Fundamentals of Magnetism II
Critical Switching curve:
Coupled vs. Uncoupled Magnetic Systems

Leonard Spinu
University of New Orleans
MAGNETIZATION SWITCHING

Magnetization changes by rotation = Switching

- Static switching – SW switching
  - Stonner-Wohlfarth model
  - Single domain particles
  - DC applied field

- Dynamic switching (control of switching field and time)
  - Switching time
  - M(t) – LLG equation
  - H(t) (Pulsed field)

Two stable states (logical “0” and “1”)

ΔE = KV
Stoner-Wohlfarth Model

- Single Domain Particles at T=0K
- No Interactions
- Uniaxial anisotropy
- Coherent Rotation
- Very useful in modeling magnetic particles in
  - Magnetic storage
  - Biomagnetism
  - Rock Magnetism
  - Paleomagnetism
Stoner-Wohlfarth Model
A MECHANISM OF MAGNETIC HYSTERESIS IN HETEROGENEOUS ALLOYS

By E. C. STONER, F.R.S. AND E. P. WOHLFARTH

Physics Department, University of Leeds

(Received 24 July 1947)
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THEORY OF MAGNETIC HYSTERESIS IN FILMS AND ITS APPLICATION TO COMPUTERS

by

J. C. SLONCZEWSKI

FIGURE 3. THE ORIENTATION OF M, INDICATED BY ARROWS, DEPENDS ON H. Hx AND HY ARE IN UNITS OF 2K.
The free energy of a uniaxial anisotropy single domain particle

$$W(\theta, H) = K_1 \sin^2 \theta - \mathbf{M} \cdot \mathbf{H}$$

$$\frac{\partial W}{\partial \theta} = 0; \quad \frac{\partial^2 W}{\partial \theta^2} = 0 \quad \Rightarrow \quad h_x = \sin^3 \theta; \quad h_y = -\cos^3 \theta$$

$${\mathbf{h}} = 1.25, \quad \theta = 45^\circ$$

CC = the locus of in-plane fields at which the irreversible magnetization reversal occurs

J. C. Slonczewski, IBM Research Center Memorandum R. M. Report No. 003.111.224.
Classical and quantum magnetization reversal studied in nanometer-sized particles and clusters

Edited by

to be published in: Advances in Chemical Physics
invitation from Stuart A. Rice
Second version of 31 Dec. 2000

Author

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Fig. 1.5 Schematic drawing of a planar micro-bridge-DC-SQUID on which a ferromagnetic particle is placed. The SQUID detects the flux through its loop produced by the sample magnetization. Due to the close proximity between sample and SQUID a very efficient and direct flux coupling is achieved.

Fig. 3.3 Temperature dependence of the switching field of a 3 nm Co cluster, measured in the plane defined by the easy and medium hard axes ($H_y - H_z$ plane in Fig. 2.6). The data were recorded using the blind mode method (Sect. 1.2.6) with a waiting time of the applied field of $\Delta t = 0.1$s. The scattering of the data is due to stochastic and in good agreement with Eq. 3.10.
Two-dimensional magnetic switching of micron-size films in magnetic tunnel junctions

A. Anguelouch, B. D. Schrag, and Gang Xiao
Department of Physics, Brown University, Providence, Rhode Island 02912

Yu Lu, P. L. Trouilloud, R. A. Wanner, and W. J. Gallagher
IBM T. J. Watson Research Center, Yorktown Heights, New York 1

S. S. P. Parkin
IBM Almaden Research Center, San Jose, California 95120

FIG. 1. (a) Schematic of the sample layer structure. The two cobalt layers will align antiparallel due to interlayer exchange across the thin Ru layer. Neel coupling occurs as a result of interface roughness at the tunneling barrier, as shown. The pinned axis (represented by P1 and P2) is assumed to be slightly misaligned with respect to the easy axis of the sample. (b) Magnetoresistance hysteresis loop, with zero applied hard-axis field.

FIG. 2. (a) Asteroid curve of one representative MTJ with dimensions 0.8 \( \times \) 6.4 \( \mu \text{m}^2 \). \( H_x \) and \( H_y \) are fields in the easy- and hard-axis directions. (b) Stoner–Wohlfarth critical curve for an ideal single-domain particle with uniaxial anisotropy.
Static Critical Curve: Hypocycloid
Micromagnetic studies of coherent rotation with quartic crystalline anisotropy

Ching-Ray Chang
Department of Physics, National Taiwan University, Taipei, Taiwan, Republic of China
(Received 28 August 1990; accepted for publication 23 October 1990)
Critical curves for determining magnetization directions in implanted garnet films

C. C. Shir and Y. S. Lin

IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598
(Received 28 September 1978; accepted for publication 15 December 1978)

FIG. 13. (a)-(d) Sequence of Ferrofluid patterns of domain walls and underlying magnetic bubble in response to a 25-0e rotating field with interpretations by using critical curves. Strike-out in (b), "flip" motion in (d), (f), (h), and "whip" motion in (j), (k), and (m). (a)-(h) Convergent charged wall with bubble after "flip" motion in (f), and divergent charged wall without bubble after "flip" motion in (h). (i)-(m) Charged walls in zigzag shape in (j), convergent charged wall with bubble after "whip" motion in (k), and divergent charged wall after "whip" motion in (m).
Introducing our new sensitive method based on reversible susceptibility's singularities detection for determination of the critical curve of 2D magnetic systems.

\[ \chi_{ij} = \lim_{\Delta H_j \to 0} \frac{\Delta M_i}{\Delta H_j} \]

\[ \chi_T = \chi_{xx} = \left( \frac{dM_x}{dH_x} \right)_{H_z=H_y=0} \]
Letter to the Editor

Transverse susceptibility as the low-frequency limit of ferromagnetic resonance

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The TS tensor components

\[ \chi_{xx} = \frac{M^2}{F_{\theta\theta}F_{\varphi\varphi} - F_{\theta\varphi}^2} \left( \sin^2 \theta_M \sin^2 \varphi_M F_{\theta\theta} \right. \\
+ \frac{\sin 2\theta_M \sin 2\varphi_M}{2} F_{\theta\varphi} + \cos^2 \theta_M \cos^2 \varphi_M F_{\varphi\varphi} \left. \right) \]

\[ \chi_{yy} = \frac{M^2}{F_{\theta\theta}F_{\varphi\varphi} - F_{\theta\varphi}^2} \left( \sin^2 \theta_M \cos^2 \varphi_M F_{\theta\theta} \right. \\
- \frac{\sin 2\theta_M \sin 2\varphi_M}{2} F_{\theta\varphi} + \cos^2 \theta_M \sin^2 \varphi_M F_{\varphi\varphi} \left. \right) \]

\[ \chi_{zz} = \frac{M^2}{F_{\theta\theta}F_{\varphi\varphi} - F_{\theta\varphi}^2} \left( \sin^2 \theta_M F_{\varphi\varphi} \right) \]
Probing 2D switching using susceptibility experiments

\[ \frac{\Delta \chi_T}{\chi_T} \sim \frac{\Delta f_{\text{res}}}{f_{\text{res}}} \]

\[ H_{\text{DC}} \]

\[ H_{\text{AC}} \]

\[ D(\theta_M, \varphi_M) = F_{\theta\theta} F_{\varphi\varphi} - F_{\theta\varphi}^2, \quad \chi_{xx} \propto \frac{1}{D(\theta_M, \varphi_M)}. \]
Probing two-dimensional magnetic switching in Co/SiO$_2$ multilayers using reversible susceptibility experiments

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Advanced Materials Research Institute and Department of Physics, University of New Orleans, New Orleans, Louisiana 70148
Vectorial mapping of exchange anisotropy in IrMn/FeCo multilayers using the reversible susceptibility tensor

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A. I. Stancu
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Seagate Research, Pittsburgh, Pennsylvania 15222, USA
Static vs. Dynamic

Switching of magnetization by nonlinear resonance studied in single nanoparticles

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(a) Static

(b) 3.2 GHz
Ellipsoidal permalloy Particle

\[ 4\pi M_s = 10.8 \text{kG} \]
\[ N_x = 0.008 \]
\[ N_y = 0.012 \]
\[ N_z = 0.980 \]

\[ H_{appl} \]

\[ \mathbf{m}_1 \]

\[ \text{Time (ns)} \]
\[ \text{rise time} \quad \text{pulse length} \quad \text{fall time} \]

\[ \text{Scheme of the pulse to explain the parameters} \]
\[ \text{pulse rise time/pulse length/pulse fall time} \]

Switching behavior of a Stoner particle beyond the relaxation time limit

M. Bauer, J. Fassbender,* and B. Hillebrands
Static Critical Curve vs. Dynamic Critical Curve

DC Applied Field

Operate here to write ‘1’

Operate here to write ‘0’

Applied Pulse 0/2.75/0 ns

Applied Pulse 0/1.4/0 ns
The Nobel Prize in Physics 2007
Discovery of Giant Magnetoresistance

Albert Fert

Peter Grunberg
(a) basic magnetic tunnel junction structure consisting of two ferromagnetic metals separated by a thin insulating layer

(b) exchange-coupling one of the magnetic layers to an antiferromagnetic layer (by “pinning” the layer) the TMR response reflects the hysteresis of the other so-called “free” layer and has a response more suitable for memory

(c) magnetic offset caused by fields emanating from the pinned layer \(\Rightarrow\) SAF pinned layer; the lower layer in this artificial antiferromagnet is pinned via exchange bias. This flux closure increases the magnetic stability of the pinned layer and reduces coupling to the free layer

(d) both pinned and free = antiferromagnetically coupled pairs \(\Rightarrow\) used in toggle-MRAM
The synthetic antiferromagnet (SAF) = a nanostructured sandwich of two ferromagnetic thin films antiferromagnetically coupled through a non-magnetic metallic spacer.

Many technological applications:
- soft underlayer for perpendicular recording
- hard disk reading heads
- magnetic sensors
- MRAM
- Toggle-MRAM

**MRAM**

- Top electrode
- Free
- AlOx
- Fixed
- Ru
- Pinned
- AF pinning layer
- Template
- Seed
- Base electrode

- Low resistance contact
- Switches between two magnetic states in applied field. Stores information.
- Tunnel barrier. Affects resistance and MR ratio.
- Synthetic Antiferromagnet (SAF). AF coupling through Ru layer makes the structure stable in applied magnetic fields. Relative thickness of Fixed and Pinned used to center loop.
- Pins the bottom magnetic layers.
- Seeds growth, determines crystal structure
- Low resistance contact
Comparison between the Non-Coupled and Coupled Magnetic Structures

<table>
<thead>
<tr>
<th></th>
<th>Non-Coupled</th>
<th>Coupled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetization vectors</td>
<td></td>
<td><img src="image" alt="Magnetization vectors" /></td>
</tr>
<tr>
<td></td>
<td>$M = M \angle \theta$</td>
<td>$M_1 = M_1 \angle \theta_1$ \n $M_2 = M_2 \angle \theta_2$</td>
</tr>
<tr>
<td>Major easy axes</td>
<td>$\theta = 0$</td>
<td>$\theta_1 = 0$, $\theta_2 = \alpha$</td>
</tr>
</tbody>
</table>
| Total free energy     | Anisotropy energy $+$  
                       | + magnetization energy                     | Anisotropy energy $1 +$  
                       |                                                  | + magnetization energy $1$ (due to applied field) $+$  
                       |                                                  | + magnetization energy $2$ (due to applied field) $+$  
                       |                                                  | + magnetization energy (due to interacting field) |
| Stable states         | $\partial F / \partial \theta = 0$, subject to the condition  
                       | $\frac{\partial^2 F}{\partial \theta^2} > 0$ | $\partial F / \partial \theta_1 = \partial F / \partial \theta_2 = 0$, subject to the conditions  
                       |                                                  | $\frac{\partial^2 F}{\partial \theta_1^2} > 0$ and  
                       |                                                  | $\left( \frac{\partial^2 F}{\partial \theta_1 \partial \theta_2} \right)^2 - \frac{\partial^2 F}{\partial \theta_1^2} \frac{\partial^2 F}{\partial \theta_2^2} < 0$ |
| Critical states       | Limiting case of stable states  
                       | $\partial F / \partial \theta = 0$ \n $\frac{\partial^2 F}{\partial \theta^2} = 0$ | Limiting case of stable states  
                       |                                                  | $\partial F / \partial \theta_1 = \partial F / \partial \theta_2 = 0$ \n $\frac{\partial^2 F}{\partial \theta_1 \partial \theta_2} = 0$ or  
                       |                                                  | $\left( \frac{\partial^2 F}{\partial \theta_1 \partial \theta_2} \right)^2 - \frac{\partial^2 F}{\partial \theta_1^2} \frac{\partial^2 F}{\partial \theta_2^2} = 0$ |

Convenient description

![Convenient description](image)
Analysis of Static and Quasidynamic Behavior of Magnetostatically Coupled Thin Magnetic Films

Abstract: When two superposed films exert fields on each other, their static and dynamic behaviors change. The following method of analysis is used to study the behavior: The stable states are found by minimizing the total free energy of the films. Then constant-field contours are plotted in the $\theta_1,\theta_2$ plane ($\theta$'s being the stable orientations of the magnetization vectors). In examining the plot, one can predict multiple stable states, switching, threshold, hysteresis, and the detailed paths of magnetization change as a function of applied field.

The solution is carried out by a numerical process which permits evaluation of the following effects: the variation of the degrees of symmetries of the anisotropy energies, the relative orientation between the films, the coupling strength, and the drive-line layout. An example is carried out in sufficient detail for illustrative purposes.

Table 2  **Comparison between single film and pair of magnetostatically coupled films.**

<table>
<thead>
<tr>
<th></th>
<th>Single film</th>
<th>Coupled films</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Magnetization vectors</strong></td>
<td>$\mathbf{M} = M \perp \theta$</td>
<td>$\mathbf{M}_1 = M_1 \perp \theta_1; \mathbf{M}_2 = M_2 \perp \theta_2$</td>
</tr>
<tr>
<td><strong>Major easy axes</strong></td>
<td>$\theta = 0$</td>
<td>$\theta_1 = 0, \theta_2 = \alpha$</td>
</tr>
<tr>
<td><strong>Total free energy</strong></td>
<td>Anisotropy energy + magnetization energy</td>
<td>Anisotropy energies + magnetization energies (due to applied field) + magnetization energy (due to interacting field)</td>
</tr>
<tr>
<td><strong>Stable states (corresponding to minimum energy) are determined by solving equations shown</strong></td>
<td>$\frac{\partial E}{\partial \theta} = 0$, subject to the condition $\frac{\partial^2 E}{\partial \theta^2} &gt; 0$</td>
<td>$\frac{\partial E}{\partial \theta_1} = \frac{\partial E}{\partial \theta_2} = 0$ subject to the conditions $\frac{\partial^2 E}{\partial \theta_1^2} &gt; 0$ and $\left( \frac{\partial^2 E}{\partial \theta_1 \partial \theta_2} \right)^2 - \frac{\partial^2 E}{\partial \theta_1^2} \frac{\partial^2 E}{\partial \theta_2^2} &lt; 0$</td>
</tr>
<tr>
<td><strong>Critical states</strong></td>
<td>Limiting case of stable states</td>
<td>Limiting case of stable states</td>
</tr>
<tr>
<td></td>
<td>$\frac{\partial E}{\partial \theta} = 0$</td>
<td>$\frac{\partial E}{\partial \theta_1} = \frac{\partial E}{\partial \theta_2} = 0$</td>
</tr>
<tr>
<td></td>
<td>$\frac{\partial^2 E}{\partial \theta^2} = 0$</td>
<td>$\frac{\partial^2 E}{\partial \theta_1^2} = 0$ or $\frac{\partial^2 E}{\partial \theta_2^2} = 0$ or</td>
</tr>
<tr>
<td></td>
<td>$\left( \frac{\partial^2 E}{\partial \theta_1 \partial \theta_2} \right)^2 - \frac{\partial^2 E}{\partial \theta_1^2} \frac{\partial^2 E}{\partial \theta_2^2} = 0$</td>
<td></td>
</tr>
<tr>
<td><strong>Convenient description</strong></td>
<td>Critical curve and equilibrium line in $h_x-h_y$ plane (see Appendix I and Fig. 8)</td>
<td>Constant field contours in $\theta_1-\theta_2$ plane (see Section of that title and also Fig. 4)</td>
</tr>
</tbody>
</table>

Constant Angle Contour Maps
Critical-field curves for switching toggle mode magnetoresistance random access memory devices (invited)

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(Presented on 9 November 2004; published online 17 May 2005)

FIG. 4. Critical-field curves obtained keeping the parameters \( m = 1 \) and \( k = 1 \) for \( t = 1, \) \( h_{x_0}/h_{y_0} = 1.5, \) and \( h_j = 1.5, \) and changing the thickness ratio \( t_\alpha \) to (a) \( t_\alpha = 1, \) (b) \( t_\alpha = 0.8, \) and (c) \( t_\alpha = 0.6. \) For the same assumption as in Fig. 3 about \( M_r, H_{Ku}, \) and \( t_\alpha, \) the set of normalized parameters chosen here corresponds to the parameter set of \( H_{Ku} = 4 \) Oe, \( M_r(N_{xy} - N_{xz}) = 4 \) Oe, and \( J = rM_r^2/(2N_{xy}) = 0.02 \) ergs/cm\(^2\).
SAF Critical Curve

\[ m = 1, \ k = 1, \ t = 1, \ h_f = 0.00 \]
Critical Curves (CC) – Symmetric SAF

$m = 1, \ k = 1, \ t = 1, \ h_f = 2$

$h_x = 3.30, \ h_y = 0.20
Critical Curves (CC) – Asymmetric SAF

\[
m = 1, \quad k = 0.81193, \quad t = 0.8, \quad h_J = 4
\]

\[
h_x = 8.77, \quad h_y = 0.50
\]
Critical Curve and Susceptibility Tensor for a coupled structure

\[
\frac{\partial F}{\partial \theta_1} = \frac{\partial F}{\partial \theta_2} = 0
\]

\[
\left( \frac{\partial^2 F}{\partial \theta_1 \partial \theta_2} \right)^2 - \left( \frac{\partial^2 F}{\partial \theta_1^2} \right) \left( \frac{\partial^2 F}{\partial \theta_2^2} \right) = 0
\]

\[
\chi_{xx} = \lim_{H_{ac} \to 0} \frac{\Delta M_x}{H_{ac}} = \lim_{H_{ac} \to 0} \frac{\Delta M_{1,x} + \Delta M_{2,x}}{H_{ac}}
\]

\[F = \text{energy density per unit area} = F_{\text{Zeeman}} + F_{\text{anisotropy}} + F_{\text{coupling}} = \frac{J_h}{M_{1,s}} m_1 m_2
\]

\[m = \frac{M_{2,s}}{M_{1,s}}; \quad t = \frac{t_2}{t_1}; \quad t_i = \text{thickness of layer } i
\]

\[
\chi_{xx} = \frac{F_{\theta_2,\theta_2} \cos^2 \theta_1 - 2mt F_{\theta_1,\theta_2} \cos \theta_1 \cos \theta_2 + m^2 t^2 F_{\theta_1,\theta_1} \cos^2 \theta_2}{F_{\theta_1,\theta_1} F_{\theta_2,\theta_2} - F_{\theta_1,\theta_2}^2}
\]
Depending on the coupling strength between the two ferromagnetic layers in a coupled (i.e. SAF) structure, the critical curve evolves from a simple astroid at zero coupling to more complicated critical curves for larger coupling field values.
Measurements of the critical curve for a coupled structure

\( h_{\text{sweep}, \text{ea}} = 72^\circ \)
\( t = 1, \ h_j = 0.5 \)
Measurements of the critical curve for a coupled structure
Switching behavior of one SW particle: using field pulses starting from origin.

- M. Bauer et. al. PRB 61, 3410, 2000
- H. Pham et.al. JAP 97, 10P106, 2005

Switching behavior of SAF: using a bias and a pulse field
Dynamic CC of an asymmetric SAF – sweep rate dependence

The dynamical CC of a SAF element shows that only a digit or word field, which are the field applied at and with respect to the easy axis in toggle MRAM structure, can switch the magnetization

→ using the static CCs instead of dynamic CCs in characterization of a SAF can lead to inadvertent switching of half-selected memory cells
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→ using the static CCs instead of dynamic CCs in characterization of a SAF can lead to inadvertent switching of half-selected memory cells
Dynamic CC of a symmetric SAF – sweep rate dependence

\[ \nu_H = \frac{h_{\text{max}}}{t_{\text{rise}}} \]

Dynamic critical curve of a synthetic antiferromagnet

Huy Pham, Dorin Cimpoesu, Andrei-Valentin Plamadă, Alexandru Stancu, and Leonard Spinu

\[ T_H = 3 \text{ ns} \]
\[ \alpha = 0.008 \]
Dynamic CC of a symmetric SAF – damping dependence

\[ \nu_H = \frac{h_{\text{max}}}{t_{\text{rise}}} \]

**Dynamic critical curve of a synthetic antiferromagnet**

Huy Pham,\(^1\) Dorin Cimpoesu,\(^2\)\(^,\)\(^+\) Andrei-Valentin Plamadă,\(^2\) Alexandru Stancu,\(^2\) and Leonard Spinu\(^1\)\(^,\)\(^+\)

\( \alpha = 0.008 \)

\( t = 1, \ h_j = 2 \)

\( T_H = 3 \text{ ns} \)

\( \nu_H = 1 \text{ Oe/ps} \)
Edmund Clifton Stoner (1899-1968)
Erich Peter Wohlfarth (1924-1988)
References