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Session II

Precession Preliminaries

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Precession Preliminaries — 2

I. Torque Equation
Torque and angular momentum. Torque and the magnetic induction. Gyromagnetic ratio. The first torque equation of motion. Circular precession and instantaneous circular precession.

II. Stifness Fields - More than Meets the Eye
The simple ellipsoidal sample example. Stiffness fields. Stiffness frequencies. The practical gyromagnetic ratio -2.8 GHz/kG. Stiffness fields and static equilibrium

III. Ferromagnetic Resonance
Harmonic solutions for the dynamic magnetization response. Kittel frequency as the geometric mean of the stiffness fields.
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IV. Ellipticity
Ellipticity as the ratio (square root) of the stiffness fields. Rotational symmetry. Equal stiffness fields

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Precession Preliminaries — 3

V. Thin Film Example - The P_A (Ellipticity) Factor
In-plane magnetized Permalloy film. FMR frequency vs. field. dispersion curve. Slopes and divergences. The PA factor.

VI. Stiffness Fields, Equilibrium, and Damping.
Drive to equilibrium. Internal field and stiffness field equivalence. Three damping models: Landau-Lifshitz, Modified Bloch-Bloembergen, Codrington-Olds-Torrey.

VII. Instantaneous Frequency
The angular frequency is not constant.
The angular frequency oscillates" between the stiffness frequencies.

Torque equation — 4

The torque equation of motion →

$$\mathbf{T}_p = \text{torque per unit volume} = \mathbf{M}\times\mathbf{B} = d\mathbf{L}/dt$$
$$\mathbf{M} = \gamma_e \mathbf{L}$$
$$\frac{d\mathbf{M}}{dt} = -|\gamma_e|\mathbf{M}\times\mathbf{B} = -|\gamma_e|\mathbf{M}\times\mathbf{H}\ (\text{Gaussian cgs})$$
$$= -|\gamma_e|\mu_0\,\mathbf{M}\times\mathbf{H}\ (\text{SI})$$

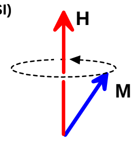
Torque is driven by the magnetic induction B

$|\mathbf{M}| = M_s$ = saturation magnetization (emu/cm^3 or A/m)
$\mathbf{B}$ = magnetic induction (G [Gauss] or T [Tesla])
$\mathbf{H}$ = total effective magnetic field (Oe or A/m)
γ_e = electron gyromagnetic ratio (negative!)
μ_0 = Permeability of free space (SI)

First archival record (that I know of) for the TORQUE EQUATION Landau & Lifshitz (1935) [Some (not me!) call it the Landau Lifshitz equation.]

$$\dot{s}/\mu_0 = [fs]$$

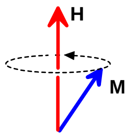
So trivial it was not even numbered!



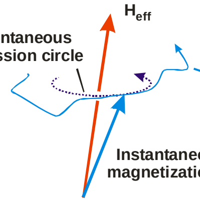
Instantaneous Precession — 5

$\mathbf{H}$ = total EFFECTIVE magnetic field $\Rightarrow$ $\mathbf{H}_{eff}$

PRECESSION IS COUNTER-CLOCKWISE (CCW)



Standard circular precession picture is too simple.



One has "simple circular precession." BUT, H_{eff} is a moving target (Jim Rantschler).

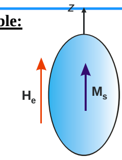
Stiffness Fields — 6

For simplicity, focus on an ellipsoidal sample:

External field: H_e

Saturation magnetization: M_s

Demag factors: $N_x, N_y, N_z,$



Torque equation of motion small signal transverse response gives:

$$\frac{d\mathbf{M}}{dt} = -|\gamma_e|\mathbf{M}\times\mathbf{H}$$

$$\begin{pmatrix} \dfrac{dm_x}{dt} \\ \dfrac{dm_y}{dt} \end{pmatrix} = -|\gamma_e| \begin{pmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ m_x & m_y & M_s \\ -4\pi m_x N_x & -4\pi m_y N_y & H_e - 4\pi M_s N_z \end{pmatrix}_{x,y}$$

$$\begin{pmatrix} (H_e - 4\pi M_s N_z + 4\pi M_s N_y) m_y \\ -(H_e - 4\pi M_s N_z + 4\pi M_s N_x) m_x \end{pmatrix}$$

Define stiffness fields:

$$H_x = H_e + 4\pi M_s (N_x - N_z)$$
$$H_y = H_e + 4\pi M_s (N_y - N_z)$$

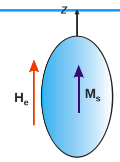
Stiffness Fields and Stiffness Frequencies (7)

Ellipsoidal sample:

External field: H_e

Saturation magnetization: M_s

Demag factors: $N_x, N_y, N_z,$



$$\frac{d\mathbf{M}}{dt} = -|\gamma_e|\mathbf{M}\times\mathbf{H} \quad\longrightarrow\quad \frac{dm_x}{dt} = -|\gamma_e|H_y m_y, \quad \frac{dm_y}{dt} = +|\gamma_e|H_x m_x$$

The H_x and H_y are STIFFNESS FIELDS: $H_x = H_e + 4\pi M_s(N_x - N_z)$

VERY IMPOTANT CONCEPT! $H_y = H_e + 4\pi M_s(N_y - N_z)$

Stiffness frequencies $\omega_x = |\gamma_e|H_x, \ \omega_y = |\gamma_e|H_y$ $\qquad |\gamma_e|/2\pi \approx 2.8$ GHz/kOe

will also be useful: $\omega_x = |\gamma_e|\mu_0 H_x, \omega_y = |\gamma_e|\mu_0 H_y$ $\qquad |\gamma_e|\mu_0/2\pi \approx 28$ GHz/T

Stiffness Fields and Tickle Fields (8)

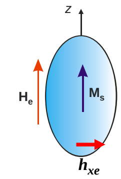
Stiffness fields:

$H_x = H_e + 4\pi M_s(N_x - N_z)$

$H_y = H_e + 4\pi M_s(N_y - N_z)$

Stiffness fields are exactly that:

H_x (for example) is a "stiffness field" that opposes the action of any transverse applied field h_{xe}.



Static equilibrium:

$$\begin{pmatrix} \frac{dm_x}{dt} \\ \frac{dm_y}{dt} \end{pmatrix} = -|\gamma_e| \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ m_x & m_y & M_s \\ h_{xe} - 4\pi m_x N_x & -4\pi m_y N_y & H_e - 4\pi M_s N_z \end{pmatrix}_{x,y} = 0$$

$$\begin{pmatrix} m_y H_y \\ -m_x H_x + M_s h_{xe} \end{pmatrix} = 0 \quad\longrightarrow\quad \frac{m_x}{M_s} = \frac{h_{xe}}{H_x}$$

Stiffness Fields are "Static" Animals (9)

Stiffness fields:

$H_x = H_e + 4\pi M_s(N_x - N_z)$

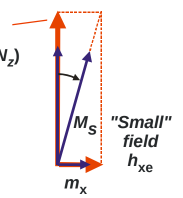
$H_y = H_e + 4\pi M_s(N_y - N_z)$

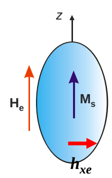
Stiffness field H_x

$H_x = H_e + 4\pi M_s\ (N_x - N_z)$

In small signal limit:

$m_x = \frac{h_{xe}}{H_x} M_s$





This is a static problem.
No dynamics involved!

Stiffness Fields/Frequencies and Ferromagnetic Resonance (10)

Back to dynamics: Component equations give ferromagnetic resonance

$$\begin{pmatrix} dm_x/dt \\ dm_y/dt \end{pmatrix} = -|\gamma_e| \begin{pmatrix} H_y m_y \\ -H_x m_x \end{pmatrix} = \begin{pmatrix} -\omega_y m_y \\ +\omega_x m_x \end{pmatrix} \Rightarrow \begin{matrix} \dot{m}_x = -\omega_y m_y \\ \dot{m}_y = +\omega_x m_x \end{matrix}$$

Harmonic solution:

$m_x = m_{x0}(H_e)\cos[\omega_{FMR}(H_e)t + \phi]$

$m_y = m_{y0}(H_e)\sin[\omega_{FMR}(H_e)t + \phi]$

Secular equation:

$$\begin{pmatrix} -\omega_{FMR} & +\omega_y \\ -\omega_x & +\omega_{FMR} \end{pmatrix} \begin{pmatrix} m_{x0} \\ m_{y0} \end{pmatrix} = 0$$

FMR frequency: $\omega_{FMR} = \sqrt{\omega_x \cdot \omega_y} = |\gamma_e|\sqrt{H_x \cdot H_y}$ **Kittel resonance frequency**

Kittel frequency is the geometric mean of the stiffness frequencies.

Stiffness Fields/Frequencies and Ellipticity (11)

Component equations give Larmor elliptical precession:

$m_x = m_{x0}(H_e)\cos[\omega_{FMR}(H_e)t]$

$m_y = m_{y0}(H_e)\sin[\omega_{FMR}(H_e)t]$

$(\phi = 0)$

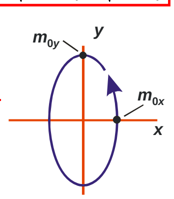
$$\begin{pmatrix} -\omega_{FMR} & +\omega_y \\ -\omega_x & +\omega_{FMR} \end{pmatrix} \begin{pmatrix} m_{x0} \\ m_{y0} \end{pmatrix} = 0$$

$\omega_{FMR} = |\gamma_e|\sqrt{H_x \cdot H_y} = \sqrt{\omega_x \cdot \omega_y}$

$-\omega_{FMR} m_{x0} + \omega_y m_{y0} = 0$

$-\omega_x m_{x0} + \omega_{FMR} m_{y0} = 0$

$$\frac{m_{x0}}{m_{y0}} = \frac{\omega_y}{\omega_{FMR}} = \frac{\omega_{FMR}}{\omega_x} = \sqrt{\frac{\omega_y}{\omega_x}} = \sqrt{\frac{H_y}{H_x}}$$ **Ellipticity:**



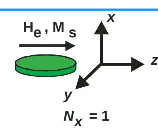
Square root of the ratio of the stiffness fields (or frequencies)

When we get to LL or G damping:

Decay rate: $\eta_{FMR} = \alpha\frac{\omega_x + \omega_y}{2}$ **Arithmetic mean of the two stiffness frequencies**

Thin Film Example (12)

In-plane magnetized thin film case:



$N_x = 1$

$N_y = N_z = 0$

H_x (out of plane) $= H_e + 4\pi M_s$

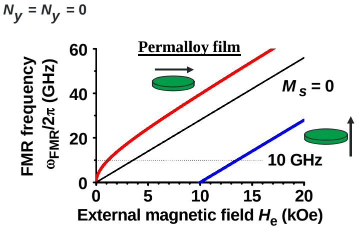
H_y (in plane) $= H_e$

$\omega_{FMR} = |\gamma_e|\sqrt{H_e(H_e + 4\pi M_s)}$

Two important points:

1: Hard H_x $(= H_e + 4\pi M_s)$ gives a frequency enhancement.

2: ω_{FMR} not linear in H_e.



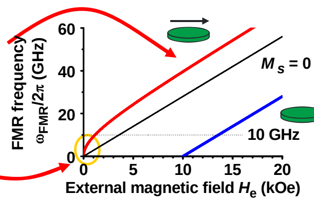
Perpendicular case: $H_x = H_y = H_e - 4\pi M_s, \quad \omega_{FMR} = |\gamma_e|(H_e - 4\pi M_s)$

Non-linear Dispersion - the Ellipticity Factor — 13

HIGH FEELD SLOPE IS $|\gamma_e|$.

$$\omega_{FMR} = |\gamma_e|\sqrt{H_e(H_e + 4\pi M_s)}$$
$$\approx |\gamma_e| H_e + |\gamma_e| 2\pi M_s$$

SLOPE DIVERGES AT ZERO FIELD (and frequency).



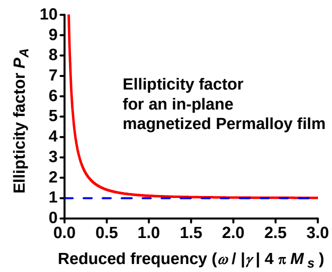
This brings us to another very iportant parameter:

$$\left.\frac{\partial\omega_{FMR}}{\partial|\gamma_e|H_e} = \frac{\omega_x + \omega_y}{2\sqrt{\omega_x\omega_y}}\right|_{FMR} = \frac{(\omega_x + \omega_y)|_{FMR}}{2\omega_{FMR}} = P_A = \text{ellipticity factor}$$

$$\frac{(\omega_x + \omega_y)}{2} = \text{arithmetic mean of the two stiffness frequencies}$$

Non-linear Dispersion - the Ellipticity Factor — 14

$$\left.\frac{\partial\omega_{FMR}}{\partial|\gamma_e|H_e} = \frac{\omega_x + \omega_y}{2\sqrt{\omega_x\omega_y}}\right|_{FMR} = \frac{(\omega_x + \omega_y)|_{FMR}}{2\omega_{FMR}} = P_A = \text{ellipticity factor}$$



Q: Why is P_A so important?

A: It shows up everywhere in relaxation analyses, physical and phenomenological.

Stiffness Fields, Internal Fields, Equilibrium, and Damping — 15

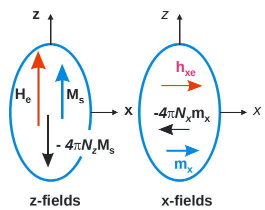
Close these "preliminaries with two points:

Point 1: Stiffness fields, demag fields, equilibrium and damping.

Point 2: Instantaneous frequency.

1:

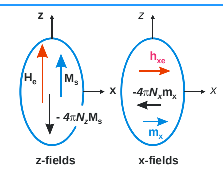
Internal fields and stiffness fields give equivalent equilibrium conditions.



Internal Fields and Equilibrium — 16

Internal fields:

$$H_{zi} = H_e - 4\pi M_s N_z$$
$$h_{xi} = h_{xe} - 4\pi m_x N_x$$
$$h_{yi} = h_{ye} - 4\pi m_y N_y$$



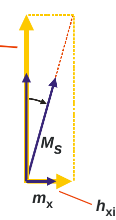
Static Equilibrium:

$$\frac{m_x^{(eq)}}{M_s} = \frac{h_{xi}}{H_{zi}}$$

Internal field H_{zi}

In small TIPPING limit:

$$m_x^{(eq)} = \frac{h_{xi}}{H_{zi}} M_s$$



Stiffness Fields, Internal Fields, and Equilibrium — 17

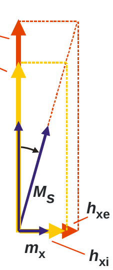
Stiffness field H_x

Internal field H_{zi}

Look at stiffness field and internal field geometries together:

BOTH APPROACHES GIVE SAME EQUILIBRIUM:

$$m_x^{(eq)} = \frac{h_{xe}}{H_x} M_s = \frac{h_{xi}}{H_{zi}} M_s$$



$$\frac{m_x^{(eq)}}{M_s} = \frac{h_{xi}}{H_{zi}} \text{(internal)} = \frac{h_{xe}}{H_x} \text{(stiffness)}$$

This all seems pretty trivial.
But:

Drive to Equilibrium — 18

$$\frac{m_x^{(eq)}}{M_s} = \frac{h_{xi}}{H_{zi}} \text{(internal)} = \frac{h_{xe}}{H_x} \text{(stiffness)}$$

Rearranging the "drive to equilibrium" gives the three basic types of phenonenological damping (Gilbert excluded):

Landau Lifshitz damping term: $-\lambda_{LL}\left(\frac{H_x}{M_s} m_x - h_{xe}\right)$

$-\lambda_{LL}\left(\frac{H_{zi}}{M_s} m_x - h_{xi}\right)$ (equivalent)

Modified Bloch-Bloembergen: $-\lambda_{BB}\left(m_x - \frac{M_s}{H_x} h_{xe}\right)$

Codrington-Olds-Torrey: $-\lambda_{COT}\left(\frac{H_x}{H_{zi}} m_x - \frac{M_s}{H_{zi}} h_{xe}\right)$

Part of a separate "drive to equilibrium" lecture.

Instantaneous Precession 19

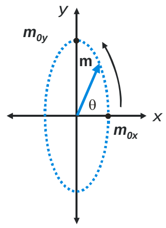
2:

The elliptical precession leads to a widely varying instantaneous frequency!

A. Precession can be highly elliptical.
B. The angular frequency is NOT constant.

Look at:

$$\frac{dm_x}{dt} = -\omega_y m_y \qquad \frac{dm_y}{dt} = +\omega_x m_x$$



It is easy to show that

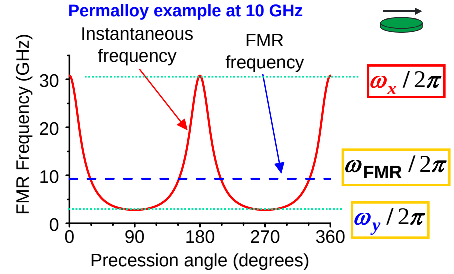
at $\theta = 0$, $\frac{d\theta}{dt} = \omega_x$

and

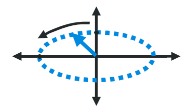
at $\theta = 90°$, $\frac{d\theta}{dt} = \omega_y$.

Instantaneous Precession 20

Permalloy example at 10 GHz



$\omega_x = |\gamma_e|(H_e + 4\pi M_s)$
$\omega_y = |\gamma_e| H_e$
$\omega_{FMR} = |\gamma_e| \sqrt{H_e (H_e + 4\pi M_s)}$



Kepler's law for precession