Fundamentals of Magnetism

Part I

Magnetostatics, Anisotropy, Domains, Coherent Rotation, Incoherent Processes and Thermal Effects

Kevin O’Grady
The University of York
and
J.W. Harrell
University of Alabama
Magnetic Pole Density and Fields

\[ \mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) \]

\[ \nabla \cdot \mathbf{B} = 0 \quad \Rightarrow \quad \nabla \cdot \mathbf{H} = -\nabla \cdot \mathbf{M} = \rho_m \]

\( \rho_m \) = magnetic charge density

For point pole, \( \mathbf{H} = \frac{q_m \mathbf{r}}{4\pi r^2} = -\nabla \varphi_m \),

where \( \varphi_m = \frac{q_m}{4\pi r} \) = magnetic potential gradient = \( \nabla (\Sigma N/l) \)

For a volume distribution,

\[ \varphi_m = \int \frac{\rho_m dV}{4\pi r} \]

What is ‘flux’ and ‘flux density’?

What is a ‘magnetic field’?

\[ \mathbf{E} = -\nabla V \]

\[ \nabla \cdot \mathbf{D} = \varepsilon_0 \nabla \cdot \mathbf{E} + \nabla \cdot \mathbf{P} \]

\( \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \)

\( \rho_{\text{total}} = \rho_b = \rho_{\text{free}} \)
Field From a Sheet Uniformly Magnetised Perpendicular to Surface

Inside:

- \( M = M \)
- \( H = -4\pi M \) (demag)
- \( B = 0 \)

Outside:

- \( M = H = B = 0 \)

Why?

\[ B = \mu_0(H+M) = \mu_0(-M+M) = 0 \]
Thin Rod Uniformly Magnetised Along Axis

\[ \sigma_m = M \]
\[ \rho_m = 0 \]
\[ -\sigma_m = -M \]

Outside: \[ M = 0, B = \mu_0 H \]

Inside: \[ M = +M_2, H_z < 0, B_z > 0 \]
Demagnetization Field, $H_d$

$H_{\text{int}}$ (or $H_{\text{Total}}$) = $H + H_d$

For a body with arbitrary shape, $H_d$ is not constant; however, for an ellipsoid $H_d = \text{constant}$.

Generally, $H_d = -\mathbf{\tilde{N}} \cdot \mathbf{M}$, where

$$\mathbf{\tilde{N}} = \begin{pmatrix} N_a & 0 & 0 \\ 0 & N_b & 0 \\ 0 & 0 & N_c \end{pmatrix}$$

so, $H_d = -N_a M_x \mathbf{i} - N_b M_y \mathbf{j} - N_c M_z \mathbf{k}$

If $\mathbf{M}$ is along a principle axis, then

$H_d = -N \mathbf{M}$ \quad ($N = N_a, N_b, or N_c$)

In general,

$N_a + N_b + N_c = 1$ \quad (4\pi \text{ in cgs})
Special Cases

**Thin oblate spheroid** (pancake)
\[ N_b = N_c = 0 \]
\[ N_a = 1 \quad (4\pi) \]

**Sphere**
\[ N_a = N_b = N_c = N \]
\[ 3N = 1 \]
\[ N = 1/3 \quad (4\pi/3) \]

**Thin prolate ellipsoid** (cigar)
\[ N_c = 0, \quad 2N_a = 1 \]
\[ N_a = 1/2 \quad (2\pi) \]

\[ r = c/a \]
\[ (a = b) \]
Demagnetisation Energy and Fields

\[ \mathbf{H}_d = -\mathbf{\tilde{N}} \cdot \mathbf{M} = -\left( N_a M_x \mathbf{i} + N_b M_y \mathbf{j} + N_c M_z \mathbf{k} \right) \]
\[ = -\left( N_a \cos \alpha \mathbf{i} + N_b \cos \beta \mathbf{j} + N_c \cos \gamma \mathbf{k} \right) \mathbf{M} \]

Energy associated with demagnetisation field -

\[ \mathbf{E}_d = \frac{1}{2} \mu_0 \mathbf{M} \cdot \mathbf{H}_d \]
\[ = \frac{1}{2} \mu_0 \left( N_a \cos^2 \alpha + N_b \cos^2 \beta + N_c \cos^2 \gamma \right) \mathbf{M}^2 \]

Perpendicular Media -
There is a global \( \mathbf{H}_d \)
\( \mathbf{H}_d \) is non-uniform
in places \( \mathbf{H}_d = 0 \)
in places \( \mathbf{H}_d \) is positive!!
The effect of the demagnetizing field on the M-H loop can be described by the following equation:

\[ H_{\text{in}} = H_{\text{app}} - NM \]

- The loop is sheared with a slope of 1 (\(4\pi\)).
- The only ‘true’ point is at \(H_c\) where \(M=0\).
- In practice for \(N=1\) (\(4\pi\)), exchange coupling reduces the loop shear.
ANISOTROPY

Shape Anisotropy

\[ u_d = \frac{1}{2} \mu_0 \Delta NM_s^2 \sin^2 \theta = K_s \sin^2 \phi \]
\[ \Delta N = N_a - N_c \]

(Note: These are sample shapes, not energy surfaces.)

- For \( c/a > 10 \), \( H_c (T=0) > 1T \)
- Not achieved because of incoherent reversal

- Elongated particles for tape have \( c/a \sim 4 \) and \( H_c = 0.4T \)
- CoFe is used to maximise \( M_s \)
Crystalline anisotropy

- Due to spin-orbit coupling and chemical bonding of orbitals with local environment (crystalline electric field)
- Must have non-spherical atomic orbitals ($L_z \neq 0$) and non-spherical crystalline field.
- Dipole-dipole interactions are not strong enough to cause significant crystalline anisotropy.

- Occurs in all crystals but is usually weak in cubic materials.
- Very strong in hexagonal crystals e.g. Co and Ba – ferrite.
- Even stronger in some tetragonal crystals e.g. FePt
Other sources of anisotropy

**Induced**

- Heat in a field, stress, plastic deformation (e.g., rolling), etc.

**Exchange anisotropy**

- coupling between FM and AFM materials
Uniaxial Anisotropy

General case: \( u = K_1 \sin^2 \theta + K_2 \sin^4 \theta + \ldots \)

Second and higher-order terms are usually negligible

1\(^{\text{st}}\) order positive

\( K_1 > 0, K_2 = 0 \):

\( E_k = K_1 \sin^2 \theta \)

Mathematically the same as shape anisotropy for prolate spheroid.
1\textsuperscript{st}-order negative

$K_1 < 0, K_2 = 0$:

$E_k = -|K_1| \sin^2 \theta = + |K_1| \cos^2 \theta$ (+ constant)

Mathematically the same as shape anisotropy for oblate spheroid.
Torque curves

Torque (per unit volume) exerted on crystal by $\mathbf{M}$

$$L = -\frac{dE}{d\theta}$$

First order:

$$E_k = K_1 \sin^2 \theta$$

$$L = -2K_1 \sin \theta \cos \theta = -K_1 \sin 2\theta$$

- Unless the anisotropy is purely uniaxial torque curves are difficult (impossible) to interpret
- In practice only possible for single crystals
Examples of Uniaxial Anisotropy

- Uniaxial anisotropy occurs in elongated particles used in tapes.
- The $M_s^2$ term is why these particles are made from Fe$_{60}$Co$_{40}$.
- It also occurs in materials with strong crystal asymmetry.
- These include hcp Cobalt and Barium Ferrite.
- Complex mixed anisotropies can occur in materials with elongation perpendicular to a c-axis, e.g. Ba-Ferrite platelets.
Stoner-Wohlfarth Theory

- This theory explains the behaviour of single domain particles at $T = 0$.

- The particles must be uniaxial and align by moment rotation over an anisotropy barrier.

The energy is then:

$$E = KV \sin^2 \phi - \mu H \cos \theta$$

$$\therefore \frac{dE}{d\theta} = 2K \sin \theta \cos \theta - \mu_0 H M_s \sin(\phi - \theta)$$

For $H$ perpendicular to $EA$, $\phi = 90^\circ$

$$\therefore 2K \sin \theta \cos \theta = \mu_0 H M_s \cos \theta$$

$$\therefore 2K \frac{M}{M_s} = \mu_0 H M_s$$

$$\therefore \frac{M}{M_s} = \frac{\mu_0 H M_s}{2K}$$

Therefore, the system saturates at:

$$H = \frac{2\mu_0 K}{M_s} = H_K$$

Anisotropy Field
The Aligned Case

- The Anisotropy Field $H_K = \mu_0 2K/M_s$ is needed to pull the moment to $90^\circ$.

- It can then fall into either direction along the easy axis.

- This gives a square loop switching at $H_K$.

- For a small misalignment of $10^\circ$ the switching field $H_s$ falls by 30%.

- At $90^\circ$ there is no hysteresis.
• The minimisation
\[
\frac{dE}{d\theta} = 2KV \sin \theta \cos \theta - \mu_0 HM_s V \sin(\varphi - \theta) = 0
\]

• This gives a critical field and a critical angle that will cause switching.

Dividing by 2KV and setting \( h = H/H_K \) gives:
\[
\sin \theta \cos \theta - h \sin(\varphi - \theta) = 0 \quad (1)
\]

Now,
\[
\frac{d^2 E}{d\theta^2} = \cos^2 \theta - \sin^2 \theta + h \cos(\varphi - \theta) = 0 \quad (2)
\]

Solving (1) and (2) simultaneously gives:
\[
h_c^2 = 1 - \frac{3}{4} \sin^2 2\theta_c
\]
\[
\tan^3 \theta_c = -\tan \varphi
\]
The Energy Barrier

Minimisation:

\[ 0 = \sin \theta (2KV \cos \theta + \mu_0 m_s VH) \]

\[ E_{\text{min}} = \mu_0 m_s VH \]

\[ \cos \theta = -\frac{\mu_0 m_s VH}{2KV} = -\frac{\mu_0 m_s H}{2K} \]

\[ E_{\text{max}} = KV \left(1 - \cos^2 \theta\right) - \mu_0 m_s VH \cos \theta \]

\[ E_{\text{max}} = KV \left(1 - \frac{\mu_0^2 m_s^2 H^2}{4K^2}\right) + \frac{\mu_0^2 m_s^2 H^2}{2K^2} \]

The Energy Barrier, \( \Delta E \):

\[ \Delta E = E_{\text{max}} - E_{\text{min}} \]

\[ \Delta E = KV \left(1 + \frac{\mu_0^2 m_s^2 H^2}{4K^2}\right) - \mu_0 m_s VH \]

\[ = KV \left(1 + \frac{\mu_0^2 m_s^2 H^2}{4K^2} - \frac{\mu_0 m_s H}{K}\right) \]

Since

\[ H_K = \frac{2K}{\mu_0 m_s} \]

We can write

\[ \Delta E = KV \left(1 + \frac{H^2}{H_K^2} - 2H \frac{H}{H_K}\right) \]

And so

\[ \Delta E = KV \left(1 - \frac{H}{H_K}\right)^2 \]
Coercivity and Switching Field

\[ \alpha = 20^\circ \]

\[ H_c = H_{sw} \]

\[ \alpha = 70^\circ \]

\[ H_c < H_{sw} \]

See magnetization reversal applet at

http://bama.ua.edu/~tmewes
Incoherent Reversal – Small Particles

- Particles with a single-domain remanent state may reverse \textit{incoherently}, depending on size, shape, and material properties.
- Reversal modes can be complex and often best dealt with using micromagnetic simulations.
- Common reversal modes are \textit{fanning}, \textit{curling} and \textit{domain wall nucleation and propagation}.

\begin{itemize}
  \item coherent reversal
  \item fanning
  \item curling
  \item dw nucleation & propagation
\end{itemize}
Thermal Activation

• All the models reviewed so far apply \textbf{ONLY at } T = 0.\textbf{.}

• In real materials, the moments fluctuate about the easy axis in zero field to a degree depending on $\Delta E (= KV)$.

• Thus for particles with a small barrier reversal can be activated by thermal energy with a relaxation time

\[ \tau^{-1} = f_0 \exp \left[ \frac{\Delta E}{k_B T} \right] \]

• For a measurement time of 100s and $f_0 = 10^9 s^{-1}$, this gives a critical barrier for stability.

\begin{align*}
\text{100s: } \Delta E &= 25k_B T \\
\text{10 years: } \Delta E &= 40k_B T
\end{align*}

Particles with $\Delta E < 25k_B T$ are \textbf{SUPERPARAMAGNETIC} \\
$\Delta E > 25k_B T$ are \textbf{BLOCKED}
Effects of Thermal Energy

\[ KV \left(1 - \frac{H}{H_K}\right)^2 = 25k_B T \]

For \( t = 100s \)

The critical size

\[ V_p = \frac{25k_B T}{K} \quad H = 0 \]

\[ D_p = \sqrt[3]{\frac{150k_B T}{\pi K}} \]

The blocking temperature

\[ T_B = \frac{KV}{25k_B} \quad H = 0 \]

The coercivity

\[ KV \left(1 - \frac{H_c}{H_K}\right)^2 = 25k_B T \]

\[ H_c = \frac{2K}{M_s} \left[1 - \left(\frac{25k_B T}{KV}\right)^{1/2}\right] \]

\[ \frac{H_c}{H_c(0)} = 1 - \left(\frac{V_p}{V}\right)^{1/2} \]

\[ \frac{H_c}{H_c(0)} = 1 - \left(\frac{D_p}{D}\right)^{3/2} \]

\[ \frac{H_c}{H_c(0)} = 1 - \left(\frac{T}{T_B}\right)^{1/2} \]
Measurement of Blocking Temperatures

- At low T all grains are blocked and Mr/Ms is a maximum

\[ \frac{M_r}{M_s} = 1 \quad \text{aligned} \quad \frac{M_r}{M_s} = 0.5 \quad \text{random} \]

- The median blocking temperature \(<T_B>\) is at the point where \(\frac{M_r}{M_s}\) is half its maximum value

- The distribution of blocking temperatures \(f(T_B)\) is given by the differential of the temperature decay of remanance

- The susceptibility/temperature curve is related to \(f(T_B)\) but \(<T_B>\) is not at the peak
Time Dependence

- Thermal energy can reverse moments and leads to time dependence.

- Because there is always a distribution of $\Delta E$ the time dependence is not exponential.

- The ‘decay’ is found generally to be linear in $\ln(t)$.

$$M(t) = M(0) \pm S \ln(t)$$

- The coefficient $S(H)$ varies with $H$, peaking around $H_c$.

- This causes a sweep-rate dependence of $H_c$. 
Cubic Anisotropy

\[
\frac{E_k}{V} = K_1(\alpha_1^2 \alpha_2^2 + \alpha_2^2 \alpha_3^2 + \alpha_3^2 \alpha_1^2) + K_2(\alpha_1^2 \alpha_2^2 \alpha_3^2) + \ldots
\]

\[
\alpha_1 = \cos \gamma_1 = \sin \theta \cos \phi \\
\alpha_2 = \cos \gamma_2 = \sin \theta \sin \phi \\
\alpha_3 = \cos \gamma_3 = \cos \theta
\]

Can write 1\textsuperscript{st} order term as

\[
\frac{E_k}{V} = K_1 \sin^2 \theta \left( \frac{1}{4} \sin^2 \theta \sin^2 2\phi + \cos^2 \theta \right)
\]

- In Fe (100) is easy and (111) is hard, \( K_1 > 1 \)
- In Ni (111) is easy and (100) is hard, \( K_1 < 0 \)
Switching in Cubic Materials

- A cubic material switching at $T = 0$ is similar to the uniaxial case.

- The difference is that to get from (100) to (T00) the moment does not have to cross the (111) hard direction.

- There is an intermediate route via (110) and a very complex energy surface.

This reduces the anisotropy field to:

$$H_K = 0.64 \frac{K}{M_s}$$

The energy barrier is reduced to:

$$\Delta E = \frac{KV}{4} \quad (K>0)$$

$$\Delta E = \frac{KV}{12} \quad (K<0)$$
Magnetisation Curves

Iron ($K_1 > 0$)

Nickel ($K_1 < 0$)

- The reduction in $\Delta E$ makes Ni soft
- In Fe shape anisotropy is often dominant due to the $M_s^2$ term

$$K_s = \frac{1}{2} \mu_0 M_s^2 (N_c - N_a)$$
Temperature Dependence of Anisotropy

Crystalline anisotropy (low temperature regime):

\[
\frac{K_1(T)}{K_1(0)} \approx \left( \frac{M_s(T)}{M_s(0)} \right)^{l(l+1)/2}
\]

\(l = \) symmetry

Cubic: \(K_1 \sim M^{10}\)

Crystalline uniaxial: \(K_1 \sim M^{3}\)

Shape anisotropy: \(K_s \sim M^{2}\)

\(K_s\) has no \(T\) dependence
Magnetic Domains

**Magnetic domain**: Region in which $\mathbf{M}$ is approximately uniform in direction.

**Domain wall**: Boundary between adjacent domains in which $\mathbf{M}$ changes direction.
Terminology

- Magnetic poles is a term analogous to magnetic charges.

- If a north pole is brought near a susceptible material, a south pole is induced causing attraction.

- The magnetic field (of force) is represented by lines of flux.

- The flux is the flow (of the ether) and the strength of the field is the density of lines/unit area $B$

$$B = \mu_0 (H + M)$$
Why Domains?

Large demagnetisation field and energy in external field.

Reduced demagnetisation field and external field.
Domain Wall Energy

Domain wall costs exchange energy and anisotropy energy (and possibly magnetoelastic energy)

\[ E_{ex} = -2JS^2 \sum \cos \theta_{ij}, \quad E_{anis} = K_u \sin^2 \phi \]

**Narrow wall**: large \( \theta_{ij} \), high \( E_{ex} \), low \( E_{anis} \)

**Wide wall**: small \( \theta_{ij} \), low \( E_{ex} \), high \( E_{anis} \)

\[ \varphi_{Fe} = 1.5^\circ \]
\[ \varphi_{Ni} = 0.62^\circ \]
Wall Energy Minimization -

\[
\frac{d\sigma}{dN} = \frac{-JS^2 \pi^2}{N^2 a^2} + K a = 0
\]

\[
N = \sqrt{\frac{JS^2 \pi^2}{Ka^3}}
\]

\[
\delta_{dw} = N a = \sqrt{\frac{JS^2 \pi^2}{Ka}}
\]

\[
A \equiv \frac{JS^2}{a}
\]

\[
\delta_{dw} = \pi \sqrt{\frac{A}{K}}
\]

\[
\sigma = \frac{JS^2 \pi^2}{Na^2} + K N a = \frac{A \pi^2}{\pi \sqrt{\frac{A}{K}}} + K \pi \sqrt{\frac{A}{K}}
\]

\[
\sigma_{dw} = 2 \pi \sqrt{AK}
\]

Example: hcp Co

\[
K = 4 \times 10^5 \text{ J/m}^3
\]

\[
A = 1 \times 10^{-11} \text{ J/m}
\]

\[
\Rightarrow \delta_{dw} = 16 \text{ nm}
\]
Bloch and Néel Walls in Thin Films

- **Thin films**: Néel walls
- **Thick films**: Bloch walls

Wall type determined by magnetostatic energy:
- Thin films: Néel walls
- Thick films: Bloch walls
Closure Domains

• For **cubic anisotropy**, magnetostatic energy can be further minimized with closure domains without adding anisotropy energy. (Slight increase in domain wall energy.)

• In systems with strong uniaxial anisotropy e.g Co closure domains cannot form due to the hardness of the hard axis.
Domain Wall Pinning

Domain wall motion limited by non-uniformities in wall energy due to non-magnetic inclusions or high-anisotropy defects (crystalline or magnetoelastic).

\[
\frac{d\sigma_{dw}}{dx} = 4 \frac{d}{dx} (AK)^{1/2}
\]

If \( \delta_{dw} \ll \text{defect size} \),

\[
\left( \frac{d\sigma_{dw}}{dx} \right)_{\text{max}} = 2M_s H_c
\]

\[
H_c = \frac{1}{2M_s} \left( \frac{d\sigma_{dw}}{dx} \right)_{\text{max}}
\]
Switching Field Distributions (SFD)

- Only an isolated particle switches at a single field.

- The SFD results from the distribution of $\Delta E$.

  \[ \Delta E = K V (1 - \frac{H}{H_K})^2 \]

- The SFD is due to:
  - Particle size distribution, usually lognormal
  - Distribution of $K$ and $M_s$
  - Distribution of orientation
  - Dipole-Dipole exchange interactions
Measurement of SFD

- We can measure $f(\Delta E)$ from $f(T_B)$
- To get the SFD it is best to measure the remenance curve.
- This measures the irreversible switching only
- The differential gives the SFD for granular or domain wall pinned systems.
Nucleation and Propagation

- When a material reverses a reverse domain must nucleate.

- If $H_n > H_{dw}$ then the domain sweeps through the sample giving a square loop.

- If $H_n < H_{dw}$ the loop will be round as $H_{dw}$ is overcome gradually.

- In perpendicular films the loop can be round as $H_d$ reduces.
Summary

• Magnetisation produces effective surface and volume charges. Demagnetisation field and energy depends on sample shape.

• Magnetic anisotropy determines preferred orientations of the magnetisation. Source of anisotropy can be shape, crystalline, stress, exchange, …

• Magnetisation can form domains to minimize magnetostatic energy. Domain wall width is determined by minimising exchange and anisotropy energy.

• Quasistatic magnetisation reversal in small particles can be described by coherent reversal model. Larger systems reverse incoherently.
References


