

Fundamentals of Magnetism

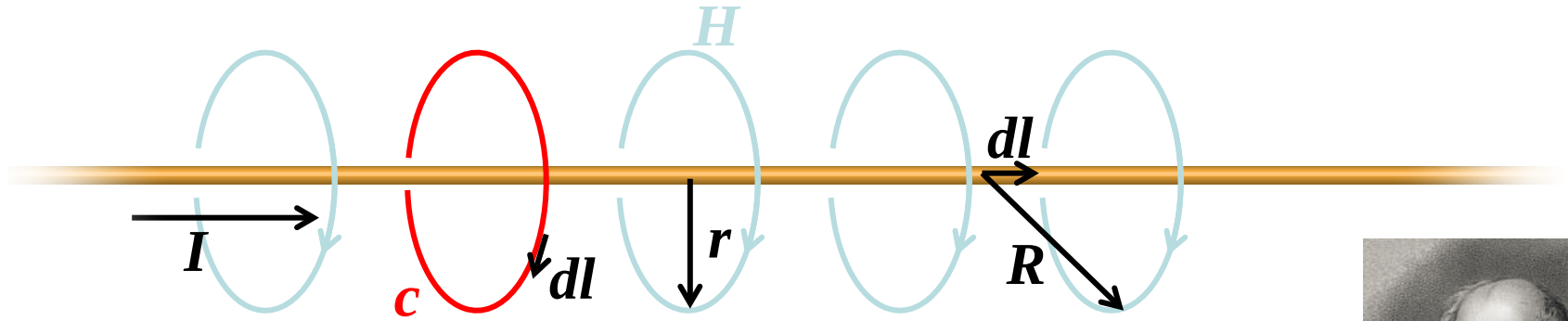
Albrecht Jander

Oregon State University

Part I: H, M, B, χ, μ

Part II: $M(H)$

Magnetic Field from Current in a Wire



Ampere's Circuital Law:

Biot-Savart Law:

$$\oint_c \vec{H} \cdot d\vec{l} = I$$

$$d\vec{H} = \frac{Id\vec{l} \times \hat{R}}{4\pi|\vec{R}|^2}$$

$$H = \frac{I}{2\pi r} \text{ [A/m]}$$



André-Marie Ampère
(1775–1836)

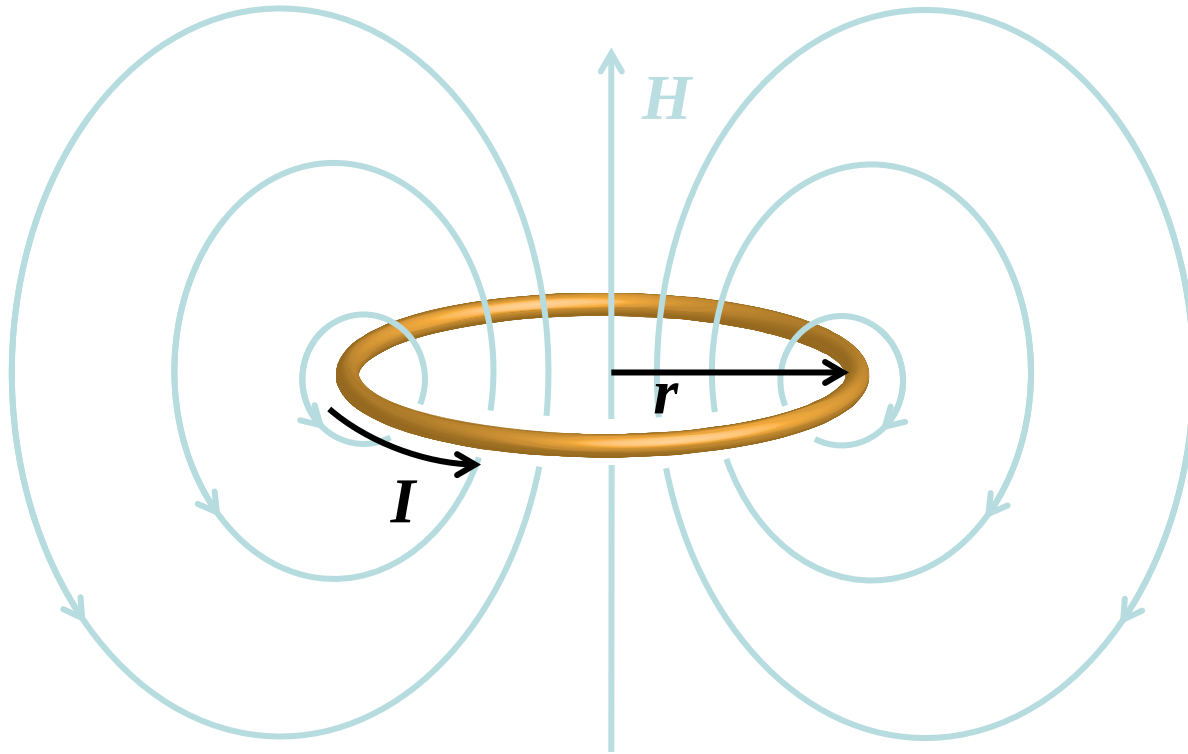


Jean-Baptiste Biot
(1774–1862)



Félix Savart
(1791–1841)

Magnetic Field from Current Loop



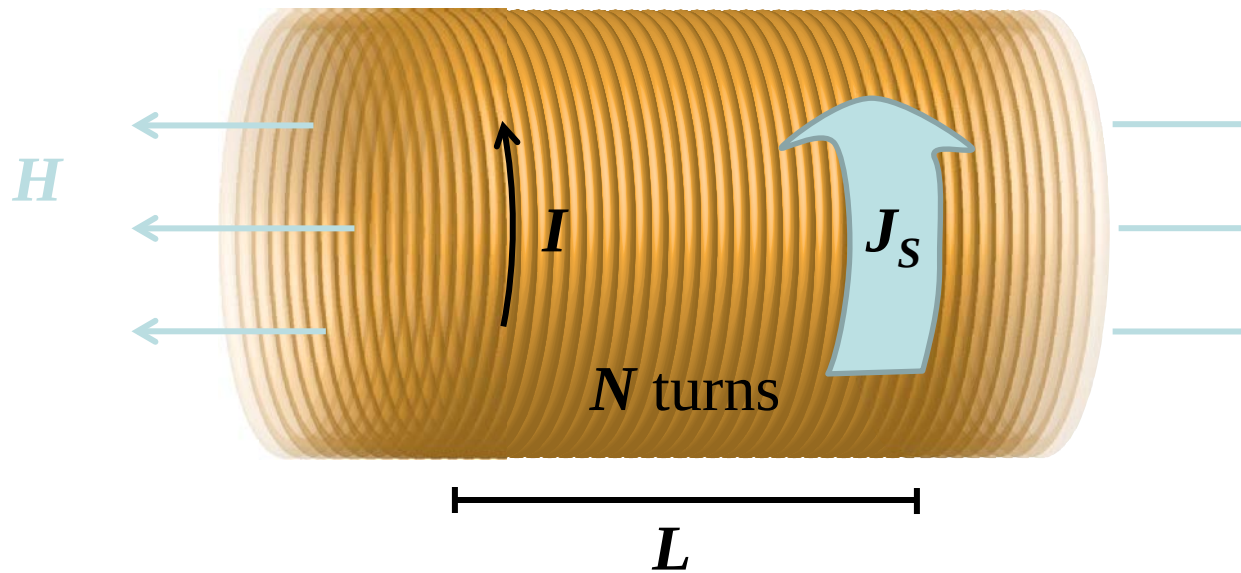
Use Biot-Savart Law:

$$d\vec{H} = \frac{I d\vec{l} \times \hat{R}}{4\pi |\vec{R}|^2}$$

In the center:

$$H = \frac{I}{2r} \text{ [A/m]}$$

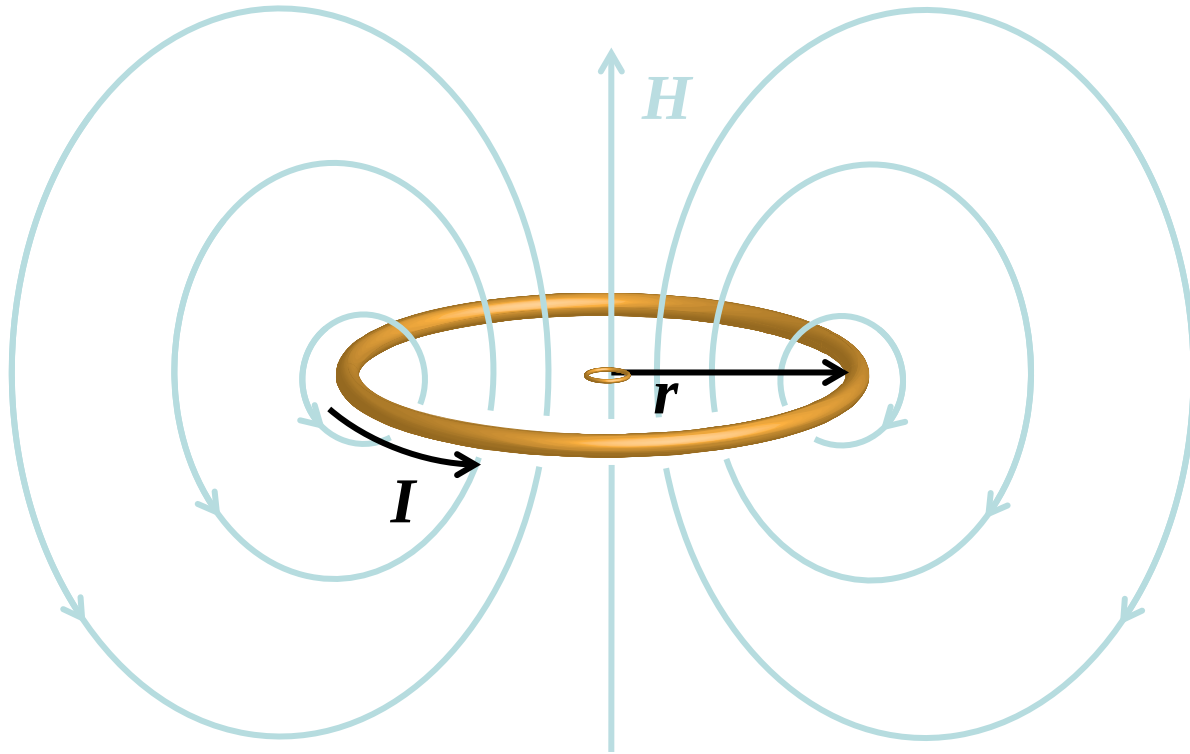
Magnetic Field in a Long Solenoid



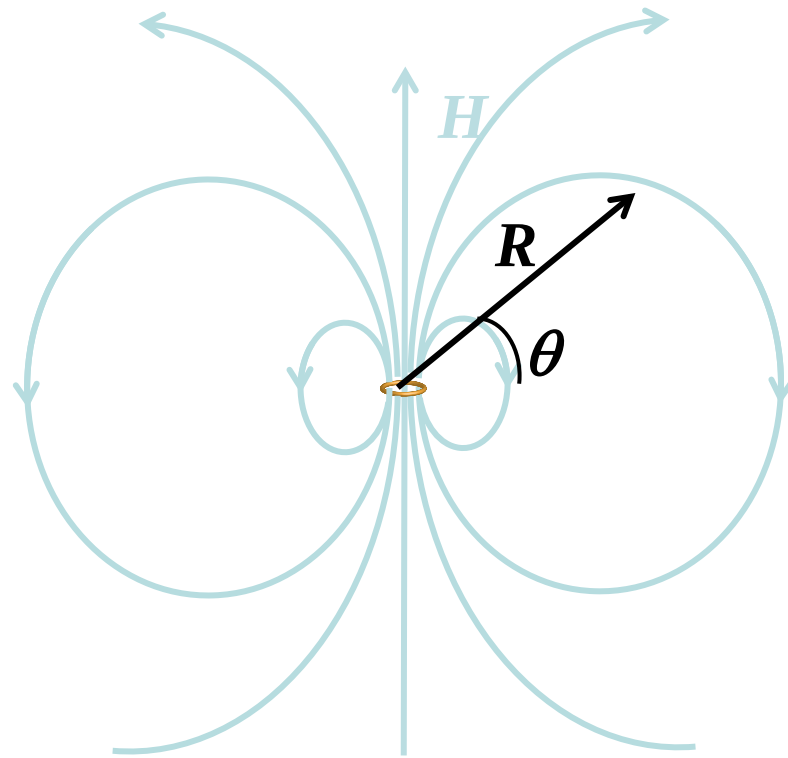
Inside, field is uniform with:

$$H = \frac{NI}{L} \text{ [A/m]} \qquad H = J_S \text{ [A/m]}$$

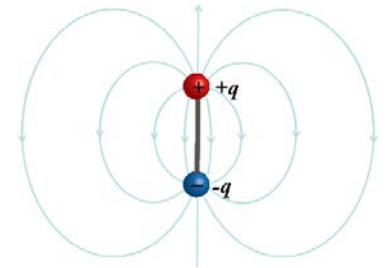
Magnetic Field from “Small” Loop (Dipole)



Magnetic Field from Small Loop (Dipole)



Compare to
Electric dipole:

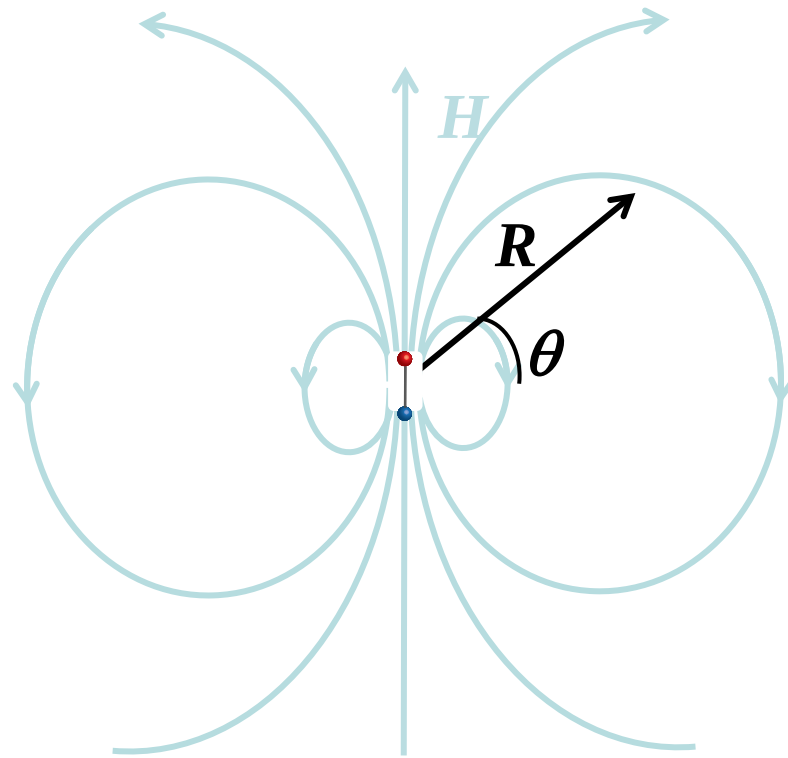


For $R \gg r$

$$H = \frac{IA}{4\pi R^3} [2\hat{r} \cos(\theta) + \hat{\theta} \sin(\theta)] \quad [\text{A/m}]$$

Magnetic (dipole) moment: $\vec{m} = I\vec{A} \quad [\text{Am}^2]$

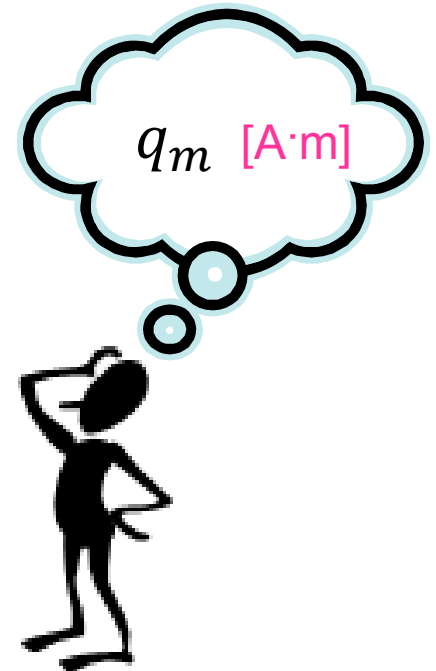
Magnetic Field of a Dipole



For $R \gg r$

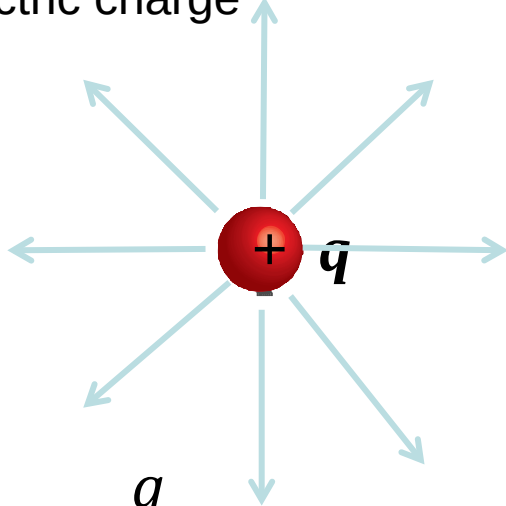
$$H = \frac{q_m d}{4\pi R^3} [2\hat{r} \cos(\theta) + \hat{\theta} \sin(\theta)] \quad [\text{A/m}]$$

Magnetic (dipole) moment: $\vec{m} = q_m \vec{d} \quad [\text{Am}^2]$



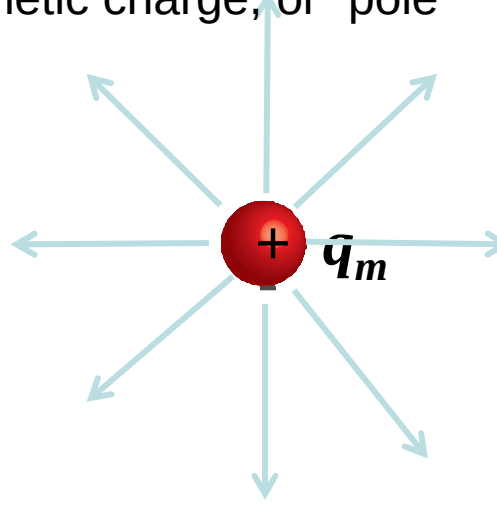
Magnetic Poles and Dipoles

Electric charge



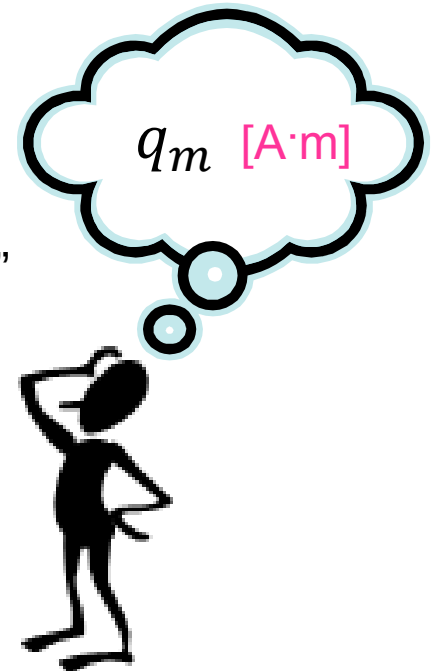
$$D = \frac{q}{4\pi R^2}$$

Magnetic charge, or "pole"

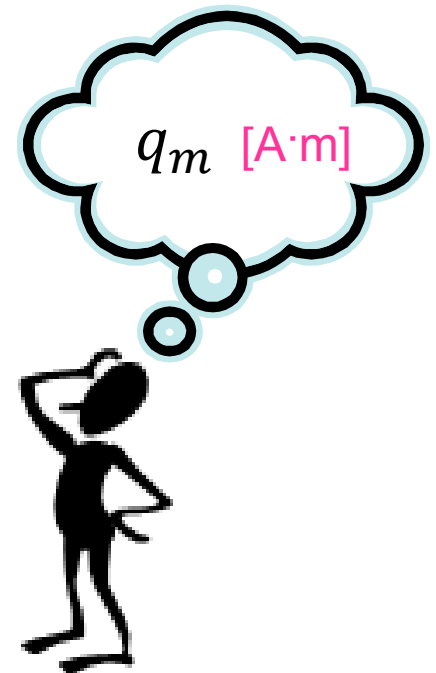
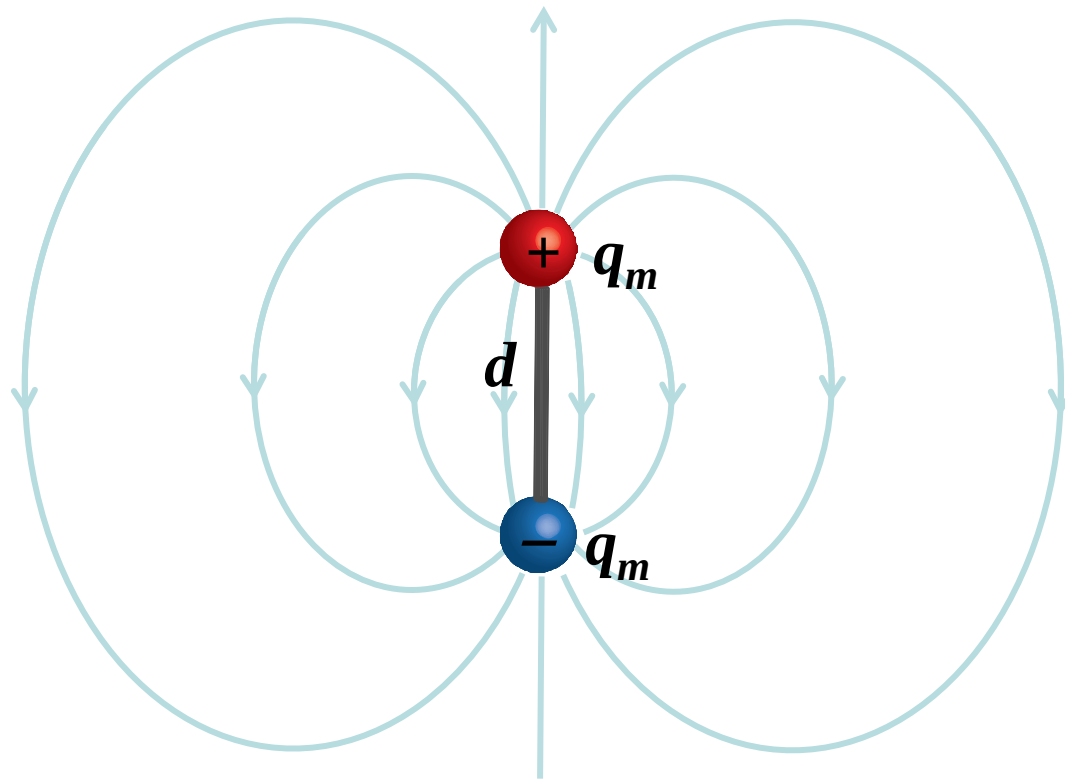


Magnetic field from a magnetic "monopole"

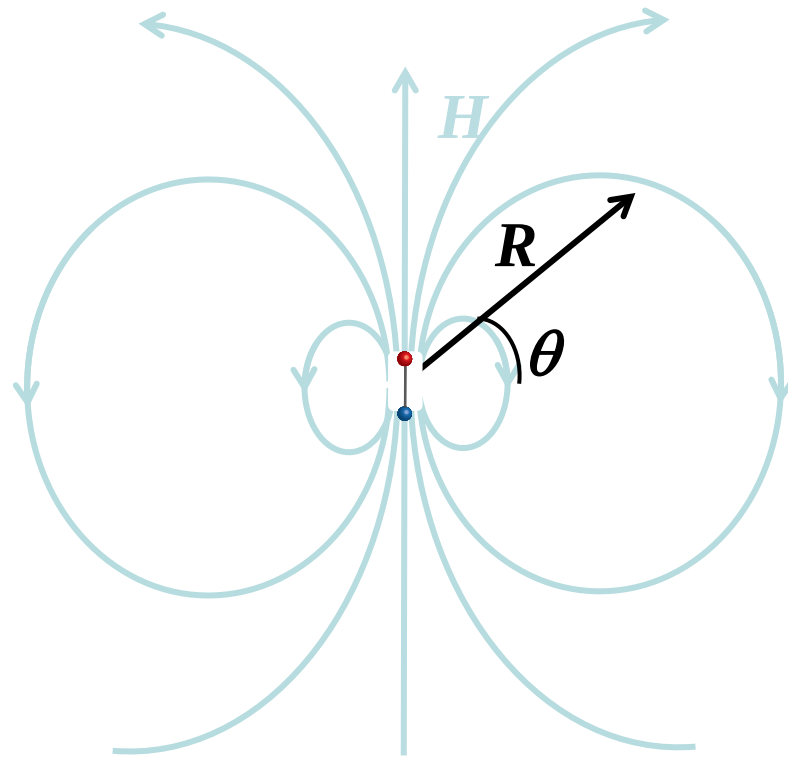
$$H = \frac{q_m}{4\pi R^2} \quad [\text{A/m}]$$



Magnetic Field of a Dipole



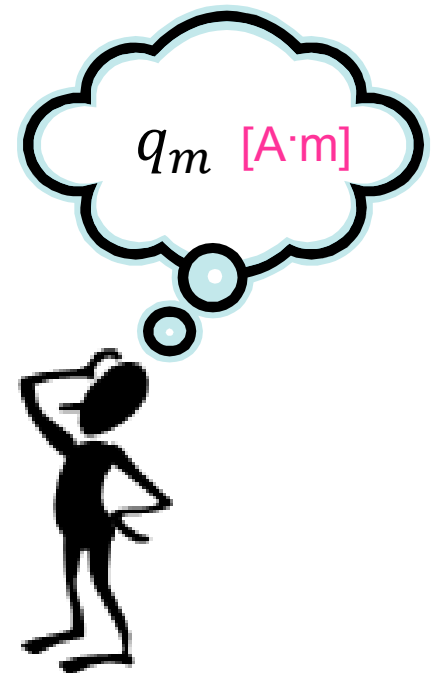
Magnetic Field of a Dipole



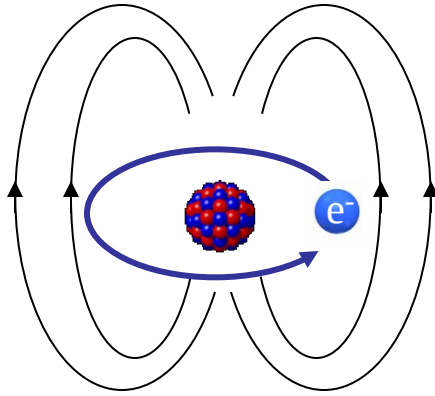
For $R \gg r$

$$H = \frac{q_m d}{4\pi R^3} [2\hat{r} \cos(\theta) + \hat{\theta} \sin(\theta)] \quad [\text{A/m}]$$

Magnetic (dipole) moment: $\vec{m} = q_m \vec{d} \quad [\text{Am}^2]$



Orbital Magnetic Moment



- Electrons orbiting a nucleus are like a circulating current producing a magnetic field.

- For an electron with charge q_e orbiting at a radius R with frequency f , the **Orbital Magnetic Moment** is

$$m = IA = -q_e f \pi R^2 \quad [\text{Am}^2]$$

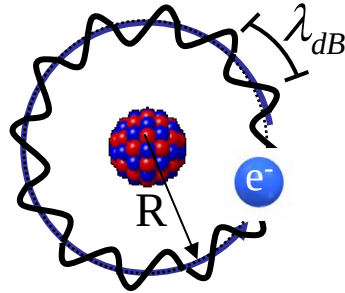
- It also has an **Orbital Angular Momentum**

$$L = m_e v R = m_e 2\pi R f R$$

- Note: $\frac{m}{L} = \frac{-q_e}{2m_e} = \text{gyromagnetic ratio}$

Orbital Moment is Quantized

Bohr model of the atom



• DeBroglie wavelength is:

$$\lambda_{dB} = \frac{h}{m_e v}$$

• Bohr Model: orbit must be integer number of wavelengths

$$2\pi R = N\lambda_{dB} = N \frac{h}{m_e v} = N \frac{h}{m_e 2\pi R f}$$

• Thus the orbital magnetic moment is quantized:

$$m = -q_e f \pi R^2 = N \frac{h}{2\pi} \frac{q_e}{2m_e} = N \frac{\hbar q_e}{2m_e}$$

• Magnetic moment restricted to multiples of

Bohr Magneton

$$\mu_B = \frac{\hbar q_e}{2m_e} = 9.2742 \times 10^{-24} [Am^2]$$



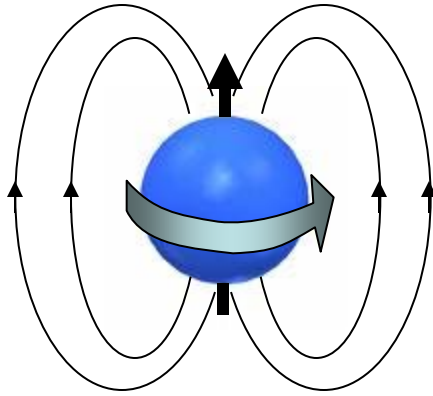
Niels Bohr
(1885-1962)



Scanned at the American
Institute of Physics

Louis de Broglie
(1892-1987)

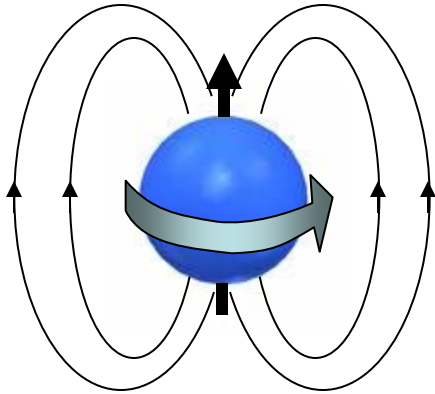
Spin



- **Spin** is a property of subatomic particles (just like charge or mass.)
- A particle with **spin** has a magnetic dipole moment and angular momentum.
- Spin may be thought of **conceptually** as arising from a spinning sphere of charge. (However, note that neutrons also have spin but no charge!)



Pauli and Bohr contemplate the “spin” of a tippy-top



Electron Spin

- *When measured in a particular direction, the measured angular momentum of an electron is*

$$L_z = \pm \frac{\hbar}{2}$$

- *When measured in a particular direction, the measured magnetic moment of an electron is*

$$m_z = s_z \frac{\hbar q_e}{m_e} = \pm \mu_B$$

- We say “spin up” and “spin down”

- Note:

$$\frac{m}{L} = \frac{-q_e}{m_e}$$

Compare:

Spin

Orbital

$$m_z = \pm \mu_B$$

$$m_z = N\mu_B$$

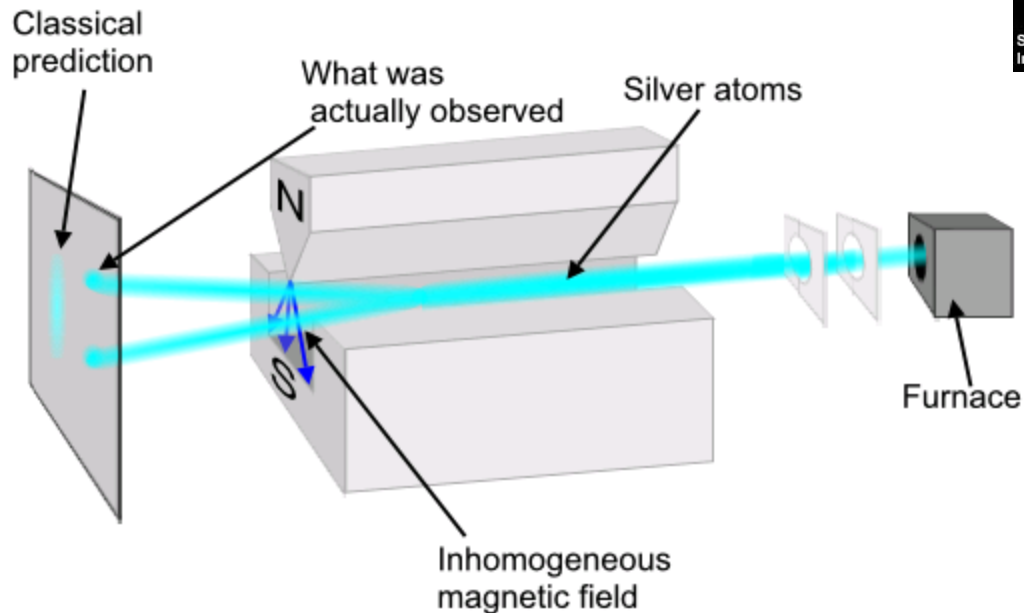
$$\frac{m}{L} = \frac{-q_e}{m_e}$$

$$\frac{m}{L} = \frac{-q_e}{2m_e}$$

$$g = 2 \quad \frac{m}{L} = g \frac{-q_e}{2m_e} \quad g = 1$$

Stern-Gerlach Experiment - 1922

Demonstrated that magnetic moment is quantized with $\pm\mu_B$



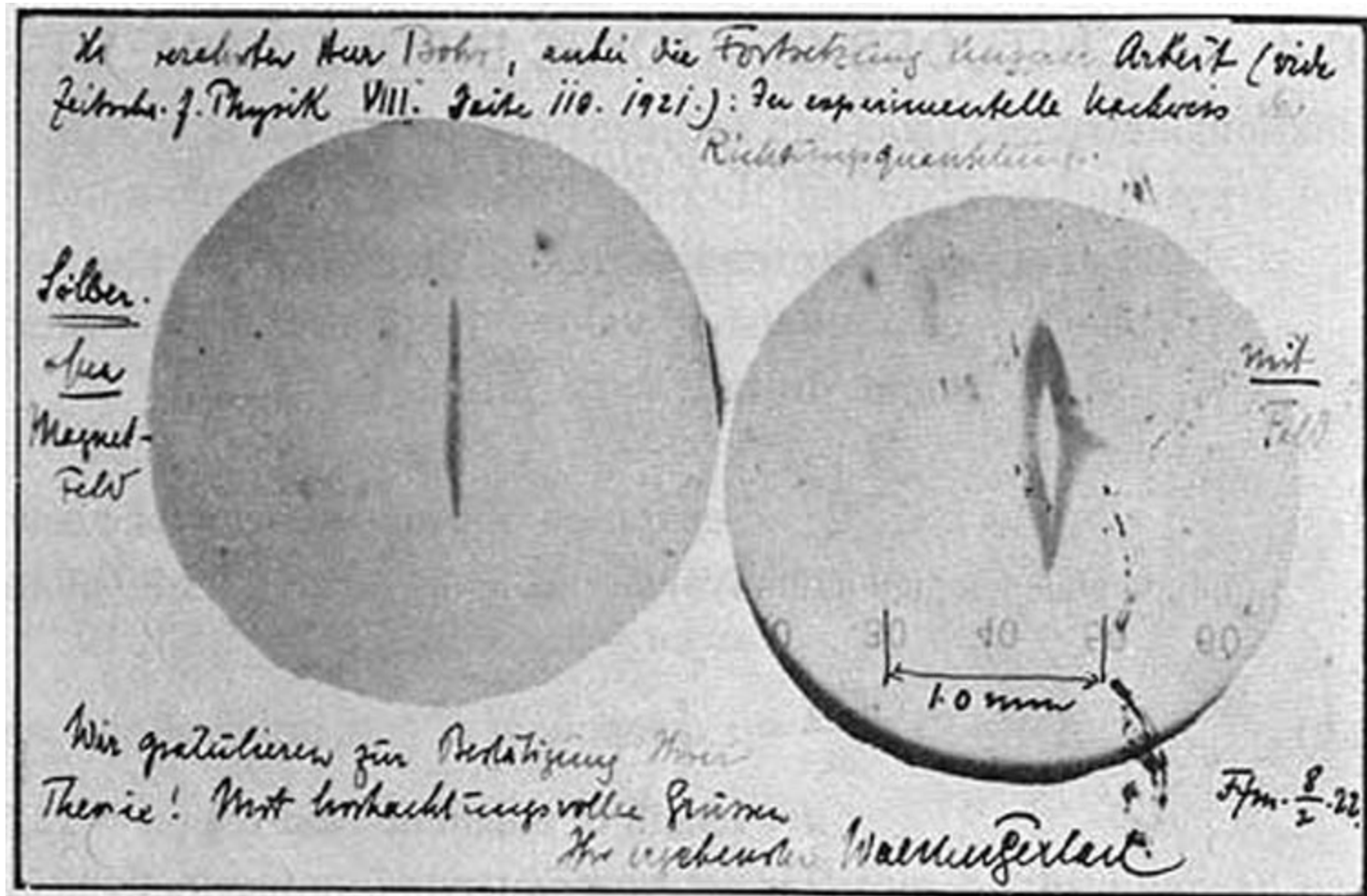
Otto Stern (1888-1969)



Walter Gerlach
(1889-1979)

Walther Gerlach, Otto Stern (1922). "Das magnetische Moment des Silberatoms". *Zeitschrift für Physik A Hadrons and Nuclei* 9.

Stern-Gerlach Experiment



Gerlach's postcard, dated 8 February 1922, to Niels Bohr. It shows a photograph of the beam splitting, with the message, in translation: "Attached [is] the experimental proof of directional quantization. We congratulate [you] on the confirmation of your theory." (Physics Today December 2003)

Quantum Numbers of Electron Orbitals

- Electrons bound to a nucleus move in orbits identified by quantum numbers (from solution of Schrödinger's equation):

n	$1 \dots$	principal quantum number, identifies the “shell”
ℓ	$0 \dots n-1$	angular momentum q. #, type of orbital, e.g. s,p,d,f
m_ℓ	$-\ell \dots \ell$	magnetic quantum number
m_s	$+1/2$ or $-1/2$	spin quantum number

For example, the electron orbiting the **hydrogen** nucleus (in the ground state) has:

$$n=1, \ell=0, m_\ell=0, m_s=+1/2$$

In spectroscopic notation: $1s^1$

Spectroscopic notation

$\ell=0$ s

$\ell=1$ p

$\ell=2$ d

$\ell=3$ f

n ℓ # of electrons
 in orbital

$1s^2 2s^2$

Spin and Orbital Magnetic Moment

- Total orbital magnetic moment (sum over all electron orbitals)

$$m_{tot_orbital} = \mu_B \sum m_\ell$$

- Total spin magnetic moment

$$m_{tot_spin} = 2\mu_B \sum m_s$$

full orbitals: no net moment

E.g. Iron:



How to fill remaining d orbitals ($\ell=2$) 6 electrons for 10 spots:

	$m_\ell = -2$	$m_\ell = -1$	$m_\ell = 0$	$m_\ell = +1$	$m_\ell = +2$
$m_s = +1/2$	↑	↑	↑	↑	↑
$m_s = -1/2$					↓

$$m_{tot_orbital} = 2\mu_B$$

$$m_{tot_spin} = 4\mu_B$$

Hund's rules:

Maximize $\sum m_s$

Then maximize $\sum m_\ell$

PERIODIC TABLE

Atomic Properties of the Elements

NIST

National Institute of Standards and Technology
Technology Administration, U.S. Department of Commerce

18
VIII A

Frequently used fundamental physical constants	
For the most accurate values of these and other constants, visit physics.nist.gov/constants	
1 second = 9 192 631 770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of ^{133}Cs	
speed of light in vacuum	c 299 792 458 m s ⁻¹ (exact)
Planck constant	h 6.626 070 15 × 10 ⁻³⁴ J s (exact)
elementary charge	e 1.602 176 634 × 10 ⁻¹⁹ C
electron mass	m_e 9.109 383 56 × 10 ⁻³¹ kg
	$m_e c^2$ 0.511 MeV
proton mass	m_p 1.672 621 63 × 10 ⁻²⁷ kg
fine-structure constant	α 1/137.036
Rydberg constant	R_∞ 10 973 731.7 m ⁻¹
	R_H 3.289 842 × 10 ¹⁵ Hz
	$R_\infty hc$ 13.6057 eV
Boltzmann constant	k 1.380 658 × 10 ⁻²³ J K ⁻¹

Solids
 Liquids
 Gases
 Artificially Prepared

Physics
Laboratory
physics.nist.gov

Standard Reference
Data Group
www.nist.gov/srd

Period

7

Symbol

Atomic
Weight

Ground-state
Configuration

Atomic Number	Ground-state Level
58	1G_4
Ce	
Cerium	
140.116	
[Xe]4f ¹ 5d ¹ 6s ²	
5.5387	
Actinides	
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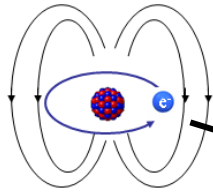
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Magnetization, $\vec{M}$



Atomic magnetic moment:

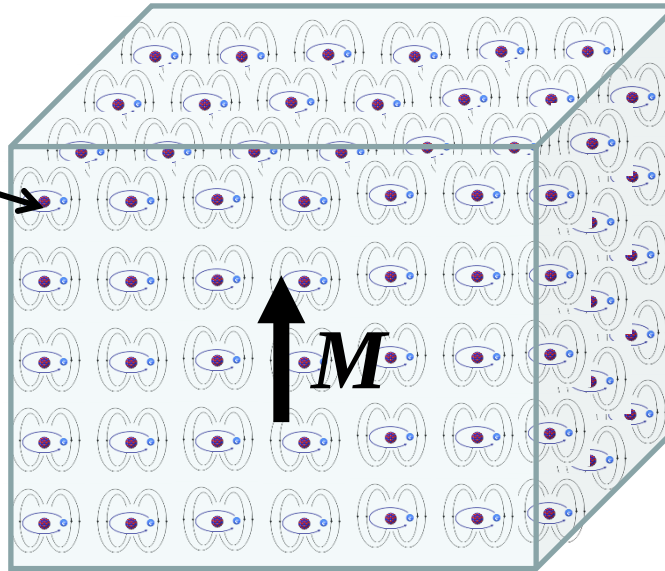
$$\vec{m}_{atom} \text{ [Am}^2\text{]}$$

Object magnetic moment:

$$\vec{m}_{obj.} = \sum \vec{m}_{atom} \text{ [Am}^2\text{]}$$

Atomic density:

$$d = \frac{N}{vol} \text{ [1/m}^3\text{]}$$

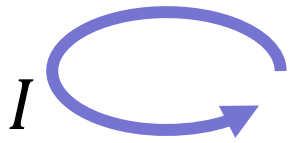


What is H here?

Magnetic moment per unit volume:

$$\vec{M} \stackrel{\text{def}}{=} \frac{\vec{m}_{obj.}}{vol} = \vec{m}_{atom} d \text{ [A/m]}$$

Equivalent Loop Currents



Atomic magnetic moment:

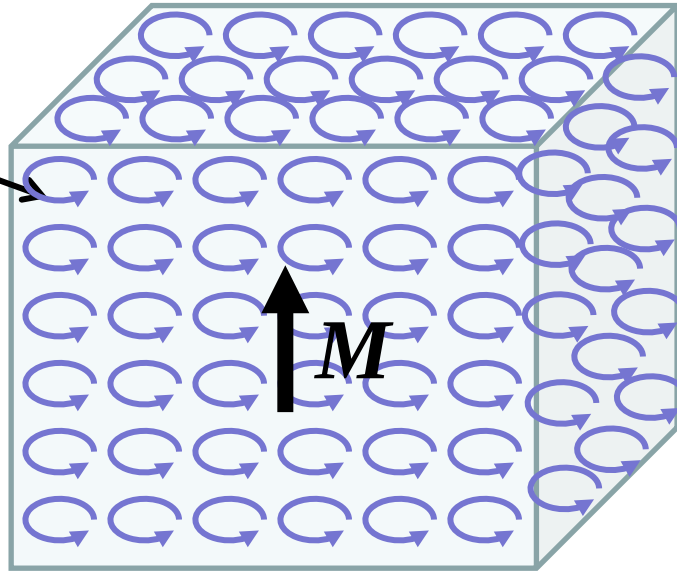
$$\vec{m}_{atom} = I\vec{A} \quad [\text{Am}^2]$$

Object magnetic moment:

$$\vec{m}_{obj.} = \sum \vec{m}_{atom} \quad [\text{Am}^2]$$

Atomic density:

$$d = \frac{N}{vol} \quad [1/\text{m}^3]$$

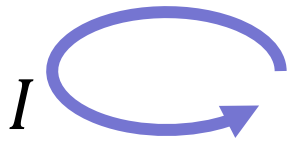


What is H here?

Magnetic moment per unit volume:

$$\vec{M} \stackrel{\text{def}}{=} \frac{\vec{m}_{obj.}}{vol} = \vec{m}_{atom} d \quad [\text{A/m}]$$

Equivalent Surface Current



Atomic magnetic moment:

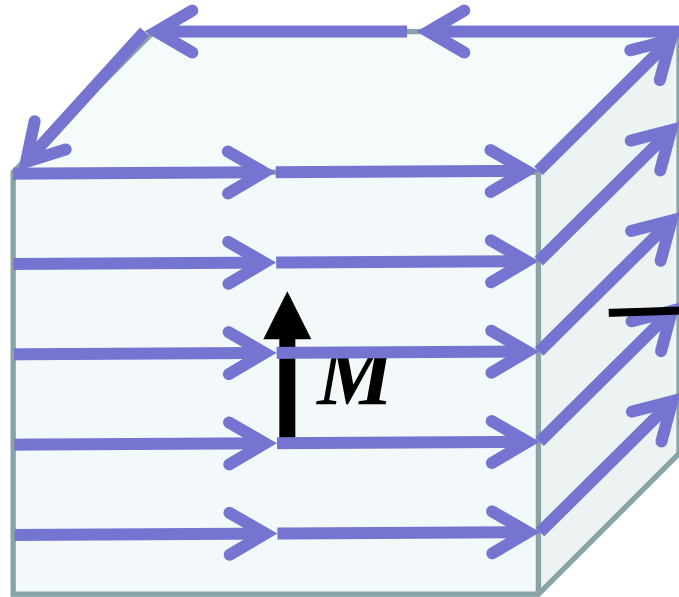
$$\vec{m}_{atom} = I \vec{A} \quad [\text{Am}^2]$$

Object magnetic moment:

$$\vec{m}_{obj.} = \sum \vec{m}_{atom} \quad [\text{Am}^2]$$

Atomic density:

$$d = \frac{N}{vol} \quad [1/\text{m}^3]$$

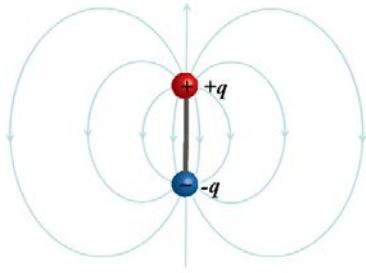


What is H here?

Equivalent surface current:

$$\vec{J}_S = \vec{M} \times \hat{n} \quad [\text{A/m}]$$

Magnetic Pole Model



Atomic magnetic moment:

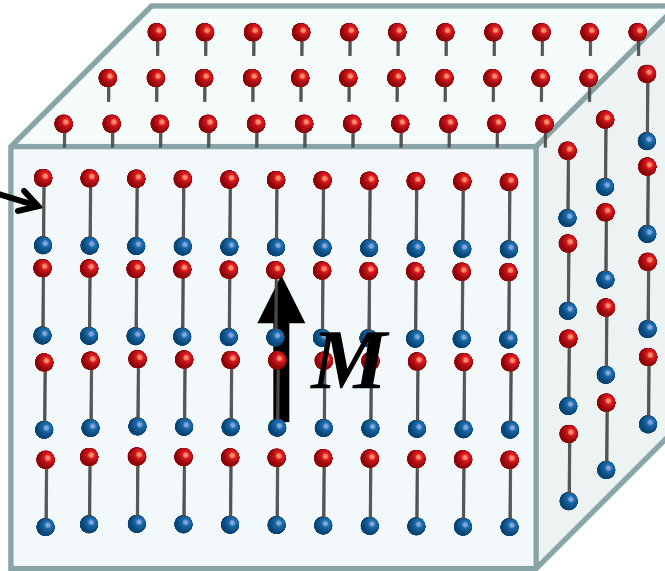
$$\vec{m}_{atom} = q_m \vec{d} \text{ [Am}^2\text{]}$$

Object magnetic moment:

$$\vec{m}_{obj.} = \sum \vec{m}_{atom} \text{ [Am}^2\text{]}$$

Atomic density:

$$d = \frac{N}{vol} \text{ [1/m}^3\text{]}$$

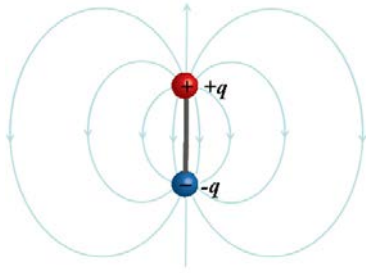


Magnetic moment per unit volume:

$$\vec{M} \stackrel{\text{def}}{=} \frac{\vec{m}}{vol} \text{ [A/m]}$$

What is H here?

Equivalent Surface Pole Density



Atomic magnetic moment:

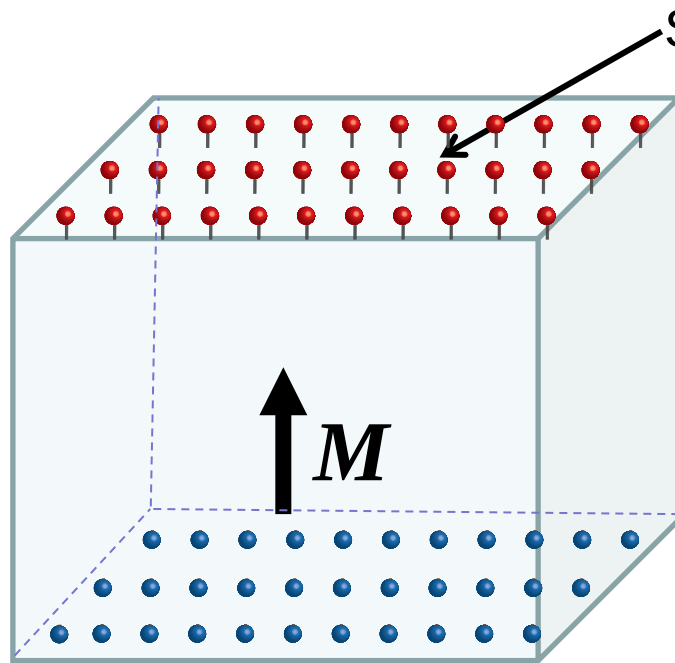
$$\vec{m}_{atom} = q_m \vec{d} \quad [\text{Am}^2]$$

Object magnetic moment:

$$\vec{m}_{obj.} = \sum \vec{m}_{atom} \quad [\text{Am}^2]$$

Atomic density:

$$d = \frac{N}{vol} \quad [1/\text{m}^3]$$



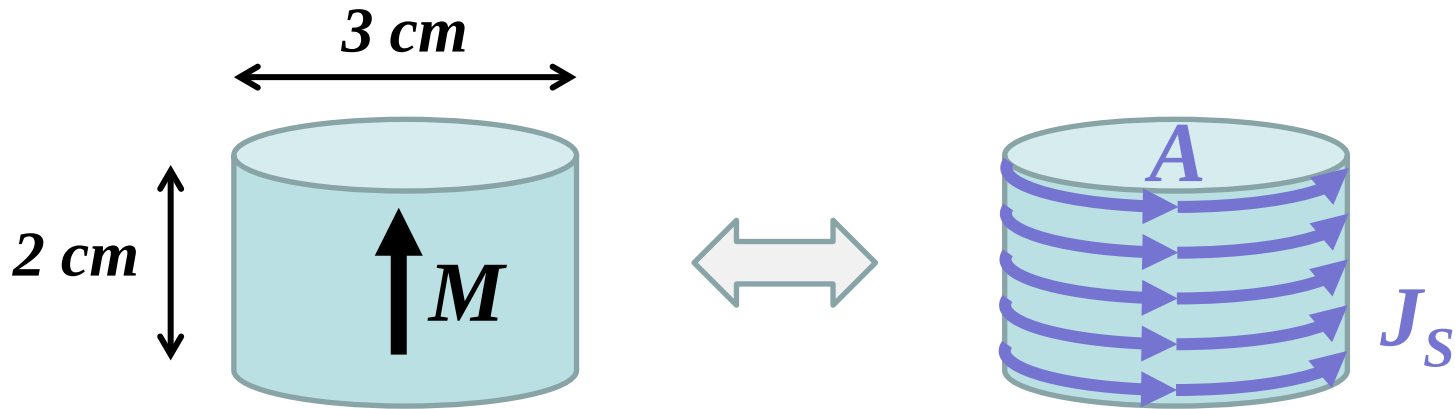
Surface charge density:

$$\rho_{ms} = \frac{q_m}{area} \quad \frac{[\text{Am}]}{[\text{m}^2]}$$

Magnetic pole density:

$$\rho_{ms} = \vec{M} \cdot \hat{n} \quad [\text{A/m}]$$

Example: NdFeB Cylinder



$$M = 10^6 \text{ [A/m]}$$

$$\text{vol} \simeq 14 \cdot 10^{-6} \text{ [m}^3\text{]}$$

$$m = 14 \text{ [Am}^2\text{]}$$

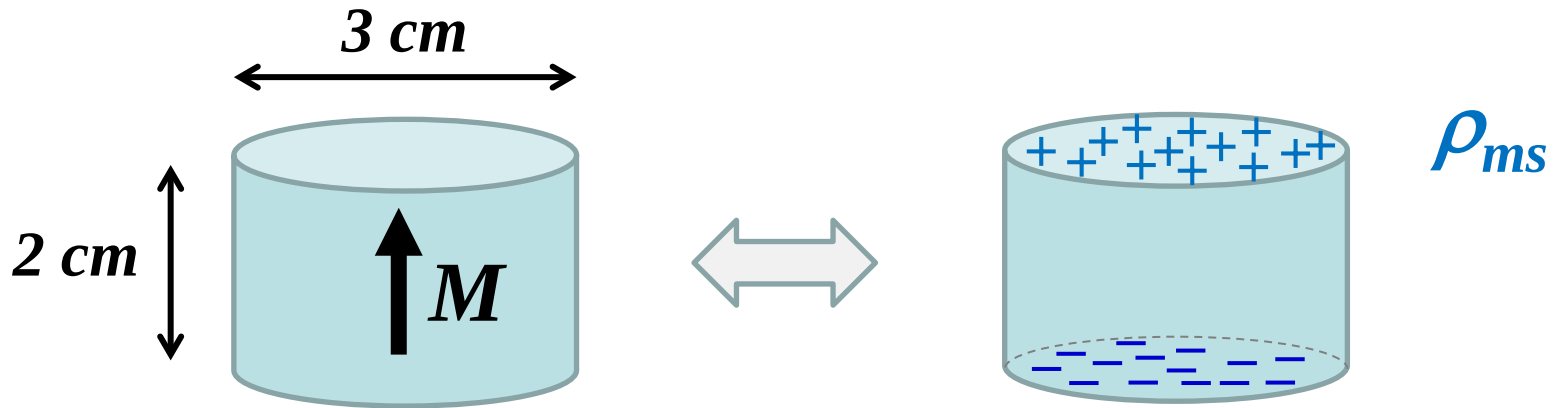
$$J_S = 10^6 \text{ [A/m]}$$

$$I = 20000 \text{ [A]}$$

$$A \simeq 7 \cdot 10^{-4} \text{ [m}^2\text{]}$$

$$m = IA = 14 \text{ [Am}^2\text{]}$$

Example: NdFeB Cylinder



$$M = 10^6 \text{ [A/m]}$$

$$\text{vol} \simeq 14 \cdot 10^{-6} \text{ [m}^3\text{]}$$

$$m = 14 \text{ [Am}^2\text{]}$$

$$\rho_{ms} = 10^6 \text{ [A/m]}$$

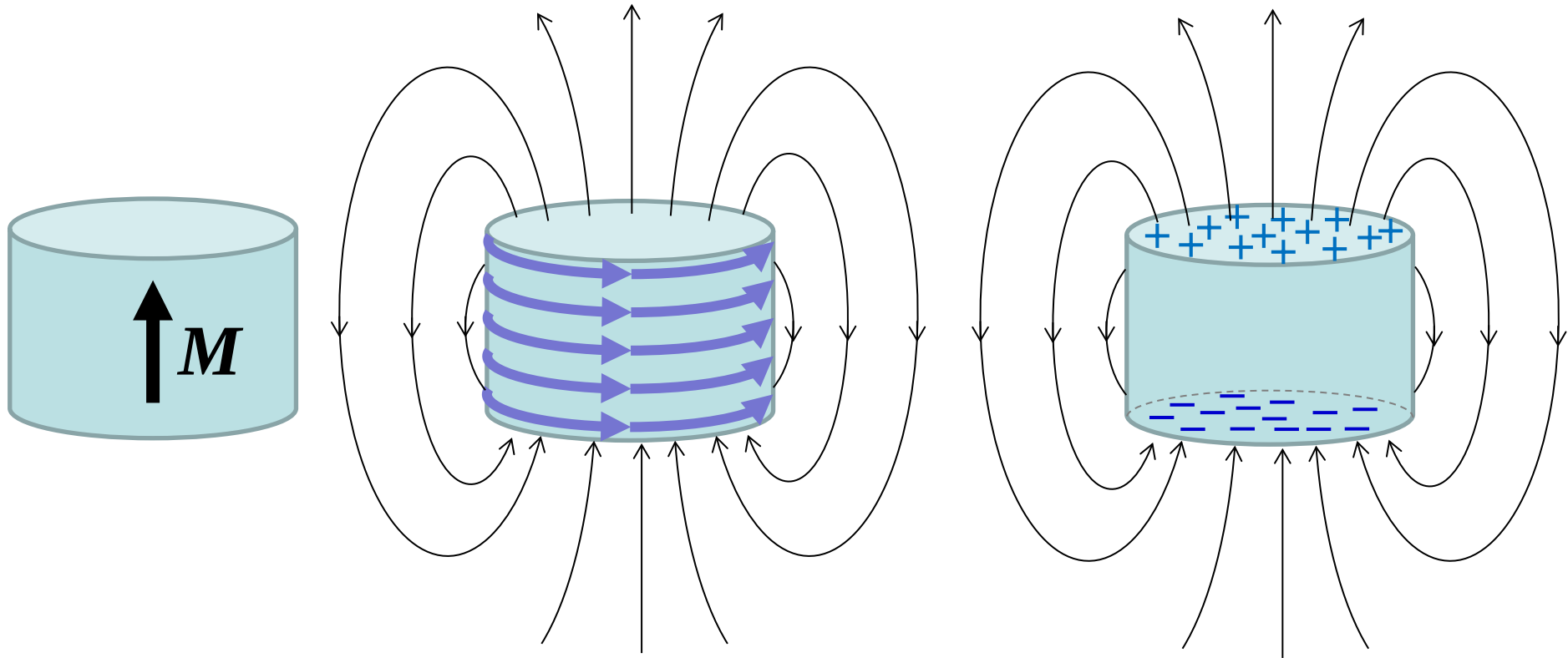
$$A \simeq 7 \cdot 10^{-4} \text{ [m}^2\text{]}$$

$$q_m = 700 \text{ [Am]}$$

$$d = 2 \cdot 10^{-2} \text{ [m]}$$

$$m = q_m d = 14 \text{ [Am}^2\text{]}$$

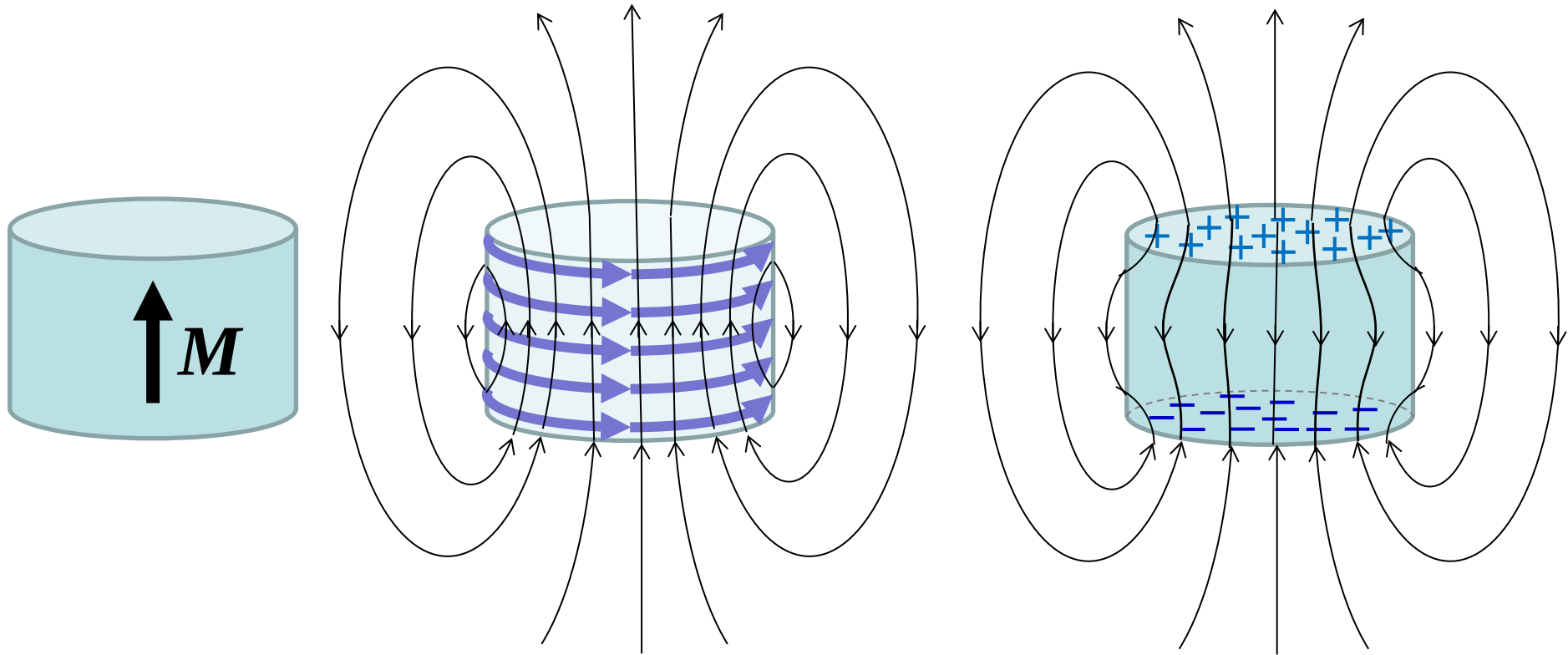
Example: NdFeB Cylinder



Result is the same external to magnetic material.

Hint: we are “far” away from the dipoles!!!

Example: NdFeB Cylinder



Result is different inside the magnetic material.

The Constitutive Relation

$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

$\mathbf{B}$ = Magnetic flux density [Tesla]

$\mathbf{H}$ = Magnetic field, “Magnetizing force” [A/m]

$\mathbf{M}$ = Magnetization [A/m]

μ_0 = Magnetic constant, “Permeability of free space” [Tesla-m/A] [Henry/m] [N/A²]

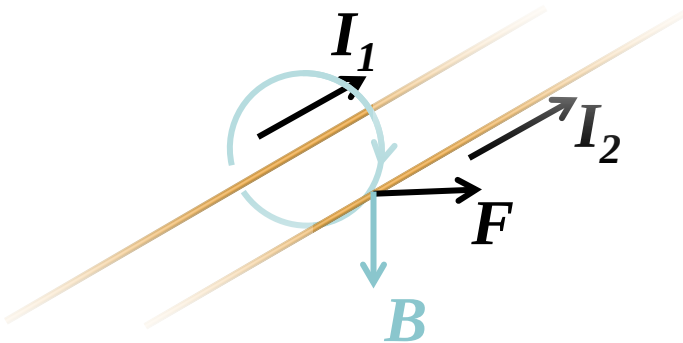
Note, in vacuum, $\mathbf{M}$ is zero:

$$\vec{B} = \mu_0 \vec{H}$$

What is μ_0 ?

μ_0 comes from the SI definition of the Ampere:

The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per meter of length.



$$B = \mu_0 H = \frac{\mu_0 I_1}{2\pi r}$$

$$\vec{F} = I_2 \vec{L} \times \vec{B}$$

Maxwell's Equations (Magnetostatics)

Differential form

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = \vec{J}$$

Integral form

$$\oiint \vec{B} \cdot d\vec{s} = 0$$

$$\oint \vec{H} \cdot d\vec{l} = I$$

With:

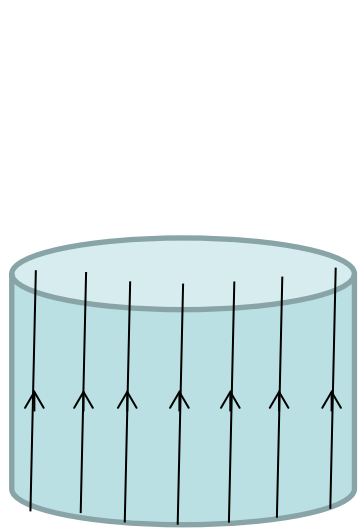
$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

$$\nabla \cdot \vec{B} = \mu_0(\nabla \cdot \vec{H} + \nabla \cdot \vec{M})$$

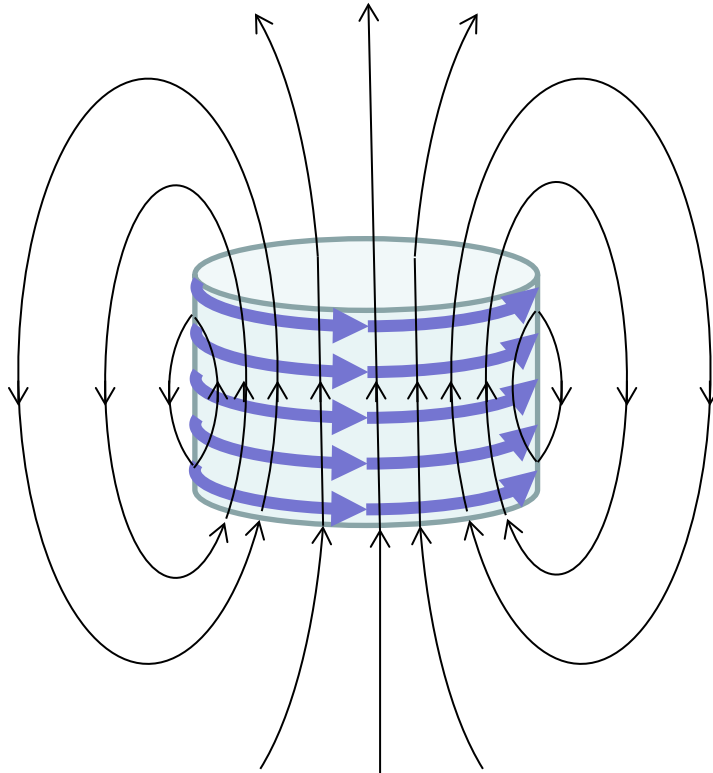
$$\nabla \cdot \vec{H} = -\nabla \cdot \vec{M} = \rho_m \quad [\text{A/m}^2]$$

Magnetic charge density
i.e. magnetic charge/volume

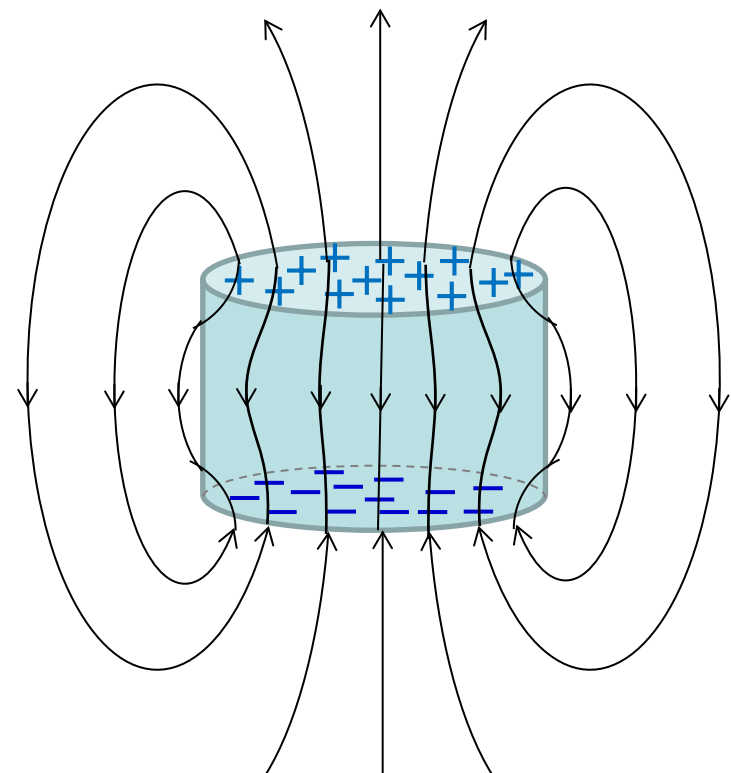
Example: NdFeB Cylinder



M



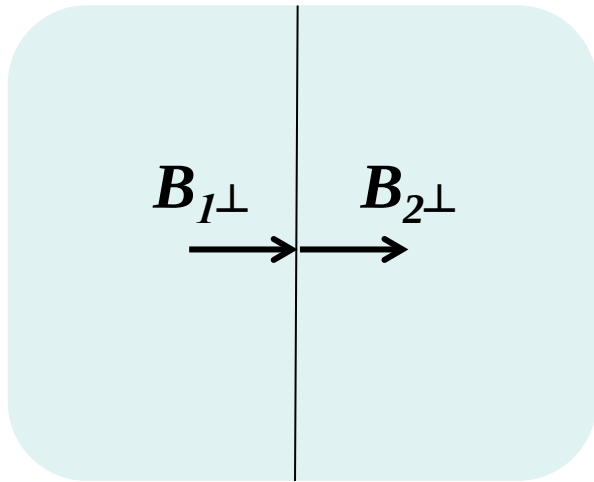
B



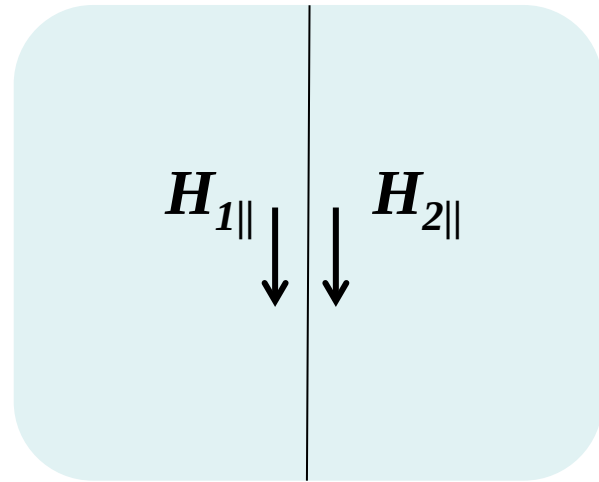
H

$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

Boundary Conditions

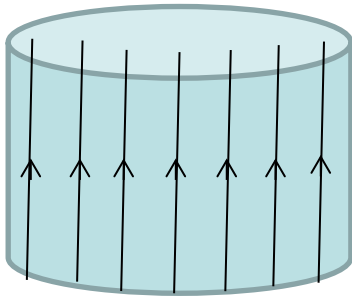


The perpendicular component of B is continuous across a boundary

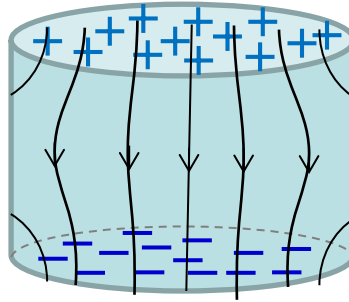


The transverse component of H is continuous across a boundary (unless there is a true current on the boundary)

Demagnetizing Fields



M



H_d

$$H_d \propto M$$

$$H_d = -NM$$



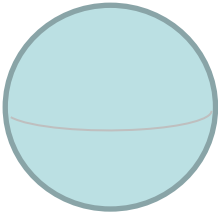
Demagnetizing Factor

Demagnetizing Factors: Special Cases

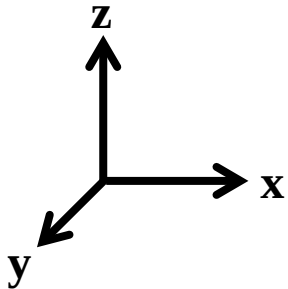
$$H_d = -NM$$

$$N_x + N_y + N_z = 1$$

Sphere



$$N_x = N_y = N_z = \frac{1}{3}$$

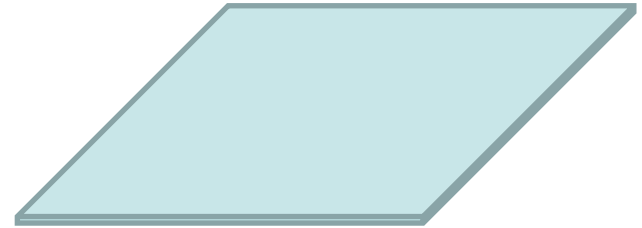


Long Rod



$$N_x = N_y = \frac{1}{2}$$
$$N_z = 0$$

Thin Sheet



$$N_x = N_y = 0$$
$$N_z = 1$$

Demagnetizing Factors

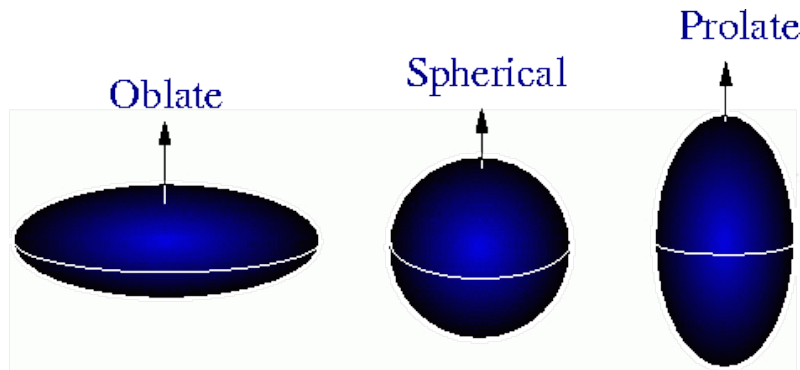


Table 2.2. Demagnetizing Factors for Rods and Ellipsoids Magnetized Parallel to the Long Axis (after Bozorth^{G.10})

Dimensional Ratio k	Rod	Prolate Ellipsoid	Oblate Ellipsoid
1	0.27	0.3333	0.3333
2	0.14	0.1735	0.2364
5	0.040	0.0558	0.1248
10	0.0172	0.0203	0.0696
20	0.00617	0.00675	0.0369
50	0.00129	0.00144	0.01472
100	0.00036	0.000430	0.00776
200	0.000090	0.000125	0.00390
500	0.000014	0.0000236	0.001567
1000	0.0000036	0.0000066	0.000784
2000	0.0000009	0.0000019	0.000392

“Demag” fields in ellipsoids of revolution are uniform.

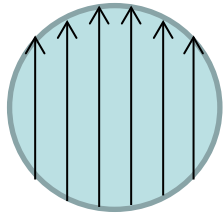
$$H_d = -NM$$

$$N_x + N_y + N_z = 1$$

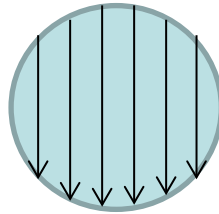
Example: Ba-Ferrite Sphere

Barrium Ferrite:

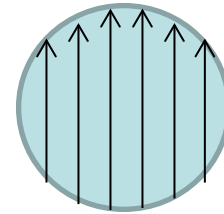
$$\mathbf{M} \sim 150 \text{ [kA/m]}$$



M



H



B

$$N_x = N_y = N_z = \frac{1}{3}$$

$$H_d = -NM$$

$$H_d = -50 \text{ [kA/m]}$$

$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

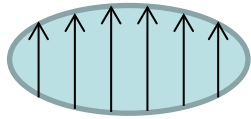
$$\vec{B} = \mu_0(100 \text{ [kA/m]})$$

$$\vec{B} = 0.126 \text{ [Tesla]}$$

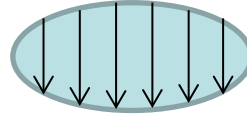
Example: Ba-Ferrite Ellipsoid

Barrium Ferrite:

$$\mathbf{M} \sim 150 \text{ [kA/m]}$$



M

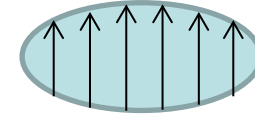


H

$$N_x = N_y = 0.45$$

$$H_d = -NM$$

$$H_d = -67.5 \text{ [kA/m]}$$



B

$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

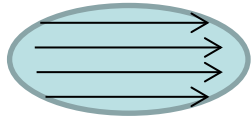
$$\vec{B} = \mu_0(82.5 \text{ [kA/m]})$$

$$\vec{B} = 0.104 \text{ [Tesla]}$$

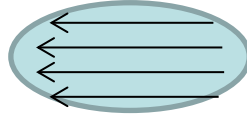
Example: Ba-Ferrite Ellipsoid

Barrium Ferrite:

$$M \sim 150 \text{ [kA/m]}$$



M

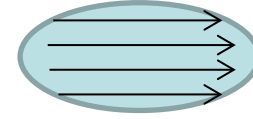


H

$$N_z = 0.1$$

$$H_d = -NM$$

$$H_d = -15 \text{ [kA/m]}$$



B

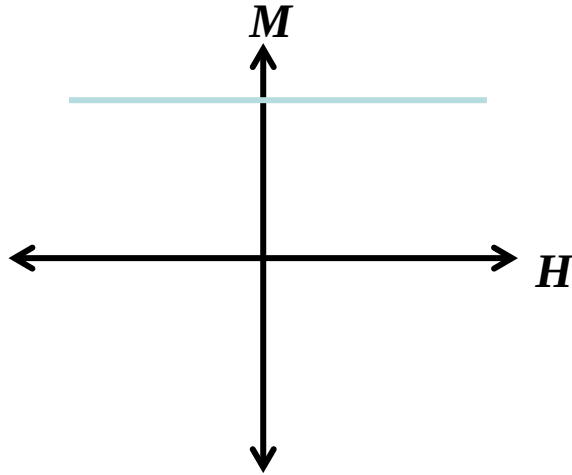
$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

$$\vec{B} = \mu_0(135 \text{ [kA/m]})$$

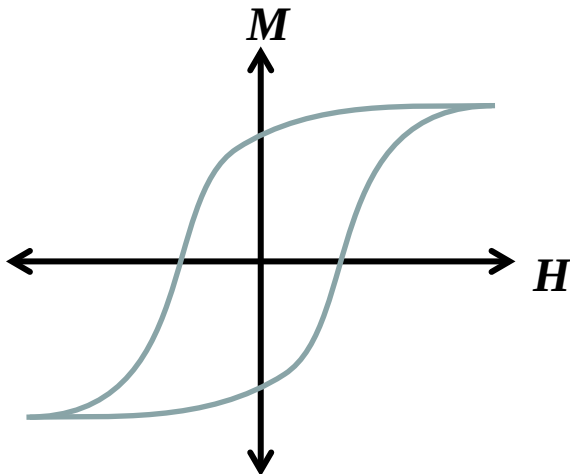
$$\vec{B} = 0.17 \text{ [Tesla]}$$

Susceptibility and Permeability

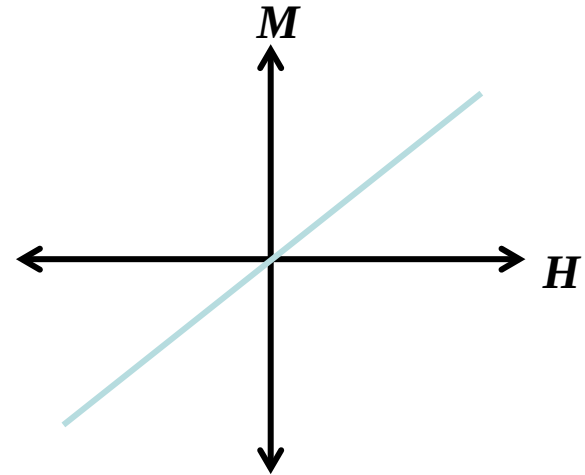
Ideal permanent magnet



Real, useful magnetic material



Ideal, linear soft magnetic material



$$M = \chi H$$

⤴ Susceptibility [unitless]

$$\vec{B} = \mu_0(\vec{H} + \chi\vec{H})$$

$$\vec{B} = \mu_0(1 + \chi)\vec{H}$$

$$\vec{B} = \mu_0\mu_r\vec{H}$$

⤴ Rel. Permeability [unitless]

Example, Long Rod in a Long Solenoid

$$I = 1 \text{ A}$$

$$N/L = 1000/\text{m}$$

$$\mu_r = 1000 \quad \chi = 999$$



$$H = \frac{NI}{L} = 1 \text{ [kA/m]}$$

$$M = \chi H = \chi \frac{NI}{L} = 999 \text{ [kA/m]}$$

$$B = \mu_0(H + M) = \mu_0(1000 \text{ kA/m}) = 1.25 \text{ [Tesla]}$$

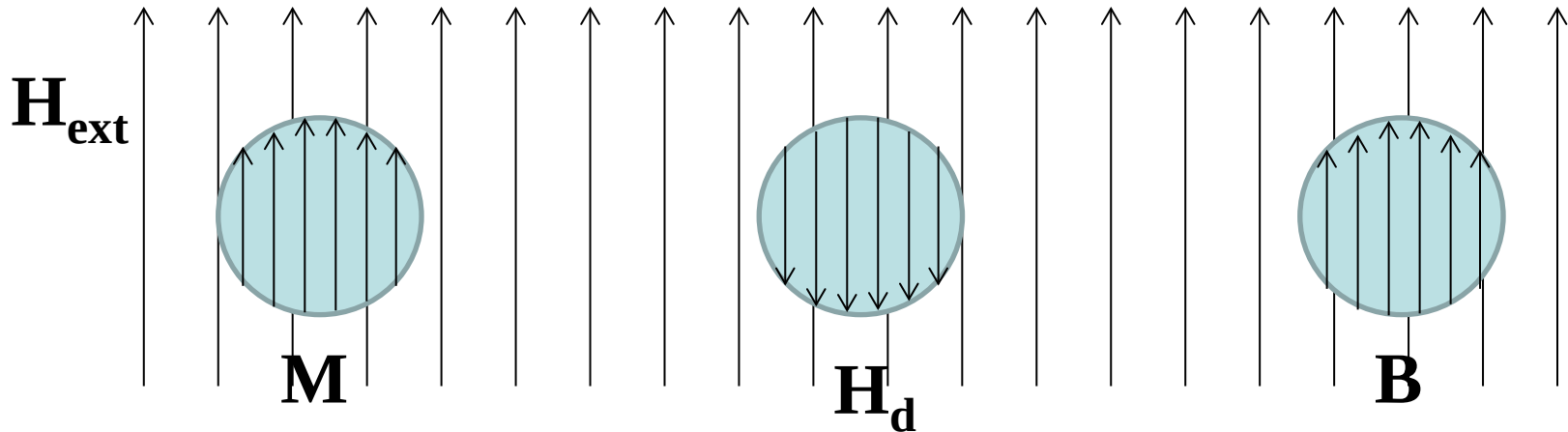
or:

$$B = \mu_0 \mu_r H = 1.25 \text{ [Tesla]}$$

Note: without the rod:

$$B = \mu_0 H = 0.00125 \text{ [Tesla]}$$

Example, Soft Magnetic Ball in a Field

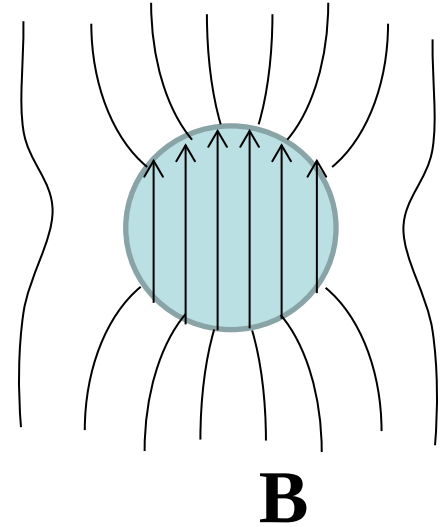


$$M = \chi H = \chi(H_{ext} + H_d) = \chi(H_{ext} - NM)$$

$$M = \frac{\chi}{1 + N\chi} H_{ext}$$

$$\chi_{eff} = \frac{\chi}{1 + N\chi} < 3!!!$$

Example, Soft Magnetic Ball in a Field

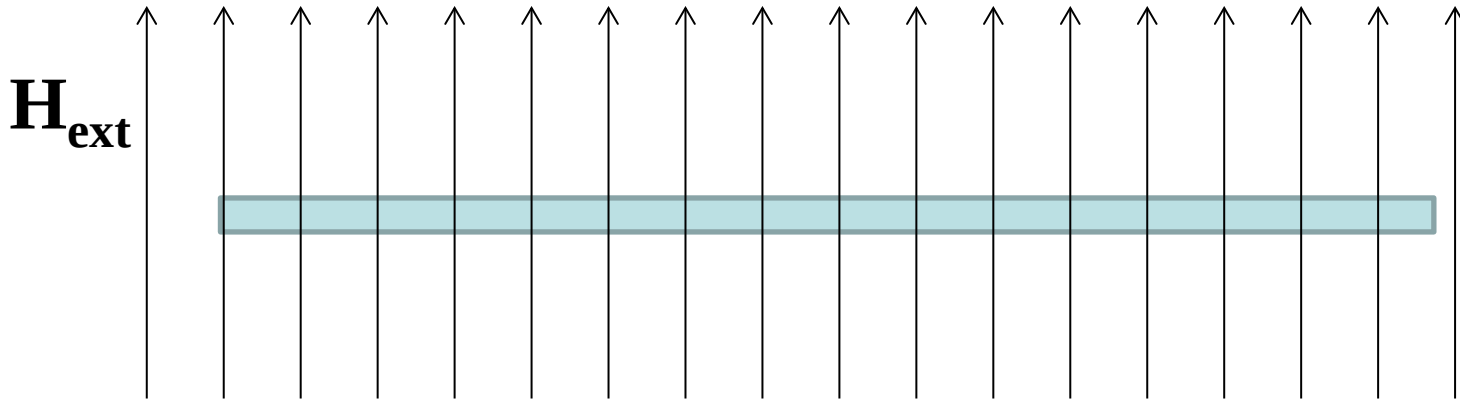


$$\mathbf{M} = \chi \mathbf{H} = \chi(\mathbf{H}_{ext} + \mathbf{H}_d) = \chi(\mathbf{H}_{ext} - N\mathbf{M})$$

$$\mathbf{M} = \frac{\chi}{1 + N\chi} \mathbf{H}_{ext}$$

$$\chi_{eff} = \frac{\chi}{1 + N\chi} < 3!!!$$

Example, Thin Film in a Field



$$M = \chi H_{inside} = \chi(H_{ext} + H_d) = \chi(H_{ext} - M)$$

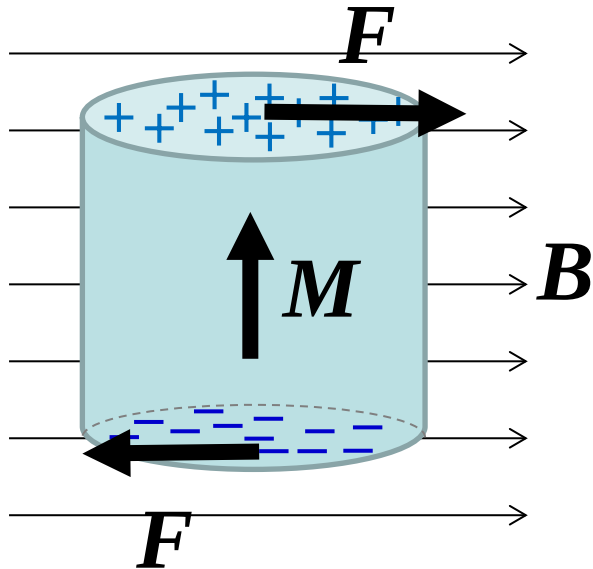
$$M = \frac{\chi}{1 + \chi} H_{ext}$$

$$H_{inside} = \frac{1}{1 + \chi} H_{ext}$$

$$B_{inside} = B_{ext}$$

Why?

Torque and Zeeman Energy



Force:

$$\vec{F} = q_m \vec{B}$$

Torque:

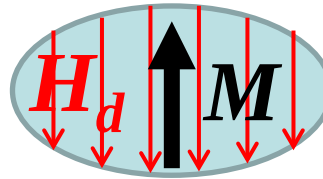
$$\vec{\mathcal{T}} = \vec{m} \times \vec{B}$$

Energy:

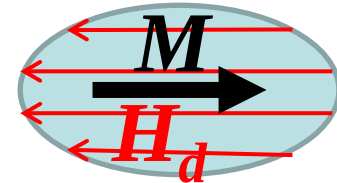
$$E = -\vec{m} \cdot \vec{B}$$

Self Energy and Shape Anisotropy

$$E = \frac{1}{2} \iiint_{vol} \mu_0 \vec{M} \cdot \vec{H}_d dv$$

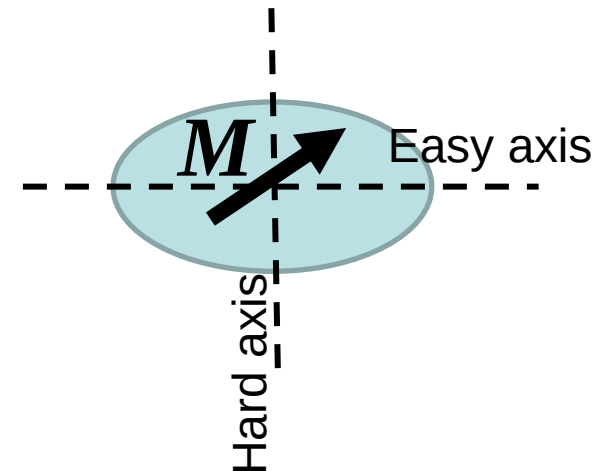
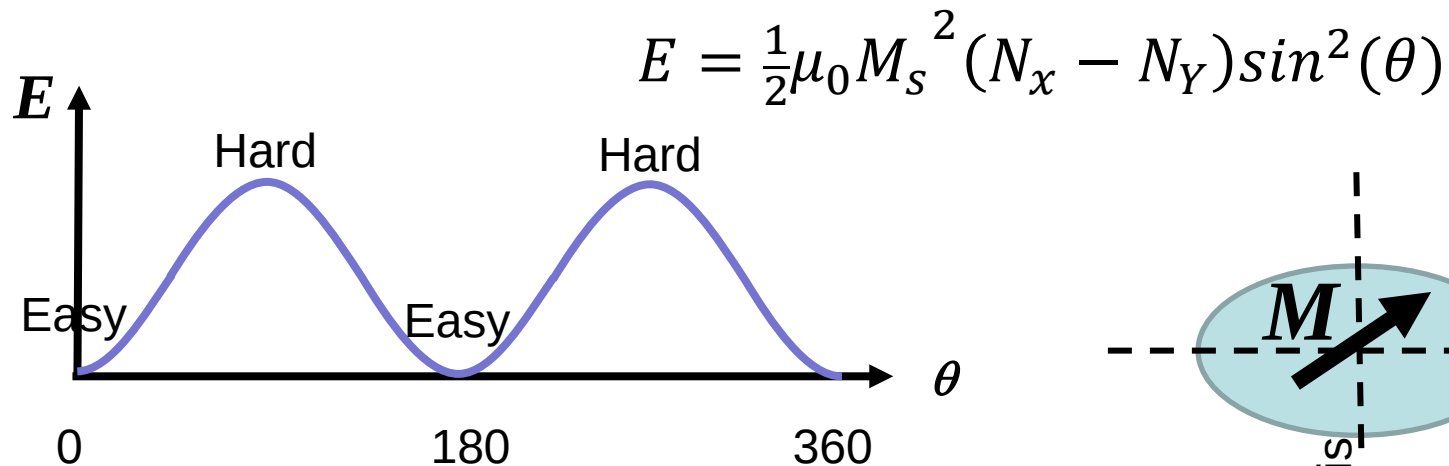


Higher energy

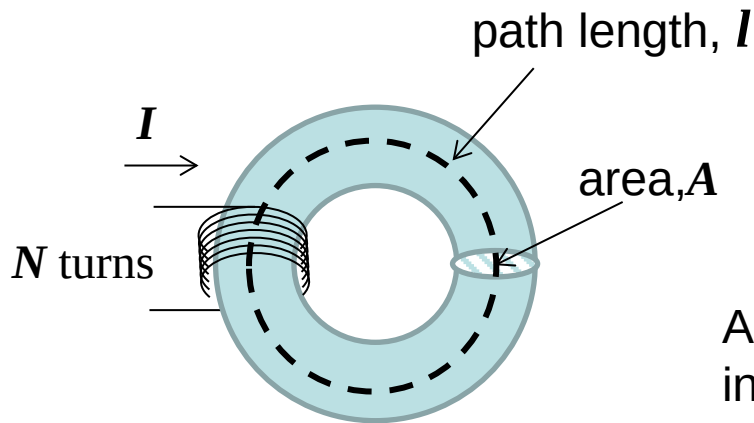


Lower energy

“Shape Anisotropy”



Magnetic Circuits



$$\oint \vec{H} \cdot d\vec{l} = NI$$

Assuming flux is uniform and contained inside permeable material:

$$B = \mu_0 \mu_r H = \frac{\phi}{A}$$

$$NI = \phi \mathcal{R}$$

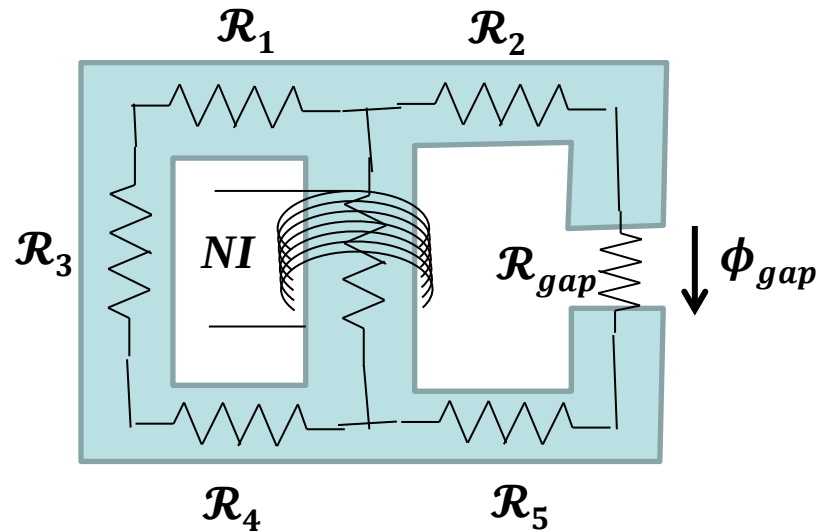
Magneto-motive force, "MMF"

Flux

Reluctance,

$$\mathcal{R} = \frac{l}{\mu A}$$

Magnetic Circuits



Solve using electric circuit theory.

Units Confusion

CGS Units

B [Gauss]

H [Oersted]

M [emu/cc]

$4\pi M$ [Gauss]

$$B = H + 4\pi M$$

$$B \cdot H \text{ [dyne/cm}^3\text{]}$$

$$N_x + N_y + N_z = 4\pi$$

SI Units

B : [Tesla]

H : [A/m]

M : [A/m]

$$B = \mu_0(H + M)$$

$$B \cdot H \text{ [J/m}^3\text{]}$$

$$N_x + N_y + N_z = 1$$

Books on Magnetism

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Bozorth, *Ferromagnetism*, 1951 (reprinted by IEEE Press, 1993.)