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Session III

Phenomenological Damping

$$\dot{s}/\mu_0 = [fs] + \lambda\left(f - \frac{(fs)\,s}{s^2}\right)$$

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Part II. Phenomenological Damping 2

A. Some references

B. Physics versus Phenomenology
Roadmap to physical mechanisms. Spin-lattice and spin-spin.
Is the magnetization conserved?

C. Historical View of Phenomenological Damping
Landau-Lifshitz (LL) damping – the original and the perverted form.
Damping parameter α_{LL} as a magnetic loss tangent.
Bloch-Bloembergen (BB).
Codrington-Olds-Torrey (COT) – a vectorized BB equation.
Gilbert (G) damping. Modified Bloch-Bloembergen (MBB).

D. Landau-Lifshitz and Gilbert Compared
Gilbert decay spiral - a geometric view. Critical damping.
LL decay spiral - no critical damping. Conversions.
The so-called "LLG" equation.

E. Drive to Equilibrium Revisited
Equations are inconsistent. Which one(s) is (are) most fundamental?

A. Some Useful References 3

Some useful (perhaps) references:

L. Landau and E. Lifshitz, "On the Theory of the Dispersion of Magnetic Permeability in Ferromagnetic Bodies," Phys. Z. Sowjetunion 8, 153 (1936). (in English!)

F. Bloch, "Nuclear Induction," Phys. Rev. 70, 460 (1946).

N. Bloembergen, "On the Ferromagnetic Resonance in Nickel and Supermalloy," Phys. Rev. 78, 572 (1950).

R. K. Wangsness, "Magnetic Resonance for Arbitrary Field Strengths," Phys. Rev. 98, 927 (1955).

T. L. Gilbert, "A Lagrangian Formulation of the Gyromagnetic Equation of the Magnetization Field," Phys. Rev. 100, 1243 (1955).

T. L. Gilbert, "Formation, Foundations, and Applications of the Phenomenological Theory of Ferromagnetism," Ph.D. Thesis, Illinois Institute of Technology, Chicago, Illinois, June, 1956.

T. L. Gilbert, "A Phenomenological Theory of Damping in Ferromagnetic Materials," IEEE Trans. Magnetics 40, 3443 (2004). (Advances in Magnetics)

A. Some Useful References 4

Some more useful (Perhaps) references:

R. K. Wangsness, "Magnetic Resonance and Minimum Entropy Production - Macroscopic Equations of Motion," Phys. Rev. 104, 857 (1956).

R. K. Wangsness, "Effective Parameters in Ferrimagnetic Resonance," Phys. Rev. 111, 813 (1958).

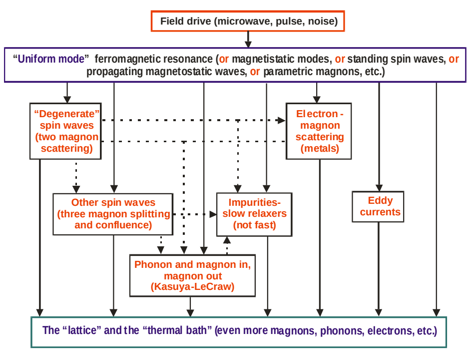
H. B. Callen, "A Ferromagnetic Dynamical Equation," J. Phys. Chem. Solids 4, 256 (1958).

B. Lax and K. J. Button, *Microwave Ferrites and Ferrimagnetics* (McGraw Hill, New York, 1962).

J. C. Mallinson, "On Damped Gyromagnetic Precession," IEEE Trans. Magnetics 23, 2003 (1987).

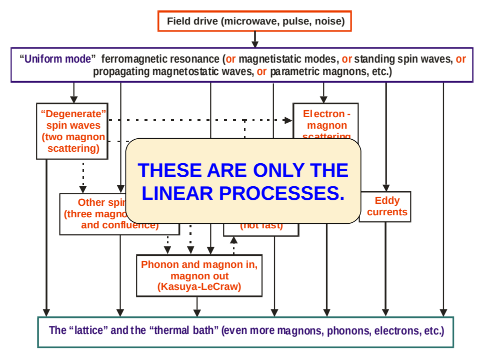
B. Physics versus Phenomenology 5

Patton's roadmap (separate talk!) to physical mechanisms:



B. Physics versus Phenomenology 6

Patton's roadmap (separate talk!) to physical mechanisms:



B. Physics versus Phenomenology — 7

Patton's roadmap (separate talk!) to physical mechanisms:

Field drive (microwave, pulse, noise)

"Uniform mode" ferromagnetic resonance (or magnetistatic modes, or standing spin waves, or

THERE IS NO α MECHANISM HERE!

PHENOMENOLOGICAL DAMPING IS NOT A MECHANISM.

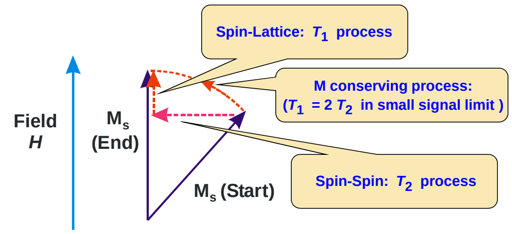
ONE CAN NEVER "MEASURE" α!

The "lattice" and the "thermal bath" (even more magnons, phonons, electrons, etc.)

B. Physics versus Phenomenology — 8

Additional important point: Some processes conserve the magnetization M_s. Some do not.

$$\boxed{\text{Moment} = M_s \bullet \text{Volume} - 2\,\mu_B \sum_{k\neq 0} n_k}$$



The HOLY GRAIL of some (most) phenomenological equations is to CONSERVE THE MAGNETIZATION.

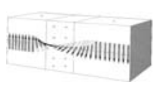
C. Historical View of Phenomenological Equations — 9

LANDAU LIFSHITZ 1936 (LL)

$$\dot{\mathbf{s}}/\mu_0 = [\mathbf{fs}] + \lambda\left(\mathbf{f} - \frac{(\mathbf{fs})\,\mathbf{s}}{s^2}\right)$$

In original form [Eq. (21), p. 163]

Paper is in English (please read it).
Developed to analyze domain wall motion.



In modern form:

$$\boxed{\frac{d\mathbf{M}}{dt} = -|\gamma_e|(\mathbf{M}\times\mathbf{H}) + \lambda_{LL}\left[\mathbf{H} - \frac{(\mathbf{H}\cdot\mathbf{M})\mathbf{M}}{M_s^2}\right]}$$

(Gaussian units)

$|\mathbf{M}| = M_s$ = saturation magnetization (emu/cm^3)
$\mathbf{H}$ = total effective magnetic field (Oe)
γ_e = electron gyromagnetic ratio (negative!)
λ_{LL} = Landau-Lifshitz damping "constant"

C. Historical View of Phenomenological Equations - Landau - Lifshitz — 10

$$\dot{\mathbf{s}}/\mu_0 = [\mathbf{fs}] + \lambda\left(\mathbf{f} - \frac{(\mathbf{fs})\,\mathbf{s}}{s^2}\right) \qquad \frac{d\mathbf{M}}{dt} = -|\gamma_e|(\mathbf{M}\times\mathbf{H}) + \lambda_{LL}\left[\mathbf{H} - \frac{(\mathbf{H}\cdot\mathbf{M})\mathbf{M}}{M_s^2}\right]$$

Landau and Lifshitz did not start with the familiar: $\mathbf{M}\times(\mathbf{M}\times\mathbf{H})$

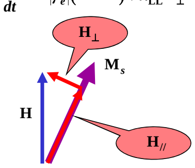
Look at: $\frac{(\mathbf{H}\cdot\mathbf{M})\mathbf{M}}{M_s^2} = \mathbf{H}\cdot\frac{\mathbf{M}}{M_s}\frac{\mathbf{M}}{M_s} = \mathbf{H}_{//}$ ← Component of H parallel to M

All Landau-Lifshitz is really saying is: $\frac{d\mathbf{M}}{dt} = -|\gamma_e|(\mathbf{M}\times\mathbf{H}) + \lambda_{LL}\mathbf{H}_\perp$

I have it on "good authority" that Landau wrote this equation with one simple idea in mind:
TO CONSTRUCT A SIMPLE PERPENDICULAR DRIVE TERM.
WHAT COULD BE SIMPLER?

That's all I did!

Lev Landau was 27 years old



C. Historical View of Phenomenological Equations - Landau - Lifshitz — 11

What they really said:

There are two kinds of the interaction between the magnetic moments in the crystal: exchange-interaction and relativistic interaction. The latter is in general much weaker than the former. The exchange interaction cannot change the magnetic moment. Therefore in the presence of the field the magnetic moment would act as a free moment, i. e. would rotate around f and we should have for $\dot{s}$ (˙ denotes differentiation by time) the equation

$$\dot{\mathbf{s}}/\mu_0 = [\mathbf{fs}]$$

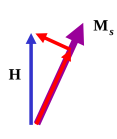
Torque equation (not even numbered)

with $\mu_0 = e/mc$ (and not $e/2mc$, because the moments s in ferromagnetic bodies are spin-moments). The approach of s to f is due only to the relativistic interaction. Since this interaction is weaker than the exchange-interaction, we can assume that the coefficient before the term [fs] is not altered, and we can simply add a term giving the approach of s to f. Thus we come to an equation of the form

$$\dot{\mathbf{s}}/\mu_0 = [\mathbf{fs}] + \lambda\left(\mathbf{f} - \frac{(\mathbf{fs})\,\mathbf{s}}{s^2}\right). \qquad (21)$$

LL damping term

The second term here is a vector directed from s to f. The constant λ is $\lambda \ll s$ in accordance with the fact that the relativistic interaction is weak. We disregard here altogether the variation of the absolute value of s.



C. Historical View of Phenomenological Equations - Landau - Lifshitz — 12

Original ("modern") form: $\frac{d\mathbf{M}}{dt} = -|\gamma_e|(\mathbf{M}\times\mathbf{H}) + \lambda_{LL}\left[\mathbf{H} - \frac{(\mathbf{H}\cdot\mathbf{M})\mathbf{M}}{M_s^2}\right]$

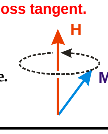
Cross product form: $\frac{d\mathbf{M}}{dt} = -|\gamma_e|(\mathbf{M}\times\mathbf{H}) - \frac{\lambda_{LL}}{M_s^2}[\mathbf{M}\times(\mathbf{M}\times\mathbf{H})]$

α form: $\frac{d\mathbf{M}}{dt} = -|\gamma_e|(\mathbf{M}\times\mathbf{H}) - \frac{\alpha_{LL}|\gamma_e|}{M_s}[\mathbf{M}\times(\mathbf{M}\times\mathbf{H})]$

This is the famous (infamous) α everyone seems to be seeking these days. $\lambda_{LL} = \alpha_{LL}|\gamma_e|M_s \quad \alpha_{LL} = \frac{\lambda_{LL}}{|\gamma_e|M_s}$

α is "like" a magnetic loss tangent.

Remember: γ_e : electron gyromagnetic ratio
γ_e is negative. Precession is counterclockwise.



C. Historical View of Phenomenological Equations - Landau - Lifshitz 13

Original form:
$$\frac{d\mathbf{M}}{dt} = -|\gamma_e|(\mathbf{M}\times\mathbf{H}) + \lambda_{LL}\left[\mathbf{H} - \frac{(\mathbf{H}\cdot\mathbf{M})\mathbf{M}}{M_s^2}\right]$$

Cross product form:
$$\frac{d\mathbf{M}}{dt} = -|\gamma_e|(\mathbf{M}\times\mathbf{H}) - \frac{\lambda_{LL}}{M_s^2}\left[\mathbf{M}\times(\mathbf{M}\times\mathbf{H})\right]$$

α form:
$$\frac{d\mathbf{M}}{dt} = -|\gamma_e|(\mathbf{M}\times\mathbf{H}) - \frac{\alpha_{LL}|\gamma_e|}{M_s}\left[\mathbf{M}\times(\mathbf{M}\times\mathbf{H})\right]$$

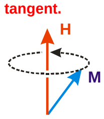
This is the famous (infamous) α everyone seems to be seeking these days.

$$\lambda_{LL} = \alpha_{LL}|\gamma_e|M_s \qquad \alpha_{LL} = \frac{\lambda_{LL}}{|\gamma_e|M_s}$$

There is a unit problem here!
These are Gaussian unit equations.
$|\gamma_e|M_s$ is NOT a frequency.
So λ_{LL} is not a bonafide relaxation frequency.

...etic loss tangent.

...ckwise.



C. Historical View of Phenomenological Equations 14

BLOCH BLOEMBERGEN 1950 (BB)

$$\left.\frac{d\mathbf{M}}{dt}\right|_{x,y} = -|\gamma_e|(\mathbf{M}\times\mathbf{H})\big|_{x,y} - \frac{1}{T_2}m_{x,y}$$

Transverse components

$$\left.\frac{d\mathbf{M}}{dt}\right|_{z} = -|\gamma_e|(\mathbf{M}\times\mathbf{H})\big|_{z} - \frac{1}{T_1}(M_z - M_s)$$

Longitudinal component

N. Bloembergen,
"On the ferromagnetic resonance in nickel and Supermalloy,"
Phys. Rev. 78, 572 (1950).
Equation 1, page 572.

1. These are small signal limit equations.
2. Static field H_e and static magnetization in *z*-direction.
3. Adapted from NMR (Bloch equations).
   T_2 = spin-spin relaxation time.
   T_1 = spin-lattice relaxation time.
4. Magnetization is NOT conserved.
5. In the small signal limit, one can MAKE M conserved by setting $T_1 = 2\,T_2$.

C. Historical View of Phenomenological Equations - BB 15

BB " designed " for NMR and EPR, not ferromagnets.
BB has a serious physical problem for ferromagnetic systems.

Equations only relax M toward the z-direction.

$$\left.\frac{d\mathbf{M}}{dt}\right|_{x,y} = -|\gamma_e|(\mathbf{M}\times\mathbf{H})\big|_{x,y} - \frac{1}{T_2}m_{x,y}$$

Drives $m_{x,y}$ to zero

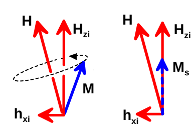
$$\left.\frac{d\mathbf{M}}{dt}\right|_{z} = -|\gamma_e|(\mathbf{M}\times\mathbf{H})\big|_{z} - \frac{1}{T_1}(M_z - M_s)$$

Drives M_z to M_s

This represents a "false" equilibrium!

1. M is dynamic.
2. Internal fields define the instantaneous equilibrium direction.

BB can lead to negative loss (among other things).



Precession around H

BB false equilibrium along z

C. Historical View of Phenomenological Equations 16

CODRINGTON, OLDS, AND TORRY 1955 (COT)

$$\left(\frac{d\mathbf{M}}{dt}\right) = -|\gamma_e|(\mathbf{M}\times\mathbf{H}) + \frac{1}{T_1}\left[\left(\frac{M_s|\mathbf{H}| - \mathbf{M}\cdot\mathbf{H}}{|\mathbf{H}|}\right)\frac{\mathbf{H}}{|\mathbf{H}|}\right] + \frac{1}{T_2}\left[\frac{(\mathbf{M}\times\mathbf{H})\times\mathbf{H}}{|\mathbf{H}|^2}\right]$$

M component along H

M component perpendicuar to H

A nice vectorized equation of motion in the spirit of BB.

R. S. Codrington, J. D. Olds, and H. C. Torrey, "Paramagnetic resonance in organic free radicals at low fields," Phys. Rev. 95, 607 (1954). (abstract only - not archived!)
First discussed and quantified in print by R. K. Wangsness, "Magnetic resonance and minimum entropy production. Macroscopic equations of motion," Phys. Rev. 104, 857 (1956). (nice paper!)

True longitudinal (along H) and transverse (perpendicular to H) relaxation.
Warning: T_1 and T_2 have their traditional meanings only in the small signal limit.

C. Historical View of Phenomenological Equations - COT 17

$$\left(\frac{d\mathbf{M}}{dt}\right) = -|\gamma_e|(\mathbf{M}\times\mathbf{H}) + \frac{1}{T_1}\left[\left(\frac{M_s|\mathbf{H}| - \mathbf{M}\cdot\mathbf{H}}{|\mathbf{H}|}\right)\frac{\mathbf{H}}{|\mathbf{H}|}\right] + \frac{1}{T_2}\left[\frac{(\mathbf{M}\times\mathbf{H})\times\mathbf{H}}{|\mathbf{H}|^2}\right]$$

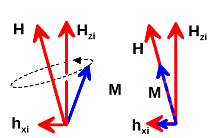
M component along H

M component perpendicuar to H

COT solves the BB problem.

COT pushes M towards the instantaneous internal field direction.

But remember: H ia the internal field. It is a moving target.
1. Dynamic applied fields
2. Demagnetizing fields



Precession around H

COT equilibrium along H

C. Historical View of Phenomenological Equations - COT 18

$$\left(\frac{d\mathbf{M}}{dt}\right) = -|\gamma_e|(\mathbf{M}\times\mathbf{H}) + \frac{1}{T_1}\left[\left(\frac{M_s|\mathbf{H}| - \mathbf{M}\cdot\mathbf{H}}{|\mathbf{H}|}\right)\frac{\mathbf{H}}{|\mathbf{H}|}\right] + \frac{1}{T_2}\left[\frac{(\mathbf{M}\times\mathbf{H})\times\mathbf{H}}{|\mathbf{H}|^2}\right]$$

M component along H

M component perpendicuar to H

F8. Paramagnetic Resonance in Organic Free Radicals at Low Fields.* R. S. Codrington, J. D. Olds, and H. C. Torrey, *Rutgers University.*—We have measured the magnetic resonance line shapes of the solid organic free radicals: diphenylpicryl hydrazyl, tri-p-anisylaminium perchlorate and tri-p-aminophenylaminium perchlorate as a function of magnetic field at the frequencies 0.8, 1.5, 4, 8, and 15 Mc. At 15 Mc all show two Lorentz shaped resonances symmetrically disposed about zero field with full widths between inflection points of 2.0, 0.68, and 0.33 oersteds, respectively. As the frequency is lowered, the resonances merge in each case to form a single absorption maximum at zero field. For each radical the entire shape is accurately predicted by a set of Bloch equations modified so that longitudinal relaxation is parallel and transverse relaxation perpendicular to the instantaneous field direction. The usual Bloch equations give completely erroneous results at low frequencies. In every case the oscillating field was sufficiently small that no longitudinal

Key word is "instantaneous."

C. Historical View of Phenomenological Equations - COT 19

$$\left(\frac{d\mathbf{M}}{dt}\right) = -|\gamma_e|(\mathbf{M}\times\mathbf{H}) + \frac{1}{T_1}\left[\left(\frac{M_s|\mathbf{H}| - \mathbf{M}\cdot\mathbf{H}}{|\mathbf{H}|}\right)\frac{\mathbf{H}}{|\mathbf{H}|}\right]$$ M component along H

$$+\frac{1}{T_2}\left[\frac{(\mathbf{M}\times\mathbf{H})\times\mathbf{H}}{|\mathbf{H}|^2}\right]$$ M component perpendicuar to H

F8. Paramagnetic Resonance in Organic Free Radicals at Low Fields.* R. S. CODRINGTON, J. D. OLDS, AND H. C. TORRE... mag- netic ... icals: diphe... e and tri-p-... n of magn... 5 Mc. At 15... metri- cally ... en in- flectio... y. As the frequency is lowered, the resonances merge in each case to form a single absorption maximum at zero field. For each radical the entire shape is accurately predicted by a set of Bloch equations modified so that longitudinal relaxation is parallel and transverse relaxation perpendicular to the instantaneous field direction. The usual Bloch equations give completely erroneous results at low frequencies. In every case the oscillating field was sufficiently small that no longitudinal

Form is inconsistent with Landau Lifshitz damping

Key word is "instantaneous."

C. Historical View of Phenomenological Equations 20

GILBERT 1955 (G)

$$\frac{d\mathbf{M}}{dt} = -|\gamma_e|(\mathbf{M}\times\mathbf{H}) + \frac{\alpha_G}{M_s}\left[\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right]$$

Very similar to the LL form:

$$\frac{d\mathbf{M}}{dt} = -|\gamma_e|(\mathbf{M}\times\mathbf{H}) - \frac{\alpha_{LL}|\gamma_e|}{M_s}[\mathbf{M}\times(\mathbf{M}\times\mathbf{H})]$$

Just replace precessional component of $\frac{d\mathbf{M}}{dt}$, $-|\gamma_e|(\mathbf{M}\times\mathbf{H})$,

with the total $\frac{d\mathbf{M}}{dt}$.

1. There is an "exact" algebraic equivalence between these two equations (if we jerk γ_e around).
2. However, there is a drastic physical inequivalence.
   - G gives critical damping at $\alpha_G = \pi/4$
   - LL gives NO critical damping.

More on these comparisons shortly.

C. Historical View of Phenomenological Equations 21

MODIFIED BLOCH BLOEMBERGEN 1956 (MBB)

$$\left.\frac{d\mathbf{M}}{dt}\right|_{x,y} = -|\gamma_e|(\mathbf{M}\times\mathbf{H})|_{x,y} - \frac{1}{T_2}\left(m_{x,y} - \frac{M_s}{H_{x,y}}h_{x,y}\right)$$

Transverse components only

Valid only in the small signal limit!

Our old friends the stiffness fields:

$H_x = H_e + 4\pi M_s(N_x - N_z)$

$H_y = H_e + 4\pi M_s(N_y - N_z)$

Recall static equilibrium:

$m_{x,y}^{(eq)} = \frac{M_s}{H_{x,y}}h_{x,y}$

The MBB driving term is just our drive to equiibrium:

$$-\left(m_{x,y} - \frac{M_s}{H_{x,y}}h_{x,y}\right)$$

$$\left.\frac{d\mathbf{M}}{dt}\right|_{x,y} = -|\gamma_e|(\mathbf{M}\times\mathbf{H})|_{x,y} - \frac{1}{T_2}\left(m_{x,y} - m_{x,y}^{(eq)}\right)$$

C. Historical View of Phenomenological Equations 22

MODIFIED BLOCH BLOEMBERGEN 1956 (MBB)

$$\left.\frac{d\mathbf{M}}{dt}\right|_{x,y} = -|\gamma_e|(\mathbf{M}\times\mathbf{H})|_{x,y} - \frac{1}{T_2}\left(m_{x,y} - \frac{M_s}{H_{x,y}}h_{x,y}\right)$$

Transverse components only

Limited to the small signal limit

Our old frie... ...rium:

$H_x = H$...

$H_y = H$...

Form is inconsistent with Landau Lifshitz damping

...n:

$$-\left(m_{x,y} - \frac{M_s}{H_{x,y}}h_{x,y}\right)$$

$$\left.\frac{d\mathbf{M}}{dt}\right|_{x,y} = -|\gamma_e|(\mathbf{M}\times\mathbf{H})|_{x,y} - \frac{1}{T_2}\left(m_{x,y} - m_{x,y}^{(eq)}\right)$$

C. Historical View of Phenomenological Equations 23

TYPE	Drive to equilibrium	Equation
LL	H comp. ⊥ M	$\frac{d\mathbf{M}}{dt} = -\lvert\gamma_e\rvert(\mathbf{M}\times\mathbf{H}) - \frac{\alpha_{LL}\lvert\gamma_e\rvert}{M_s}[\mathbf{M}\times(\mathbf{M}\times\mathbf{H})]$
BB	[illegible]	$\left.\frac{d\mathbf{M}}{dt}\right\rvert_{x,y} = -\lvert\gamma_e\rvert(\mathbf{M}\times\mathbf{H})\rvert_{x,y} - \frac{1}{T_2}m_{x,y}$
COT	M comp. ⊥ H	$\left(\frac{d\mathbf{M}}{dt}\right)_{\perp} = -\lvert\gamma_e\rvert(\mathbf{M}\times\mathbf{H}) + \frac{1}{T_2}\left[\frac{(\mathbf{M}\times\mathbf{H})\times\mathbf{H}}{\lvert\mathbf{H}\rvert^2}\right]$
G	$\lvert d\mathbf{M}/dt\rvert \perp \mathbf{M}$	$\frac{d\mathbf{M}}{dt} = -\lvert\gamma_e\rvert(\mathbf{M}\times\mathbf{H}) + \frac{\alpha_G}{M_s}\left[\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right]$
MBB	$m_{x,y} - m_{x,y}^{(eq)}$	$\left.\frac{d\mathbf{M}}{dt}\right\rvert_{x,y} = -\lvert\gamma_e\rvert(\mathbf{M}\times\mathbf{H})\rvert_{x,y} - \frac{1}{T_2}\left(m_{x,y} - \frac{M_s}{H_{x,y}}h_{x,y}\right)$

1. Drive terms amount to different statements of equilibrium:

2. Forms are mutually inconsistent (another talk)

If λ_{LL} is a constant, then $1/T_2$ (COT) is field dependent.

If λ_{LL} is a constant, then $1/T_2$ (MBB) is (a) field dependent and (b) different for x and y.

C. Historical View of Phenomenological Equations 24

TYPE	Drive to equilibrium	Equation
LL	H comp. ⊥ M	$\frac{d\mathbf{M}}{dt} = -\lvert\gamma_e\rvert(\mathbf{M}\times\mathbf{H}) - \frac{\alpha_{LL}\lvert\gamma_e\rvert}{M_s}[\mathbf{M}\times(\mathbf{M}\times\mathbf{H})]$
BB	[illegible]	$\left.\frac{d\mathbf{M}}{dt}\right\rvert_{x,y} = -\lvert\gamma_e\rvert(\mathbf{M}\times\mathbf{H})\rvert_{x,y} - \frac{1}{T_2}m_{x,y}$
CO...		
...		
M...		

Which one to use?

NONE OF THEM.

Magnetization dynamics is (usually) too complicated for a single damping parameter.

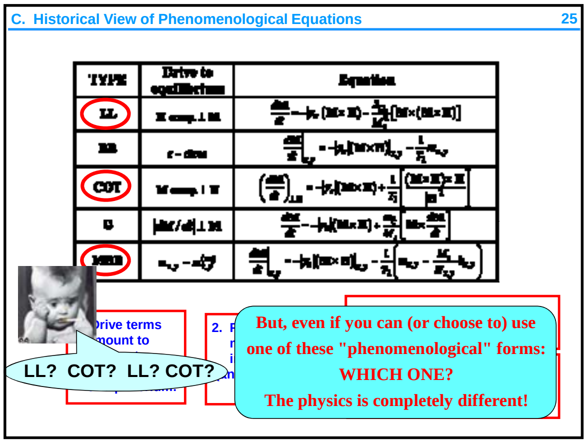
There are some possible exceptions (but be careful).

1. Two magnon free permalloy films.
2. Impurity relaxation in ferrites and metals.
3. Nanostructures ???

1. Dr... am... dif... sta... of ...

...en ...ndent.

...en ...ependent ...and y.


C. Historical View of Phenomenological Equations
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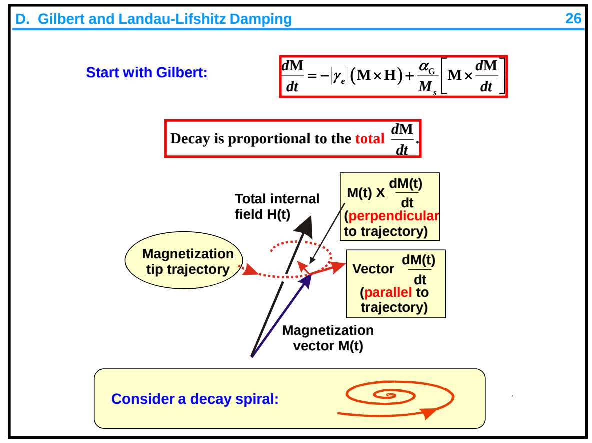
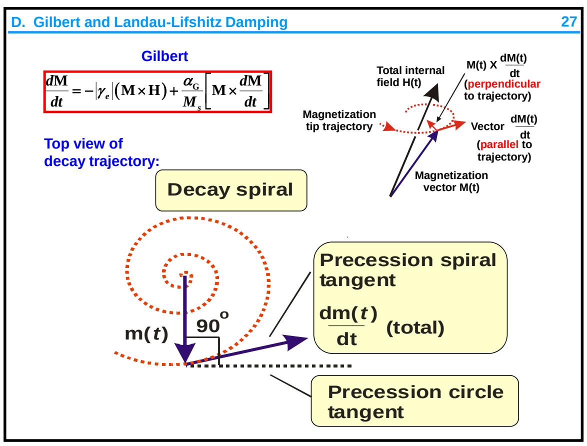
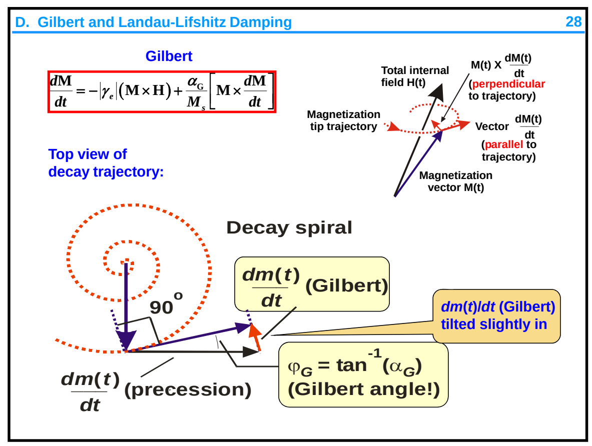
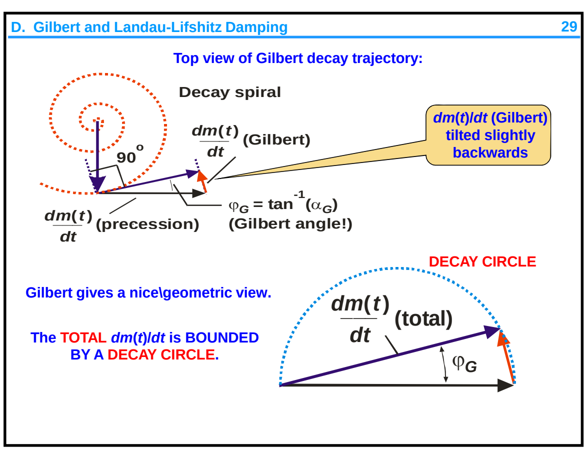
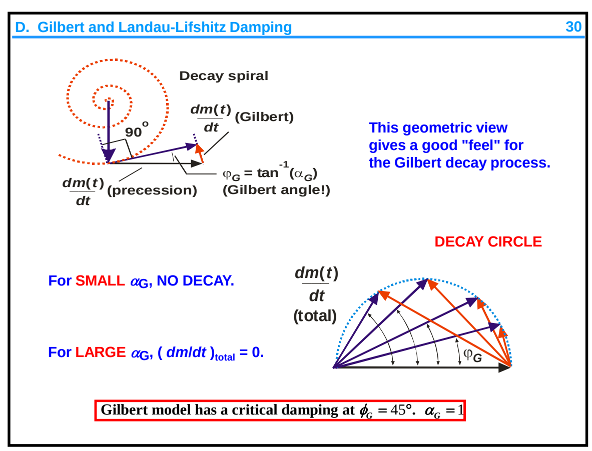
LL? COT? LL? COT?
But, even if you can (or choose to) use one of these "phenomenological" forms:
WHICH ONE?
The physics is completely different!


D. Gilbert and Landau-Lifshitz Damping
26
Start with Gilbert:
$\frac{dM}{dt} = -|\gamma_e|(M \times H) + \frac{\alpha_G}{M_s}\left[M \times \frac{dM}{dt}\right]$
Decay is proportional to the total $\frac{dM}{dt}$.
Total internal field H(t)
M(t) X dM(t)/dt (perpendicular to trajectory)
Magnetization tip trajectory
Vector dM(t)/dt (parallel to trajectory)
Magnetization vector M(t)
Consider a decay spiral:


D. Gilbert and Landau-Lifshitz Damping
27
Gilbert
$\frac{dM}{dt} = -|\gamma_e|(M \times H) + \frac{\alpha_G}{M_s}\left[M \times \frac{dM}{dt}\right]$
Top view of decay trajectory:
Decay spiral
Precession spiral tangent
dm(t)/dt (total)
m(t)
90°
Precession circle tangent


D. Gilbert and Landau-Lifshitz Damping
28
Gilbert
$\frac{dM}{dt} = -|\gamma_e|(M \times H) + \frac{\alpha_G}{M_s}\left[M \times \frac{dM}{dt}\right]$
Top view of decay trajectory:
Decay spiral
dm(t)/dt (Gilbert)
dm(t)/dt (Gilbert) tilted slightly in
90°
$\varphi_G = \tan^{-1}(\alpha_G)$ (Gilbert angle!)
dm(t)/dt (precession)


D. Gilbert and Landau-Lifshitz Damping
29
Top view of Gilbert decay trajectory:
Decay spiral
dm(t)/dt (Gilbert)
dm(t)/dt (Gilbert) tilted slightly backwards
90°
$\varphi_G = \tan^{-1}(\alpha_G)$ (Gilbert angle!)
dm(t)/dt (precession)
DECAY CIRCLE
Gilbert gives a nice\geometric view.
dm(t)/dt (total)
φ_G
The TOTAL dm(t)/dt is BOUNDED BY A DECAY CIRCLE.

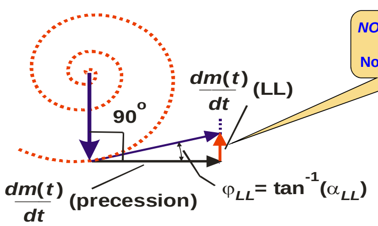

D. Gilbert and Landau-Lifshitz Damping
30
Decay spiral
dm(t)/dt (Gilbert)
90°
$\varphi_G = \tan^{-1}(\alpha_G)$ (Gilbert angle!)
dm(t)/dt (precession)
This geometric view gives a good "feel" for the Gilbert decay process.
DECAY CIRCLE
For SMALL α_G, NO DECAY.
dm(t)/dt (total)
For LARGE α_G, $(dm/dt)_{total} = 0$.
Gilbert model has a critical damping at $\phi_G = 45°$. $\alpha_G = 1$

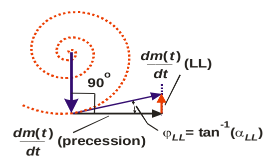
Consider Landau-Lifshitz damping

$$\frac{d\mathbf{M}}{dt} = -|\gamma_e|(\mathbf{M}\times\mathbf{H}) - \frac{\alpha_{LL}|\gamma_e|}{M_s}[\mathbf{M}\times(\mathbf{M}\times\mathbf{H})]$$

DECAY SPIRAL GEOMETRY IS TOTALLY DIFFERENT!

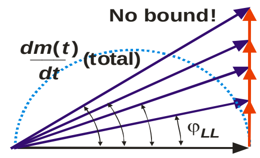
NO dm(t)/dt tilt-back.
NO semicircle.
No critical damping.





Landau-Lifshitz

For LARGE α_G, *dm/dt* just keeps GETTING BIGGER AND BIGGER.



This is UNPHYSICAL.

Analytical small signal decay rates

Gilbert: $\frac{d\mathbf{M}}{dt} = -|\gamma_e|(\mathbf{M}\times\mathbf{H}) + \frac{\alpha_G}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)$

Landau-Lifshitz: $\frac{d\mathbf{M}}{dt} = -|\gamma_e|(\mathbf{M}\times\mathbf{H}) - \frac{\alpha_{LL}|\gamma_e|}{M_s}[\mathbf{M}\times(\mathbf{M}\times\mathbf{H})]$

Exponential decay rates: $m(t) \approx e^{-\eta t}$

Gilbert:

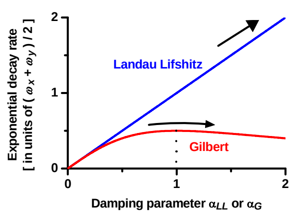
$$\eta_G = \frac{\alpha_G}{1+\alpha_G^2}\frac{(\omega_x+\omega_y)}{2}$$

Landau-Lifshitz:

$$\eta_{LL} = \alpha_{LL}\frac{(\omega_x+\omega_y)}{2}$$

Max at $\alpha_G = 1$

Bigger and bigger



Algebraic equivalence?

Start with Gilbert: $\frac{d\mathbf{M}}{dt} = -|\gamma_e|(\mathbf{M}\times\mathbf{H}) + \frac{\alpha_G}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)$

Step I. Do an M×

$$\mathbf{M}\times\frac{d\mathbf{M}}{dt} = -|\gamma_e|[\mathbf{M}\times(\mathbf{M}\times\mathbf{H})] + \frac{\alpha_G}{M_s}\left[\mathbf{M}\times\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)\right]$$

Use: $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$ on $\mathbf{M}\times\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)$

$$\mathbf{M}\times\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right) = \mathbf{M}\left(\mathbf{M}\cdot\frac{d\mathbf{M}}{dt}\right) - \frac{d\mathbf{M}}{dt}(\mathbf{M}\cdot\mathbf{M}) = -M_s^2\frac{d\mathbf{M}}{dt}$$

$$\mathbf{M}\times\frac{d\mathbf{M}}{dt} = -|\gamma_e|[\mathbf{M}\times(\mathbf{M}\times\mathbf{H})] - \alpha_G M_s\frac{d\mathbf{M}}{dt}$$

Step II. Use "raw" Gilbert and solve for $\mathbf{M}\times\frac{d\mathbf{M}}{dt}$.

$$\mathbf{M}\times\frac{d\mathbf{M}}{dt} = \frac{M_s}{\alpha_G}\left[\frac{d\mathbf{M}}{dt} + |\gamma_e|(\mathbf{M}\times\mathbf{H})\right]$$

Step III. Combine

$$\mathbf{M}\times\frac{d\mathbf{M}}{dt} = -|\gamma_e|[\mathbf{M}\times(\mathbf{M}\times\mathbf{H})] - \alpha_G M_s\frac{d\mathbf{M}}{dt} \text{ with } \mathbf{M}\times\frac{d\mathbf{M}}{dt} = \frac{M_s}{\alpha_G}\left[\frac{d\mathbf{M}}{dt} + |\gamma_e|(\mathbf{M}\times\mathbf{H})\right]$$

Isolate $\frac{d\mathbf{M}}{dt}$: $\frac{d\mathbf{M}}{dt}\left(\frac{M_s}{\alpha_G} + \alpha_G M_s\right) = \frac{d\mathbf{M}}{dt}\left(\frac{1+\alpha_G^2}{\alpha_G}\right)M_s = -\frac{|\gamma_e|M_s}{\alpha_G}(\mathbf{M}\times\mathbf{H}) - |\gamma_e|[\mathbf{M}\times(\mathbf{M}\times\mathbf{H})]$

$$\left\{\begin{matrix}\text{LL}\\ \text{from}\\ \text{G}\end{matrix}\right\} \quad \frac{d\mathbf{M}}{dt} = -\frac{|\gamma_e|}{1+\alpha_G^2}(\mathbf{M}\times\mathbf{H}) - \frac{|\gamma_e|}{(1+\alpha_G^2)}\frac{\alpha_G}{M_s}[\mathbf{M}\times(\mathbf{M}\times\mathbf{H})].$$

Same FORM as LL $(\alpha_{LL}\to\alpha_G)$. BUT one also needs $|\gamma_e| \to |\gamma_e|/(1+\alpha_G^2)$.

OK, LL and G are algebraically equivalent!

Gilbert: $\frac{d\mathbf{M}}{dt} = -|\gamma_e|(\mathbf{M}\times\mathbf{H}) + \frac{\alpha_G}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)$

$$\left\{\begin{matrix}\text{LL}\\ \text{from}\\ \text{G}\end{matrix}\right\} \quad \frac{d\mathbf{M}}{dt} = -\frac{|\gamma_e|}{1+\alpha_G^2}(\mathbf{M}\times\mathbf{H}) - \frac{|\gamma_e|}{(1+\alpha_G^2)}\frac{\alpha_G}{M_s}[\mathbf{M}\times(\mathbf{M}\times\mathbf{H})]$$

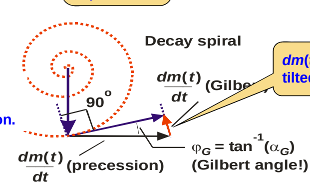
Q: Are we allowed to jerk γ_e around?

A: we must!
The $\frac{|\gamma_e|}{1+\alpha_G^2}$ trick "solves the lack of a critical damping in the "raw" LL equation.

But it also slows down the precession.

Solves the critical damping problem

Recall that Gilbert does, in fact, slow down the precession.



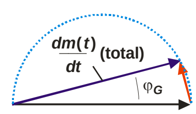
dm(t)/dt (Gilbert) tilted slightly in

Useful to compare the decay circle diagrams.

Gilbert: $\frac{d\mathbf{M}}{dt} = -|\gamma_e|(\mathbf{M}\times\mathbf{H}) + \frac{\alpha_G}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)$

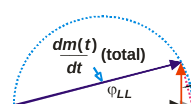
$$\left\{\begin{matrix}\text{LL}\\ \text{from}\\ \text{G}\end{matrix}\right\} \quad \frac{d\mathbf{M}}{dt} = -\frac{|\gamma_e|}{1+\alpha_G^2}(\mathbf{M}\times\mathbf{H}) - \frac{|\gamma_e|}{(1+\alpha_G^2)}\frac{\alpha_G}{M_s}[\mathbf{M}\times(\mathbf{M}\times\mathbf{H})]$$

Gilbert diagram:



Net effect must be the same

Converted LL diagram:



Shrink "converted" decay by $\frac{1}{1+\alpha_G^2}$ too.

Shrink precession by $\frac{1}{1+\alpha_G^2}$

I think this is what people (those that know) mean when they say "LLG."

LL from G or "LLG": $\dfrac{d\mathbf{M}}{dt} = -\dfrac{|\gamma_e|}{1+\alpha_G^2}(\mathbf{M}\times\mathbf{H}) - \dfrac{|\gamma_e|}{(1+\alpha_G^2)}\dfrac{\alpha_G}{M_s}[\mathbf{M}\times(\mathbf{M}\times\mathbf{H})]$

Really the Gilbert equation in wolf's clothing!

1. Has critical damping.
2. Has LL form (useful for numerics).
3. Appears to jerk around γ_e (but not really).

One can also go from LL to G (for kicks, call this "GLL").

Landau-Lifshitz: $\dfrac{d\mathbf{M}}{dt} = -|\gamma_e|(\mathbf{M}\times\mathbf{H}) - \dfrac{\alpha_{LL}|\gamma_e|}{M_s}[\mathbf{M}\times(\mathbf{M}\times\mathbf{H})]$

G from LL: $\dfrac{d\mathbf{M}}{dt} = -|\gamma_e|(1+\alpha_{LL}^2)(\mathbf{M}\times\mathbf{H}) + \dfrac{\alpha_{LL}}{M_s}\left(\mathbf{M}\times\dfrac{d\mathbf{M}}{dt}\right)$

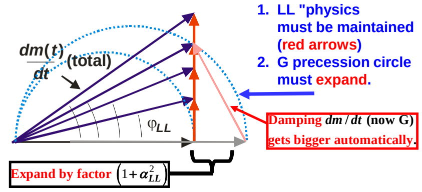
"GLL" is bad news!

G from LL: $\dfrac{d\mathbf{M}}{dt} = -|\gamma_e|(1+\alpha_{LL}^2)(\mathbf{M}\times\mathbf{H}) + \dfrac{\alpha_{LL}}{M_s}\left(\mathbf{M}\times\dfrac{d\mathbf{M}}{dt}\right)$

Now you have to SPEED UP the precession

Interestingly, the α_{LL} term survives intact.

Here is what's happening:



1. LL "physics must be maintained (red arrows)
2. G precession circle must expand.

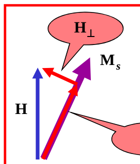
Recap of LL and G

Landau-Lifshitz:

$$\frac{d\mathbf{M}}{dt} = -|\gamma_e|(\mathbf{M}\times\mathbf{H}) + \lambda_{LL}\left[\mathbf{H} - \frac{(\mathbf{H}\cdot\mathbf{M})\mathbf{M}}{M_s^2}\right]$$

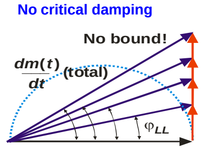
$$\frac{d\mathbf{M}}{dt} = -|\gamma_e|(\mathbf{M}\times\mathbf{H}) - \frac{\alpha_{LL}|\gamma_e|}{M_s}[\mathbf{M}\times(\mathbf{M}\times\mathbf{H})]$$

$$\lambda_{LL} = \alpha_{LL}|\gamma_e|M_s \qquad \alpha_{LL} = \frac{\lambda_{LL}}{|\gamma_e|M_s}$$



Driven by perpendicular component of field.

No critical damping

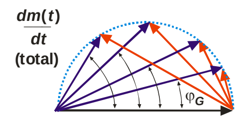


Recap of LL and G

Gilbert: $\dfrac{d\mathbf{M}}{dt} = -|\gamma_e|(\mathbf{M}\times\mathbf{H}) + \dfrac{\alpha_G}{M_s}\left[\mathbf{M}\times\dfrac{d\mathbf{M}}{dt}\right]$

Critical damping

Nice decay circle picture



"LLG": $\dfrac{d\mathbf{M}}{dt} = -\dfrac{|\gamma_e|}{1+\alpha_G^2}(\mathbf{M}\times\mathbf{H}) - \dfrac{|\gamma_e|}{(1+\alpha_G^2)}\dfrac{\alpha_G}{M_s}[\mathbf{M}\times(\mathbf{M}\times\mathbf{H})]$