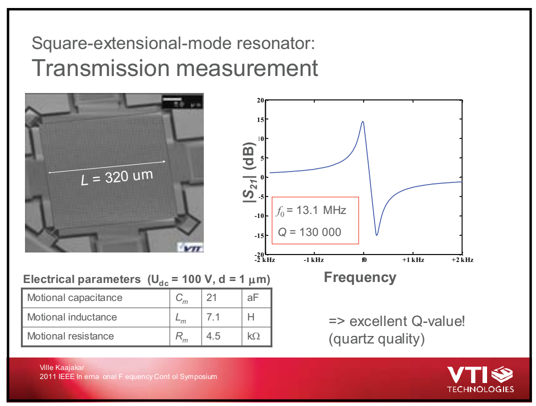
Square-extensional-mode resonator:
Transmission measurement
L = 320 um
$|S_{21}|$ (dB)
f_0 = 13.1 MHz
Q = 130 000
-2 kHz
-1 kHz
f0
+1 kHz
+2 kHz
Frequency
Electrical parameters (U_{dc} = 100 V, d = 1 μm)
Motional capacitance C_m 21 aF
Motional inductance L_m 7.1 H
Motional resistance R_m 4.5 kΩ
=> excellent Q-value!
(quartz quality)
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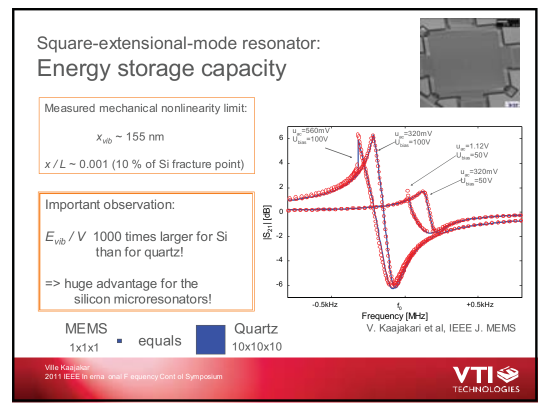
Square-extensional-mode resonator:
Energy storage capacity
Measured mechanical nonlinearity limit:
x_{vib} ~ 155 nm
x / L ~ 0.001 (10 % of Si fracture point)
Important observation:
E_{vib} / V 1000 times larger for Si than for quartz!
=> huge advantage for the silicon microresonators!
u_{ac}=560mV U_{bias}=100V
u_{ac}=320mV U_{bias}=100V
u_{ac}=1.12V U_{bias}=50V
u_{ac}=320mV U_{bias}=50V
$|S_{21}|$ [dB]
-0.5kHz
f_0
+0.5kHz
Frequency [MHz]
V. Kaajakari et al, IEEE J. MEMS
MEMS
1x1x1
equals
Quartz
10x10x10
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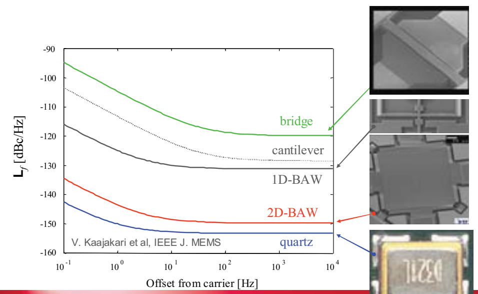
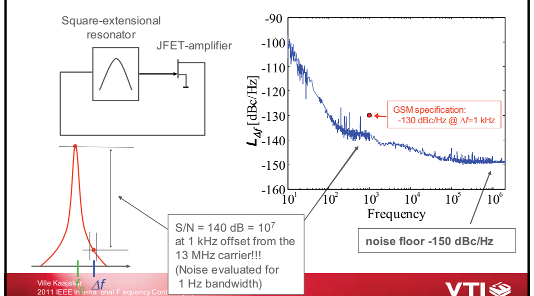
MEMS vs. quartz noise performance



Demonstrator:
Prototype resonator and off-the-self electronics

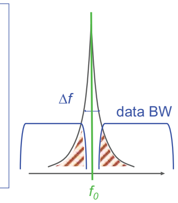


Silicon *micro*resonators - suitable for reference oscillator application?

Critical issues to achieve in micro-size:

(1) Long-term stability
- material stability
- surface stability

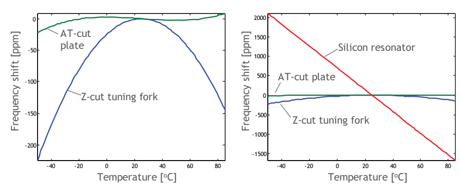
(2) Noise performance
- spectral purification in transceivers (sensitivity and selectivity)



Recap of stability and noise performance

- Microresonators susceptible to drift
  - Large surface to volume area
  - Careful packaging and good design needed to meet aging specifications (drift < 1ppm/year)
- Demonstrated quality factors compatible to bulk resonators
- Silicon BAW based oscillators can have phase noise comparable to quartz
- Next: the **"accuracy problem"**

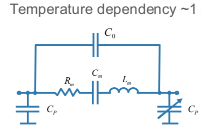
Accuracy challenge #1: silicon temperature dependency



Silicon Young's modulus changes about -60 ppm/K.
In comparison: quartz resonator can be more than 100x better.

TCXO

- TCXO = temperature compensated crystal oscillator
- Temperature measured and oscillator actively tuned with a series capacitor to cancel temperature dependency
- Temperature dependency ~1 ppm from -45 to 85 °C!



$$\omega_{osc} = \omega_0 \sqrt{1 + \frac{C_m}{C_0 + C_p/2}}$$

$$\text{range}: \omega_0 < \omega_{osc} < \omega_A$$

$$k_{eff}^2 = \frac{\eta^2}{kC_0} = \frac{C_m}{C_0}$$

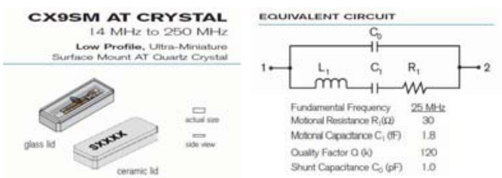
$$\omega_A = \omega_0 \left(1 + \frac{k_{eff}^2}{2}\right)$$

Maximum tuning range depends on electromechanical coupling coefficient!

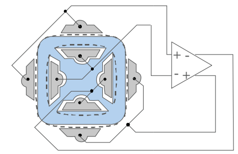
Quartz TCXO

- Uncompensated temperature drift ~ 20 ppm
- Coupling coefficient k_{eff}^2 ~ 1800 ppm
- Maximum tuning range ~900 ppm
  This is more than sufficient!

$$k_{eff}^2 = \frac{C_m}{C_0}$$



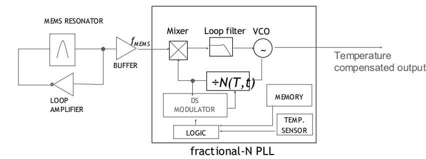
SiTime oscillator



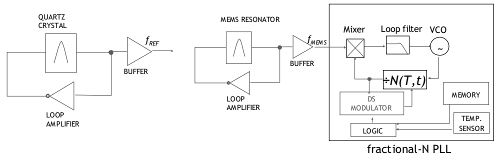
$$\eta = 7 \cdot 10^{-7}\ \mathrm{N/V}$$
$$k = 250\,\mathrm{kN/m}$$
$$C_0 = 0.1\,\mathrm{pF}$$
$$k_{eff}^2 = \frac{\eta^2}{kC_0} = 20\ \mathrm{ppm}$$

Silicon TCXO

- Uncompensated temperature drift ~ 3000 ppm
- Coupling coefficient k_{eff}^2 ~ 20 ppm
- Maximum tuning range ~ 10 ppm
  This is far from sufficient!
- Solution: compensate temperature digitally outside the oscillator loop with a phase locked loop

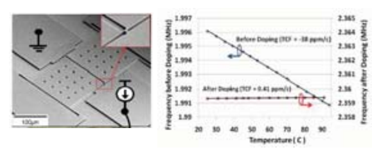


Silicon vs. quartz resonators



- MHz MEMS makes sense if fractional-N PLL is already in the system.
- Standalone MEMS oscillators have few advantages over quartz oscillators (smaller size and better shock performance).

Recent research:
Temp. compensation of silicon by doping



TEMPERATURE COMPENSATED SINGLE-DEVICE ELECTROMECHANICAL OSCILLATORS
Arash Hajjam, Amir Rahafrooz, and Siavash Pourkamali
MEMS2011

•Phosphorus doped resonators show reduced temperature dependency
•How sensitive the temperature dependency is to doping variations?

Accuracy challenge #2:
Manufacturing variations

- Typical manufacturing variations in lithography and DRIE processing >±100 nm
  - Flexural resonators with 3 um spring width have 3/2*Δw/w = 3/2*±100/3000 = ±50,000 ppm frequency variations
  - BAW resonators (L = 300 um) have ΔL/L = ±0.1/300 = ±300 ppm frequency variations
- Quartz resonators are trimmed to have initial accuracy better than ±50 ppm (typically ±20 ppm)
- MEMS manufacturing variations have to be compensated
  - physical trimming (difficult with wafer level processes especially after wafer level packaging)
  - electronic trimming (same PLL as used for temperature compensation)

Real time clocks (RTCs)

- A real-time clock (RTC) is a computer clock (most often in the form of an integrated circuit) that keeps track of the current time. Although the term often refers to the devices in personal computers, servers and embedded systems, RTCs are present in almost any electronic device which needs to keep accurate time.

Case for MEMS in RTC

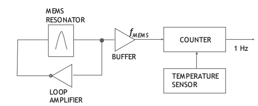
$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$k_{eff}^2 = \frac{\eta^2}{kC_0}$$

Can be ~10,000x smaller at kHz compared to MHz

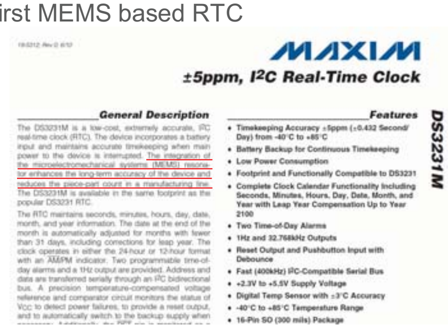
- Noise/jitter is not a major concern
  => Low signal level not a major concern
- Good electromechanical coupling at low frequencies
  => Easy to make a good oscillator
- Plenty of experience in kHz resonators (gyroscopes!)
  => Manufacturing processes readily available.
- kHz quartz is difficult to miniaturize
  => A real advantage over the incumbant

MEMS RTC architecture



- Low power MEMS oscillator core
- Temperature sensor to compensate the MEMS drift
- Temperature dependent counter to obtain 1 Hz pulses

First MEMS based RTC



MAXIM

±5ppm, I²C Real-Time Clock

DS3231M

General Description

The DS3231M is a low-cost, extremely accurate, I²C real-time clock (RTC). The device incorporates a battery input and maintains accurate timekeeping when main power to the device is interrupted. The integration of the microelectromechanical systems (MEMS) resonator enhances the long-term accuracy of the device and reduces the piece-part count in a manufacturing line. The DS3231M is available in the same footprint as the popular DS3231 RTC.

The RTC maintains seconds, minutes, hours, day, date, month, and year information. The date at the end of the month is automatically adjusted for months with fewer than 31 days, including corrections for leap year. The clock operates in either the 24-hour or 12-hour format with an AM/PM indicator. Two programmable time-of-day alarms and a 1Hz output are provided. Address and data are transferred serially through an I²C bidirectional bus. A precision temperature-compensated voltage reference and comparator circuit monitors the status of VCC to detect power failures, to provide a reset output, and to automatically switch to the backup supply when

Features

- Timekeeping Accuracy ±5ppm (±0.432 Second/Day) from -40°C to +85°C
- Battery Backup for Continuous Timekeeping
- Low Power Consumption
- Footprint and Functionally Compatible to DS3231
- Complete Clock Calendar Functionality Including Seconds, Minutes, Hours, Day, Date, Month, and Year with Leap Year Compensation Up to Year 2100
- Two Time-of-Day Alarms
- 1Hz and 32.768kHz Outputs
- Reset Output and Pushbutton Input with Debounce
- Fast (400kHz) I²C-Compatible Serial Bus
- +2.3V to +5.5V Supply Voltage
- Digital Temp Sensor with ±3°C Accuracy
- -40°C to +85°C Temperature Range
- 16-Pin SO (300 mils) Package

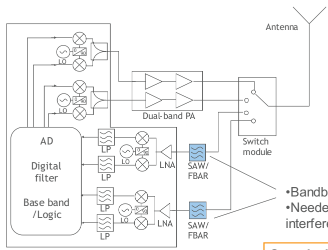
Recap of time references

- Quartz is a special material:
  - temperature dependency is almost zero
  - the remaining small dependency can be compensated by tuning the resonator. Good coupling makes this easy.
- Silicon has a HUGE temperature dependency
  - Meeting even XO performance requires complex compensation circuitry
  - Promising research on doping
  - Compensation of manufacturing variations at wafer level requires innovation
- MEMS is well suited for RTC applications
- Next: **Part 4: Filter applications**

Outline

- Part 1: modeling of microresonators
  - Lumped models for distributed resonator
  - Electrostatic vs. piezoelectric actuation
- Part 2: figures of merit
  - Quality factor
  - Electromechanical coupling coefficient
- Part 3: Oscillator applications
- **Part 4: Filter applications**

Typical dual band GSM transreceiver



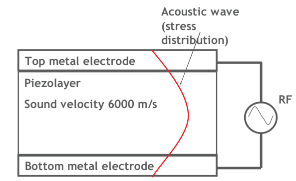
•Bandbass filters
•Needed to block interference before LNA

Complexity grows linearly with number of bands!

FBAR

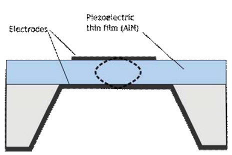
- Film bulk acoustic resonator (FBAR).
- Also known is Bulk Acoustic Wave (BAW) resonator.
- Thin film vibrating (mainly) in thickness mode.
- Excellent performance at 1 - 6 GHz.
- Used commercially as band pass filters in cell phones.

Operation principle of FBAR



- Electric field generates an acoustic wave. This wave resonates between the air interfaces.
- Resonance frequency = ½ velocity of sound / thickness.
- The sound wave may be either longitudinal or shear.

FBAR bridge resonator

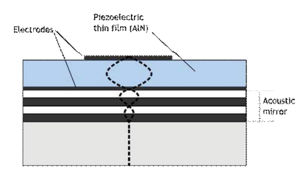


There is an air below the bridge.

Nearly perfect isolation.

Avago manufactures this type.

BAW mirror resonator

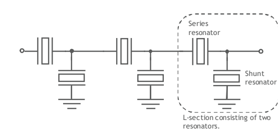


In a mirror resonator there is an acoustic interference mirror which reflects sound back to the piezolayer.

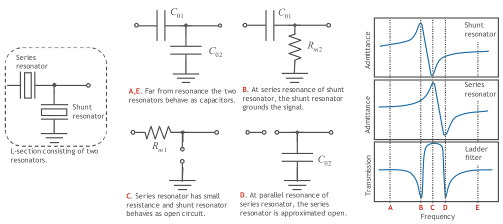
Lower Q due to losses but better power handling capacity.

Infineon and Epcos manufacture this type.

FBAR based ladder filter



Ladder filter operation



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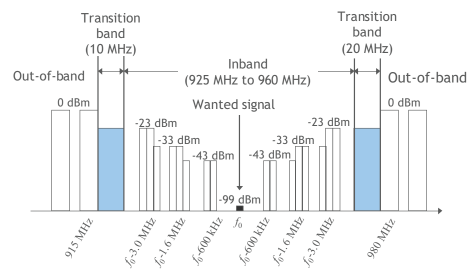
GSM receiver specifications



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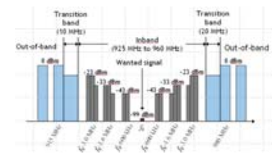
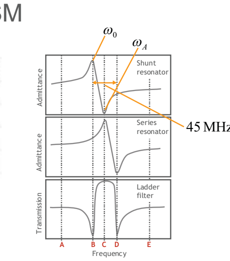
FBAR coupling for GSM





$$C_0 = \varepsilon_R \varepsilon_0 \frac{A}{h} = 3.1\,\text{pF}$$

$$C_m = \frac{\eta^2}{k} = 200\,\text{fF}$$

$$k_{eff}^2 = \frac{C_m}{C_0} = 6.5\%$$

$$k_{eff}^2 \approx 2\frac{\omega_A - \omega_0}{\omega_0} \approx \frac{45\ \text{MHz}}{940\ \text{MHz}} \approx 4.8\%$$

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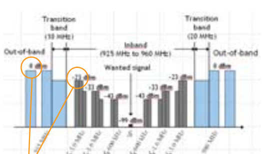
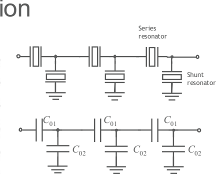
Out-of-band attenuation





We need at least -23 dB attenuation!

$$N = 4$$

$$C_{01} = C_{02}$$

$$|S_{21}|^2 = \left|\frac{C_{01}}{C_{01} + C_{02}}\right|^{2N} = -24\ \text{dB}$$

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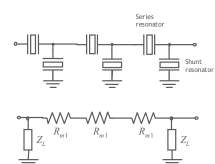
Insertion loss in FBAR filter



$$N = 4$$

$$Z_0 = 50\,\Omega$$

$$R_m = 0.84\,\Omega$$

$$|S_{21}|^2 = \left|\frac{2Z_0}{2Z_0 + NR_m}\right|^2 = -0.14\,\text{dB}$$

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FBAR performance

- Q > 1,000 at 1 GHz
- $R_m < 1\ \Omega$!
- Piezoelectric (no bias needed)
- Temperature dependency: ~-10-15 ppm/K (close to zero temperature dependency is possible at cost of other performance parameters).
- Excellent power handling capacity (~ 1 W).
- Frequency set by the film thickness. Difficult to obtain multiple frequencies on a single chip.

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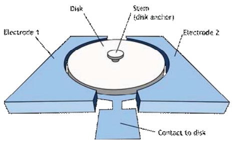
Silicon GHz resonators



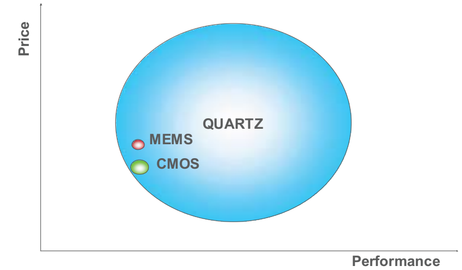
J.Wang, Z. Ren, and C.-C. Nguyen, “1.156-GHz self-aligned vibrating micromechanical disk resonator,” IEEE Trans. Ultrason., Ferroelect., Freq. Contr., vol. 51, no. 12, pp. 1607–1628, Dec. 2004.

- Electrostatic actuation (DC bias voltage needed)
- One-to-one replacement of SAW not demonstrated
- **Frequency determined by lateral (mask) dimensions – a filter bank possible**
- Demonstrated to 1 GHz with high R_m (over 1 MΩ). This challenge is fundamental.
- Coupling coefficient is low (<0.01%). This challenge is fundamental.

Recap of filter applications

- FBAR resonators have high electromechanical coupling coefficient. Needed for
  - Meeting the bandwidth requirements
  - Meeting the insertion loss requirements
- Electrostatically actuated silicon resonators at ~1 GHz have poor electromechanical coupling coefficient
  - Not suitable for current system architectures. Revolution in radio systems?
  - Piezoelectric coupling?
- Next: **Relax, we are done!**

Current MEMS timing market is miniscule - Future?



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