Hi, I was investigating about magnetism in the human body and I used a speaker with a plug connected to it and then I started touching my body with the plug to hear how it sounds. I realized that when I put the plug in my nipples it made a louder sound which means that the magnetics were bigger in that area. I have asked about this but I get no answer why, there is no coverage about this subject on the internet either, please if you know about this let me know, my theory is that our nipples are our bridge of expelling magnetics and electric signal to control the energy outside our bodies, hope this helps with some research, thank you...
Science and technology area in Grenoble
A few words about Institut NEEL

**Facts**

- Core of Condensed Matter Physics
- 180 full-staff scientists, 130 technical staff
- 40 PhD / year

**Internal structure**

- Research teams + technological support groups
- Three scientific departments
  - Condensed Matter – Low Temperatures
  - Condensed Matter – Functional Materials
  - Nanosciences
Institut Néel
Grenoble, France


http://perso.neel.cnrs.fr/olivier.fruchart/slides
Modern applications of (Nano)magnetism

Where does 'nano' contribute?

Materials

- Magnets
  - (→ motors and generators)
- Transformers
- Magnetocaloric

Data storage

- Hard disk drives
- Tapes
- MRAM

Sensors

- Compass
- Field mapping
- HDD read heads

Nanoparticles

- Ferrofluids
- MRI contrast
- Hyperthermia
- Sorting & tagging
### General table of contents

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### Part I: BASICS and MACROSPINS — Units and notations

#### SI system vs. cgs-Gauss

<table>
<thead>
<tr>
<th>Definitions</th>
<th>SI system</th>
<th>cgs-Gauss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meter</td>
<td>m</td>
<td>cm</td>
</tr>
<tr>
<td>Kilogram</td>
<td>kg</td>
<td>g</td>
</tr>
<tr>
<td>Second</td>
<td>s</td>
<td>s</td>
</tr>
<tr>
<td>Ampere</td>
<td>A</td>
<td>Ab-Ampere</td>
</tr>
<tr>
<td>(B = \mu_0 (H + M))</td>
<td>(B = H + 4\pi M)</td>
<td></td>
</tr>
<tr>
<td>(\mu_0 = 4\pi \times 10^{-7})</td>
<td>(\mu_0 = 4\pi)</td>
<td></td>
</tr>
</tbody>
</table>

#### Conversions

<table>
<thead>
<tr>
<th>Field</th>
<th>1 A/m</th>
<th>(4\pi \times 10^{-3}) Oe (Oersted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment</td>
<td>1 A.m²</td>
<td>(10^3) emu</td>
</tr>
<tr>
<td>Magnetization</td>
<td>1 A/m</td>
<td>(10^{-3}) emu/cm³</td>
</tr>
<tr>
<td>Induction</td>
<td>1 T</td>
<td>(10^4) G (Gauss)</td>
</tr>
<tr>
<td>Susceptibility</td>
<td>(\chi = M / H)</td>
<td>1 / (4\pi)</td>
</tr>
</tbody>
</table>

#### Notations


- **Microscopic quantity** \(E\)
- **Dimensionless quantity** \(e = E / E_0\)
- **Global quantity** \(\delta = \iiint E \cdot dV\)
- **Vector quantity** \(B\)
Motivation for understanding magnetization switching

Magnetostatics

Stoner-Wohlfarth model

Thermal activation

Precessional switching

Applications of macrospins (nanoparticles)
Part I: BASICS and MACROSPINS — Basics of magnetization switching

Manipulation of magnetic materials:
→ Application of a magnetic field

Zeeman energy: \( E_Z = -\mu_0 H \cdot M \)

Magnetic induction \( B = \mu_0 (H + M) \)

Another notation: \( J = \mu_0 M \)
Part I: BASICS and MACROSPINS — Basics of magnetization switching

**Soft magnetic materials**
- Transformers
- Flux guides, sensors
- Magnetic shielding

**Hard magnetic materials**
- Permanent magnets, motors
- Magnetic recording

**Magnetization switching**
- $M_A$ vs. $H_{ext}$
Part I: BASICS and MACROSPINS — Basics of magnetization switching

Bulk material

Numerous and complex magnetic domains

FeSi soft sheet

A. Hubert, Magnetic domains

Mesoscopic scale

Small number of domains, simple shape

Microfabricated dots

Kerr magnetic imaging

A. Hubert, Magnetic domains

Nanometric scale

Magnetic single-domain

Nanofabricated dots

MFM

Sample courtesy: N. Rougemaille, I. Chioar

Nanomagnetism ~ mesoscopic magnetism
**Magnetization**

Magnetization vector \( \mathbf{M} \)

May vary over time and space

Modulus is constant (hypothesis in micromagnetism)

Mean-field approach possible: \( M_s = M_s(T) \)

**Exchange interaction**

Atomistic view:

\[ \varepsilon = -2 \sum_{i>j} J_{i,j} \mathbf{S}_i \cdot \mathbf{S}_j \]  
(total energy)

Micromagnetic view:

\[ \mathbf{S}_i \cdot \mathbf{S}_j = S^2 \cos \theta_{i,j} \approx S^2 (1 - \theta_{i,j}^2/2) \]

\[ E = A (\nabla \cdot \mathbf{m})^2 \]  
(energy per unit volume)
Part I: BASICS and MACROSPINS — Micromagnetism (2/2)

**Exchange energy**

\[ E_{\text{ex}} = A(\nabla \cdot \mathbf{m})^2 = A \sum_{i,j} \frac{\partial m_i}{\partial x_j} \]

**Magnetocrystalline anisotropy energy**

\[ E_{\text{mc}} = A f(\theta, \varphi) \]

**Zeeman energy**

\[ E_Z = -\mu_0 \mathbf{M} \cdot \mathbf{H} \]

**Dipolar energy**

\[ E_d = -\frac{1}{2} \mu_0 \mathbf{M} \cdot \mathbf{H}_d \]
Part I: BASICS and MACROSPINS — Table of contents

- Motivation for understanding magnetization switching
- Magnetostatics
- Stoner-Wohlfarth model
- Thermal activation
- Precessional switching
- Applications of macrospins (nanoparticles)
Part I: BASICS and MACROSPINS — Magnetostatics (1/6)

Analogy with electrostatics (synonym: magnetostatic energy)

Maxwell equation $\nabla \cdot \mathbf{H}_d = - \nabla \cdot \mathbf{M}$

$\mathbf{H}_d(r) = - M_S \iiint_{\text{Space}} \frac{\nabla [\mathbf{m}(r')] \cdot (r - r')}{4\pi ||r - r'||^3} \, dV'$

To lift the singularity that may arise at boundaries, a volume integration around the boundaries yields:

$\mathbf{H}_d(r) = \iiint \frac{\rho(r') \cdot (r - r')}{4\pi ||r - r'||^3} \, dV' + \iint \frac{\sigma(r') \cdot (r - r')}{4\pi ||r - r'||^3} \, dS'$

Magnetic charges:

$\rho(r) = - M_S \nabla \cdot \mathbf{m}(r) \quad \rightarrow \text{volume density of magnetic charges}$

$\sigma(r) = M_S \mathbf{m}(r) \cdot \mathbf{n}(r) \quad \rightarrow \text{surface density of magnetic charges}$

Usefull expressions:

$\mathcal{E}_d = - \frac{1}{2} \mu_0 \iiint_{\text{Sample}} \mathbf{M} \cdot \mathbf{H}_d \, dV$

$\mathcal{E}_d = \frac{1}{2} \mu_0 \iiint_{\text{Space}} H_d^2 \, dV$

- Dipolar energy is always positive
- Zero energy means a minimum

Notice:

- $\mathbf{H}_d$ depends on sample shape, not size
- Synonym: dipolar $\leftrightarrow$ magnetostatic
Examples of magnetic charges

Notice: no charges and $J=0$ for infinite cylinder

Charges on surfaces

Surface and volume charges

Take-away message

Dipolar energy favors alignment of magnetization with longest direction of sample
Part I: BASICS and MACROSPINS — Magnetostatics (3/6)

Names

⇒ Generic names
  - Magnetostatic field
  - Dipolar field

⇒ Within sample
  - Demagnetizing field

⇒ Outside sample
  - Stray field
Part I: BASICS and MACROSPINS — Magnetostatics: demagnetizing coefficients (4/6)

**Principle**

Goal: evaluate strength of demagnetizing field and energy

Assume uniform magnetization $\mathbf{M}(\mathbf{r}) = \mathbf{M} = M_S (m_x \mathbf{u}_x + m_y \mathbf{u}_y + m_z \mathbf{u}_z) = M_S m_i \mathbf{u}_i$

**Demagnetizing coefficients** (Easy to prove, however not done here)

$$\langle \mathbf{H}_d(\mathbf{r}) \rangle = -M_S \mathbf{N} \cdot \mathbf{m} = -N_i M_S m_i$$

$$\mathcal{E}_d = K_d V \mathbf{m} \cdot \mathbf{N} \cdot \mathbf{m} = K_d VN_{ij} m_i m_j$$

**Notice:** Valid for any shape

$$K_d = \frac{1}{2} \mu_0 M_s^2$$

Dipolar constant $(\text{J/m}^3)$

Demagnetizing tensor

$$\mathbf{N}_x + \mathbf{N}_y + \mathbf{N}_z = 1$$

**Model cases: slabs, cylinders and ellipsoids** (Not easy to prove, mathematics...)

$\mathbf{H}_d$ is uniform for surfaces of polynomial boundary with order $\leq 2$

$\mathbf{H}_d = -M_S \mathbf{N} \cdot \mathbf{m}$

Along all three main directions: $H_i = -N_i M_S m_i$ and $\mathcal{E}_d = K_d VN_i m_i^2$

J. C. Maxwell, Clarendon 2, 66-73 (1872)

Dipolar energy contributes to magnetic anisotropy with second-order term

Example:

$$\mathcal{E}_d = VK_d (N_x m_x^2 + N_y m_y^2) = VK_d (N_x - N_y) \cos^2 \theta$$

IEEE Magnetics

Institut Nêel

Grenoble, France


http://perso.neel.cnrs.fr/olivier.fruchart/slides
Slabs (infinite thin films)

- \( L_x = L_y = \infty \)
- \( N_x = N_y = 0 \)
- \( N_z = 1 \)

Cylinders (infinite)

- \( L_x = L_y = D \)
- \( L_z = \infty \)
- \( N_x = N_y = \frac{1}{2} \)
- \( N_z = 0 \)

Spheres

- \( L_x = L_y = L_z = D \)
- \( N_x = N_y = N_z = \frac{1}{3} \)
Ellipsoids

General ellipsoid: main axes \((a, b, c)\)

\[ N_x = \frac{1}{2} abc \int_0^\infty \left( \frac{a^2 + \eta}{(a^2 + \eta)(b^2 + \eta)(c^2 + \eta)} \right)^{1/2} \, d\eta \]

For prolate revolution ellipsoid: \((a, c, \beta)\) with \(\alpha = c/a < 1\)

\[ N_x = \frac{\alpha^2}{1 - \alpha^2} \left[ \frac{1}{\sqrt{1 - \alpha^2}} \sinh \left( \frac{\sqrt{1 - \alpha^2}}{\alpha} \right) \right] - 1 \]

For oblate revolution ellipsoid: \((a, c, \beta)\) with \(\alpha = c/a > 1\)

\[ N_x = \frac{\alpha^2}{\alpha^2 - 1} \left[ 1 - \frac{1}{\sqrt{\alpha^2 - 1}} \arcsin \left( \frac{\sqrt{\alpha^2 - 1}}{\alpha} \right) \right] \]

Cylinders

For a cylinder with axis along \(x\)

\[ N_x = 0 \quad N_y = \frac{c}{b + c} \quad N_z = \frac{b}{b + c} \]


Part I: BASICS and MACROSPINS — Table of contents

- Motivation for understanding magnetization switching
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Framework

Approximation: \( \dot{\hat{r}} \mathbf{m} = 0 \) (uniform magnetization) (strong!)

\[ \delta = EV = V \left[ K_{\text{eff}} \sin^2 \theta - \mu_0 M_S H \cos (\theta - \theta_H) \right] \]

\[ K_{\text{eff}} = K_{mc} + (\Delta N) K_d \]

Dimensionless units:

\[ e = \delta / KV \]

\[ h = H / H_a \]

\[ H_a = 2K / \mu_0 M_s \]

L. Néel, Compte rendu Acad. Sciences 224, 1550 (1947)

Names used

- Uniform rotation / magnetization reversal
- Coherent rotation / magnetization reversal
- Macrospin etc.
Part I: BASICS and MACROSPINS — Magnetization reversal of macrospins (2/6)

Example for $\theta_H = 180^\circ$

Equilibrium states

\[ \partial_\theta e = 2 \sin \theta (\cos \theta - h) \quad \partial_\theta e = 0 \]

\[ \cos \theta_m = h \]

\[ \theta \equiv 0 \left[ \pi \right] \]

Stability

\[ \partial_{\theta \theta} e = 2 \cos 2 \theta - 2 h \cos \theta \]

\[ = 4 \cos^2 \theta - 2 - 2 h \cos \theta \]

\[ \partial_{\theta \theta} e(0) = 2(1 - h) \]

\[ \partial_{\theta \theta} e(\theta_m) = 2(h^2 - 1) \]

\[ \partial_{\theta \theta} e(\pi) = 2(1 + h) \]

Energy barrier

\[ \Delta e = e(\theta_{\text{max}}) - e(0) \]

\[ = 1 - h^2 + 2h^2 - 2h \]

\[ = (1 - h)^2 \]

Switching

\[ h = 1 \]

\[ H = H_a = 2K/\mu_0 M_S \]

\[ (1 - h)^\alpha \] with exponent 1.5 in general
Part I: BASICS and MACROSPINS — Magnetization reversal of macrospins (3/6)

‘Astroid’ curve

\[ H_{sw}(\theta_H) = \frac{1}{\left(\sin^{2/3} \theta_H + \cos^{2/3} \theta_H\right)^{3/2}} \]

\( H_{sw}(\theta) \) is only one signature of reversal modes

J. C. Slonczewski, Research Memo RM 003.111.224, IBM Research Center (1956)
Switching field = Reversal field

A value of field at which an irreversible (abrupt) jump of magnetization angle occurs.
Can be measured only in single particles.

Coercive field

The value of field at which $M.H=0$ ($\theta = \theta_H \pm \pi/2$)
A quantity that can be measured in real materials (large number of ‘particles’).
May be or may not be a measure of the mean switching field at the microscopic level.

$h_{Sw} = \frac{1}{\left(\sin^{2/3} \theta_H + \cos^{2/3} \theta_H\right)^{3/2}}$

$h_c = \frac{1}{2} |\sin(2\theta_H)|$
Experimental evidence


A. Thiaville et al., PRB61, 12221 (2000)

Extensions: 3D, arbitrary anisotropy etc.

M. Jamet et al., PRB69, 024401 (2004)
Size-dependent magnetization reversal

Astroids of flat magnetic elements with increasing size

Conclusion over coherent rotation

- The simplest model
- Fails for most systems because they are too large: apply model with great care!
- $H_c \ll H_a$ for most large systems (thin films, bulk): do not use $H_c$ to estimate $K$!

Early known as Brown's paradox

Part I: BASICS and MACROSPINS — Table of contents

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**Barrier height**

\[ \Delta e = e(\theta_{\text{max}}) - e(0) = (1 - h)^2 \]

\[ h = \mu_0 M_s H / 2K \]

\[ h = 0.2 \]

**Thermal activation**


\[ \tau = \tau_0 \exp \left( \frac{\Delta \varepsilon}{k_B T} \right) \quad \Rightarrow \quad \Delta \varepsilon = k_B T \ln (\tau / \tau_0) \]

\[ \tau_0 \approx 10^{-10} \text{ s} \]

Lab measurement: \( \tau \approx 1 \text{ s} \quad \Rightarrow \quad \Delta \varepsilon \approx 25 k_B T \)

\[ H_c = \frac{2 K}{\mu_0 M_s} \left( 1 - \sqrt{\frac{25 k_B T}{KV}} \right) \]

**Blocking temperature**

\( T_b \approx KV / 25k_B \)

**Superparamagnetism**


Notice, for magnetic recording: \( \tau \approx 10^9 \text{ s} \quad KV_b \approx 40 - 60k_B T \)
Part I: BASICS and MACROSPINS — Thermal activation (2/3)

**Formalism**

C. P. Bean & J. D. Livingston, J. Appl. Phys. 30, S120 (1959)

<table>
<thead>
<tr>
<th>Energy</th>
<th>Partition function</th>
<th>Average moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E = KV f(\theta, \varphi) - \mu_0 \mu H )</td>
<td>( Z = \sum \exp(-\beta E) )</td>
<td>( \langle \mu \rangle = \frac{1}{\beta \mu_0 Z} \frac{\partial Z}{\partial H} )</td>
</tr>
</tbody>
</table>

**Isotropic case**

\[
Z = \int_{-\infty}^{\infty} \exp(-\beta E) d\mu
\]

Note: equivalent to integration over solid angle

\[
\langle \mu \rangle = M \left[ \coth(x - \frac{1}{x}) \right]
\]

**Infinite anisotropy**

\[
Z = \exp(\beta \mu_0 \mathcal{K} H) + \exp(-\beta \mu_0 \mathcal{K} H)
\]

\[
\langle \mu \rangle = \mathcal{K} \tanh(x)
\]

Brillouin \( \frac{1}{2} \) function

**Note:**

Use the moment \( M \) of the particule, not spin \( \frac{1}{2} \).

\[
x = \beta \mu_0 \mathcal{K} H
\]

## Properties (including thermal) of usual magnetic materials

<table>
<thead>
<tr>
<th>Material</th>
<th>$T_C$ (K)</th>
<th>$M_S$ (kA/m)</th>
<th>$\mu_0 M_S$ (T)</th>
<th>$K$ (kJ/m$^3$)</th>
<th>$D_{300K}$ (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fe</td>
<td>1043</td>
<td>1730</td>
<td>2.174</td>
<td>48</td>
<td>16</td>
</tr>
<tr>
<td>Co</td>
<td>1394</td>
<td>1420</td>
<td>1.784</td>
<td>530</td>
<td>7.2</td>
</tr>
<tr>
<td>Ni</td>
<td>631</td>
<td>490</td>
<td>0.616</td>
<td>-4.5</td>
<td>35</td>
</tr>
<tr>
<td>Fe$<em>{20}$Ni$</em>{80}$ (Permalloy)</td>
<td>850</td>
<td>835</td>
<td>1.050</td>
<td>$\approx 0$</td>
<td>–</td>
</tr>
<tr>
<td>Fe$_3$O$_4$</td>
<td>858</td>
<td>480</td>
<td>0.603</td>
<td>-13</td>
<td>25</td>
</tr>
<tr>
<td>BaFe$<em>{12}$O$</em>{19}$</td>
<td>723</td>
<td>382</td>
<td>0.480</td>
<td>250</td>
<td>9.2</td>
</tr>
<tr>
<td>Nd$<em>2$Fe$</em>{14}$B</td>
<td>585</td>
<td>1280</td>
<td>1.608</td>
<td>4900</td>
<td>3.4</td>
</tr>
<tr>
<td>SmCo$_5$</td>
<td>995</td>
<td>907</td>
<td>1.140</td>
<td>17000</td>
<td>2.3</td>
</tr>
<tr>
<td>Sm$<em>2$Co$</em>{17}$</td>
<td>1190</td>
<td>995</td>
<td>1.250</td>
<td>3300</td>
<td>3.9</td>
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<tr>
<td>FePt L$_1$</td>
<td>750</td>
<td>1140</td>
<td>1.433</td>
<td>6600</td>
<td>3.1</td>
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<tr>
<td>CoPt L$_1$</td>
<td>840</td>
<td>796</td>
<td>1.000</td>
<td>4900</td>
<td>3.4</td>
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<tr>
<td>Co$_3$Pt</td>
<td>1100</td>
<td>1100</td>
<td>1.382</td>
<td>2000</td>
<td>4.6</td>
</tr>
</tbody>
</table>

- $T_C$: Curie ordering temperature
- $M_S$: Spontaneous magnetization at 300K
- $K$: First magnetocrystalline anisotropy coefficient at 300K
- $D_{300K}$: Diameter for superparamagnetic limit at 300K and time constant 1s
### Part I: BASICS and MACROSPINS — Table of contents

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Basics of precessional switching

Magnetization dynamics:

Landau-Lifshitz-Gilbert equation:

\[
\frac{d \mathbf{m}}{dt} = -|\gamma_0| \mathbf{m} \times \mathbf{H} + \alpha \mathbf{m} \times \frac{d \mathbf{m}}{dt}
\]

- \( \gamma_0 \) Gyromagnetic factor
- \( \gamma_0 = \mu_0 \gamma \)
- \( \gamma = -g_J \frac{e}{2m_e} < 0 \)
- \(|\gamma_0|/2\pi \approx 28 \text{ GHz/T} \)

- \( H_{\text{eff}} \) Effective field (including applied)
- \( H_{\text{eff}} = -\frac{\partial E_{\text{mag}}}{\mu_0 \partial \mathbf{M}} \)

- \( \alpha \) Damping coefficient \( \alpha = 10^{-1} - 10^{-3} \)

Part I: BASICS and MACROSPINS — Precessional switching (2/4)

Magnetization trajectories

\[
\begin{align*}
\mathbf{m}_x &= \frac{m_y + h/N_z}{1 + h_K/N_z} = 1 + \frac{h^2}{N_z(N_z + h_K)} \\
\omega &\approx 0.847 |\gamma_0| \sqrt{M_S(H - H_K)/2}
\end{align*}
\]
Stoner-Wohlfarth versus precessional switching

**Stoner-Wohlfarth model**: describes processes where the system follows quasistatically energy minima, e.g. with slow field variation

**Precessional switching**: occurs at short time scales, e.g. when the field is varied rapidly

---

**Relevant time scales**

- Precession period: $\frac{2\pi}{|\gamma|} = 35 \text{ ps.T}$
- Precession damping: $\frac{1}{2\pi \alpha}$ per period

Notice: Magnetization reversal allowed for $h > 0.5h_K$ (more efficient than classical reversal)
Part I: BASICS and MACROSPINS — Precessional switching (4/4)

Problems: multiple switching and ringing

Non-switched

Switched

Stoner-Wohlfarth astroid
(quasistatic limit)

M. Bauer et al., PRB61, 3410 (2000)
Motivation for understanding magnetization switching

Magnetostatics

Stoner-Wohlfarth model

Thermal activation

Precessional switching

Applications of macrospins (nanoparticles)
Part I: BASICS and MACROSPINS — Practical use of nanoparticles (1/2)

Ferrofluids

Principle

Surfactant-coated nanoparticles, preferably superparamagnetic

→ Avoid agglomeration of the particles

→ Fluid and polarizable

Example of use

Seals for rotating parts


Health and biology

Beads = coated nanoparticles, preferably superparamagnetic
→ Avoid agglomeration of the particles
⇒ Cell sorting

\[ \mathbf{F} = \nabla \mu \cdot \mathbf{B} \]

⇒ Hyperthermia

Use ac magnetic field

\[ H_c = H_{c,0} \left(1 - \sqrt{\frac{\ln(\tau / \tau_0) k_B T}{K V}} \right) \]

RAM (radar absorbing materials)

⇒ Principle
Absorbs energy at a well-defined frequency (ferromagnetic resonance)

\[ \frac{d \mathbf{l}}{dt} = \Gamma = \mu_0 \mu \times \mathbf{H} = \mu_0 \gamma \ell \times \mathbf{H} \]

\[ \frac{d \mu}{dt} = \mu_0 \gamma \mu \times \mathbf{H} \]

\[ \gamma = -g_J \frac{e}{2m_e} < 0 \]

\[ \gamma_s / 2\pi \approx 28 \text{ GHz/T} \]