The Development of the:
Allan Variance,
Modified Allan Variance,
Time Variance, and
Other Relevant Science

David W. Allan

November 5, 2018

This presentation and accompanying transcript document a lecture that David Allan presented to Elizabeth Donley on November 5, 2018 at Dave's home in Fountain Green Utah. On November 6, 2018 we conducted Dave's Oral History. For those wanting to see more of his life and work, you will find his oral history at the following link: https://ethw.org/Oral-History:David_W._Allan.

When Dave received the Keithley Award in 2018 it had the citation: "For leadership in time determination and precise timing instruments."

Elizabeth: Today is November 5th, 2018. My name is Elizabeth Donley, and I'm with the National Institute of Standards and Technology in Boulder, Colorado. I'm here on behalf of the IEEE with sponsorship from the Ultrasonics, Ferroelectrics, and Frequency Control Society to conduct an Oral History with David Allan. He's welcomed me into his home. We're going to start by diving right into the technical aspects of Dave's career, or at least some of them. He prepared some slides to go along with the group of questions that I sent to him in advance of his Oral History interview. We'll come back later in a different session to record the Oral History itself, in which we talk more about personal things, his childhood and things like that.

So thank you very much, Dave, for preparing and presenting these technical slides that summarize your career and work.

David: Delighted. So let's go ahead and get started with the slides.

Fundamental Time-Domain Problem

- Classical variance or Standard deviation <u>does not converge</u> for several important noise processes in clocks, navigation, communication systems, and in nature.

- Classical variance <u>cannot distinguish</u> the different important <u>kinds of noise</u> and these have very different properties important to optimization, estimation, smoothing and prediction.

- These various kinds of noise are common.

David: So this slide basically takes us to a very fundamental time domain problem that really was the trigger for my Master's thesis back in 1965. You have this fundamental problem when you go to school and take statistics. You learn about the classical variance and standard deviation, and when you encounter clock noise it turns out that the classical variance does not converge for the kinds of noise we see. For example flicker-noise frequency modulation (FM), and you see this kind of noise all the time in nature, so being able to deal with flicker noise is a very important problem not only in time and frequency, but in analyzing many other natural processes. You see it; it's very ubiquitous in nature.

So what happens when you do a classical variance analysis of the clock, you end up not being able to distinguish the kinds of noise that are present. You end up with a value and an amplitude, so you don't really know what's going on with the clocks. And so this was a problem that we encountered back in 1965, which, Dr. Barnes, my great mentor and colleague put me on to in conjunction with my thesis.

David: For background, I share in the next slide a historical perspective so that you can see how things have moved from when I started at the bureau in 1960. First of all, because of the problem of characterizing these instabilities in quartz oscillators and atomic clocks, there was a special conference held at NASA Goddard, in Beltsville, Maryland, sponsored by the IEEE and by NASA, and inviting people to present papers and discuss this problem of how you characterize the short-term and long-term instabilities of these precision oscillators, with flicker-noise and other problems.

Jim Barnes and I presented a paper at that conference. Jim had developed a special auto-correlation function that was convergent for flicker noise, and he demonstrated that by using the second difference of phase he had solved the problem basically of how to deal with flicker noise. This paper was very well received. A lot of excellent comments were made at the conference around Jim's presentation.

David: The next year, Jim did his PhD thesis using these concepts, and I did my Master's thesis the same year to try and understand how the classical variance changed as a function of these different noise processes that we see in clocks. Both Jim's and my theses were published in a special issue of the Proceedings of the IEEE in February of 1966. That issue has a lot of really useful papers in it. Jim's and my paper were privileged to be a part of that set.

I learned later that my thesis in this publication is one of the most cited to ever come out of the bureau.

Elizabeth: Neat.

David: Then, we have the definition of the second coming forth in 1967, which is a fundamental step in the atomic clock community and for the world. In '71, Jim chaired an IEEE committee and they were asked to write a paper, characterizing frequency stability. This was the first IEEE documentation of how to characterize clocks using the two-sample variance that came out of

	my thesis, along with spectral density techniques that had been developed by the time and frequency community.
	In 1974, Byron Blair published NBS Monograph 140 and I share this because it has two chapters that I wrote. One on characterizing clocks and one on the time-scale algorithm that was developed in 1968 to combine the readings of atomic clocks. That has a very important basis for time-keeping at the bureau and so that Monograph is one place where you can find the code that I wrote. There were 92 lines of code in Fortran IV using a PDP-8S computer with eight kilobytes of memory. How we laugh at that today. Jim taught me how do an optimum weighting scheme for including the clocks, which was extremely useful.
Elizabeth:	I guess you had to be pretty efficient with your coding.
David:	I had to use some variables three times.
Elizabeth:	Well, that might be a good way to learn about time scales.
David:	So 1981 was another major breakthrough because there was a problem that I'll point out with the two-sample variance that we were able to solve. Jim and I published the paper at that time to share that breakthrough with the Frequency Control Symposium community. Later, I was asked to chair a committee on standards for the IEEE. This standard for characterizing clocks was published in 1988 and was accepted by the international community. It's been upgraded since. John Vig kindly sent me the most recent: "IEEE Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology—Random Instabilities" (IEEE Std 1139™-2008). The material in this tutorial will help when they do another update.
	In the late 1980s, the telecom community seeing what we had done for both time and frequency and for the navigation community, asked me if I would help them come up with a metric for telecom. Mark Weiss helped me, and we got a whole bunch of data from them and analyzed it in detail to see what might work as a good metric. Out of that, we developed the time variance, TVAR = $\sigma_Y^2(\tau)$. It's been a very useful metric that later was adopted internationally and by the telecom community, and was also shown to be a useful metric for the time transfer systems.
	This technical note 1337 on CHARACTERIZATION OF CLOCKS AND OSCILLATORS was edited by Sullivan, Howe, Walls, and myself and has been a valuable set of reference papers. We wrote a front section to give cohesion to the papers we had chosen to include. It was published in 1990.
Elizabeth:	Yeah, it was while Don Sullivan was Division Chief.
David:	It has been like a bible over the years.
Elizabeth:	Yeah, I was handed a copy twelve years later.
David:	Then over the years, I worked on the CCIR committee (International Radio Consultative Committee). Dick Sydnor and I were asked to pull together a handbook for the ITU. That came out in 1997 after I was retired. We were privileged to have some of the best authors in the world to write different chapters in that HANDBOOK. So it's also a useful resource document, but not very well known.
	I was asked also when I was consulting for Hewlett Packard to write *The Science of Timekeeping* Application Note 1289 for them, which you can find on the www.Allanstime.com website on my publication list. It gives lots of really fun data, how GPS works, as well as how time is kept in the world and how relativity fits in. We include the famous John Harrison chronometer that solved the Longitude problem for ships at sea. Neil Ashby actually coauthored this with me, along with Cliff Hodge from National Physical Laboratory in Teddington, England.
Elizabeth:	Is that a publicly accessible document, like in a PDF?
David:	Yes; at this link: http://allanstime.com/Publications/DWA/Science_Timekeeping/index.html
Elizabeth:	Okay.
David:	Then what Dave Howe and his group have done since I've retired has been extremely important in improving the confidence on the estimates for these time-domain measures. This is

particularly important for long term stability, which is very valuable for primary standards because long term data are expensive. So that work has been a fundamental contribution. Then Bill Riley who helped build the rubidium clocks for GPS also did this great work in developing the Stable 32 software for clock stability analysis. Many people around the world use his software for time and frequency stability analysis.

1965 year of 12-author paper comparing H, Cs; wrote AT1

- R.F.C. Vessot, H. Peters, J. Vanier, R.E. Beehler, D. Halford, R. Harrach, D.W. Allan, D.J. Glaze, C.S. Snider, J.A. Barnes, L.S. Cutler, and L. Bodily
- AT1 time-scale algorithm generates time for NBS/NIST; with improvements over the years, and is still ticking! Algorithm removes systematics and automatically weights each clock for optimum prediction of time.
  - software clock better than best clock
  - best clock cannot take over; is optimized
  - worst clock enhances output
  - robust: rejecting clocks with mal-performance
  - optimizes performance for both short-term and long-term stability performance

David: In 1965, there were several of the best atomic clocks brought to NBS Boulder for comparison. Len Cutler and Lee Bodily brought there Hewlett Packard commercial 5060-A cesium beam standard. Bob Vessot brought his hydrogen maser from Smithsonian, and Harry Peters brought his hydrogen maser from NASA Goddard. Roger Beehler had our NBS-3 primary standard for comparison. I was responsible for the measurement system and data analysis and we used a frequency stability program that I had developed. The program gave both the N dependence and the tau dependence, and it became apparent with all involved the value of the two-sample variance. This was the launch pad for comparing clocks showing sigma-tau diagrams with N = 2 and showing the difference in stability of all these different kinds of standards -- the best in the world. A twelve authored paper came out of that.

It helped me significantly as I was looking at the clocks and designing algorithms for AT1 to look at the white noise, the flicker noise, and then optimally combine those so that the long-term and short-term stability could be optimized using a sigma-tau diagram amongst the clocks.

Elizabeth: Did you guys already know at that time the limitations and advantages of the different types of clocks, or did you learn that through this study?

David: We didn't know. We learned as we saw the characteristics of the clocks became more and more apparent. On a single sigma-tau diagram you could see the white-noise FM (frequency modulation) and the flicker noise FM. Then typically over the years we observed random-walk FM for the long-term stability caused by environmental perturbations on a clock. That's why Len

Cutler's clock is so valuable because he was able to remove a lot of those systematics and environmental perturbation effects. For us, the time scale environmental chamber Howard Machlan built for our atomic clocks gave us environmental immunity and much better long-term stability.

So one can see the different kinds of noise processes for both short-term and long-term data analysis using the two-sample variance.

Elizabeth: Yeah, comparing different clocks to each other.

David: The goal is to remove temperature effects and other environmental perturbations. So you always want to look at what causes a perturbation and then remove it as best you can.

Elizabeth: Mm-hmm (affirmative)

David: So this algorithm in 1968 for me has some really nice features. The software clock is better than the best clock. The best clock cannot take over. It's optimized. The worst clock even enhances the output as well. It rejects outliers. It's optimized both for short-term and long-term stability. So it deals with the short-term white noise FM and it deals with the long-term flicker-noise FM as well the random-walk FM. These show up nicely in a sigma-tau diagram. As the characteristics of the clock change over time, the AT-1 algorithm breathes with those changes appropriately.

Using the standard deviation to analyze a data set is like taking a black and white picture of a rainbow; you get only intensity and no color.

The time-domain analysis techniques I will share give full color and allow for optimization in estimation, smoothing, and prediction for power-law spectral densities.

These tools have been extremely useful for the time and frequency community, as well as for the navigation and telecom communities and many others.

Black and white

All of the color is Lost

David: So to see the value of this approach of time domain analysis, I prepared this little slide, which I think gives a good feel. You have a full spectrum of rainbow colors. Observing the tau dependence of a clock allows you to know all of its different colors of noise, which you get from a sigma-tau diagram. Whereas if you compute a standard deviation or classical variance analysis of the data, all you end up with is a black and white picture. You get intensity, but you don't know what you have. You don't know what's going on in the clock. So by having all of these

different colors, you can see the difference between environmental effects, measurement noise, clock noise, etc. So the two-sample variance has been a powerful tool.

> ## The Techniques I will share allow you to determine both the level and kind (color) of noise
>
> - Optimum estimation, smoothing and prediction depend upon the kind (color) of noise.
>
> - Power-law spectra are very common in nature, and the transforms from time-domain to frequency-domain and visa-versa have been developed as well.
>
> - Here is a link with the transform information details: http://www.allanstime.com/Publications/DWA/Conversion_from_Allan_variance_to_Spectral_Densities.pdf
>
> - Using these time-domain techniques allows one to design optimum instrumentation.
>
> - I will illustrate with time and frequency experiments and then extrapolate to some other applications.

David: So a sigma-tau diagram allows us to determine both the level and the kind (color) of the noise. It also allows us to do optimum estimation, smoothing, and prediction. So once you know what's going on with the clocks, then you can do algorithms to do estimation and optimal smoothing of the data. For GPS, that's extremely important because they go across a tracking station in twelve-hour orbits, and they have to predict time forward several hours before they cross another tracking station, and to get another update for the on-board clock parameters. So predicting forward is extremely important for GPS.

Power law spectrum are very common in nature, and the transforms for these have been developed as well to the time-domain and vise-versa. Jim Barnes, Len Cutler, and several others have worked on these over the years. Tables exist so that we have a very nice, what I call, a super-fast Fourier transform for these power-law spectral densities which well model many natural processes – especially atomic clocks and precision oscillators.

I was asked by folks at Northrup Grumman, where I gave an invited talk, to write this publication to do the conversions between time-domain and frequency-domain. It's available as a PDF. It is basically the work of lots of scientists who have developed these over the years.

The other thing it allows you to do, once you know the characteristics of a system, then you do optimum instrumentation. You can develop better GPS receivers. You can develop better measuring systems for your clock. You want to make sure the measurement noise is better than the clock noise, for example.

It seems the only thing that is constant is change!

- Emeritus Professor of Philosophy, Chauncey C. Riddle, has the best definition of time that I know: "Time seems to be the possibility of <u>change</u>."
- As I will show you, the two-sample or Allan variance, as it is called, is the optimum estimator of "<u>change</u>" in frequency of an ideal atomic clock.
- Similarly, TVAR is the optimum estimator of "<u>change</u>" in an ideal measurement system or time transfer system.

David: Here I have a great quote from a dear friend, Chauncey Riddle, who was a philosophy professor for forty years at BYU. He wrote the foreword for my book and is one of the brightest professors I've ever known. He defines time better than anyone I know: "Time seems to be the possibility of change." I think that that has profound significance, since everything is changing. Chapter 25 of my book basically works around his concept and shares with others some ideas on what really is time. There's a lot more that we can say. Time, as generated by humankind, is actually time-interval – an accumulation of intervals from some origin. That was my job for NBS/NIST for most of my 32 years in Boulder, CO, to generate time for the USA.

The point of this concept of change is that as you look at the two-sample variance in metrology for an ideal atomic-clock it is the optimum estimator of change in a variance sense from one sampling time interval to the next. You cannot do better in terms of measuring the white-noise FM.

Similarly, when we developed the time variance for the telecom community, one can also show that it is an optimum estimator of change in an ideal time measurement system in a variance sense. So if one is characterizing a time measurement system, a time distribution system, or a telecom network, this is the ideal measure of change if you have white PM, which is the ideal noise for such systems.

Furthermore, flicker-noise (1/f spectral density) is ubiquitous in nature

- Observed in semi-conductor junctions, resistors, neurons, music, traffic flow, stock-market values, the height of the River Nile at flood stage over 2000 years, in a mountain-spring flow, auto-pollen microscope focusing, etc.

- Taking the classical variance as the integral of the spectrum, for flicker or 1/f noise, the integral is infinite at both limits (0 and ∞)

- There is always a high frequency limit, f_h, in a measurement, but no-one has observed a low-frequency limit for these 1/f processes.

- 1/f noise is right on the edge of convergence; it reminds one of free-choice.

David: Flicker noise, being so ubiquitous in nature, here are some examples. You see it in semiconductor junctions, resistors, neurons, music. As an unusual example, the river Nile at flood stage, is modeled by a flicker noise process over the two thousand years for which data are available. We have a Big Spring that provides water for the community where we live as well as for irrigation. I obtained measurements for the annual flow over fifty-two years and the flow is well modeled by flicker noise. I attended an international conference in 1977 in Tokyo, where people came from all over the world talking about flicker noise and where it is observed. It was a really fun conference. I was asked to come and talk about flicker noise in atomic clocks. Professor Musha, who invited me from the University of Tokyo, shared how traffic flow is a flicker-noise process.

So for flicker noise, calculating the classical variance as the integral of the spectrum, the integral blows up at zero and infinity. The high frequency is not a problem, of course, because it is limited by the measurement system bandwidth. But the low-frequency is a problem for 1/f spectral densities. One can show that its divergence is logarithmic, and that if instead of 1/f, one had f^-0.9999... then the integral converges. It's almost like free choice.

So the classical variance grows without limit for flicker noise and also has no well-defined mean value.

> # Flicker-noise not only has a variance that grows without limit, but it also has no well defined mean value.
>
> - In 1964, James A. Barnes derived an auto-correlation function that works for flicker noise.
> - Also in 1964, he showed that the time average of the second-difference of the time deviations gave an efficient measure for 1/f noise.
> - 1965, based on Dr. Barnes work and Sir Michael James Lighthill's book, I wrote my master thesis – deriving a variance that characterized the level and kind for a useful variety of colors of noise processes including 1/f (pink noise).

Elizabeth: Did people worry about this very much before you started thinking about it for atomic clocks?

David: They did. Yes. Atkinson and Fay at NBS wrote a paper about it. They saw flicker noise in quartz-crystal oscillators, which were our best timekeepers before atomic clocks, and they said, "What are we going to do to measure flicker-noise." So mathematically, they worried and were concerned about what could be done to have a good time-domain measure of this noise, since the standard deviation did not converge and data with such a noise model had no well-defined mean value.

My wife asked me to describe flicker noise, and I did my best. It's in my book, and for those who have a little bit of math, I have an article on the book's web site www.ItsAboutTimeBook.com that describes "Natures Natural Noise Processes -- Flicker-Noise and Timing Errors in Clocks:" So the work that Jim did is very fundamental. He derived this special auto-correlation function, which I used in my thesis as well that works for flicker noise. In 1964, he presented these results at the IEEE NASA conference. In '65, we both used this special auto-correlation function in our theses and were able to solve this flicker-noise convergence problem.

Sir Michael James Lighthill,
(23 January 1924 – 17 July 1998)

- He was a British applied mathematician, known for his pioneering work in the field of aeroacoustics. (jet engine silencing)
- Held Lucasian Professor of Mathematics or Newton's chair before Stephen Hawking
- He said of Hawking's cosmology, "To me, it is not quite science, but more like creation myth."
- His book, *An Introduction to Fourier Analysis and Generalised Functions* (Cambridge Monographs on Mechanics) was key to my master's thesis, p.43! Standing on the shoulders of Giants!

David: Also, my thesis would not have been possible had not Jim given me the book by Sir Michael James Lighthill. Lighthill has become one of my heroes. He was a brilliant applied mathematician. He solved the problem of jet silencing sufficient so that airports could be close to cities. It doesn't work above Mach 1, so military aircraft are often a lot louder. He held the Newton's Chair before Stephen Hawking. Lighthill's book for me is a classic: *Introduction to Fourier analysis and generalized functions*. The chart on page 43 of that book I have turned to probably over a hundred times Very very powerful. It is his derived transform relationships that allows us to go from the time-domain to the frequency-domain so elegantly -- making what we often call a super-fast Fourier transform.
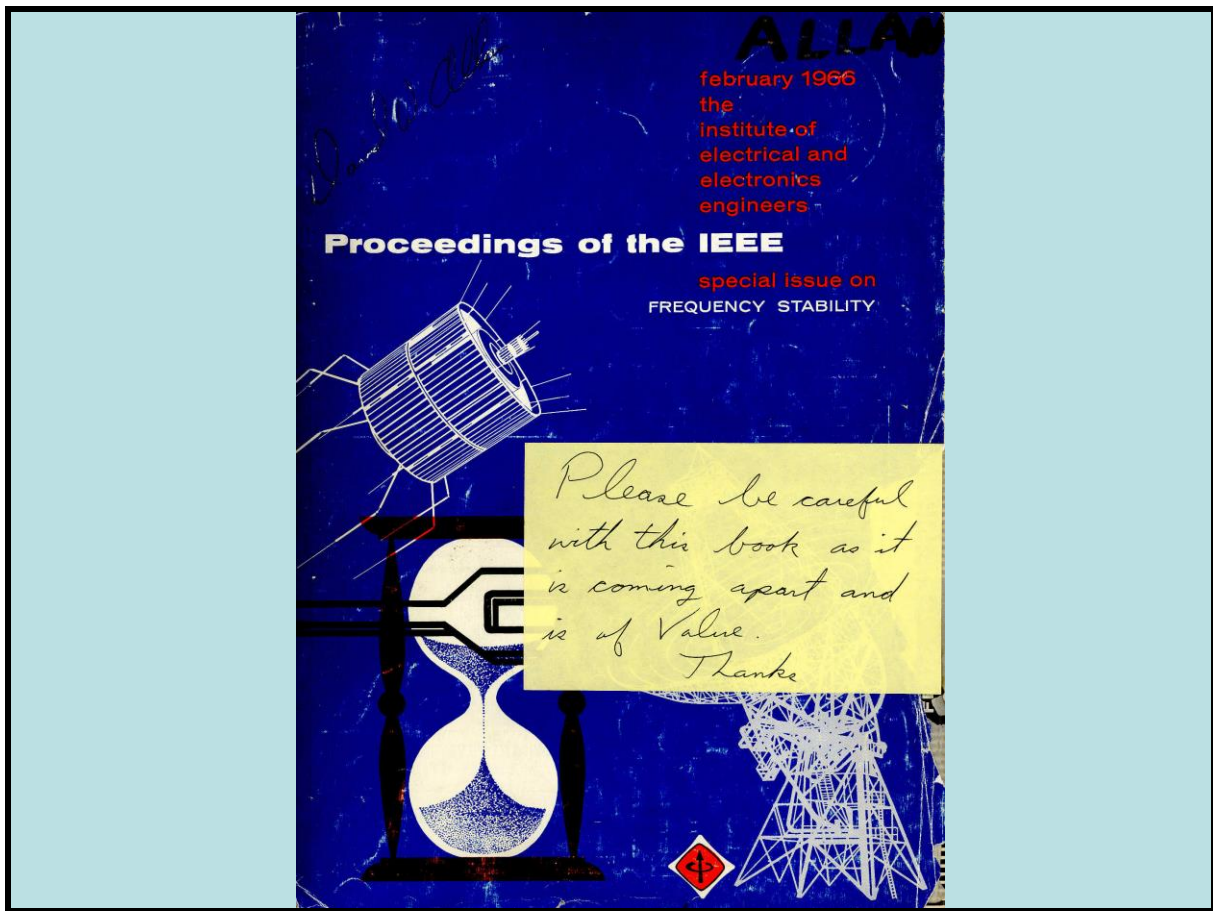
David: So this slide shows the cover of the 1966 Special Issue of the Proceedings of the IEEE on "Frequency Stability," which contains our two theses and several papers coming out of the 1964 NASA/IEEE Beltsville, Maryland conference on short-term stability. This is the publication of my thesis that IEEE/UFFC celebrated the 50th anniversary in 2016 at the International Frequency Control Symposium in New Orleans.

Fractional frequency $y(t) = \frac{v(t) - v_o}{v_o}$

- $x(t) = \frac{\varphi(t)}{2\pi v_o}$
- **So phase modulation, PM, is the same as time modulation with a scalar multiplier.**
- **And white-noise FM is the same as random-walk-noise PM, because:**
- $x(t) = \int_0^t y(t') \, dt'$
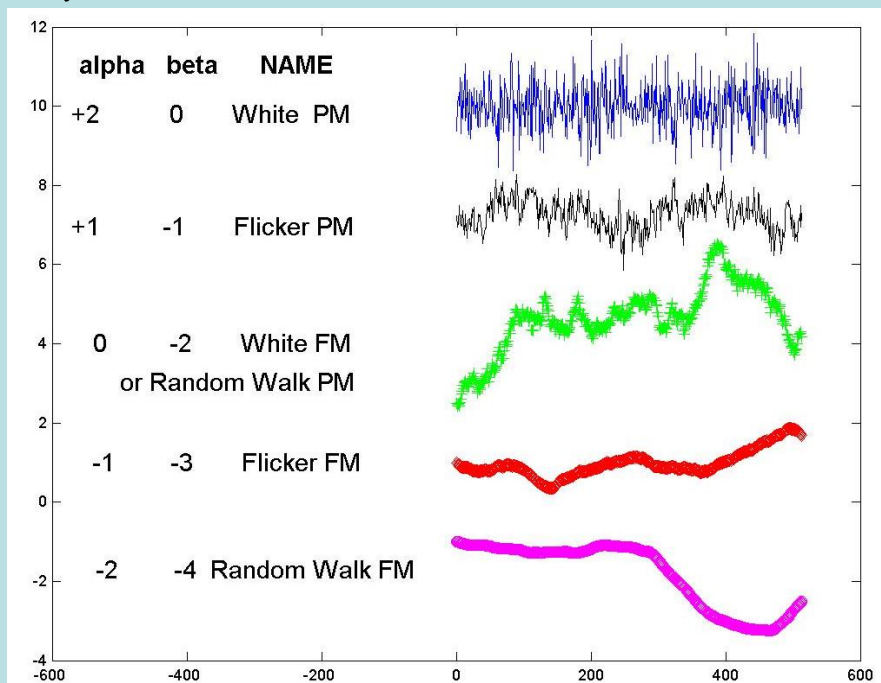- This is exactly the noise of an atomic clock

David: In this next slide is illustrated the fractional-frequency stability and is denoted by y(t). The Greek letter $v(t)$ denotes the clock's frequency, and v_o denotes the ideal frequency. We normalize y(t) by setting it equal to $[v(t) - v_o]/v_o$. So if I have a perfect standard, then y(t) is zero, which you never have. That's the point of characterizing change. You always have change. So the variations in y(t) allow us then to look at what's going on in a clock averaged over different averaging times, tau. You can write the time-error of a clock as x(t). And x(t) = the phase error, phi(t), divided by $2 \pi v_o$. So we see that the phase modulation is the same as the time modulation with a scalar multiplier. And white-noise FM is the same as random-walk-noise PM, because x(t) is the integral of y(t') over the interval zero to t. This integral relationship is generally true. And we know that white-noise FM is exactly the noise of an ideal atomic clock. We have found the definitions on this slide to be extremely useful in time and frequency metrology.

POWER-LAW SPECTRA:

$$S_y(f) \propto f^\alpha \qquad\qquad S_x(f) \propto f^\beta$$

alpha	beta	NAME
+2	0	White PM
+1	-1	Flicker PM
0	-2	White FM or Random Walk PM
-1	-3	Flicker FM
-2	-4	Random Walk FM

David: This next slide gives a real good feel, for the visual character of the different kinds of noise that we see in clocks. You see that the eye can be a good spectrum analyzer. Looking at a time-series of a data set can often tell you a lot. On the top-left is denoted the spectral density of y and is proportional f^α, and on the top-right is denoted the spectral density of x, which is proportional to f^β.

From the previous slide, because x(t) is the integral of y(t'), we see that alpha = beta + 2. We have found these five power-law noise models very useful in time and frequency metrology and in other disciplines as well. As one looks down the chart, you see the low-frequency energy increasing, and the more divergent is the timing error. We see white PM for the short-term stability of active masers and quartz-crystal oscillators. Because of the semi-conductor junction noise in a quartz-crystal oscillator, we see flicker PM in the short-term as well. White FM, is the noise of an ideal passive atomic clock. Because of the excellent work of Len Cutler and his team at HP in removing the systematics, we have observed white FM over six decades of tau for those standards. We next see a model for flicker-noise FM, which causes what is often called the flicker-floor on a sigma-tau plot. The last noise depicted is for random-walk FM, which is often caused by environmental effects on the clocks.

So these are the different kinds of power-law spectral density processes that well model precision clocks. They are also often useful models in many other natural processes. For example, the navigation and telecommunications have taken advantage of the measures we have developed to characterize the instabilities in their systems, and the 50th anniversary celebration of the publication of the Allan variance produced a special publication of the

IEEE/UFFC illustrated many other disciplines benefiting from the use of this metric and the noise models.

We can make a natural law regarding flicker noise. There is no perfect clock, and on a sigma-tau (ADEV) diagram, there will always be a flicker floor --a level of Flicker-noise FM limiting the clock's performance the result of which will be from natural phenomenon. For longer averaging times, tau, we often see random-walk-noise FM, which is often caused, as mentioned, by environmental influences on the clock.

My colleagues have also derived an equation, which gives the law of potential clock stability, which is ADEV(τ) = Δv/[v Signal/Noise] where Δv is the quantum transition line-width at a transition frequency v. The Signal to Noise ratio is improved as the square-root of Nτ, where N is the number of atoms or molecules available for the measurement per second and τ is the averaging time for the measurement in seconds. From this law one can see immediately why optical clocks have such an advantage over microwave clocks with the frequency, v, being about 100,000 times higher. From this law, we also see why the cesium fountains were a major improvement because Δv could be made much smaller because of the much larger interrogation times as compared to thermal beam cesium-beam standards. The interrogation times went from the order of a millisecond down to about a second.
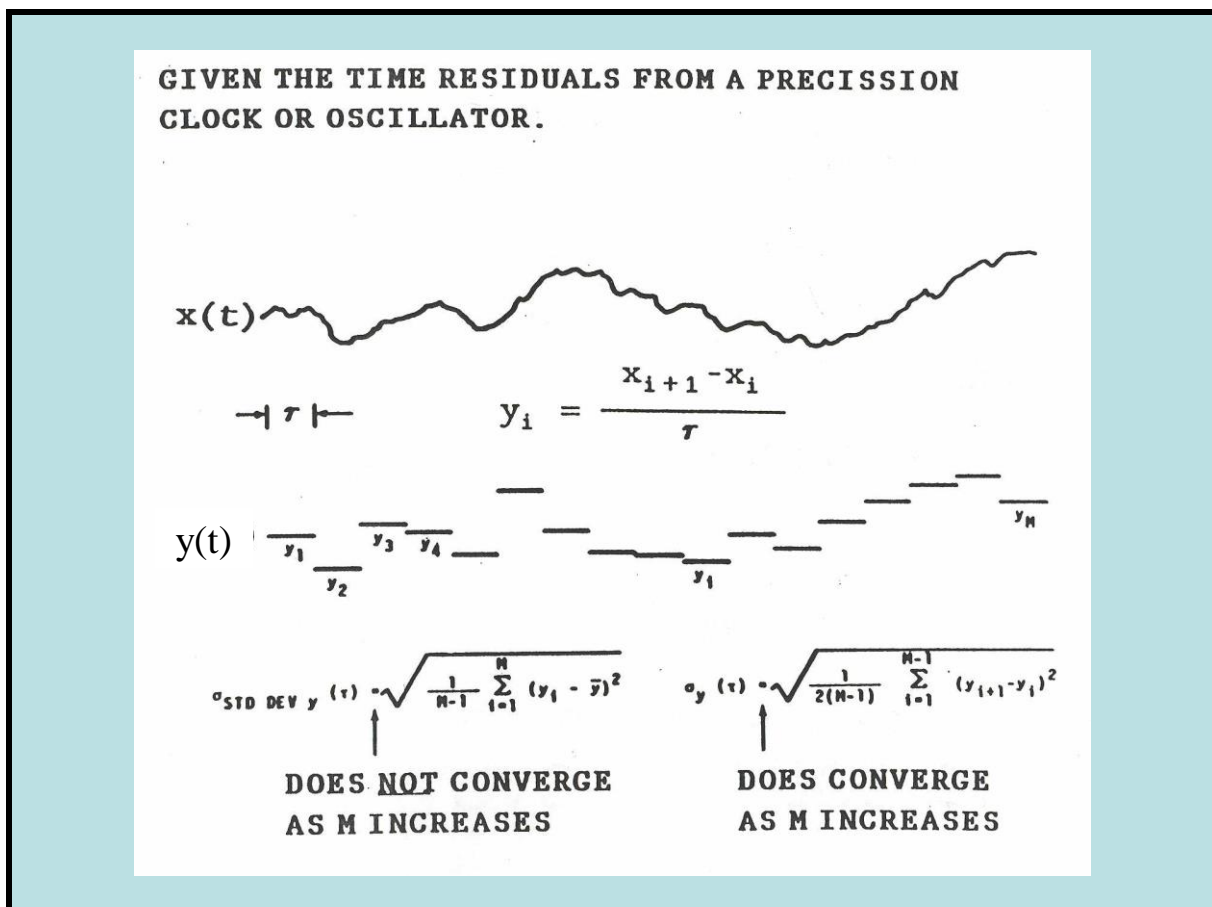
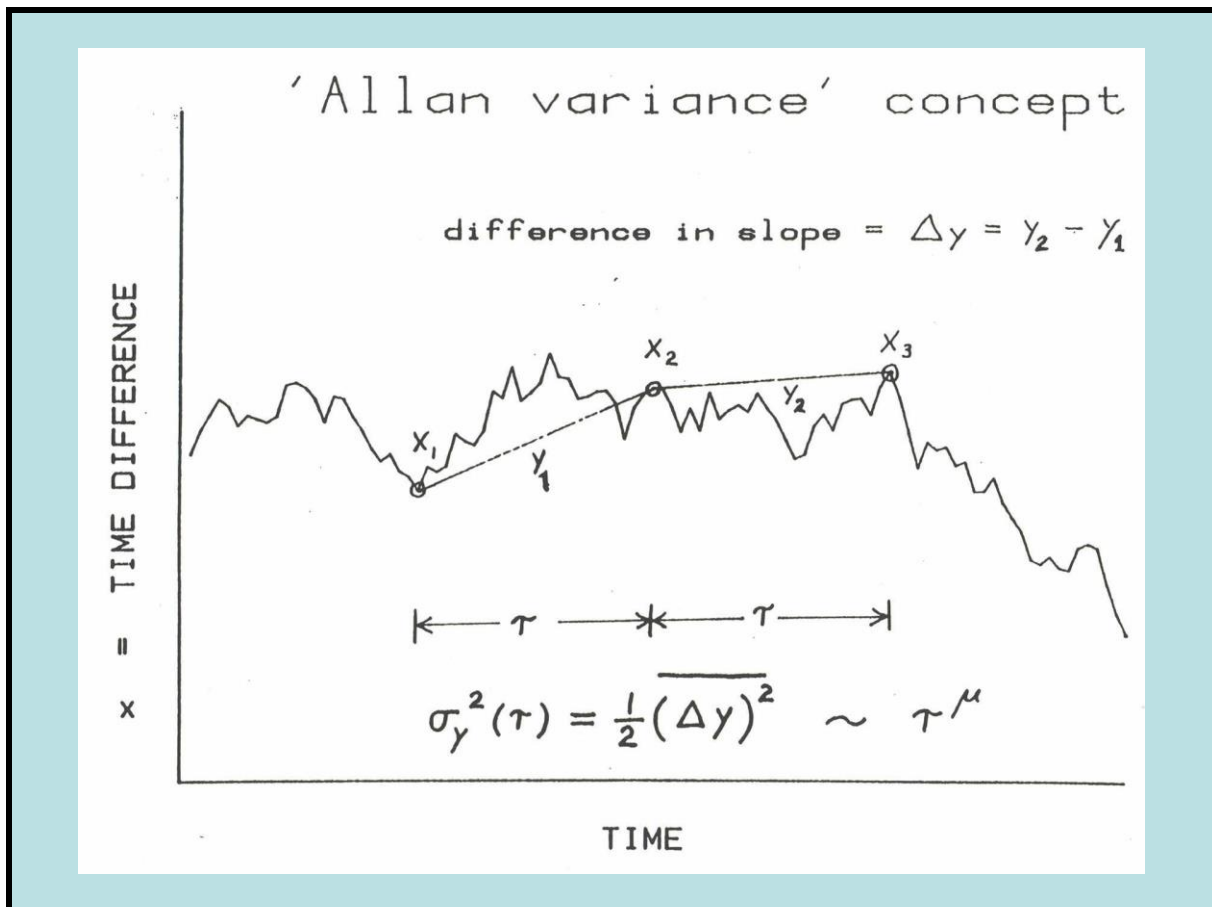David: This next slide is a pictorial of the time error of a clock denoted as x(t); we also depict a sample time τ, and we denote the ith fractional frequency measurement yi = (x(i+1) - x(i))/τ. The y(t) plot below the x(t) plot denotes the fractional frequencies measured, each over an interval τ, for this pictorial. For illustrative purposes, let us assume x(t) is a flicker-noise FM process. We

can compute the standard deviation of this data set as shown, and we can also compute ADEV(τ) for this same data set. And we see that the standard deviation (the square-root of the classical variance for N samples) does not converge for two reasons; flicker-noise FM does not have a well-defined mean value and the standard deviation grows logarithmically as N increases. While ADEV(τ) not only gives us a convergent metric, but also the level of the noise as well as the kind or color of the noise as a function of τ.

You can never measure instantaneous frequency, because τ can never be zero, and there will always be some measurement bandwidth limiting the sampling window for the data.


David: This slide is a pictorial of the 'Allan variance' concept. It shows a simulated clock's time error, x(t). We show an example of three-time error measurements, x1, x2, and x3, taken at τ interval spacings. We show the fractional-frequencies computed, y1 and y2, for these two intervals, and we show the "difference in slope", $\Delta y = y2 - y1$. Now we compute the average of the squares for all the Δy's having the sampling interval τ and divide by 2. This gives us a value for AVAR(τ) for this data set. Then if we do this for all τ values available and notice how AVAR varies with τ on a log-log plot of AVAR vs. τ, then the slope will give us the color of the noise, and we also have the intensity of the noise as well. My family and friends asked me to write an article in lay-person's terms as to what is the Allan variance: https://itsabouttimebook.com/allan-variance-in-laymans-terms/.

The ratio on the ordinate is called the bias function, $B_1(N)$. We are using the divergent nature of the classical variance to **ascertain** kind (color) of the noise, and to know if it is convergent:
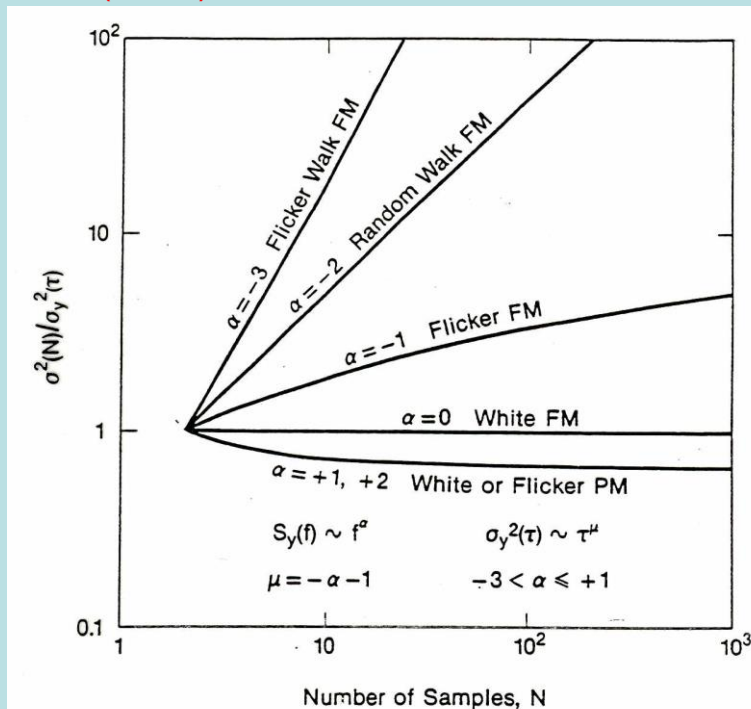
David: In the next slide, we normalize the classical variance for N samples by dividing by AVAR(τo), where τo is the minimum sampling interval. As we plot this ratio against N we see how the classical variance diverges with the different kinds of noise processes and we also see where it is convergent. For flicker-noise FM, the divergence is logarithmic. For random-walk FM the divergence goes as N, and for flicker-walk FM it is proportional to N squared. We have an interesting phenomenon if there is dead-time in the data, then this ratio is 1 for both white-noise PM and for flicker-noise PM. This phenomenon is explained in a paper that Jim Barnes and I wrote in 1990, NIST Technical Note 1318, "Variances Based on Data with Dead Time Between the Measurements." The zero-dead time appears like a delta function, as shown in TN 1318.
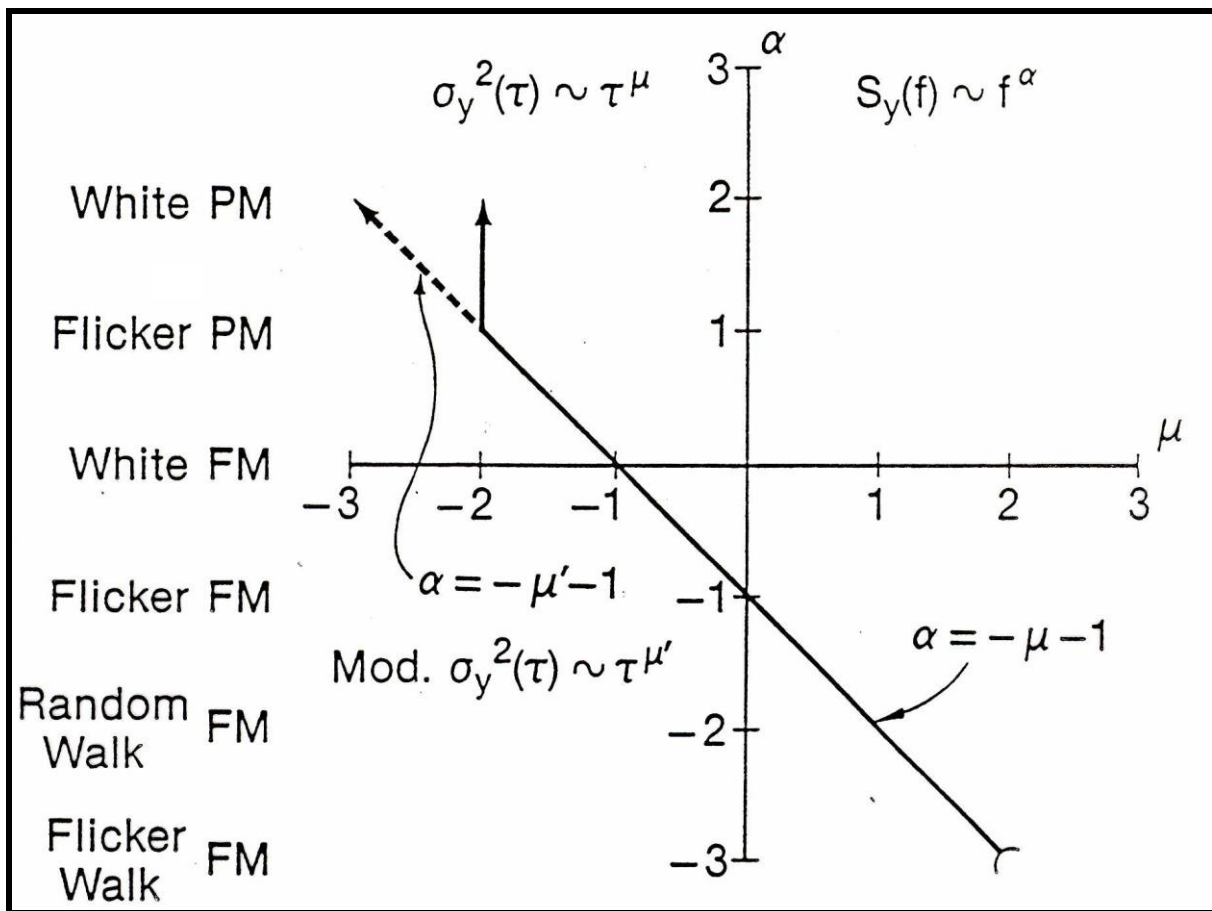
David: The next slide shows the simple and elegant relationship between the time-domain and the frequency-domain as obtained from page 43 of Lighthill's book. If AVAR is proportional to τ^mu and the spectral density is proportional to f^alpha, then we may derive that alpha = - mu - 1, and we have this super-fast Fourier-transform relationship from -2 < mu < 3. At mu = -2, we have the ambiguity problem for AVAR; there alpha is greater than or equal to 1. We solved this ambiguity problem 16 years later in 1981 with the Modified Allan variance -- obtaining alpha = - mu' - 1 for all useful values of mu' modeling clock noise: -4 < mu' < 3.

When we discovered this ambiguity problem back in 1965, Bob Vessot, who was the reader for my thesis for the 1966 Proceedings saw that I had a mistake in my master's thesis at CU as published, there. Jim and I went back to Lighthill's book and sorted this out to get the dark line plotted in this graph. Bob was especially interested in this problem, because of the excellent short-term stability of his hydrogen masers having white-noise PM modulation. He knew and we knew that this kind of noise was bandwidth dependent. Even though we had the ambiguity problem for AVAR at mu = -2, one could differentiate between white-noise PM and flicker-noise PM by modulating the bandwidth, but that is a nuisance. It wasn't until 1981 that we discovered how to do software bandwidth modulation and developed the Modified Allan variance, as shown in this slide, which has wider spectreal properties and covers all the noise processes well modeling precision oscillators as well as time and frequency time-transfer systems.

SINCE

$$\sigma_y^2(\tau) \sim \tau^\mu,$$

THEN

$$\sigma_y(\tau) \sim \tau^{\mu/2}.$$

THE LOG OF THIS IS
$$LOG(\sigma_y(\tau)) \sim \mu/2 \; LOG(\tau);$$

THEREFORE, THE SLOPE ON
A LOG LOG PLOT OF
$\sigma_y(\tau)$ VS. τ IS $\mu/2$

SINCE
$$\alpha = -\mu - 1,$$
$$(-2 \le \mu < 2)$$

WE CAN DEDUCE THE
SPECTRAL TYPE OF
NOISE KNOWING α

David: As shown in the next slide, the typical procedure since AVAR is proportional to τ^mu, is to take the square-root of AVAR -- giving you ADEV, and then make a log-log sigma vs. tau plot of the analyzed data. Since the slope on a log-log plot is mu/2, we can use that to ascertain the kind of noise present along with the level of noise for the clocks being analyzed. And knowing mu then allows us to know alpha, since alpha = - mu - 1, and thus deduce the spectral density for power-law spectral densities. It is important to realize that when we compare two independent clocks, the resulting sigma-tau plot is the square-root of (AVAR of clock 1 + AVAR of clock 2).

FREQUENCY STABILITY
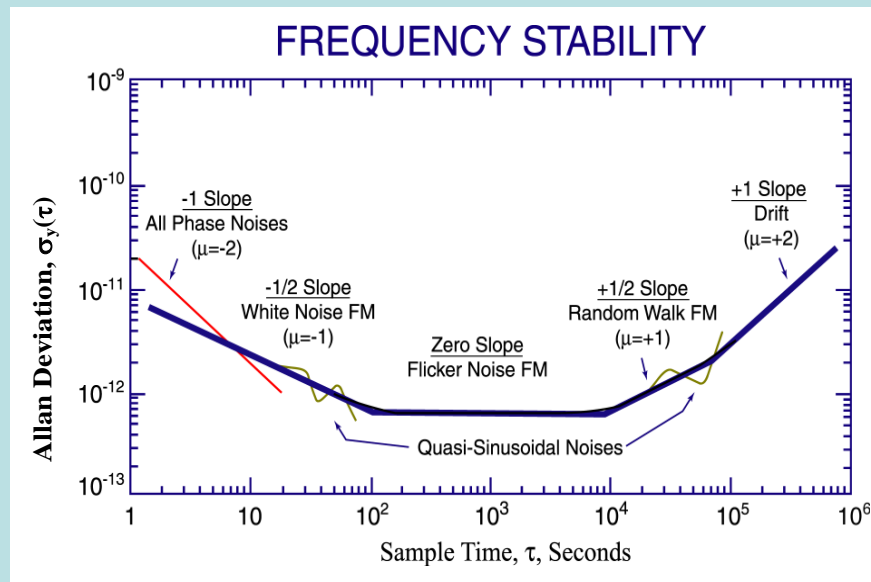
23

David: In this next ADEV plot (sigma-tau diagram), we have a typically occurring bath-tub like curve resulting from the different power-law spectral density processes often used in modeling atomic clocks. The dark line plotted on the left represents the white noise FM coming from a passive atomic clock. So we see the $\tau^{-1/2}$ slope. The next segment is the zero-slope flicker-noise FM region of tau values, sometimes called the flicker floor. The third segment is the model for random-walk FM -- giving rise to a $\tau^{+1/2}$ slope. The last segment is what one observes if there is frequency drift in the data giving a τ^{+1}. This same slope will occur if there is flicker-walk noise in the data, alpha = - 3, if there is no frequency drift in the data then. If there is frequency drift in the data the sigma-tau values will fall very close to a line with slope of τ^{+1}, whereas, if the noise is modeled by flicker-walk then the sigma-tau values will vary around that slope. The red line on the left is what one observes if we have flicker-noise PM or white-noise PM on an ADEV plot and illustrates the ambiguity problem at mu = -2.

QUARTZ OSCILLATOR (DIANA) VS COMMERCIAL CESIUM (#601)
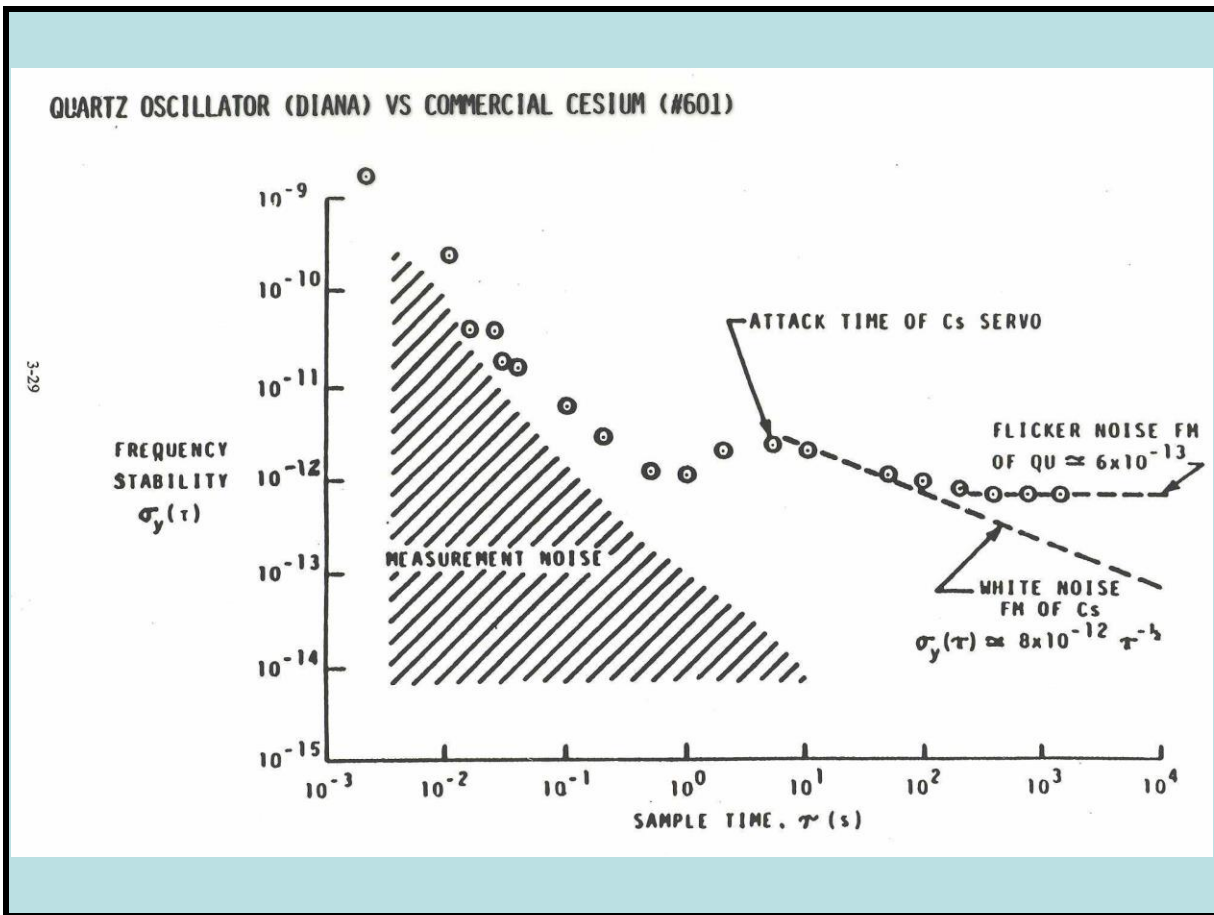
David: This next figure is an analysis taken back in the 1960s showing the value of a sigma-tau plot in comparing a high-quality quartz-crystal oscillator with a commercial cesium-beam frequency standard. The τ^{-1} like slope on the left of the diagram is the measurement noise. Next we see the response of the cesium-beam servo lock-loop with an attack time of about 10 seconds. Next we see the white-noise FM, $\tau^{-1/2}$, coming from the cesium-beam standard. Lastly, we see the flicker-noise FM of the quartz-crystal-oscillator.
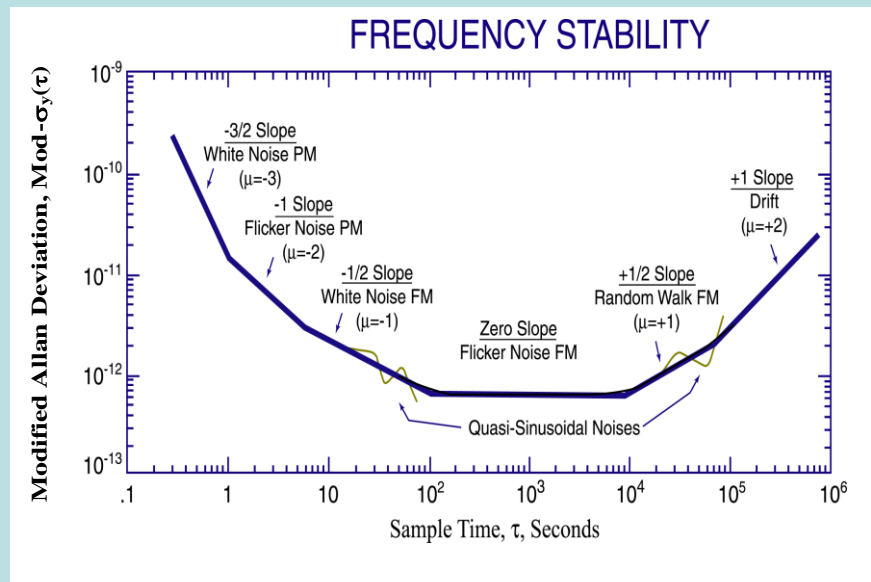
25

David: The next slide shows the benefits of an MDEV plot for distinguishing between white-noise PM and flicker-noise PM, where the slopes go from τ^-3/2 to τ^-1, respectively. This work was triggered in 1981 when Dr. James J. Snyder came out to JILA from Gaithersburg, Maryland to do studies on short-term stability of lasers. He invited Jim Barnes and I to go over to JILA and see his results. He was able to get a τ^-3/2s slope on a log-log sigma-tau diagram for the white-phase noise by averaging the phase for his laser data.

Equation 34 in my 1965 Master's thesis shows bandwidth dependence

- Bob Vessot, who helped with my thesis as well, had experimented with hardware bandwidth modulation, but this was hard.

- But we didn't know how to do it in software until 1981, when Jim Snyder came out to JILA to study laser instabilities. He found by averaging the time readings he could get the tau^-3/2 behavior with white-noise PM.

- We had lived with this ambiguity problem for 16 years, so this was a happy time to have this problem resolved.

David: So I was really excited when I saw his results; we had lived with this ambiguity problem for 16 years. A little side story. I had been invited to give a talk in New Delhi at an international conference for Third World countries on time and frequency transfer techniques, and on the flight from Denver to New Delhi, I was using my little programmable HP-65 calculator, to investigate these concepts that Dr. Snyder had shown us.

When I got there I was higher than a kite. I sent a message to Jim Barnes that we needed to share these results at the next Frequency Control Symposium.

As I review, in the next slide "Equation 34" of my thesis shows the τ dependence as well as the bandwidth dependence, but then we didn't know how to do software bandwidth phase or time modulation.

At each measurement, there is a hardware bandwidth limiting what is observed. As we average the readings, we are effectively increasing the time window of observation. This was the fundamental breakthrough.

$$\text{Mod. } \sigma_y^2(\tau) = \frac{1}{2\tau^2}\left\langle (\Delta^2 \bar{x}_i)^2 \right\rangle$$

$$\tau = n\tau_0$$

$$f_s = \frac{1}{n\tau_o}$$
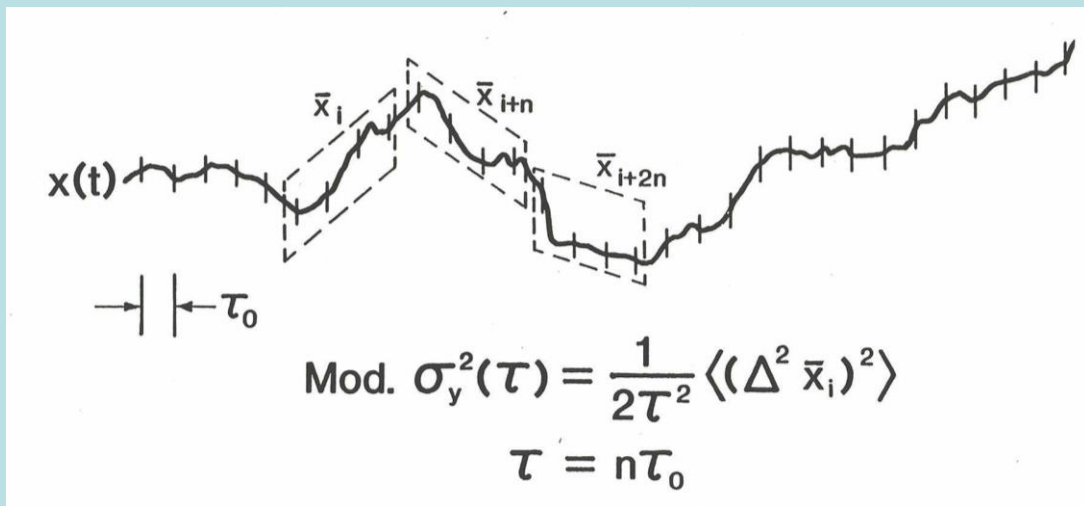
David: The next slide illustrates software bandwidth time-modulation for the example of n = 4. In general, one can think of the hardware measurement system bandwidth being denoted f_h, and it is proportional to $1/\tau s$, where τs represents the minimum sampling window due to the minimum sampling time τo. So the second-difference operator in MVAR uses these three averages each over n samples (shown is this case for n = 4), and our software bandwidth fs decreases as $1/n\tau o$ exactly as needed to remove this ambiguity problem.
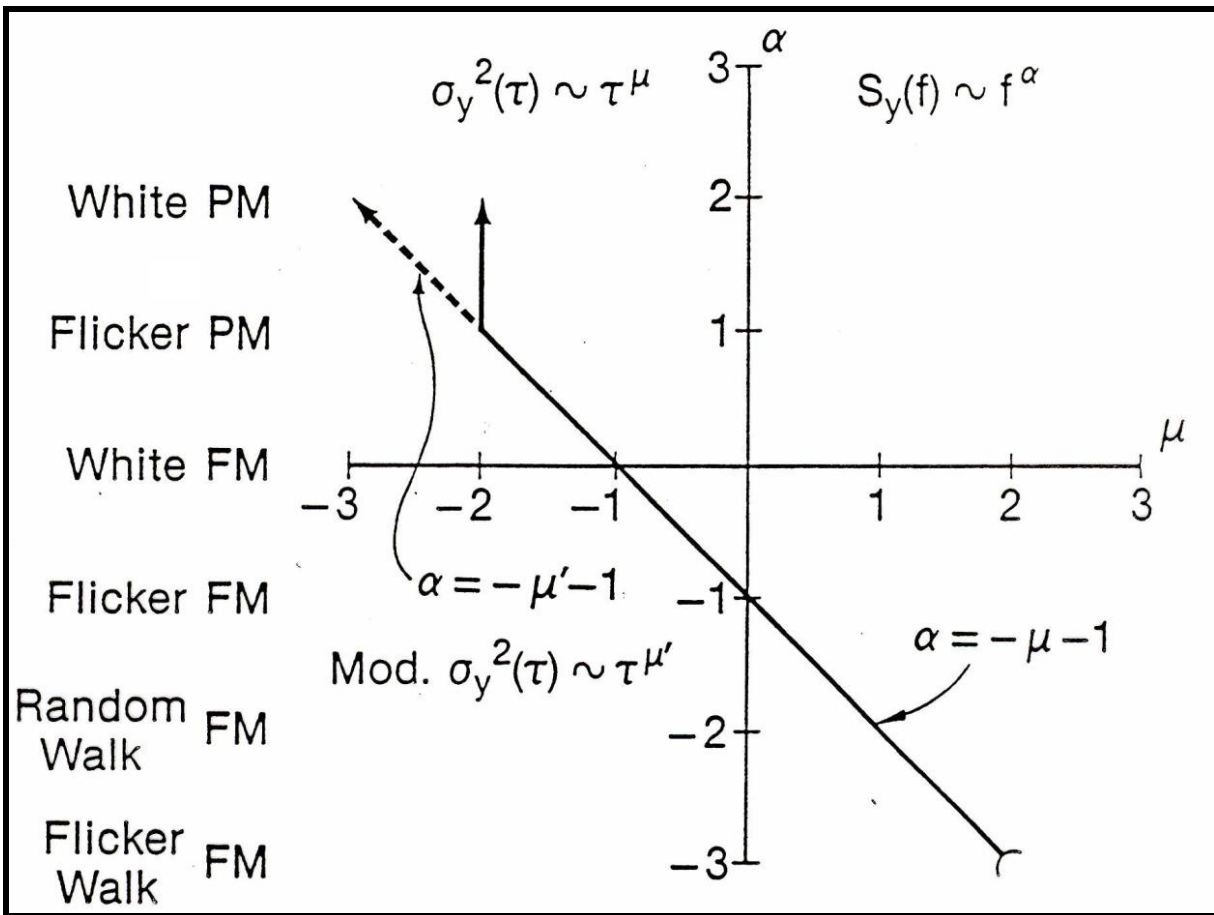
David: So this was a major breakthrough in variance analysis which we were able to share in our paper in the 1981 Frequency Control Symposium. I remember well that we were in New Delhi in February 1981, as I was able to take my wife and celebrate our wedding anniversary in Guam on our way home on the 20th. I had found a $50 coupon to buy a second ticket for her from TWA. The UN paid for mine. After celebrating in Guam, we flew that evening to Hawaii, and it was again the 20th of February having crossed the International Date Line. So we got to celebrate that anniversary twice. Who but a "time-nut" would make that happen!

In the next slide we show the virtues of the Modified Allan variance using again Lighthill's book for the time-domain to frequency-domain transformations. So as MVAR is proportional to τ^mu', we now have alpha = - mu' -1 for all values of alpha -3 < alpha < +3, which nicely removes the ambiguity between white-noise PM and flicker-noise PM.

This breakthrough

- Not only resolved the ambiguity problem for $\sigma_y^2(\tau)$ for white PM and flicker PM and gave us the Modified Allan variance with better noise color perception, useful for masers, time dissemination systems and for navigation, but also,

- In the late 1980s, it allowed us to write a time variance (TVAR) for the telecom community to help them fulfill their needs: TVAR = τ^2 MVAR/3; normalized so that TVAR = classical variance for white PM for $\tau = \tau_o$ like AVAR is normalized for white FM.

David: So as shown in the next slide this "breakthrough" not only helped the time and frequency community, it was picked up by the international navigation community as well. In 2015, I was invited to St. Petersburg, Russian to give a paper, give a plenary talk, and chair a panel around this subject at an international navigation conference. I was impressed with what they have done with the MVAR metric.

The second bullet in this same slide shares how in the late 1980s the telecom community came to me and asked for help to develop a metric that would be useful for them. They gave us a bunch of their data, and Marc Weiss and I analyzed it extensively and came up with TVAR as a desirable metric for them. As you can see from the slide, TVAR is proportional to MVAR with a 1/3 τ^2 multiplier. The 3 in the denominator normalizes TVAR to be equal to the classical variance for white-noise PM -- the ideal time measurement noise. TVAR caught on with the national telecom community and then soon with the international community. The success of TVAR with the international telecom community won my wife and me a free trip to Edinburgh, Scotland in 2011 to receive the "Time Lord" award. So they have found this metric very useful for network analysis. We are very happy to help them as well with their metrology problems, and while in Scotland we were able to visit the area where the Allan name originated.

Characteristics of Useful Measures

- Theoretically Sound
- Easy to use and intuitive
- Relates to real situations
- Yields useful spectral information for design engineers
- Useful diagnostic tool
- Optimum smoothing, estimation, and prediction
- Communicates to the Manager (Decision Makers)
- Is efficient like Ockham's razor: Simple is often the best

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David: In the next slide, we address the topic of what are the CHARACTERISTICS OF USEFUL MEASURES in metrology. As we examine the eight bullet points listed, we find that AVAR, MVAR, and TVAR all satisfy these criteria.

Elizabeth: So what makes TVAR applicable to telecom for timing?

David: They, basically need to have syntonization between nodes of a network; incorrectly, they call it synchronization. If you have synchronization, then syntonization logically follows. So TDEV (the square-root of TVAR) allowed them to characterize the timing errors between the nodes of a network. And from that they could see if they were meeting their syntonization specification.

Elizabeth: So they were wanting the frequencies correct across a network?

David: Yes, and this allowed them to pick the right kinds of frequency standards for a certain level of data flow. So TDEV became a very useful metric for lots of people both in time and frequency metrology as well as with the international telecom community.

IEEE Standard 1139 (1988)

Time Domain Variances

AVAR:

$$\sigma_y^2(\tau) = \frac{1}{2} \langle (\bar{y}_{k+1} - \bar{y}_k)^2 \rangle$$

$$= \frac{1}{2\tau^2} \langle (x_{k+2} - 2x_{k+1} + x_k)^2 \rangle$$

MVAR:

$$\text{mod.}\,\sigma_y^2(\tau) = \frac{1}{2\tau^2} \langle (\bar{x}_{k+2} - 2\bar{x}_{k+1} + x_k)^2 \rangle$$

TVAR:

$$\sigma_x^2(\tau) = \frac{\tau^2}{3}\,\text{mod.}\,\sigma_y^2(\tau)$$

David: The IEEE asked me to chair a panel to come up with recommended clock characterization standards; this standard was published in 1988. These next two slides summarize these recommendations in the time-domain and then in the frequency-domain. AVAR may be written as the average of the squared first-difference of adjacent fractional frequencies averaged over τ and all divided by 2. AVAR may also be written in terms of the second difference of the time-error values, x(t), with spacing τ, as denoted in the second equation. And we see that MVAR takes on the same equational form as AVAR in terms of the second difference of the time errors, x(t), except now each member of the second difference is a time average of the appropriate x(t)s averaged over each of the three τ intervals. I just noticed for the first time a mistake in this slide in this equation for MVAR: the last xk should have a bar over it denoting it is also averaged over τ. And as mentioned before, TVAR is defined in terms of MVAR, but is now normalized both to be equal to the classical variance for white-noise PM and to be in terms of time errors instead of frequency errors. As mentioned earlier, this standard has been updated with the most recent one being: "IEEE Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology—Random Instabilities" (IEEE Std 1139™-2008). As I have reviewed this last standard and knowing that the navigation and telecommunication communities extensively use MVAR and TVAR and that this current standard doesn't reflect that, could there be useful cross-fertilizations and interactions between these communities?

Frequency Domain Spectral Densities

$$S_y(f), \quad S_\phi(f), \quad S_{\dot{\phi}}(f), \quad \text{or} \quad S_x(f)$$

Relationships:

$$S_y(f) = \frac{f^2}{\nu_0{}^2} S_\phi(f)$$

$$S_{\dot{\phi}}(f) = (2\pi f)^2 S_\phi(f)$$

$$S_x(f) = \frac{1}{(2\pi\nu_0)^2} S_\phi(f)$$

$$S_y(f) = (2\pi f)^2 S_x(f).$$

David: The second slide coming out of this 1988 IEEE committee's work are the recommendations for the frequency-domain in terms of appropriate spectral densities. As the IEEE standards committee looks forward to updating the 2008 standard, I have some suggestions:

Given the advent of high-performance optical clocks and the challenges for comparing them remotely, the virtues of MDEV and TDEV become very relevant. For power-law spectral densities, $S_y(f) = h_\alpha f^\alpha$ these metrics are convergent for $-3 < \alpha < +3$, which nicely covers measurement noise and the clock noise processes. For example, for Two-way Satellite Time and Frequency (TWSTF) transfer, an MDEV diagram allows you to see the excellent short-term stability performance of the TWSTF technique, but then for larger tau values it is degraded by systematics. Then for even larger tau values it is used extensively for comparing remote clocks. But as I share in my ORAL HISTORY, Robin Giffard and I developed a GPS Advanced Common View technique which is better both in performance and significantly in cost of operation, and a TVAR plot reveals this. As is mentioned in the last standard, ADEV should only be used for tau values out to 10 % of the data length, which is good counsel. But I have used it many times for longer tau values with the realization that AVAR is chi-squared, which is mentioned later on in the standard, and as the degrees of freedom decrease the probability of low values for ADEV increase. In fact, at tau equal to half the data length, the most probable value is zero. Knowing of this limitation allows one to use it for larger tau values with caution and understanding of the causes of biases. Because long term data are expensive and extremely valuable in clock characterization, clearly the work Dave Howe and his group in developing sigma-total adds significantly to metrology by increasing the degrees of freedom and greatly improving the confidence of the estimates for long tau values on a sigma-tau diagram. Something we used many times for long-term stability analysis is to subtract the mean frequency from the data, which pins the end points in a time or phase plot to the same

value. Then compute x_{pp}/T, where x_{pp} is the peak-to-peak time deviations and T is the data length. Very often because flicker-noise FM has high energy at low frequencies, this plot will look like a noisy sinewave. There is some theoretical basis for this calculation, and we have used it many times experimentally. This is very useful because it gives you an estimate of stability at the data length: ADEV(T). If there is significant frequency drift, then this plot will show as parabola, and one can get a good estimate of the size of the drift by computing: $D = -8\ x_{pp}/T^2$. Observing the data is especially wise when doing these calculations to make sure they make   sense.

In summary, the Relationship between AVAR, MVAR, and TVAR:

- $\sigma_y^2(\tau) = \frac{1}{2} < (\Delta y)^2 >$

- **Since** $y(t) = \frac{x(t)-x(t-\tau)}{\tau} = \frac{\Delta x(t)}{\tau}$,

- $\sigma_y^2(\tau) = \frac{1}{2\tau^2} < (\Delta^2 x)^2 >$

- We obtain software bandwidth modulation by averaging x over τ; therefore:

- $Mod.\,\sigma_y^2(\tau) = \frac{1}{2\tau^2} < (\Delta^2 \overleftrightarrow{x})^2 >$

- $\sigma_x^2(\tau) = \frac{\tau^2}{3}\ Mod.\,\sigma_y^2(\tau)$

David: The next slide shows a summary of "the relationships between AVAR, MVAR and TVAR." And the next slide gives their denoted square-roots. The square-root values are typically used in metrology.

The square-root of AVAR, MVAR, and TVAR are denoted:

- $\sigma_y(\tau) = ADEV$

- $Mod.\,\sigma_y(\tau) = MDEV$

- $\sigma_x(\tau) = TDEV$

STATISTICAL THEOREM

- The optimum estimate of the mean of a process with a white-noise spectrum is the simple mean.
- HENCE:
  - For white PM, the optimum estimate of the phase or the time is the simple mean of the independent phase or time readings.
  - For White FM, the optimum estimate of the frequency is the simple mean of the independent frequency readings, which is equivalent to the last time reading minus the first time reading divided by the data length, if there is no dead-time between the readings.

David: The next slide is a simple but important "STATISTICAL THEOREM." And HENCE: two fundamental implications of this theorem are listed. And the next hand written slide -- using this theorem -- shows why AVAR is the optimum estimator of change for white-noise FM in a variance sense. It follows that AVAR or ADEV is the optimum metric in variance space for the white-noise FM coming from an ideal passive atomic clock. Similarly, TVAR is the optimum estimator of change in a variance sense for white-noise PM; so, wherever white-noise PM is present, TVAR or TDEV in variance space is the optimum metric. If the frequency stability is desired instead of the time stability, then the optimum estimator in variance space is MVAR or MDEV for white-noise PM.

$$\sigma_y^2(\tau) = \frac{1}{2}\left\langle (\Delta \bar{y})^2 \right\rangle$$

$$\bar{y} = \frac{x(t+\tau) - x(t)}{\tau}$$
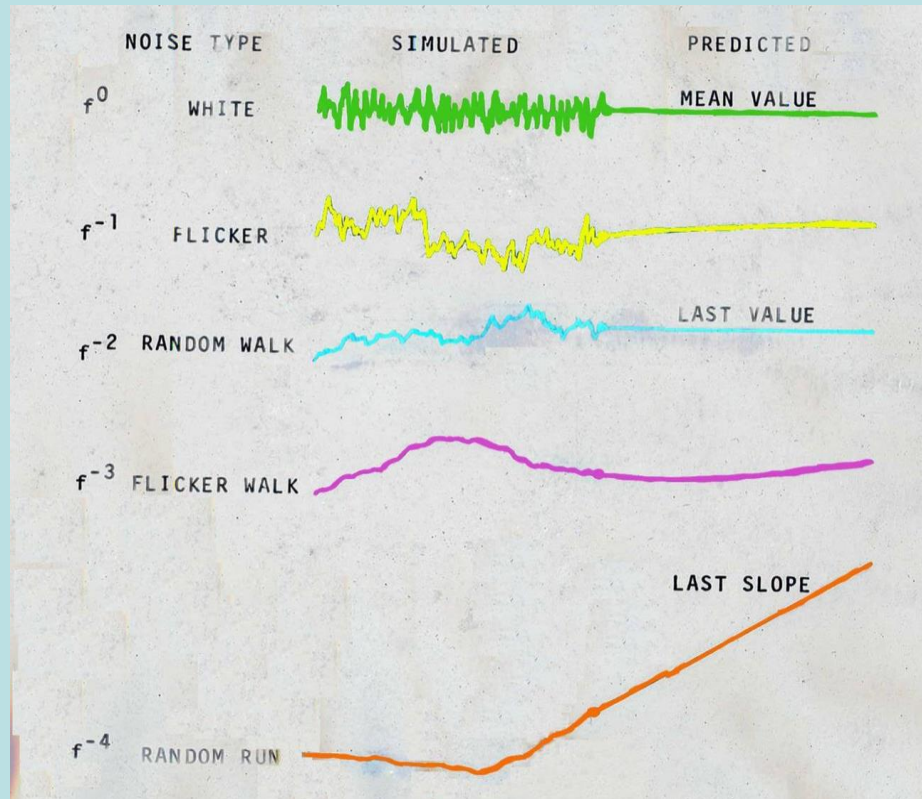
is optimum for white FM

- - - - - - - - - - - - - -

$$\sigma_x^2(\tau) = \frac{1}{6}\left\langle (\Delta^2 \bar{x})^2 \right\rangle$$

$$\bar{x} = avg. \ of \ x \ over \ \tau$$

is optimum for white PM

David We have shown that AVAR may be written as a function of the second-difference of the time deviation readings. As an example of a non-optimum estimator of change in a variance sense for white-noise FM, we have the Hadamard variance, which is now used for GPS clocks. The Hadamard variance can be written as a third-difference of the time deviation readings, and hence is useful for removing the effects of frequency drift. However, for atomic clocks it is better to remove the effects of systematics – like frequency drift – by other means and then analyze the residual variations using AVAR. In the paper that Jim Barnes and I wrote in 1967 and cited with the next slide, we show that the third difference, when used as a predictor of time error is far from optimum for the kinds of noise we see in atomic clocks.

David: In the next slide regarding "OPTIMUM PREDICTION," we give examples of how to apply this same STATISTICAL THEOREM for how to do optimum prediction in the case of noises with even power-law spectral densities. For white-noise PM, it is the mean value; for white-noise FM, which is the same as random-walk PM, it is the last value; and for random-run FM, it is the last slope for the time-error readings. The optimum predictors for the flicker-noise process are more complicated, but can be done using inverted Box and Jenkins ARIMA filtering techniques. There are also some simple and useful predictors for these flicker-noise processes. For example, for flicker-noise FM, a near optimum predictor for the time error of a clock for τp into the future is $x(t + \tau p) = 2\, x(t) - x(t - \tau p)$, where t is current time error. Jim Barnes and I show that this predictor is about 20% from optimum for flicker-noise FM, when we were discussing the prediction of time, which is a good model for the earth's random fluctuations for Fourier frequencies higher than one cycle per year. See Figure 3 in AN APPROACH TO THE PREDICTION OF COORDINATED UNIVERSAL TIME: https://tf.nist.gov/general/pdf/185.pdf .
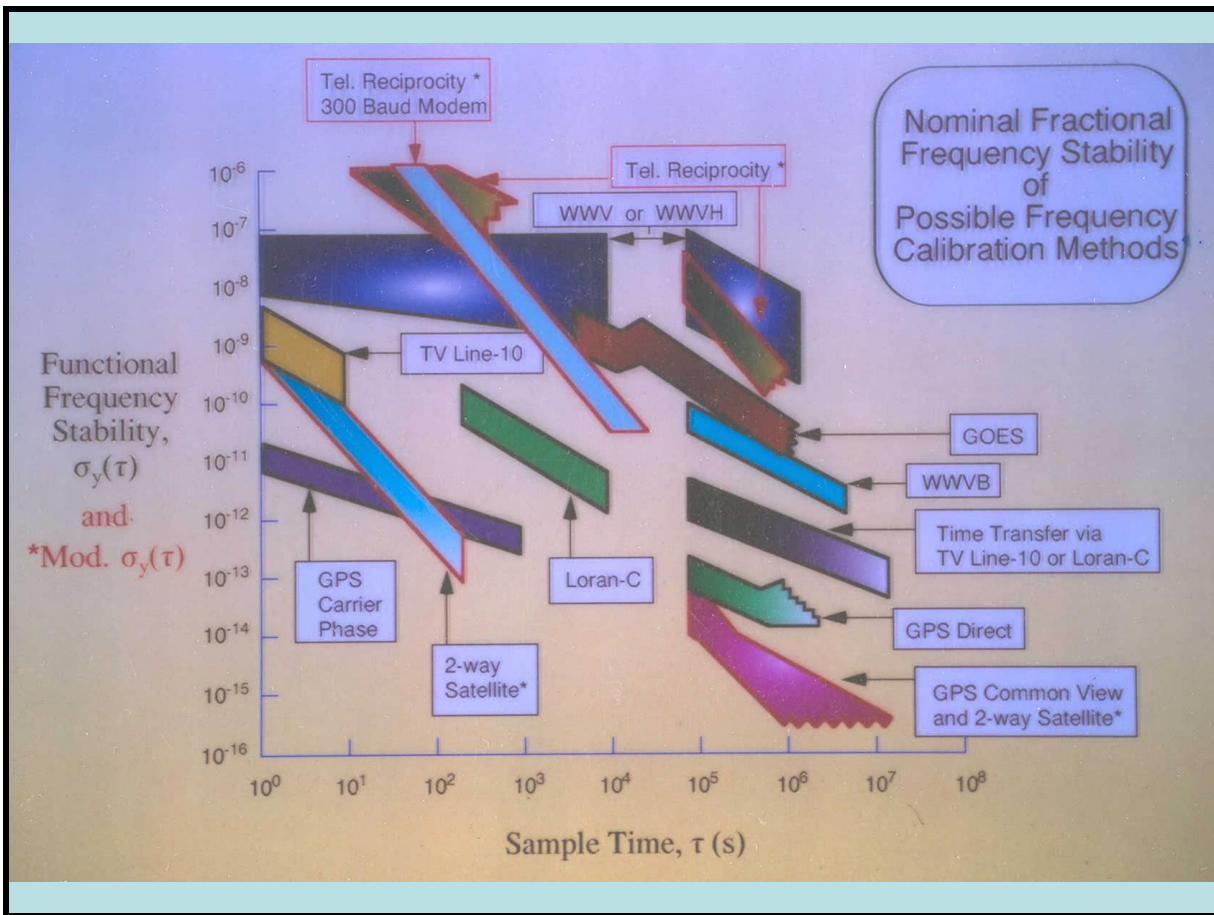
David: This next plot shows the frequency stability of a large variety of time and frequency transfer processes. Notice, I use MDEV where it is appropriate, as it is most useful for the short-term stability of two-way satellite time and frequency transfer, for Telephone ACTS reciprocity using a 300 baud modem, and for GPS common-view. It was fascinating to observe that a 300 baud modem in reciprocity ACTS mode over a short distance could reach frequency instabilities of 3 x 10^{-11}.

Clock-time Keeping Ability or predictability: x $_p$ and τ is prediction time.

$$x_p(\tau) = k \; \tau \; \sigma_y(\tau)$$

$k = 1$ *for white FM and random walk FM*

$k = 1/\sqrt{\ln 2} = 1.2$ *for flic*ker *FM*

$k = 1/\sqrt{3}$ *for white PM and flic*ker *PM*

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David: As a rule of thumb one can write the time-prediction error of a clock as τp times ADEV(τp), where τp is the time prediction interval in question. This prediction-error calculation is extremely useful in time prediction for GPS, since the GPS satellites have to predict time forward from one tracking and upload station to the next, which can be several hours long. In the next slide we write the general equation for timekeeping or time predictability and list the specific coefficients for the typical models used for atomic clocks. This equation is often used with k = 1, as mentioned above.

Time Dispersion of Various Clocks
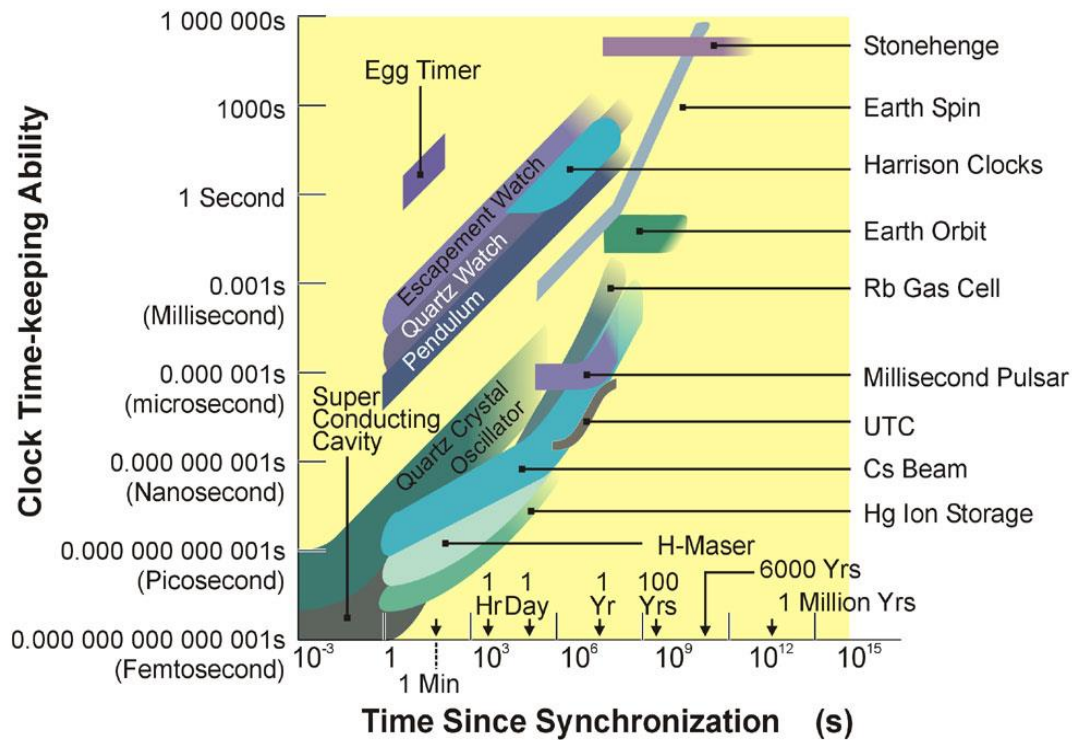
David: In the following slide, I use this above prediction equation to describe the timekeeping ability of a big variety of clocks -- including Stonehenge! I made this diagram back in the 1990s, and the clocks now are more than 100 times better than the best ones shown here. UTC continues to get better as the now more than 400 clocks are added to the international ensemble, and these clocks are improving remarkably as well with the advent of optical frequency standards. Notice that earth orbit and earth spin have very different spectral densities. Earth orbit, which used to be official time -- ephemeris time until the cesium-133 definition came in 1967 -- is just the limiting nine-year average measurement noise (white-noise PM). However, the random-variations in earth spin are characterized by flicker-noise FM at a level of ADEV = 1.4 e-9 out to a Fourier frequency of about one cycle per year, then for lower frequencies (longer time dispersion intervals) the model that fits is f^-5, which is the most dispersive of all the clock noises. Given this noise model the ages of the earth deduced by modern science gives a probability that the earth could end up spinning in the opposite direction! Sunset in the east! Just for general interest, the earth has a TDEV(τ = 1 day) of about 70 microseconds. This amounts to a prediction error of speeding up or slowing down of about 5 cm (~ 2 inches). At the same time there are earth tides of the order of 30 cm each day -- depending on where you are and the sun-moon relationship. Everything is moving and changing when measured carefully enough!

In 1987, I was asked by the IEEE to write a paper around the following question:

- Should the Classical Variance Be Used As a Basic Measure in Standards Metrology? IEEE TRANSACTIONS ON INSTRUMENTATION AND MEASUREMENT Vol. 1M-36, No. 2, June 1987 https://tf.nist.gov/general/pdf/776.pdf
- Found flicker noise in volt standards and gauge blocks.
- Used the bias function $B1(N)$ as a reliable test to see if it is safe to use the classical variance or standard deviation.
- If $B1(N) = 1$, then the data have a white spectrum.
- If $B1(N)$ is not 1, then the value can tell you the nominal color of the noise and whether the classical variance will be convergent and well behaved or not for that data set.

David: The IEEE asked me to write this paper in 1987, SHOULD THE CLASSICAL VARIANCE BE USED AS A BASIC MEASURE IN STANDARDS METROLOGY? https://tf.nist.gov/general/pdf/776.pdf I feel I gave them a good answer, but for the most part folks continue following the classical variance and standard deviation tradition. Since flicker-noise is so ubiquitous in nature, I believe this paper could improve measurement science significantly. In this slide, I give some simple guide-lines derived from this paper. It seems a shame to me that the standard deviation gives you one number for a data set, and you know nothing of color -- the most meaningful part of metrology -- allowing you to separate measurement noise from the noise in the standard and the effects of environment. Every measurement made has all three of these components, and without the color of the noise and variations one can never sort out what is really going on in a set of measurements.

Elizabeth: You should always use the Allan variance and then see what it is.

David: Yes, if you want to know what is going on in the data.

Elizabeth: Time keepers use it all the time.

The variations of a clock's output can be well modeled as:

- **SYSTEMATICS**: time offset; frequency offset; frequency drift; its sensitivity to the environment, etc.

PLUS

- **RANDOM VARIATIONS**: white noise, flicker noise, random-walk noise, etc.
- On top of these is measurement noise

David: In this next slide I share the model for variations for clocks, but this information is generally applicable. This model is general in that you always have both systematic and random variations in a data set of measurements. You can kind of put the systematics in one pocket and the random variations in the other pocket. Knowing the kind (color) of noise in the random variations allows you to better estimate the systematics. Then on top of these variations, you have measurement noise.

PERCEIVED CAUSES OF CLOCK DEVIATIONS

MEASUREMENT

CLOCK

Each of these three
Will typically have a
Different color of noise.

ENVIRONMENT

PERCEIVED PERFORMANCE	=	MEASUREMENT NOISE	+	RANDOM AND SYSTEMATIC WITHIN CLOCK	+	ENVIRONMENTAL PERTURBATIONS
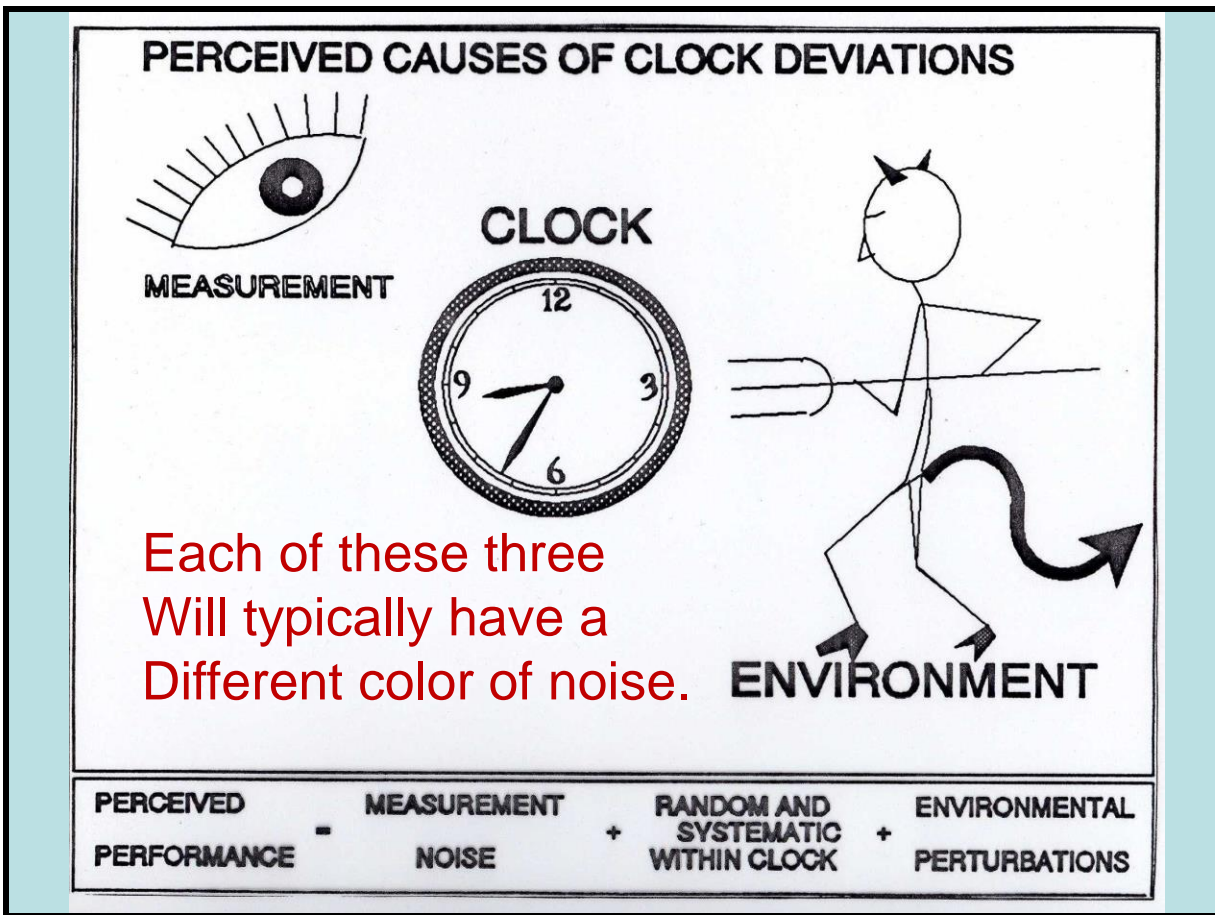
David: The next slide is a cartoon illustration of these concepts. What is perceived in any measurement is always a composite of the measurement system, the internal behavior of what is being measured with that system, and then the external perturbations affecting the measurements. Because the three components of the measurements typically have different spectral densities, the time-domain analysis tools developed are exceedingly useful. Knowing where the perturbations and variations come from allows better design of both measurement systems, devices, and shielding against adverse environmental influences, which may be affecting both the measurement device as well as the device being measured.

The time-domain techniques, I have shared, work for clocks, telecom systems, navigation devices, and in a large number of other applications

- Even though random variations tend to be dominant in the short-term and systematic variations in the long-term, but not always

- Determining kind of random-noise allows optimum prediction and parameter estimation

- Optimally estimating the systematics and subtracting them can enhance the performance enormously – as I will show.

David: The next slide shares that these time-domain techniques not only work for clocks, but in many other areas of metrology -- allowing one to do optimum prediction, estimation, and smoothing, as well estimation of the effects coming from the systematics present, which are always there.

Example:"Hidden Value" of Quartz

- Historically, the high-performance of quartz-crystal oscillators has been overlooked:
  - *frequency offset & drift can be effectively removed*
  - *temperature & acceleration transduced to frequency*
- When systematic effects are compensated, Quartz performance improves dramatically:
- Two examples of optimally measuring the systematics and removing them:
1) NBS LIRTCU removed systematics; achieved ~10^{-12} stability with calibration data available for 1 s every 12 h (1966); two NBS papers on this topic
2) 5¢ Quartz Resonator (like in your wristwatch) is stable at 3 x 10^{-8} over 140 days (1997, like 1s/yr.). I will show you a fascinating experiment in this regard.

David: Here I share some fun examples of these concepts with quartz-crystal oscillator clocks. In the first example we used computer simulation to model flicker-noise FM and the systematics in high-quality quartz oscillators; back then we only had an analog computer. Then we built a device that would automatically remove the systematics in the presence of flicker-noise FM. (Low Information Rate Time Control Unit, LIRTCU), and observed that the output behaved much like an atomic clock. It was used for a time to generate official time for NBS. https://tf.nist.gov/general/pdf/187.pdf AN ULTRA-PRECISE TIME SYNCHRONIZATION SYSTEM DESIGNED BY COMPUTER SIMULATION https://tf.nist.gov/general/pdf/180.pdf

In the second example, I purposely bought the cheapest stopwatches I could find at Target -- $6 each with tax! I knew the 5-cent quartz-crystal oscillator used had excellent performance -- because the robotic technology has come a long way in that regard. Professor Neil Ashby, who did the relativity for GPS, helped me with this experiment. This experiment is one of the best examples I have seen in a tutorial sense of how to sort out the measurement variances (different kinds), the individual clock variances, and the effects of the environment. We were able to show that a 5-cent crystal clock has a flicker floor of 3 e^-8; this only amounts to a time-dispersion error of about 0.3 seconds for the 140 days of the experiment, while the systematics were about 400 times larger. In other words, removing the systematics makes an enormous difference. When I was given the Keithley Award in May 2018, I shared this experiment in a talk I was invited to give at the International IEEE/I&M Conference in Houston, TX. I am willing to share those slides for that talk with whomever wants them. david@allanstime.com. In these slides are all the information and equations needed to sort out the different measurement noises contributing, the individual clock variances, and the effects of the environment.

Example: the development of the HP 5071-A cesium-beam clock

- Dr. Len Cutler and his team at HP
  - They optimized the short-term stability
  - They considered every systematic; measured it, and removed it automatically.
  - The first atomic-clock to be "booted up!"
- Obtained the best long-term frequency stability performance for commercial Cs. Ideal white-noise FM over six decades + of tau values.
- ~85% of the 300+ clocks making up TAI and UTC are 5071-A cesium clocks; now owned and produced by Microsemi.

David: In this slide I document the outstanding work of Dr. Leonard (Len) S. Cutler and his team at HP in removing the systematics and optimizing the performance of their HP 5071-A cesium-beam atomic clock. On an ADEV diagram it behaves with the ideal white-noise FM over many decades of averaging times. Timing centers all over the globe have purchased this model because of the outstanding performance, and today the vast majority of the some 400 clocks contributing to TAI and UTC are this kind. It was the first atomic clock that one booted up! When they first developed it, Len brought one to me in our lab in Boulder, and I was privileged to document its amazing performance. It was exciting to watch it turn-on (cold-start) and see the software learn all the lessons needed -- including magnetic-field compensation. It came on frequency in minutes with an accuracy as good as the NBS primary cesium-beam frequency standards of recent vintage. The following paper by Allan, Lepek, Cutler, Giffard, and Kusters: "The Impact of the HP 5071A on International Atomic Time," shows the outstanding performance of the 5071A cesium-beam atomic clocks: https://apps.dtic.mil/dtic/tr/fulltext/u2/a518478.pdf The last figure in this paper also shows the results obtained for GPS Advanced Common-view. The paper was presented and published in the Proceedings of the Fifth Symposium on Frequency Standards and Metrology, Woods Hole, MA, October 1995 and is listed as the first reference in this Appendix.

Elizabeth: Do you know how many years it took them to develop it?

David: Yeah, We worked with Len and his team at HP over several years. Len had all the software in his laptop for the 5071A, I am guessing they had been working on the ideas for it for about eight years.

Len and I go back a long way. In 1964, he and Al Bagley developed the HP 5060A cesium-beam clock, and NBS bought several of those for our time-scale ensemble. Len later upgraded that technology to the 5061A. In 1991 HP introduced the remarkable performance of the 5071A. Len visited the lab in Boulder often, and we would see each other at the annual FCS and PTTI meetings. He was often my jogging partner at those meetings, and I would often go out to dinner with the folks from HP. For a time, the HP frequency standards efforts went with Bomac in the Boston area. Len moved back there for a time including his grand piano; he was an excellent pianist. David Glaze, who designed NBS-4, worked with HP and then Bomac in its construction, and I remember Dave driving across the country with that standard in tow. Later, I worked on NBS-4 to improve its flicker floor and got some significant benefits by putting another magnetic shield around the beam and improving the cavity tuning. https://tf.nist.gov/general/pdf/69.pdf Helmut Hellwig and Dave Glaze co-authored this paper with me, and I believe it has important relevance today -- combining the concepts of accuracy and stability.

Lyons developed the first atomic clock and the first cesium-beam frequency standard but missed the opportunity to turn it into a clock. It took Essen and Parry at NPL to make the first cesium-beam atomic clock in June 1955. Over the years, I have observed the philosophy of different timing laboratories. For example, PTB likes to run their primary standards as clocks. In contrast, NBS/NIST typically has not used their primary standards as clocks. We have used the primary standards instead to calibrate commercial fly-wheel standards to act as memory of the primary standards. There are virtues and disadvantages to both approaches. In the above cited paper, we turned NBS-4 into a clock and got some exciting results. USNO has taken advantage of the excellent stability of optical clocks for their ensemble. If the major timing centers were to have operating optical clocks, it could improve the stability of TAI and UTC by a large factor. The big problem today is how to communicate the high level of accuracy coming from this new optical clock technology. The accuracy budget for any primary standard is a long list, and some uncertainties cannot be addressed without turning off the standard. Perhaps some new research could show us how to have sustained accuracy for a primary standard operating as a clock. PTB has been doing this for decades with good success.

When Len, Robin Giffard and team pulled together the 5071A, it was an optimization of the best chemistry, electrical engineering, mechanical engineering, physics, and computer software and hardware. The first cesium-beam atomic clock to be booted up, and in the process addressing how to make it be as accurate and stable as possible. Len addressed all the accuracy questions except the microwave cavity shift, and he was working on that one, which would have turned it into a primary standard.

David: Having grown up with the atomic clocks and GPS, I have seen magnificent growth in these technologies. It is fun to think back that many of the key people who developed GPS were in the seminars we taught at NBS in Boulder. I remember Colonel Brad Parkinson, who I consider to be the father of GPS, was in one of the seminar classes I taught, and I can still remember one of his poignant questions regarding flicker noise. Since atomic clocks are the heart of GPS, I spent many man years helping and was in many meetings with Brad. One evening we were setting around a banquet dinner table together, and one of the other people at the table was a movie producer. He said, he was wanting to come up with a movie showing the effects if GPS were destroyed. I told him I could tell him how, but I wouldn't. Brad said, "Thank you!"

In the link on this slide, I detail many of the activities with which I was involved.
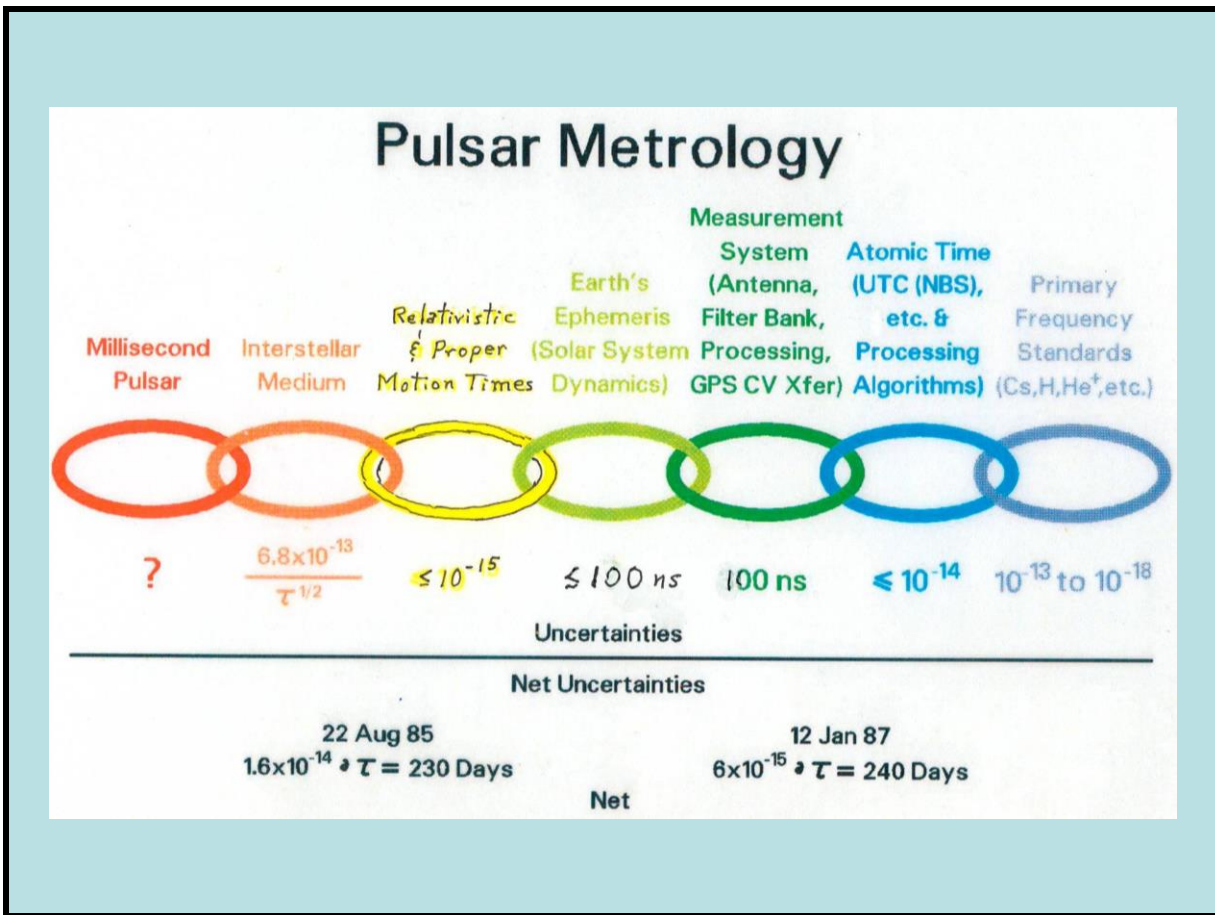
Time-keeping equation

$$x(t) = x_o + y_o t + \frac{1}{2} D t^2 + \varepsilon(t)$$

This equation is idealistic but useful; in practice what is measured is the time or frequency difference Between two clocks
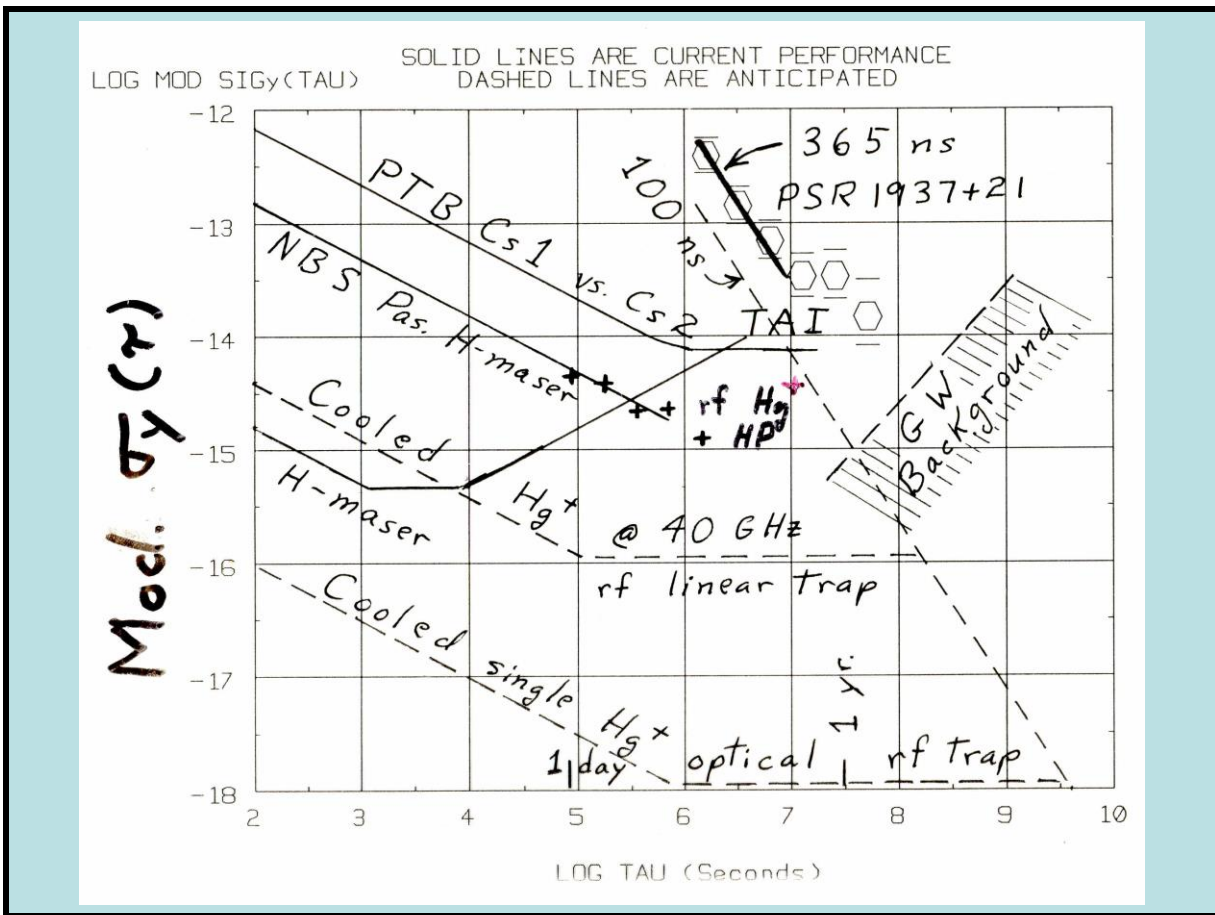
David: In the next slide is what we call the basic model for time dispersion in a clock. This equation is idealistic, but we have found it extremely useful over the years. GPS uses it to estimate the error of each of the SV clocks against GPS time, which is nominally synchronized with UTC(USNO) -- avoiding leap-second adjustments that are made to UTC to chase earth time. Since the only clock giving you correct time is the one you define to be so, you can think of this equation describing the error of any clock. First, you have its synchronization error, x_o. Then you have its syntonization error, y_o. Then you have the frequency drift, which exists in essentially all commercial standards. The frequency drift of the 5071A clocks is extremely small because of how well they have dealt with the systematics. Len Cutler, Bob Kern and I once made a formal recommendation to GPS folks to use the concepts incorporated in the 5071A along with the excellent design of the space qualified cesium standard that Bob had made at FTS, and which became the first GPS cesium-beam clock. We did this because of the excellent low frequency drift properties of this design. The short-term stability is not as good as the GPS rubidium gas-cell clocks being used, but would have been good enough for operations to have good Kalman filter estimates. The big advantage of this approach is avoiding the drift term error which grows as time squared. This has very important tactical advantages. The last term in the equation is the model for all the residual noise variations of the clock, which can be both random and systematic -- like temperature effect variations. In practice, of course, we are always comparing two clocks.

Pulsar Metrology

Millisecond Pulsar	Interstellar Medium	Relativistic & Proper Motion Times	Earth's Ephemeris (Solar System Dynamics)	Measurement System (Antenna, Filter Bank, Processing, GPS CV Xfer)	Atomic Time (UTC (NBS), etc. & Processing Algorithms)	Primary Frequency Standards (Cs, H, He^+, etc.)
?	$\dfrac{6.8\times10^{-13}}{\tau^{1/2}}$	$\leq 10^{-15}$	$\leq 100\ ns$	$100\ ns$	$\leq 10^{-14}$	10^{-13} to 10^{-18}

Uncertainties

Net Uncertainties

22 Aug 85
1.6×10^{-14} ə $\tau = 230$ Days

12 Jan 87
6×10^{-15} ə $\tau = 240$ Days

Net

David: The Pulsar Metrology slide was a real fun experiment. Don Backer, who became a good friend and colleague, and others discovered the first millisecond pulsar back in 1982 and published their results in *Nature*. This millisecond pulsar, PSR 1937 +21, is about a seventh of the way to the center of our "Milky-way" galaxy, and so it takes about 12,000 years for the signal to get to earth. I read their paper and could see some things that could be done to help them. I ended up taking a GPS common-view receiver to the Arecibo Observatory in 1984. As illustrated by the chain, you can see the different error terms contributing to the measurement. You can see our efforts paid off over the next few years. Joe Taylor (Nobel Laureate), who became a good friend and colleague as well, was involved with the measurements. I asked them about the electron content across the path. They had assumed it to be constant. I did a two frequency check using their data and using ADEV was able to show it was a random-walk process. It is as if electron clouds are walking across the path. I also so evidence in the data that it could be a binary; later it was shown to be so.

SOLID LINES ARE CURRENT PERFORMANCE
DASHED LINES ARE ANTICIPATED

LOG MOD SIGy(TAU)

Mod. $\sigma_y(\tau)$

−12
−13
−14
−15
−16
−17
−18

365 ns
PSR 1937+21
100 ns
PTB
Cs 1
vs. Cs 2
TAI
NBS
Pas. H-maser
Cooled
H-maser
rf Hg
+ HP
Hg+
@ 40 GHz
rf linear trap
Cooled single
Hg+
1 day
optical
rf trap
GW Background

LOG TAU (Seconds)

2   3   4   5   6   7   8   9   10

David: I presented the following slide at a millisecond pulsar workshop at UC Berkeley some years after we had set up our GPS common-view receiver. Here I shared that even with the anticipated $25M upgrade for the Arecibo Telescope the white-noise PM measurement limit of 100 ns would require them to average for 200 years to get one data point at the current level of anticipated atomic-clock accuracies of 1e-18. One of the hoped-for results coming out of this research was an indication of the gravity-wave background variations. They have never been observed.

GPS (with SA) versus Cesium Ensemble
(Filtered and Unfiltered)
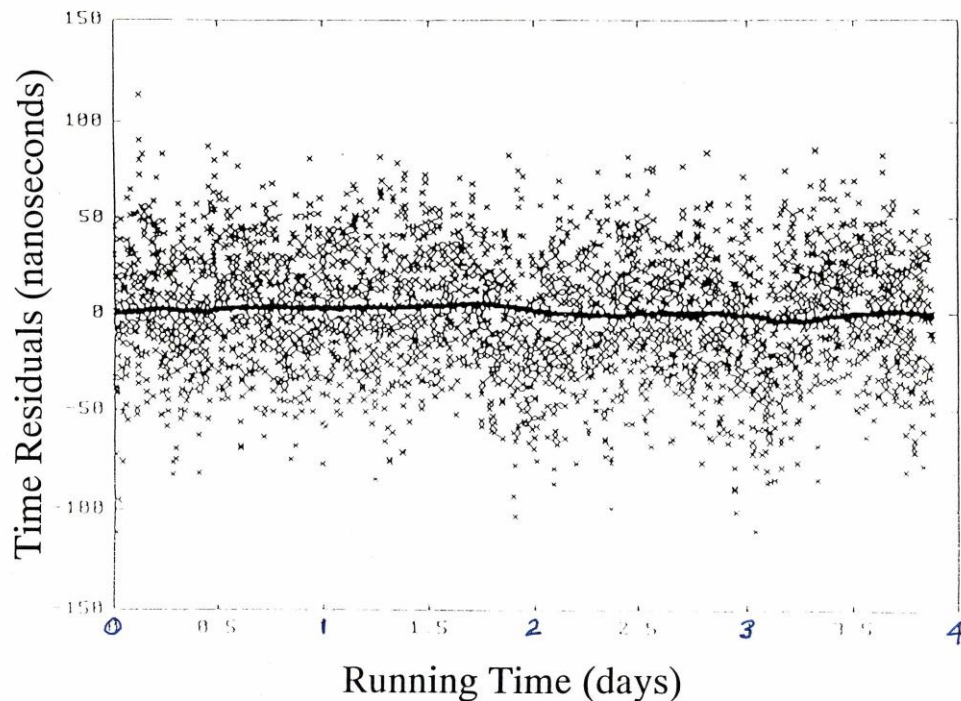
David: When I retired from NIST in 1992, I had the idea that we might be able to remove the selective availability (SA) from GPS for the civil sector. SA was a modulation purposely put on the civil L1 frequency to give degraded performance. The slide shows that the peak-to-peak scatter for SA is over 100 ns. I talked to both military folks I knew as well as commercial GPS receiver manufacturers and no-one could tell me the spectrum of SA. Wayne Dewey at True Time in Santa Rosa offered to take the data, and then I could analyze it. I used TDEV to look at the time-domain spectrum, and I was able to come up with a digital filter to filter out the SA. The dark line is the filtered estimate. We were able to test the algorithm at USNO by tracking a military receiver and obtained 1 1/2 ns rms variation over a month.

Elizabeth: What does the military guys think about it when they see these results?

David: Yeah, one fun presentation I made was at an ION-GPS meeting in 1993 at the Abravanel Hall in Salt Lake City where there were a whole bunch of blue-suits-AF folks in the audience. You could see the look on their faces, "You can't do that." If it had been a solution for position, they would have probably taken me out front and shot me! But timing is not their thing.

Elizabeth: Or were they saying, "You shouldn't do that?"

David: But the first presentation with this SA filtering and Smart-Clock technology implemented at HP was with Qualcomm. Chuck Wheatley, who was their VP of technology used to work for Rockwell on the GPS program, and he had designed both the algorithm for SA and the synthesizer to add it to the atomic clock on board to give the SA degradation. He was in the audience when I presented our results at HP, Santa Clara. He sat in the back of the room with a big smile on his face. It was really fun to see his response to how well we had removed what he had done. In the next slide you see a plot of EGPS. They sold these HP 58503 GPS receivers all over the globe for synchronizing cell-towers. You can see that they could keep the timing error

under 10 ns most of the time, and if the GPS signal was interrupted, the Smart-Clock technology would keep the errors within the specification of a microsecond over a day.
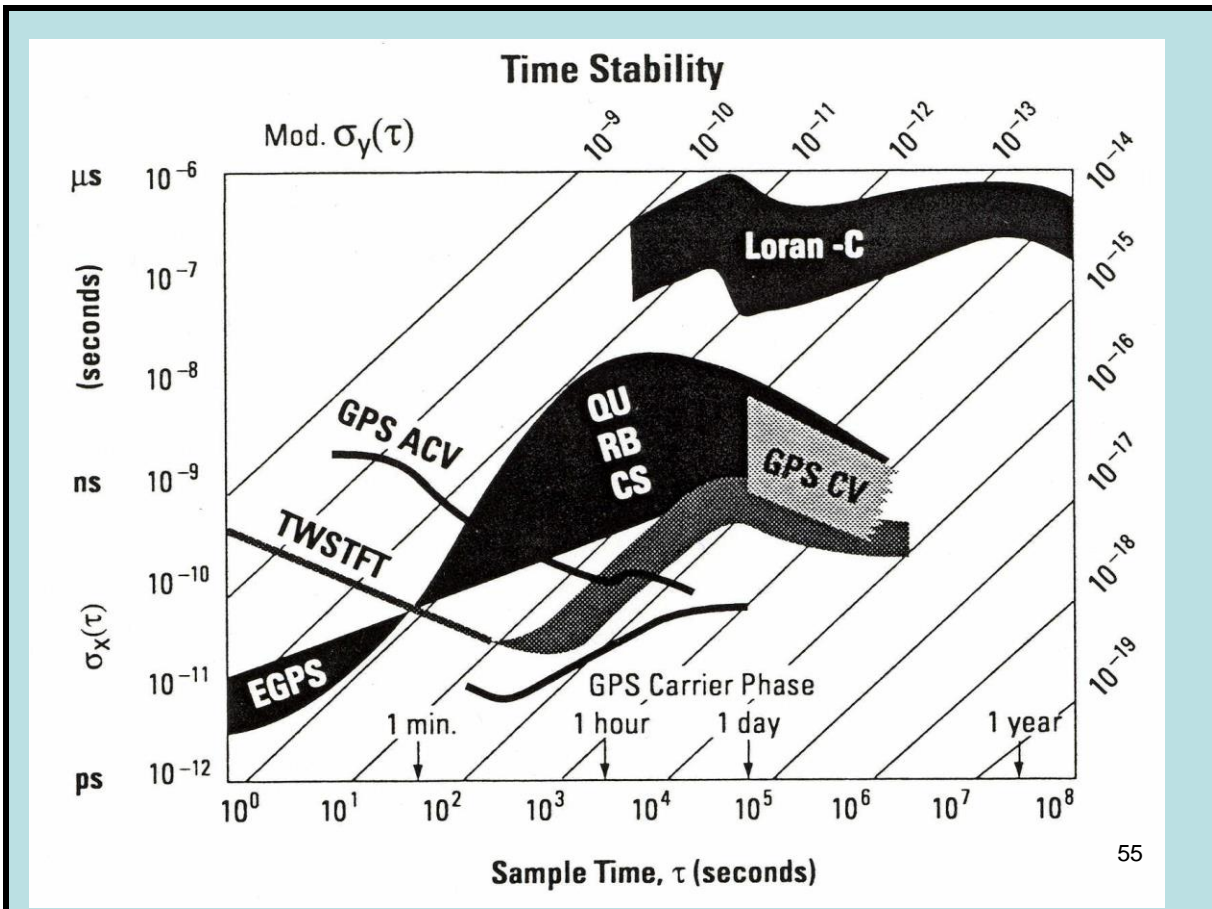
David: You can see several interesting results in this slide. You can see the diurnal and annual effects on Loran-C. You can see how two-way satellite time and frequency (TWSTFT) degrades by a couple of decades due to diurnal and environmental perturbations. But the result that I think is most exciting is the GPS Advanced Common-View (GPS ACV) that Robin Gifford and I developed on the assumption that the best you can do with GPS is to get rid of the systematics and one should be only limited by the white-noise PM measurement noise. We had a Motorola Encore GPS receiver and Robin went into the very heart of that receiver to look at all systematic issues and how to remove them.

David: We looked at multipath, ephemeris systematics per satellite, and receiver delays in the coding. Robin had figured a way to look across the spectrum of the L1 signal to estimate the ionospheric delay, since it is $1/f^2$ dependent. We did the test across the US with a receiver at USNO in Washington D.C. and one in Palo Alto, CA. We came very close to the ideal noise-model of white PM at a level of white PM noise at six nanoseconds at one second going down as tau^-1/2. If extrapolated, we could reach 1e-18 in two months. One could imagine a set of these receivers across the globe with super-optical clocks running continuously connecting all the major timing centers at this level of stability, and allowing accuracy comparisons at this level. With longer baselines there are more challenges, but the concepts and the extrapolations have general validity because of the technique and the fundamental theory behind it. Furthermore, the civil community will have two frequencies, so they can estimate the ionospheric delay like the military has always done.

Elizabeth: Is there anything that makes it difficult to implement or is there a reason why it never took off?

David:	Robin died, and my wife and I went on a mission. I think it should all be resurrected. I'm retired almost. But yes, it could be resurrected and I would think it could be extremely useful given the enormous improvements in optical standards over the past years. HP called me up and asked if I wanted Robin's information on GPS ACV, and they sent it to me, and I think I have that box they sent, if I can find it! I remember enough of what Robin and I did that I could pass it on to someone.

Effect of fm on TDEV

$$\sigma_x(\tau) = 1.4\, x_{pp} \frac{\tau_o \sin^3(\pi\, fm\, \tau)}{\tau \sin(\pi\, fm\, \tau_o)}$$
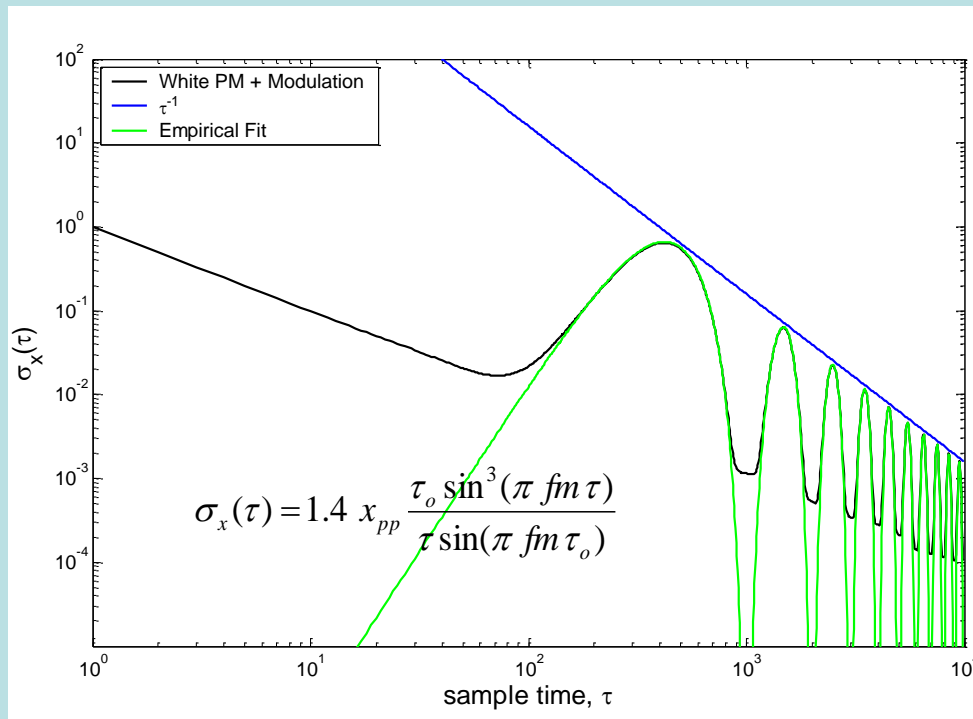
David:	This next slide shows the effects of frequency modulation on TDEV. So you have this modulation effect coming down as a cascade. At tau equal the reciprocal of the modulation frequency times n, you get a cancellation of the effect of the modulation. This is true of ADEV and MDEV as well. This is how you can detect diurnal and annual terms as shown in the previous slide, and it is useful as a low-frequency modulation detection technique. We will use this later in a very important experiment as we look at some of the unified field theory work we have done.

Granite keystone from Salt Lake Temple where we were married 20 Feb. 1959.

TRUTH IS LIGHT

Keystone to our entry

David: This is the key stone to our home in Fountain Green. And the next slide is a picture of our solar home with no furnace. It gets down to minus 30 degrees Celsius here and we're still alive; works really well. In January temperatures may reach more than 100 degrees F in our solarium, which then using convection moves that heat to the north side of our home. http://allanstime.com/SolarHome/index.html.

Our family home in Fountain Green, Utah, in 1992 with no furnace -- passive solar with six solar heating mechanisms and one passive solar air-conditioner; temperatures down to – 30º C. Personal web site shows how: http://allanstime.com/SolarHome/index.html

Elizabeth: And you designed and built the home. Right?

David: Yeah, I had to be the architect and the general contractor. I couldn't find anybody to do that so we had our family come and help. It was 1992, the year I retired, that we built it. It took us a year.
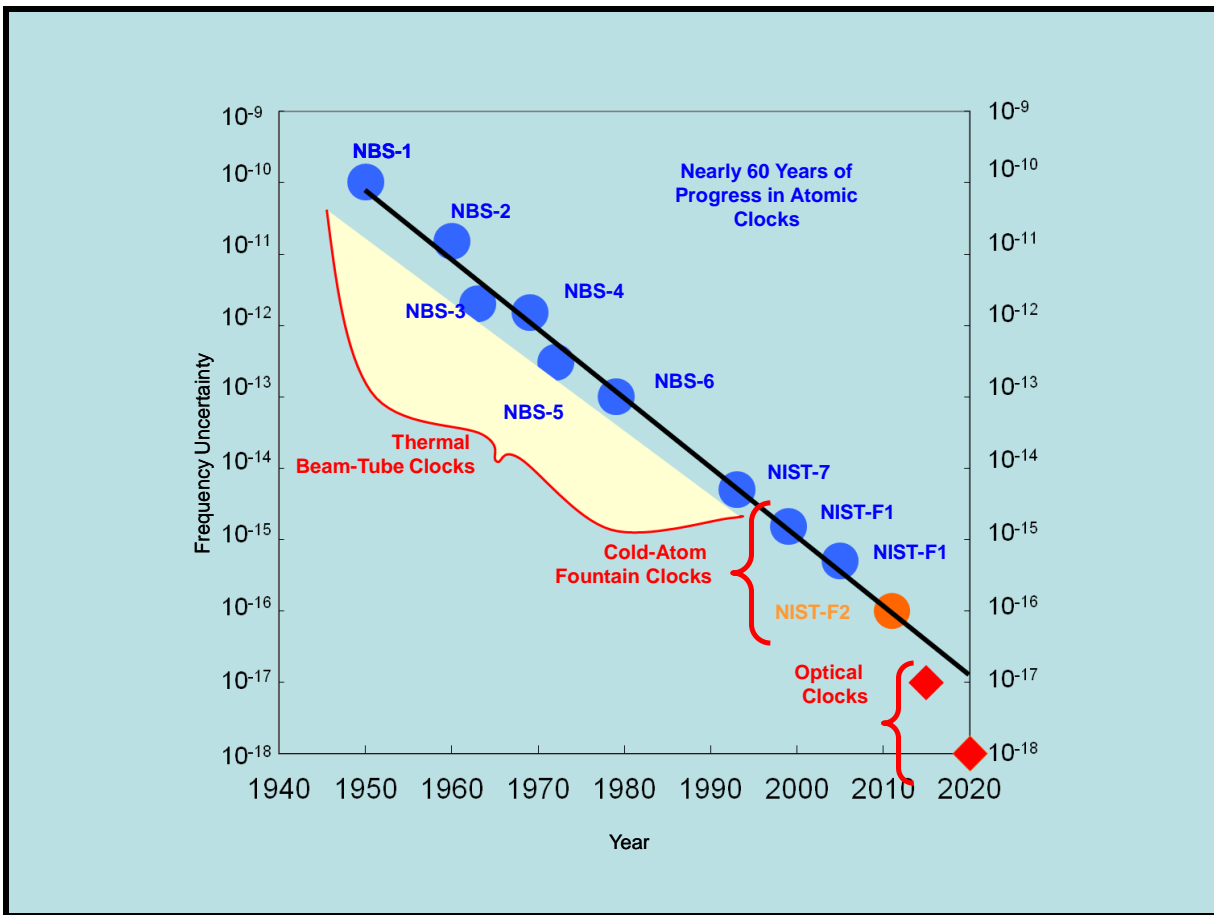
You see my Sweetheart of 60 years as of February 20th, 2019. I have one of my old reliable mountain bikes (big red) that probably has over 10,000 miles on it.

81ˢᵗ birthday

David: My family gave me this awesome mountain bike for my 81st birthday. I love to exercise on it and enjoy the mountains nearby.

David: This slide shows the amazing improvement in primary standards starting with Lyons, et. al. making NBS-1 with a frequency uncertainty of about 1e-10. The standards have improved by about a factor of 10 every seven years. With the advent of the optical frequency standards the improvement has been even faster. All but NBS-1 were built in Boulder and I knew their makers well.

Elizabeth: How many of those standards were you involved with?

David: I was basically a user of the primary frequency standards for calibrating the ensemble clocks, which were used to generate time for NBS/NIST. My 32 years at the lab took me all the way through the thermal cesium beam standards. The primary standards were next door to where I worked with the atomic-clock ensemble and measurement system. Since I helped with the seminars almost every year, I got to know the makers of all these standards, and I made some improvements on NBS-4 to obtain better long-term frequency stability in conjunction with an accuracy algorithm experiment that we conducted. I published a paper with Helmut Hellwig and David Glaze in that regard: "An Accuracy Algorithm for an Atomic Time Scale," https://tf.nist.gov/general/pdf/69.pdf .

<div style="border: 3px solid black; background-color: #b8d8d8; padding: 20px;">

<p style="color: red; text-align: center;">Atomic clocks are now so good that we don't have a way to compare them between nations.</p>

- 10^{-18} is like +/- a second in 30 billion years
- But using the techniques shared herein, I believe we can come up with a solution to this most demanding metrology problem:

  - put optical clock in space station (90 min)

  - use Bob Vessot's 3-freq. ionospheric delay removal technique (10 ps precision, white PM)

  - white PM averages as tau^-3/2, which means in 10 days could have 10^-18.

- This is like a super portable clock in the sky ☺

</div>

David: The performance of the most recent optical standards are amazing: As you know, with some clever physics, your team at NIST set the world record for atomic-clock stability using two cold-atom Ytterbium ensembles. The stability achieved gave an ADEV of $\sigma_y(\tau) = 8\text{x}10^{-17}\,\tau^{-1/2}$. Their results are published in NATURE PHOTONICS, entitled, *Ultrastable optical clock with two cold-atom ensembles,* M. Schioppo, R. C. Brown, W. F. McGrew, N. Hinkley, R. J. Fasano, K. Beloy, T. H. Yoon, G. Milani, D. Nicolodi, J. A. Sherman, N. B. Phillips, C. W. Oates, and A. D. Ludlow.

This level of stability is not only a hundred-thousand times better than is now needed for GPS, where the atomic clocks are synchronized at the nanosecond level, but also can reach $1\text{x}10^{-18}$ with two hours averaging time, tau, which is equivalent to one second in 30 billion years.

That's what you get when you live to be eighty- two years old! So in my life timekeeping has improved a billion-fold with the earth stable at the 1e-9 level down to current standards approaching 1e-18. As you well know, how to compare these standards globally is a big problem; they are so good. A concept that Professor Ashby and I have been playing around with might help in how one might make a GPS like system with millimeter positioning accuracies and with super optical clocks on board the space vehicles (SV). It is exciting to think about where this could take us. Determining the SVs position in space can be a big challenge. We have shown that using Kepler's Third law has the potential to give you millimeter positioning accuracy; it has the needed orthogonality. Bob Vessot's Gravity Probe A three frequency microwave 10 picosecond time transfer system along with the zero-g Gravity Probe B system might lead to a solution. For GLONASS, the Russians use a laser to bounce a signal off of a corner reflector and measure the round trip time. I saw that system when I was at VNIIFTRI in 2014 in conjunction with an invited talk I was asked to give at a Time and Space Symposium in Suzdal. They were able to achieve 5 cm position accuracies back then. But that system is weather dependent, whereas the one we are thinking about is not.

**HEWLETT PACKARD**

The Science of Timekeeping

Application Note 1289
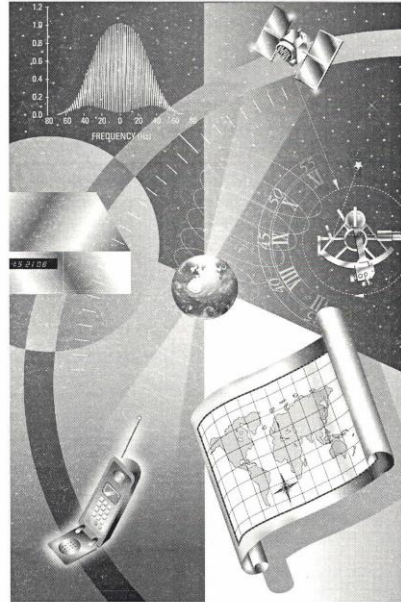
- UTC: Official World Time
- GPS: A Time Distribution Utility
- Time: An Historical and Future Perspective
- From Laboratory to Practical Use
- Broad Applications Across Society

David: While I was consulting for HP, they asked me to write *The Science of Timekeeping* application note. I invited Professor Ashby and Cliff Hodge from NPL to help. It is available on my www.allanstime.com web site under publications. It shows the history of timekeeping all the way back to the John Harrison famous chronometer. It was published in 1997. It shares many of the extremely large and important applications of precise timing -- including how GPS works and the relativity equations associated therewith, but it is written for a lay audience. http://allanstime.com/Publications/DWA/Science_Timekeeping/index.html
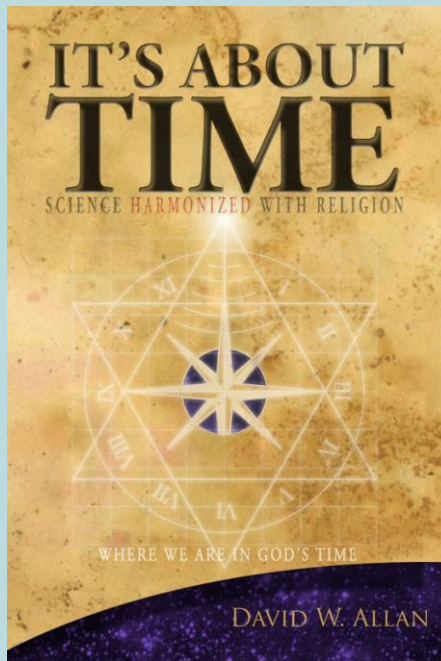
# My Book
(autobiographical)

- Explains timekeeping and how GPS works

- Can be purchased at this link: www.ItsAboutTimeBook.com. It is also available in audio formats.

- Shares advanced concepts in what is time and some Unified Field Theory experiments discovering diallel-field lines with several experiments validating.

David: I wrote this book in 2013-14. It took us a year. It is basically auto-biographical. It was a very enjoyable book to write. I use an advanced scientific method for the scientific analysis, and both inductive and deductive reasoning to show how data can show the beauty of our universe. Chapter 20 shows how time is kept in the world and how GPS works – including the relativity. Chapter 21 describes the Unified Field Theory (UFT) work that we did. I believe the ideas therein if incorporated would help the world be a better place.

Diallel-field line coupling of the planets to the sun shown in sunspot activity
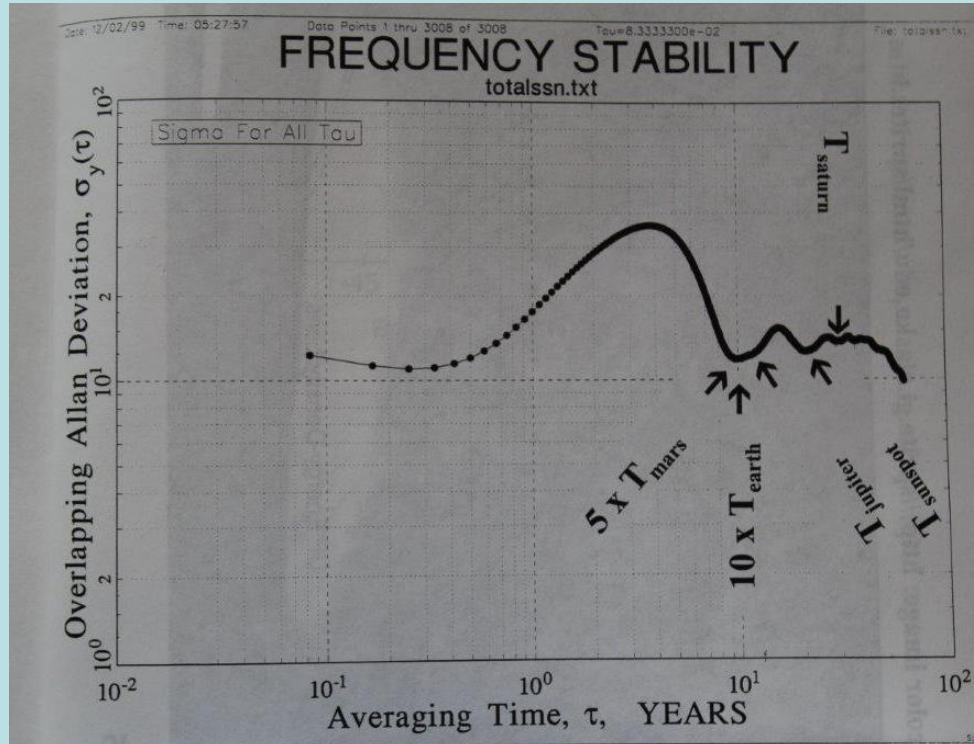
David: In our UFT experiments we document the existence of what we call diallel-field lines. This theory predicts a coupling between energy density of our planets and sunspot activity. I did an analysis of sunspot activity over 100 years and used ADEV to analyze that data. In the next slide you see evidence of the periods of the planets as predicted by this theory.

Please Allow a Personal Note

- I have been privileged to stand on the shoulders of giants over my career path.
- My life mission is to use my time and talents to serve – to try and help the world be a better place.
- My hope is that what I share will help each of you both in your lives as well as in your work.
- It seems that the more I get to serve, the happier and more blessed is my life.

David: In the next slide, I share a personal note, if you will. I've been privileged to stand on the shoulders of giants over my career path. I'm extremely grateful. Wonderful people, brilliant contributors.

I feel like my mission is to use my time and talents to serve and to help the world be a better place. My hope is that what I share will help each of you in your lives as well as in your work. It seems that the more I get to serve, the happier and more blessed is my life.

David: I have learned to be grateful for challenges for they are part of life. We can grow from these and we can view them as growth opportunities. I've had many in my life and have learned a lot from those challenges. I believe it's really important to use your mind but listen to your heart.

Time

Involves

Measurements

Extraordinaire

David: TIME involves measurements extraordinaire. The experiment we did in this home -- showing the existence of a fifth dimension was really fun. We were able to show that we could modify gravity. The experiment we did at BYU demonstrating the existence of a quantum transition in these diallel-field lines was one of the most exciting pieces of science I have ever done.

Elizabeth: Great, thank you very much for these words of wisdom.