# Fundamentals of Magnetism

## Part I

Magnetostatics, Anisotropy, Domains, Coherent Rotation, Incoherent Processes and Thermal Effects

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# Magnetic Pole Density and Fields

$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$

$\nabla \cdot \mathbf{B} = 0 \quad \Rightarrow \quad \nabla \cdot \mathbf{H} = -\nabla \cdot \mathbf{M} = \rho_m$

$\rho_m = \text{magnetic charge density}$

For point pole, $\mathbf{H} = \dfrac{\mathrm{q_m}\hat{\mathbf{r}}}{4\pi r^2} = -\nabla \varphi_m$,

where $\varphi_m = \dfrac{\mathrm{q_m}}{4\pi r} = \text{magnetic potential gradient} = \nabla(\Sigma NI)$

For a volume distribution,

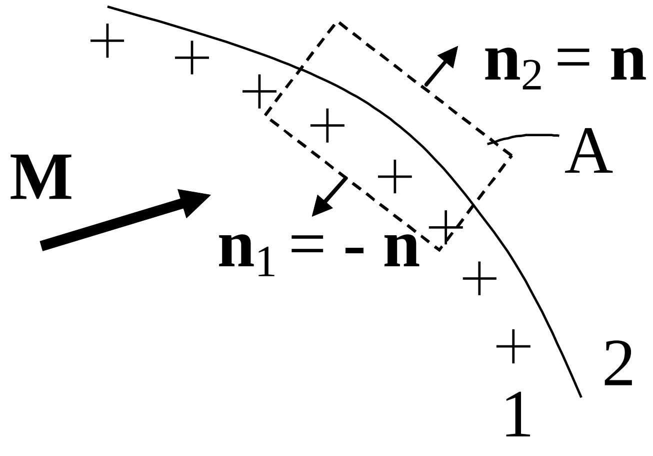
$$\varphi_m = \int \frac{\rho_m dV}{4\pi r}$$

(compare $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$,

$\nabla \cdot \mathbf{D} = \varepsilon_0 \nabla \cdot \mathbf{E} + \nabla \cdot \mathbf{P}$

$= \rho_{total} - \rho_b = \rho_{free}$)

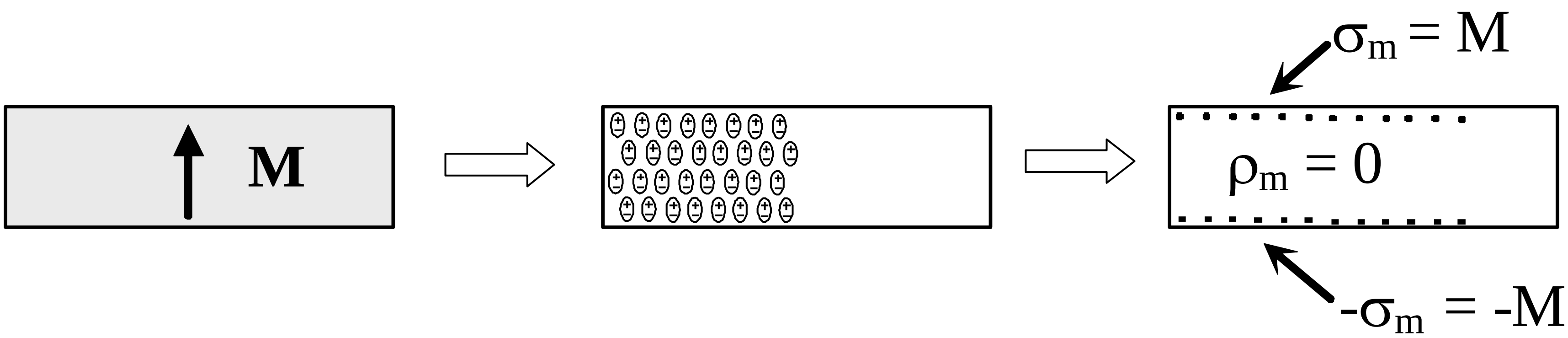
$\mathbf{E} = -\nabla V$

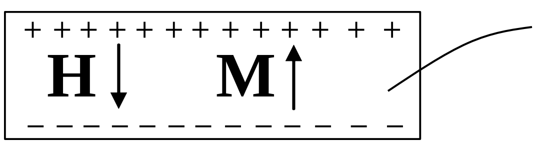



What is 'flux' and 'flux density'?

What is a 'magnetic field'?

# Field From a Sheet Uniformly Magnetised Perpendicular to Surface




$\mathbf{B} = \mu_0(\mathbf{H}+\mathbf{M})$

$= \mu_0(-\mathbf{M}+\mathbf{M}) = 0$

Inside: $\mathbf{M} = \mathbf{M}$

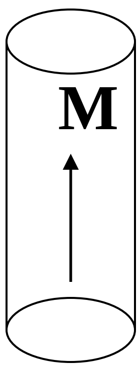
$\mathbf{H} = -4\pi\mathbf{M}$ (demag)

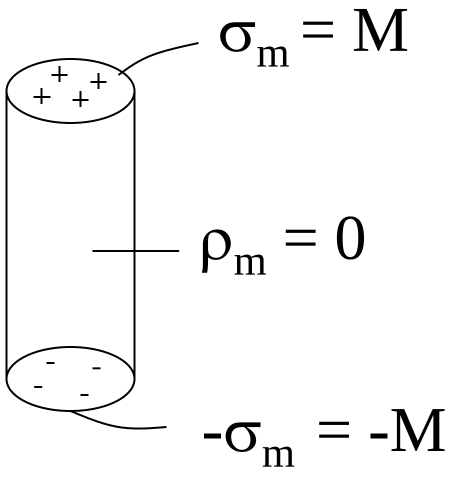
$\mathbf{B} = 0$

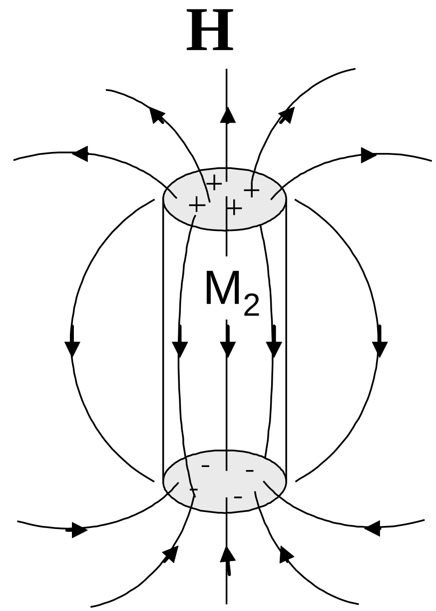
Outside: $\mathbf{M} = \mathbf{H} = \mathbf{B} = 0$

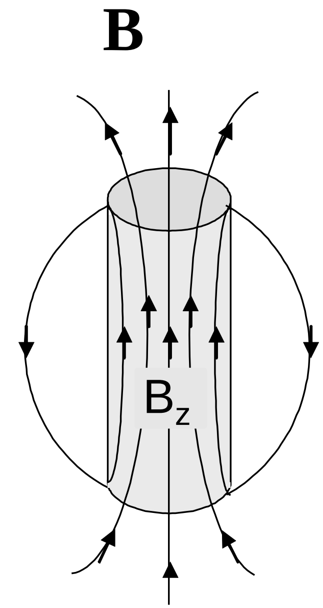
Why?

# Thin Rod Uniformly Magnetised Along Axis



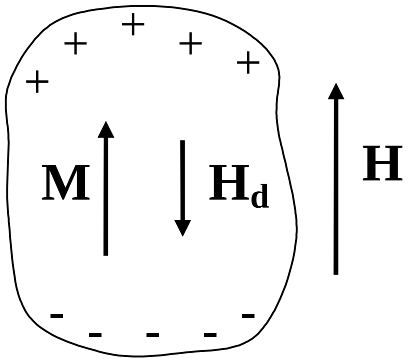




Outside: $\mathbf{M} = 0$, $\mathbf{B} = \mu_0 \mathbf{H}$

Inside: $\mathbf{M} = +M_2$, $H_z < 0$, $B_z > 0$

# Demagnetization Field, $H_d$




$\mathbf{H}_{int}$ (or $\mathbf{H}_{Total}$) = $\mathbf{H}$ + $\mathbf{H}_d$

For a body with arbitrary shape, $\mathbf{H}_d$ is not constant; however, for an ellipsoid

$\mathbf{H}_d$ = constant.

Generally, $\mathbf{H}_d = -\tilde{\mathbf{N}} \cdot \mathbf{M}$, where

$$\tilde{\mathbf{N}} = \begin{pmatrix} N_a & 0 & 0 \\ 0 & N_b & 0 \\ 0 & 0 & N_c \end{pmatrix}$$

so, $\mathbf{H}_d = -N_a M_x \hat{\mathbf{i}} - N_b M_y \hat{\mathbf{j}} - N_c M_z \hat{\mathbf{k}}$

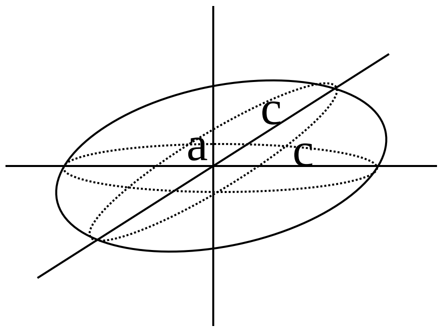
If **M** is along a principle axis, then

$$\mathbf{H}_d = -N\mathbf{M} \quad (N = N_a, N_b, or\ N_c)$$

In general,

$$N_a + N_b + N_c = 1 \quad (4\pi\ in\ cgs)$$

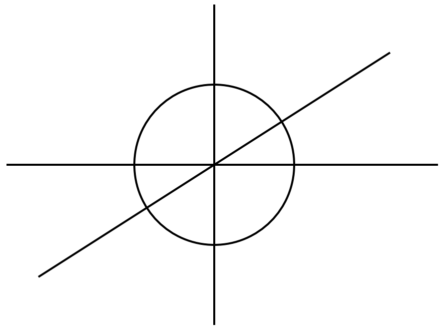
# Special Cases




**Thin oblate spheroid** (pancake)

$N_b = N_c = 0$

$N_a = 1 \ (4\pi)$

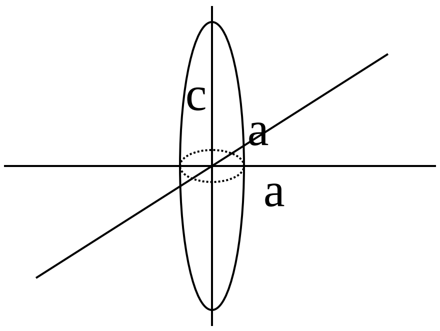


**Sphere**

$N_a = N_b = N_c = N$

$3N = 1$

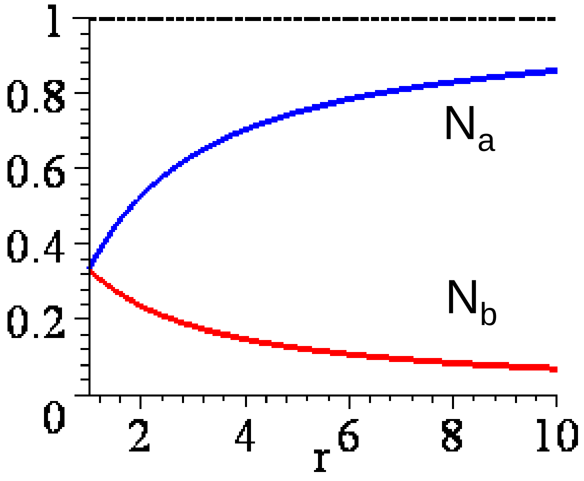
$N = 1/3 \ (4\pi/3)$




**Thin prolate ellipsoid** (cigar)

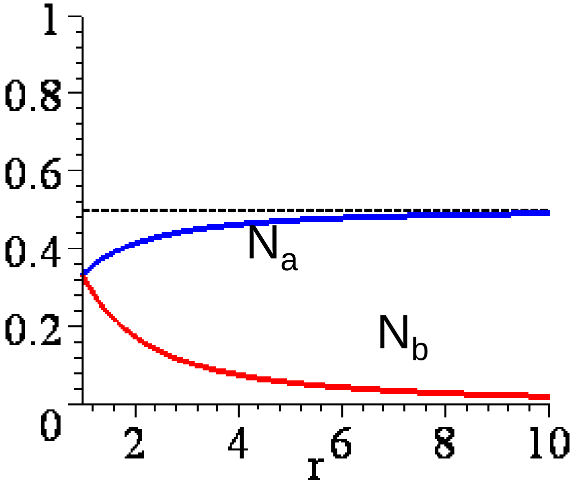
$N_c = 0, \ 2N_a = 1$

$N_a = 1/2 \ (2\pi)$



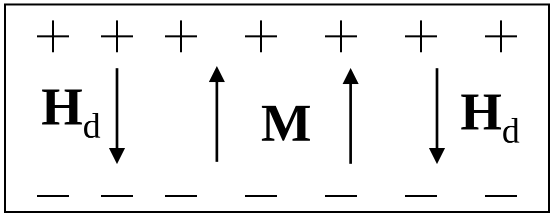

r = c/a

(a = b)



# Demagnetisation Energy and Fields

$$\mathbf{H}_d = -\tilde{\mathbf{N}}\cdot\mathbf{M} = -\left(N_a M_x \hat{\mathbf{i}} + N_b M_y \hat{\mathbf{j}} + N_c M_z \hat{\mathbf{k}}\right)$$
$$= -\left(N_a \cos\alpha\,\hat{\mathbf{i}} + N_b \cos\beta\,\hat{\mathbf{j}} + N_c \cos\gamma\,\hat{\mathbf{k}}\right)M$$




Energy associated with demagnetisation field -

$$\mathbf{E}_d = \tfrac{1}{2}\mu_0 \mathbf{M}\cdot\mathbf{H}_d$$
$$= \tfrac{1}{2}\mu_0\left(N_a \cos^2\alpha + N_b \cos^2\beta + N_c \cos^2\gamma\right)M^2$$

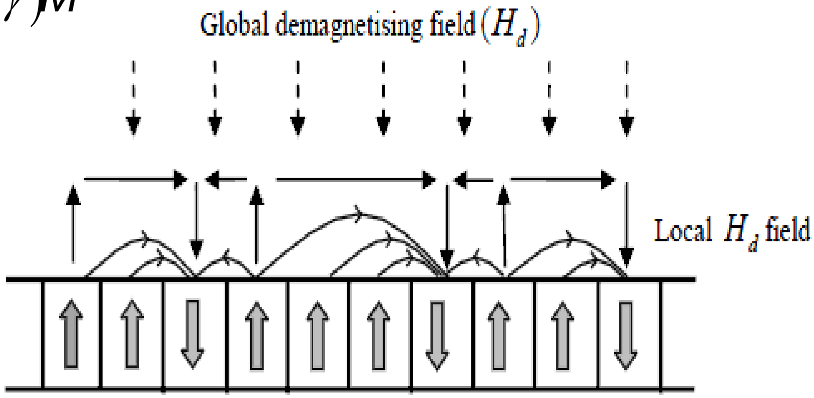
Perpendicular Media -

There is a global $H_d$

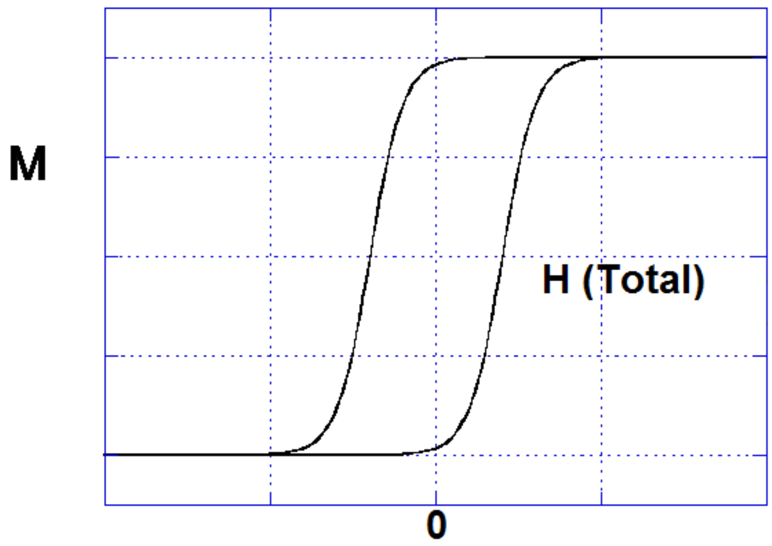
$H_d$ is non-uniform

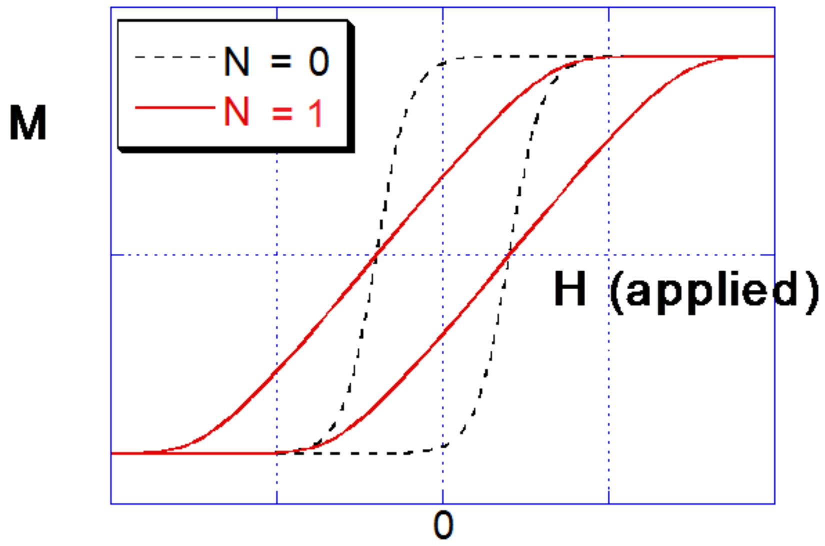
in places $H_d = 0$

in places $H_d$ is positive!!



# Effect of Demag Field on M-H Loop




$$\mathbf{H}_{in} = \mathbf{H}_{app} - N\mathbf{M}$$

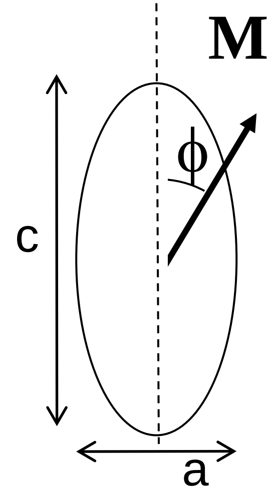
- The loop is sheared with a slope of 1 (4π).
- The only ‘true’ point is at $H_c$ where M=0.
- In practice for N=1 (4π), exchange coupling reduces the loop shear

# ANISOTROPY

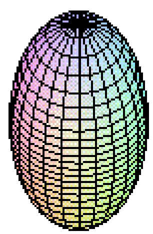
## Shape Anisotropy

$u_d = \frac{1}{2}\,\mu_0 \Delta N M_s^2 \sin^2\theta = K_s \sin^2\phi$

$\Delta N = N_a - N_c$

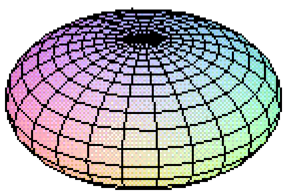



$K_s > 0$



prolate spheroid

$K_s < 0$



oblate spheroid

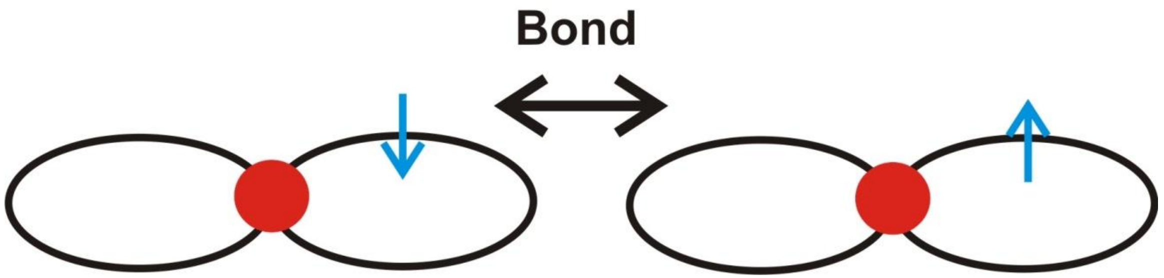
(Note: These are sample shapes, not energy surfaces.)

- For $c/a > 10$, $H_c$ (T=0) > 1T
- Not achieved because of incoherent reversal

- Elongated particles for tape have $c/a \sim 4$ and $H_c = 0.4$T
- CoFe is used to maximise $M_s$

# Crystalline anisotropy

- Due to spin-orbit coupling and chemical bonding of orbitals with local environment (crystalline electric field)
- Must have non-spherical atomic orbitals ($L_z \neq 0$) and non-spherical crystalline field.
- Dipole-dipole interactions are not strong enough to cause significant crystalline anisotropy.




- Occurs in all crystals but is usually weak in cubic materials.
- Very strong in hexagonal crystals e.g Co and Ba – ferrite.
- Even stronger in some tetragonal cystals e.g. FePt

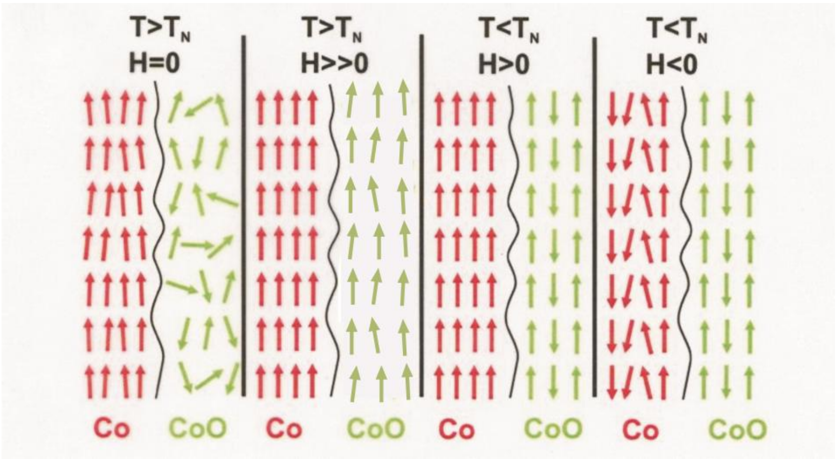
# Other sources of anisotropy

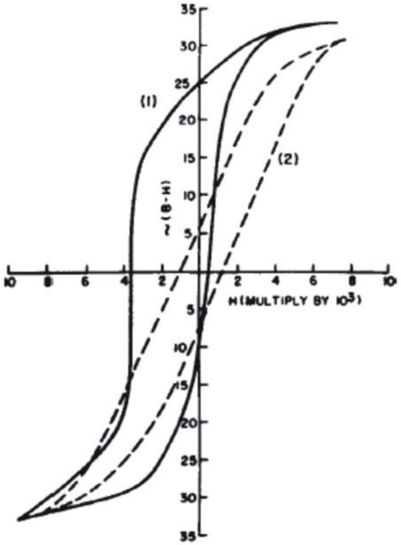
Induced

- Heat in a field, stress, plastic deformation (e.g., rolling), etc.

Exchange anisotropy

- coupling between FM and AFM materials



# Uniaxial Anisotropy

General case: $u = K_1 \sin^2 \theta + K_2 \sin^4 \theta + \ldots$

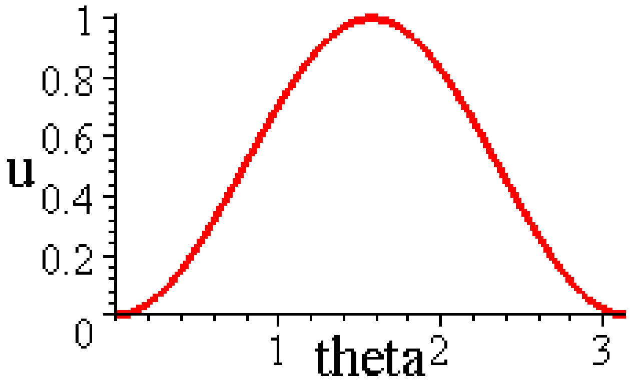
Second and higher-order terms are usually negligible

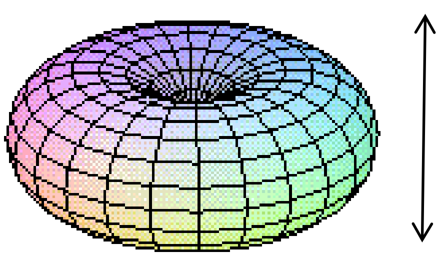
## $1^{st}$ order positive

$K_1 > 0, K_2 = 0$:

$E_k = K_1 \sin^2 \theta$

Mathematically the same as shape anisotropy for prolate spheroid.



Easy axis

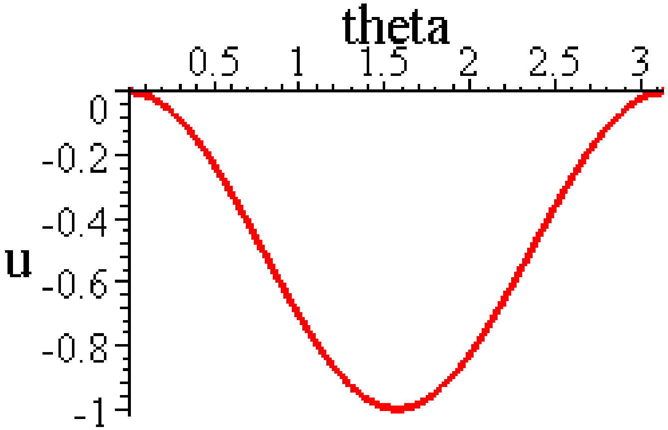
Energy surface

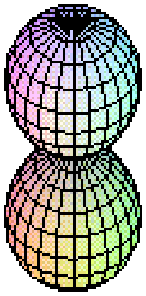
# $1^{st}$-order negative

$K_1 < 0, K_2 = 0$:

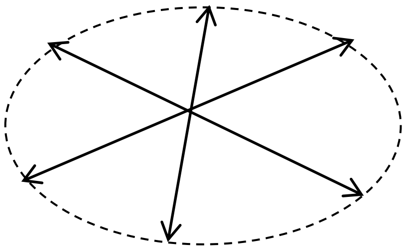
$E_k = -|K_1| \sin^2\theta = +|K_1| \cos^2\theta$ (+ constant)

Mathematically the same as shape anisotropy for oblate spheroid.



Energy surface



Easy plane
($\theta = 90^o$)

# Torque curves

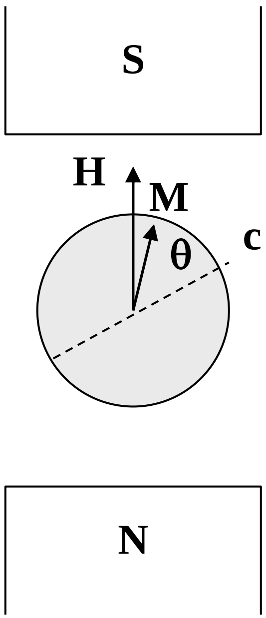
Torque (per unit volume) exerted on crystal by **M**

$$L = -\frac{dE}{d\theta}$$

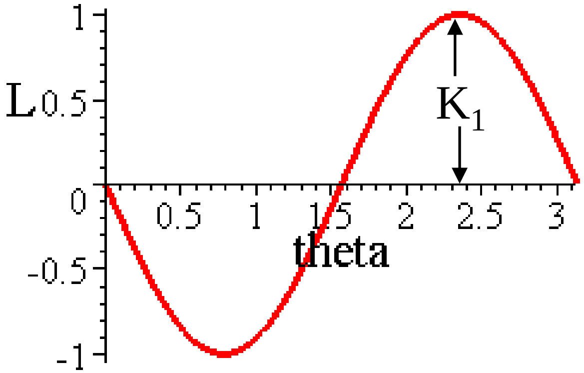
First order:

$$E_k = K_1 \sin^2\theta$$

$$L = -2K_1 \sin\theta\cos\theta = -K_1 \sin 2\theta$$




- Unless the anisotropy is purely uniaxial torque curves are difficult (impossible) to interpret
- In practice only possible for single crystals



# Examples of Uniaxial Anisotropy

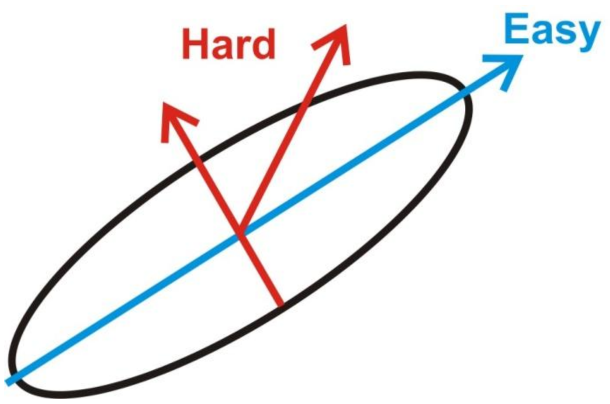
- Uniaxial anisotropy occurs in elongated particles used in tapes

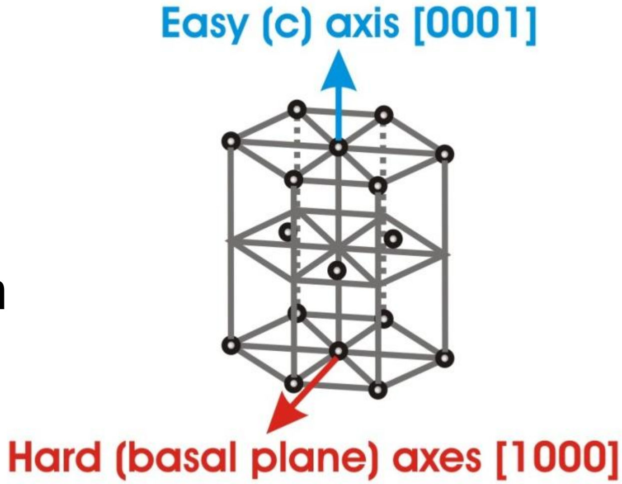
- The $M_s^2$ term is why these particles are made from $Fe_{60}Co_{40}$

- It also occurs in materials with strong crystal asymmetry

- These include hcp Cobalt and Barium Ferrite

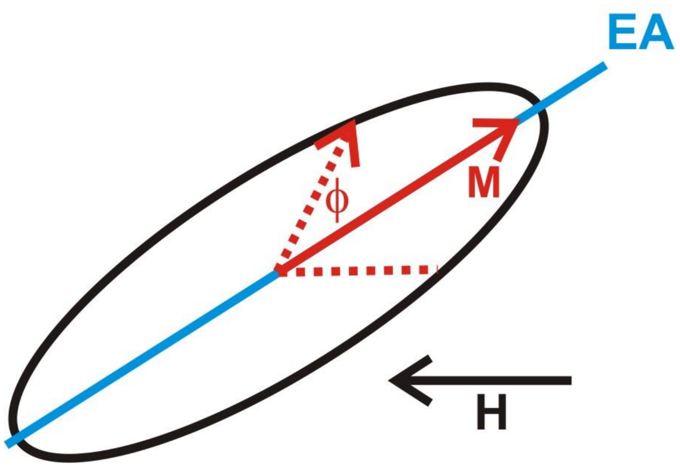
- Complex mixed anisotropies can occur in materials with elongation perpendicular to a c-axis, e.g. Ba-Ferrite platelets



# Stoner-Wohlfarth Theory

- This theory explains the behaviour of single domain particles at T = 0.
- The particles must be uniaxial and align by moment rotation over an anisotropy barrier.




The energy is then:

$$E = KV\sin^2\phi - \mu H\cos\theta$$

$$\therefore \frac{dE}{d\theta} = 2K\sin\theta\cos\theta - \mu_0 HM_s \sin(\phi - \theta)$$

For H perpendicular to EA, $\phi$ = 90°

$$\therefore 2K\sin\theta\cos\theta = \mu_0 HM_s\cos\theta$$

$$\therefore 2K\frac{M}{M_s} = \mu_0 HM_s$$

$$\therefore \frac{M}{M_s} = \frac{\mu_0 HM_s}{2K}$$

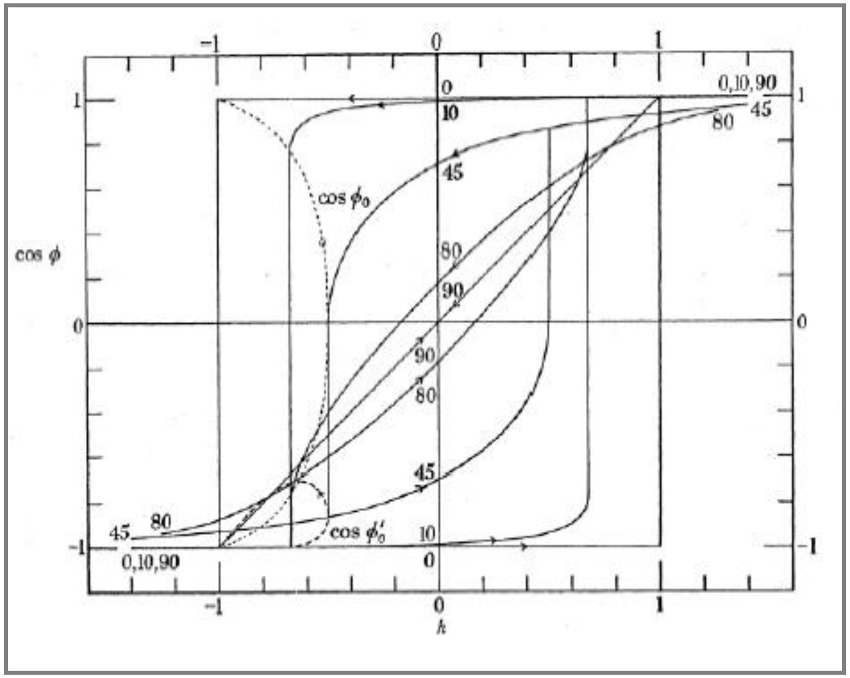
Therefore, the system saturates at:

$$H = \frac{2\mu_0 K}{M_s} = H_K$$

**Anisotropy Field**

# The Aligned Case

- The Anisotropy Field $H_K = \mu_0 2K/M_s$ is needed to pull the moment to 90°.
- It can then fall into either direction along the easy axis.
- This gives a square loop switching at $H_K$.
- For a small misalignment of 10° the switching field $H_s$ falls by **30%**
- At 90° there is no hysteresis.



# Critical Fields and Angles

- The minimisation

$$\frac{dE}{d\theta} = 2KV\sin\theta\cos\theta - \mu_0 HM_s V\sin(\varphi - \theta) = 0$$

- This gives a critical field and a critical angle that will cause switching.

Dividing by 2KV and setting h = $H/H_K$ gives:

$$\sin\theta\cos\theta - h\sin(\varphi - \theta) = 0 \qquad (1)$$

Now,

$$\frac{d^2E}{d\theta^2} = \cos^2\theta - \sin^2\theta + h\cos(\varphi - \theta) = 0 \qquad (2)$$

Solving (1) and (2) simultaneously gives:

$$h_c^2 = 1 - \frac{3}{4}\sin^2 2\theta_c$$

$$\tan^3\theta_c = -\tan\varphi$$

# The Energy Barrier

**Minimisation:**

$$0 = \sin\theta(2KV\cos\theta + \mu_0 m_s VH)$$

$$E_{min} = \mu_0 m_s VH$$

$$\cos\theta = -\frac{\mu_0 m_s VH}{2KV} = -\frac{\mu_0 m_s H}{2K}$$

$$E_{max} = KV(1-\cos^2\theta) - \mu_0 m_s VH\cos\theta$$

$$E_{max} = KV\left(1-\frac{\mu_0^2 m_s^2 H^2}{4K^2}\right) + \frac{\mu_0^2 m_s^2 H^2}{2K^2}$$

$$E_{max} = KV\left(1-\frac{\mu_0^2 m_s^2 H^2}{4K^2} + \frac{\mu_0^2 m_s^2 H^2}{2K^2}\right)$$

$$E_{max} = KV\left(1+\frac{\mu_0^2 m_s^2 H^2}{4K^2}\right)$$

**The Energy Barrier, ΔE:**

$$\Delta E = E_{max} - E_{min}$$

$$\Delta E = KV\left(1+\frac{\mu_0^2 m_s^2 H^2}{4K^2}\right) - \mu_0 m_s VH$$

$$= KV\left(1+\frac{\mu_0^2 m_s^2 H^2}{4K^2} - \frac{\mu_0 m_s H}{K}\right)$$

**Since**

$$H_K = \frac{2K}{\mu_0 m_s}$$

**We can write**

$$\Delta E = KV\left(1+\frac{H^2}{H_K^2} - \frac{2H}{H_K}\right)$$

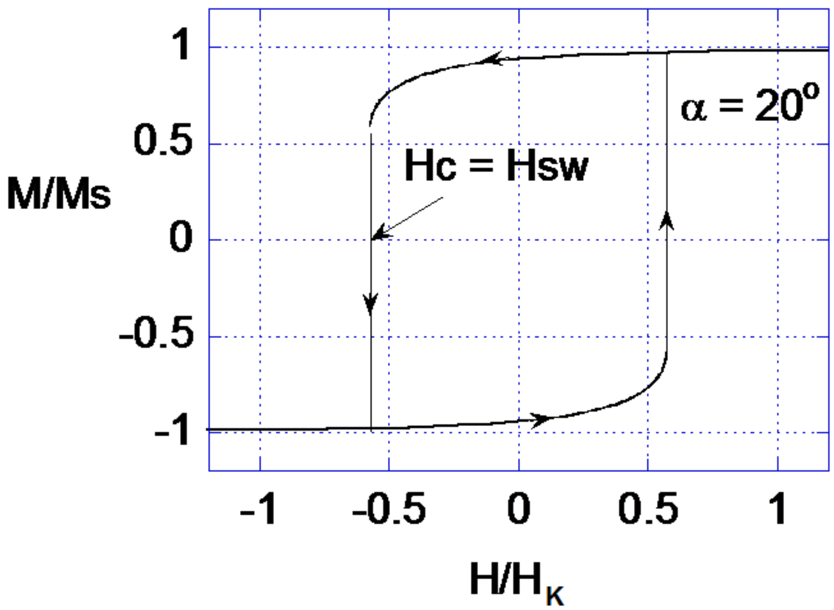
**And so**

$$\Delta E = KV\left(1-\frac{H}{H_K}\right)^2$$

# Coercivity and Switching Field

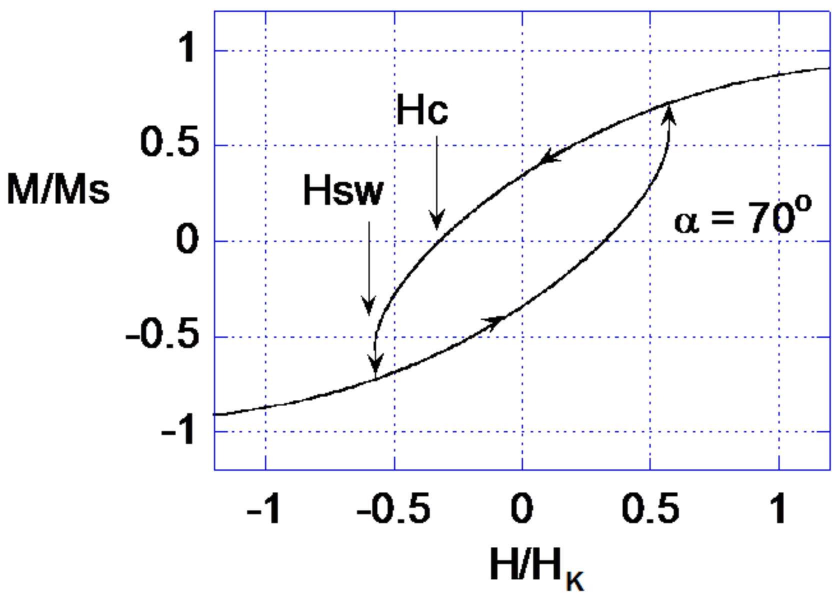
$\alpha = 20^o$

$H_c = H_{sw}$




$\alpha = 70^o$

$H_c < H_{sw}$

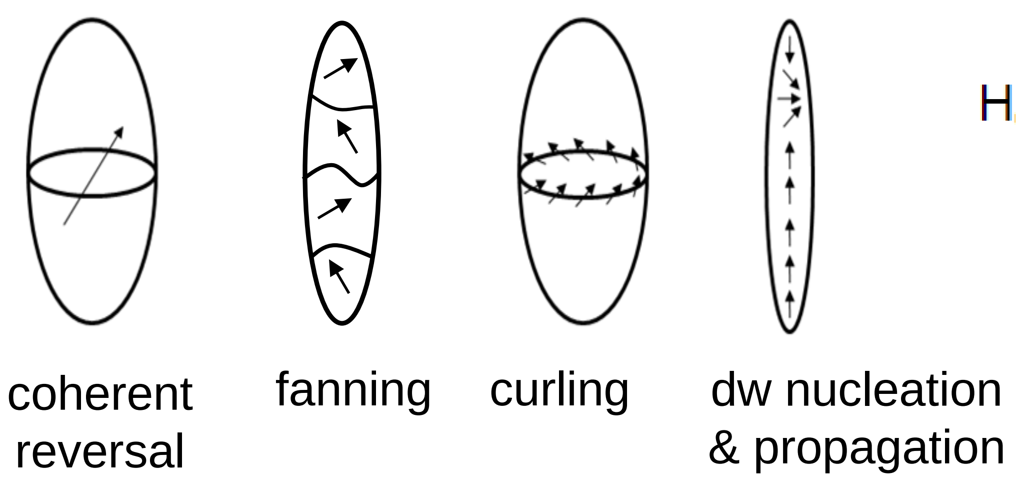


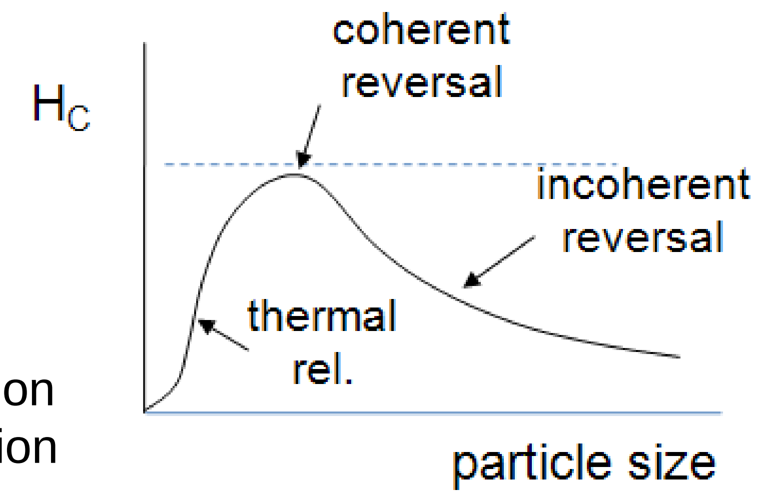

See magnetization reversal applet at

**http://bama.ua.edu/~tmewes**

# Incoherent Reversal – Small Particles

- Particles with a single-domain remanent state may reverse ***incoherently***, depending on size, shape, and material properties.
- Reversal modes can be complex and often best dealt with using micromagnetic simulations.
- Common reversal modes are **fanning**, **curling** and **domain wall nucleation and propagation**.



# Thermal Activation

- All the models reviewed so far apply **ONLY at T = 0**.
- In real materials, the moments fluctuate about the easy axis in zero field to a degree depending on ΔE (= KV).
- Thus for particles with a small barrier reversal can be activated by thermal energy with a relaxation time

$$\tau^{-1} = f_0 \exp\left[\frac{\Delta E}{k_B T}\right]$$

- For a measurement time of 100s and $f_0 = 10^9 s^{-1}$, this gives a critical barrier for stability.

**100s: $\Delta E = 25k_BT$** **10 years: $\Delta E = 40k_BT$**

Particles with $\Delta E < 25k_BT$ are SUPERPARAMAGNETIC

$\Delta E > 25k_BT$ are BLOCKED

# Effects of Thermal Energy

$$KV\left(1-\frac{H}{H_K}\right)^2 = 25k_BT$$

For t = 100s

## The critical size

$$V_p = \frac{25k_BT}{K}$$

**H = 0**

$$D_p = \sqrt[3]{\frac{150k_BT}{\pi K}}$$

## The blocking temperature

$$T_B = \frac{KV}{25k_B}$$

**H = 0**

## The coercivity

$$KV\left(1-\frac{H_c}{H_K}\right)^2 = 25k_BT$$

$$H_c = \frac{2K}{M_s}\left[1-\left(\frac{25k_BT}{KV}\right)^{1/2}\right]$$

$$\frac{H_c}{H_c(0)} = 1-\left(\frac{V_p}{V}\right)^{1/2}$$

$$\frac{H_c}{H_c(0)} = 1-\left(\frac{D_p}{D}\right)^{3/2}$$

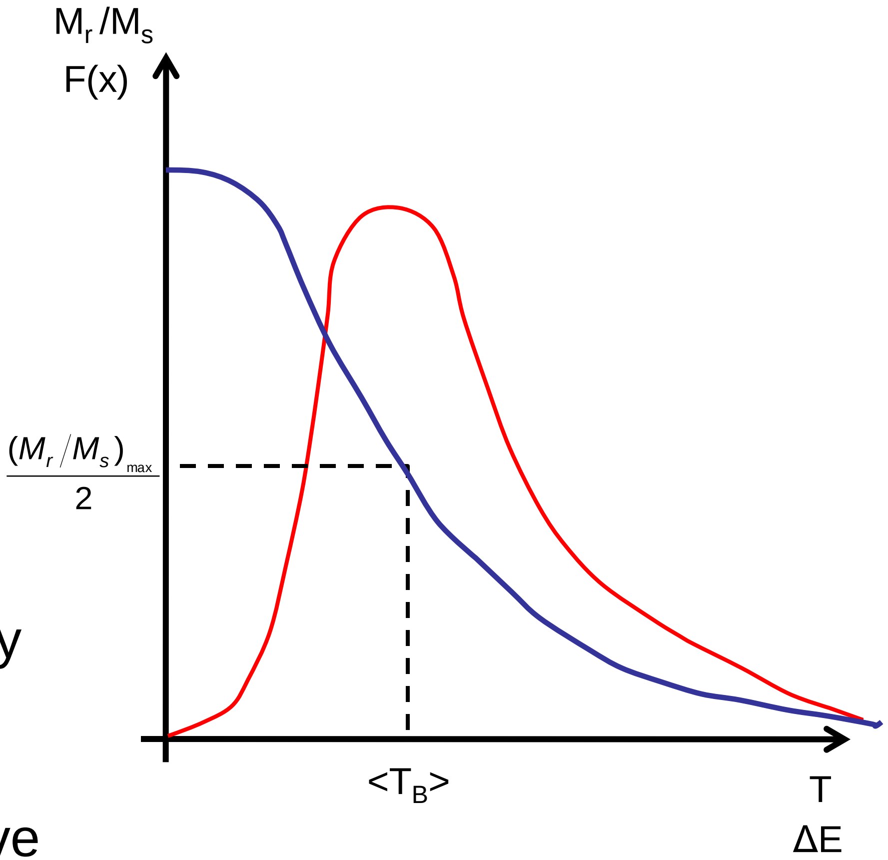
$$\frac{H_c}{H_c(0)} = 1-\left(\frac{T}{T_B}\right)^{1/2}$$

# Measurement of Blocking Temperatures

- At low T all grains are blocked and Mr/Ms is a maximum

$$M_r/M_s = 1 \text{ aligned} \qquad M_r/M_s = 0.5 \text{ random}$$

- The median blocking temperature $\langle T_B \rangle$ is at the point where $M_r/M_s$ is half its maximum value
- The distribution of blocking temperatures $f(T_B)$ is given by the differential of the temperature decay of remanance
- The susceptibility/ temperature curve is related to $f(T_B)$ but $\langle T_B \rangle$ is not at the peak

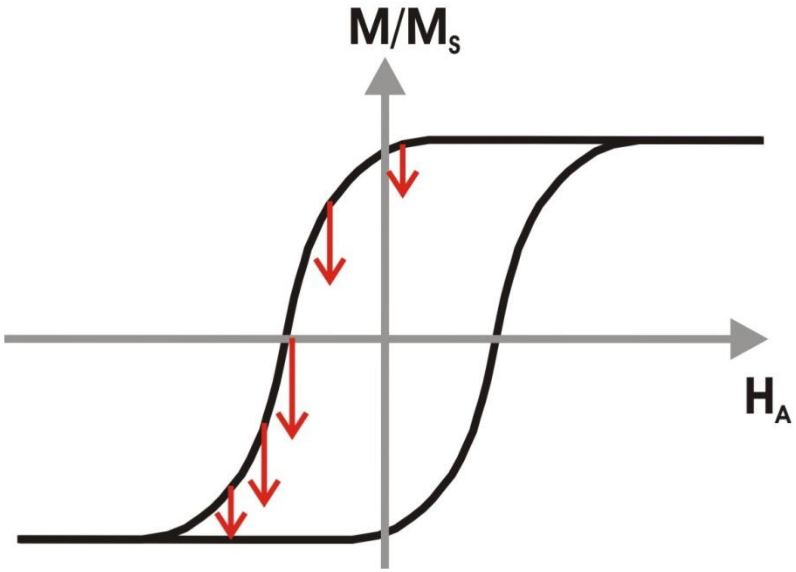


# Time Dependence

- Thermal energy can reverse moments and leads to time dependence.
- Because there is always a distribution of ΔE the time dependence is not exponential.
- The 'decay' is found generally to be linear in ln(t).

$$M(t) = M(0) \pm S \ln(t)$$

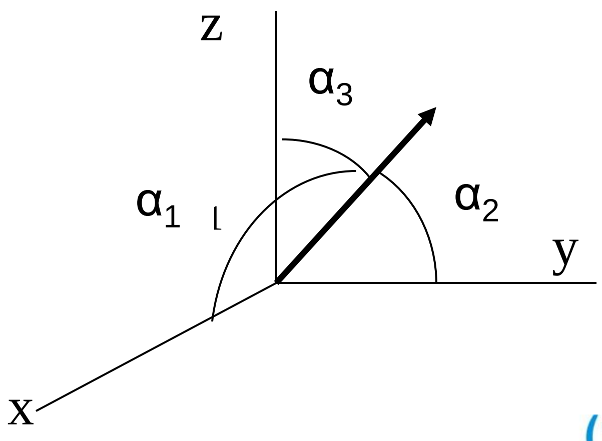
- The coefficient S(H) varies with H, peaking around $H_c$.
- This causes a sweep-rate dependence of $H_c$.



# Cubic Anisotropy

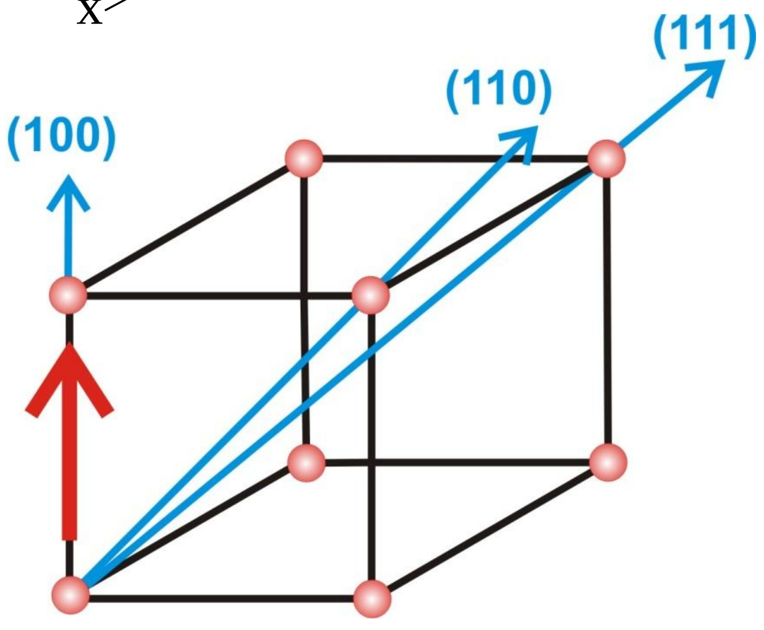
$$\frac{E_K}{V} = K_1(\alpha_1^2\alpha_2^2 + \alpha_2^2\alpha_3^2 + \alpha_3^2\alpha_1^2) + K_2(\alpha_1^2\alpha_2^2\alpha_3^2) + \ldots$$

$$\alpha_1 = \cos\gamma_1 = \sin\theta\cos\phi$$
$$\alpha_2 = \cos\gamma_2 = \sin\theta\sin\phi$$
$$\alpha_3 = \cos\gamma_3 = \cos\theta$$




Can write 1st order term as

$$\frac{E_K}{V} = K_1\sin^2\theta\left(\tfrac{1}{4}\sin^2\theta\sin^2 2\phi + \cos^2\theta\right)$$



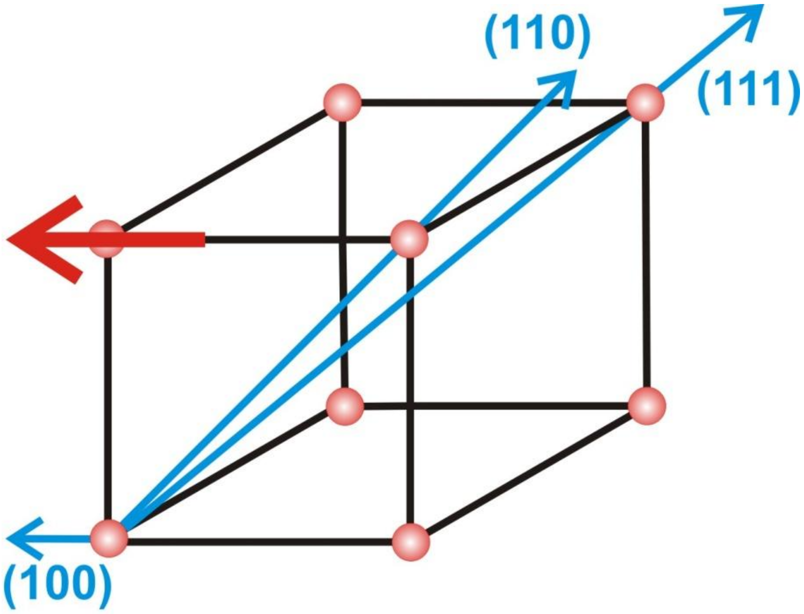

- In Fe (100) is easy and (111) is hard, $K_1 > 1$
- In Ni (111) is easy and (100) is hard, $K_1 < 0$

# Switching in Cubic Materials

- A cubic material switching at T = 0 is similar to the uniaxial case.

- The difference is that to get from (100) to ($\bar{1}$00) the moment does not have to cross the (111) hard direction.

- There is an intermediate route via (110) and a very complex energy surface.




This reduces the anisotropy field to:

$$H_K = 0.64\frac{K}{M_s}$$

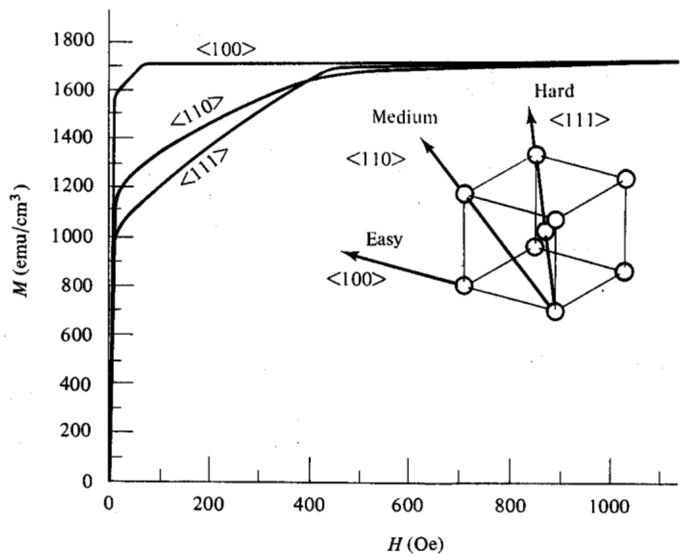
The energy barrier is reduced to:

$$\Delta E = \frac{KV}{4} \qquad \textbf{(K>0)}$$

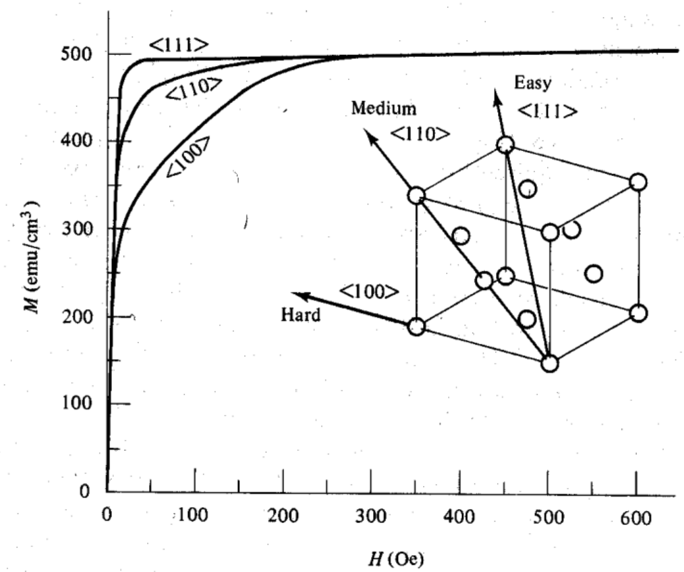
$$\Delta E = \frac{KV}{12} \qquad \textbf{(K<0)}$$

# Magnetisation Curves

## Iron ($K_1 > 0$)




## Nickel ($K_1 < 0$)




- The reduction in ΔE makes Ni soft
- In Fe shape anisotropy is often dominant due to the $M_s^2$ term

$$K_s = \frac{1}{2}\mu_0 M_s^2 (N_c - N_a)$$

# Temperature Dependence of Anisotropy

Crystalline anisotropy (low temperature regime):

$$\frac{K_1(T)}{K_1(0)} \approx \left(\frac{M_s(T)}{M_s(0)}\right)^{l(l+1)/2}$$

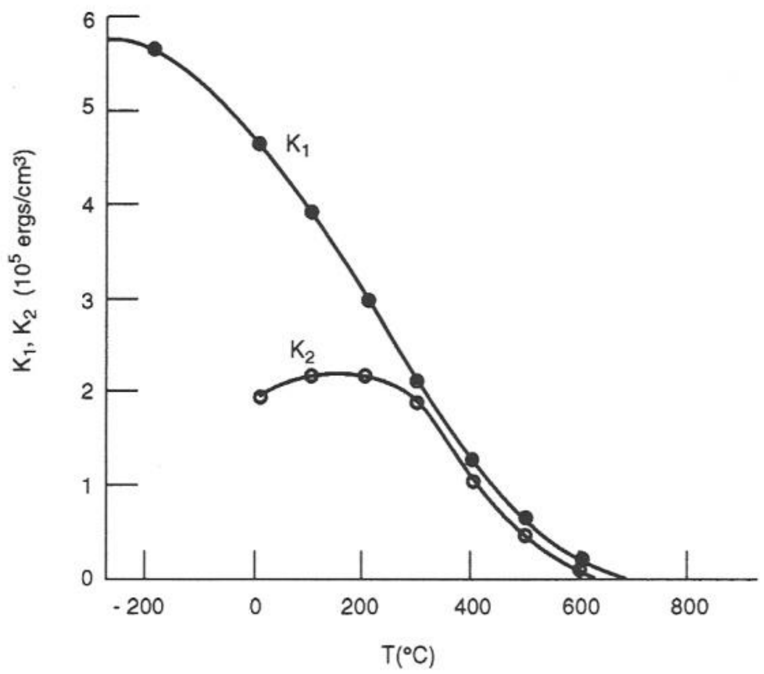
($l$ = symmetry)

Cubic: $K_1 \sim M^{10}$

Crystalline uniaxial: $K_1 \sim M^{3}$

Shape anisotropy: $K_s \sim M^{2}$

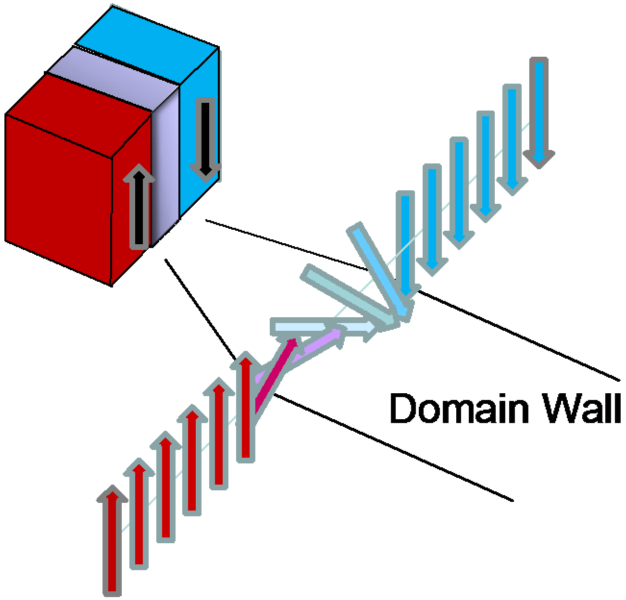
**$K_s$ has no T dependence**



# Magnetic Domains

**Magnetic domain**: Region in which **M** is approximately uniform in direction.

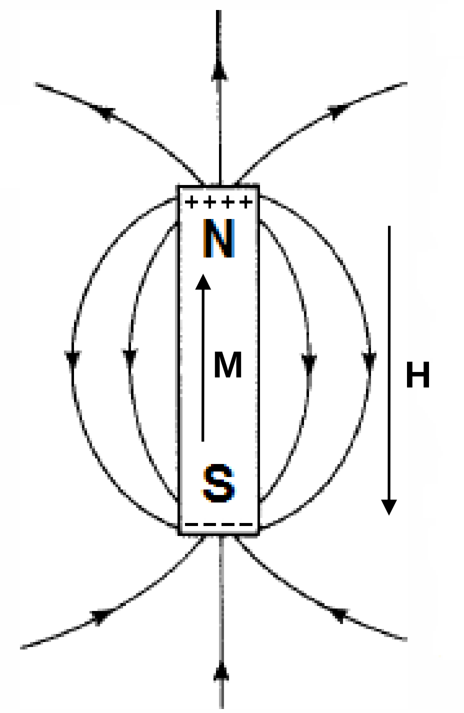
**Domain wall**: Boundary between adjacent domains in which **M** changes direction.



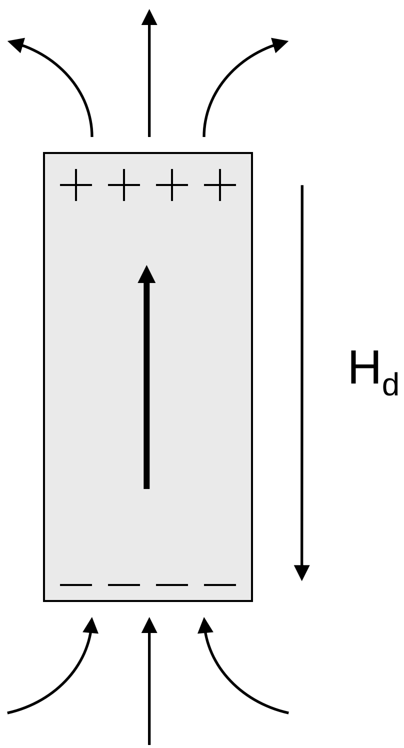
# Terminology

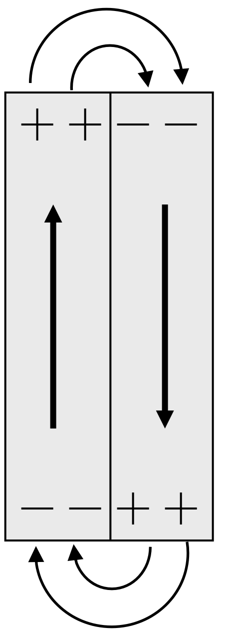
- Magnetic poles is a term analogous to magnetic charges
- If a north pole is brought near a susceptible material, a south pole is induced causing attraction
- The magnetic field (of force) is represented by lines of flux
- The flux is the flow (of the ether) and the strength of the field is the density of lines/unit area **B**

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$$



# Why Domains?



Large demagnetisation field and energy in external field.

Reduced demagnetisation field and external field.

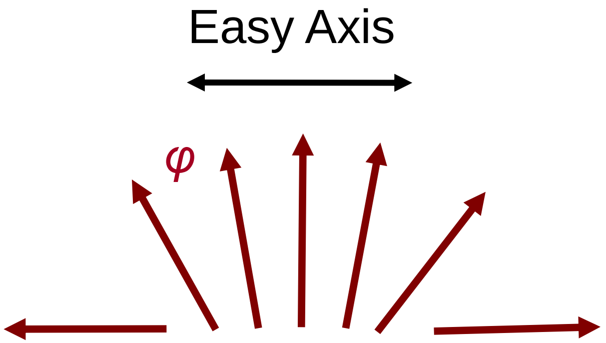
# Domain Wall Energy

Domain wall costs exchange energy and anisotropy energy (and possibly magnetoelastic energy)

$$E_{ex} = -2JS^2 \sum \cos\theta_{ij}, \quad E_{anis} = K_u \sin^2 \phi$$

**Narrow wall**: large $\theta_{ij}$, high $E_{ex}$, low $E_{anis}$

**Wide wall**: small $\theta_{ij}$, low $E_{ex}$, high $E_{anis}$




**$\varphi_{Fe} = 1.5°$**

**$\varphi_{Ni} = 0.62°$**

# Wall Energy Minimization -

$$\frac{\partial\sigma}{\partial N} = \frac{-JS^2\pi^2}{N^2a^2} + Ka = 0$$

$$N = \sqrt{\frac{JS^2\pi^2}{Ka^3}}$$

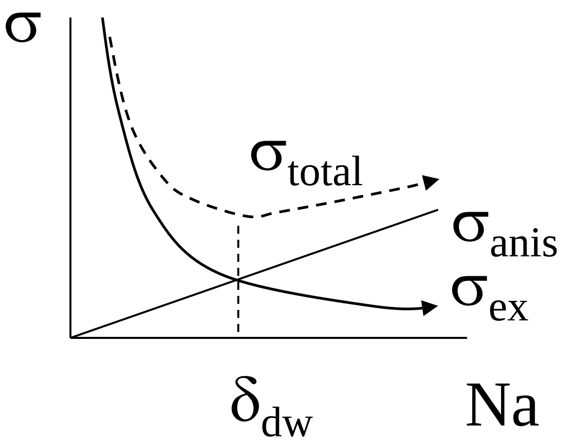
$$\delta_{dw} = Na = \sqrt{\frac{JS^2\pi^2}{Ka}}$$

$$A \equiv \frac{JS^2}{a}$$

$$\delta_{dw} = \pi\sqrt{\frac{A}{K}}$$

$$\sigma = \frac{JS^2\pi^2}{Na^2} + KNa = \frac{A\pi^2}{\pi\sqrt{\frac{A}{K}}} + K\pi\sqrt{\frac{A}{K}}$$

$$\sigma_{dw} = 2\pi\sqrt{AK}$$




Example: hcp Co

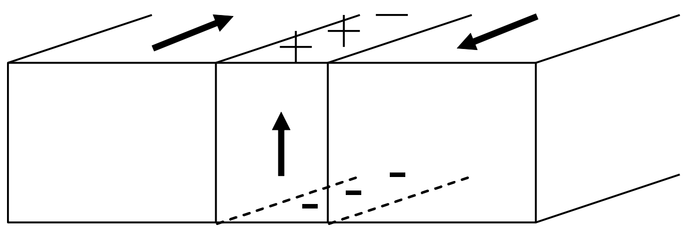
K = 4 x 10$^{5}$ J/m$^{3}$

A = 1 x 10$^{-11}$ J/m

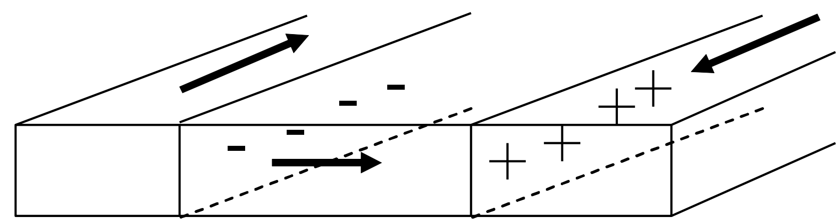
$\Rightarrow$ **$\delta_{dw}$ = 16 nm**

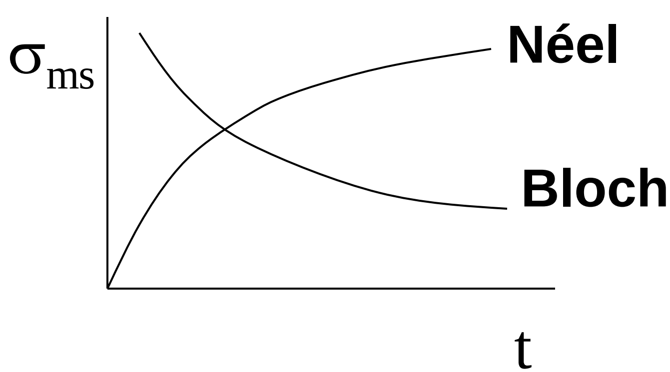
# Bloch and Néel Walls in Thin Films

**Bloch**



**Néel**

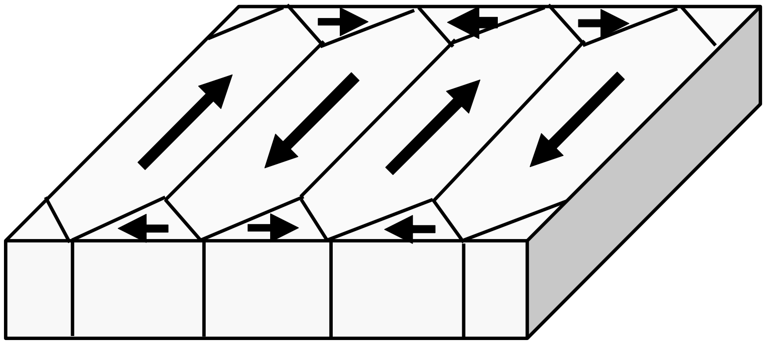





Wall type determined by magnetostatic energy

- **Thin films: Néel walls**
- **Thick films: Bloch walls**

# Closure Domains



- For **cubic anisotropy**, magnetostatic energy can be further minimized with closure domains without adding anisotropy energy.  (Slight increase in domain wall energy.)
- In systems with strong uniaxial anisotropy e.g Co closure domains cannot form due to the hardness of the hard axis.

# Domain Wall Pinning

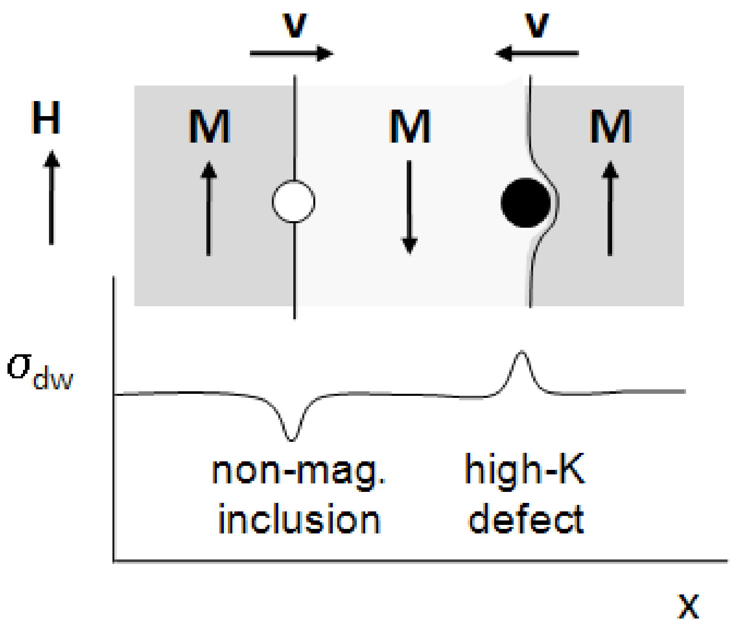
Domain wall motion limited by non-uniformities in wall energy due to non-magnetic inclusions or high-anisotropy defects (crystalline or magnetoelastic).

$$\frac{d\sigma_{dw}}{dx} = 4\frac{d}{dx}(AK)^{1/2}$$

If $\delta_{dw}$ **<< defect size**,

$$\left(\frac{d\sigma_{dw}}{dx}\right)_{max} = 2M_s H_c$$

$$H_c = \frac{1}{2M_s}\left(\frac{d\sigma_{dw}}{dx}\right)_{max}$$



# Switching Field Distributions (SFD)

- Only an isolated particle switches at a single field

- The SFD results from the distribution of **ΔE**

$$\Delta \mathbf{E} = KV(1 - \frac{\mathbf{H}}{\mathbf{H}_K})^2$$

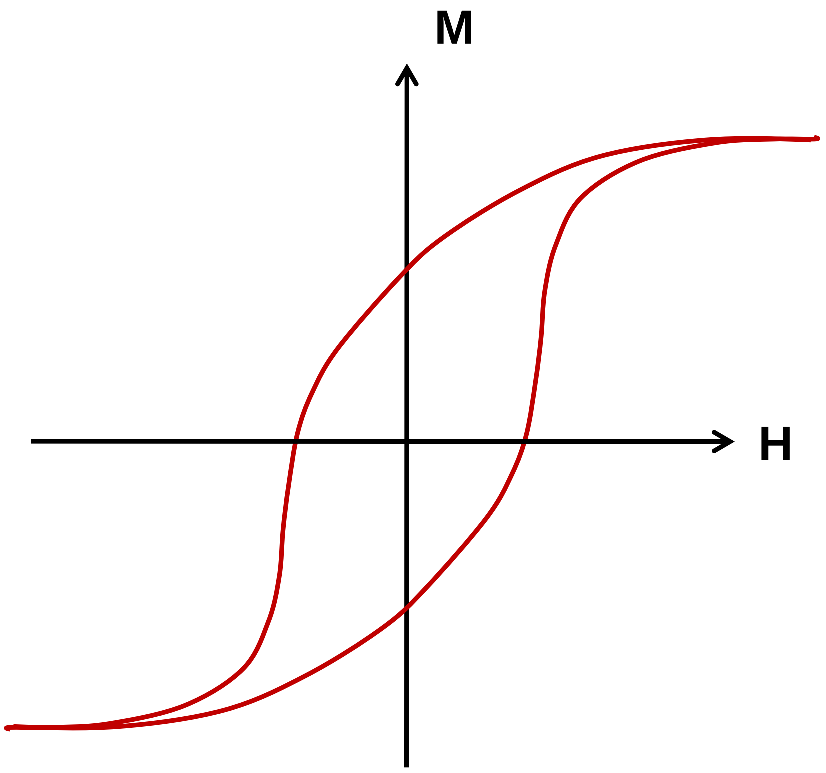
- The SFD is due to:

- Particle size distribution, usually lognormal

- Distribution of K and Ms

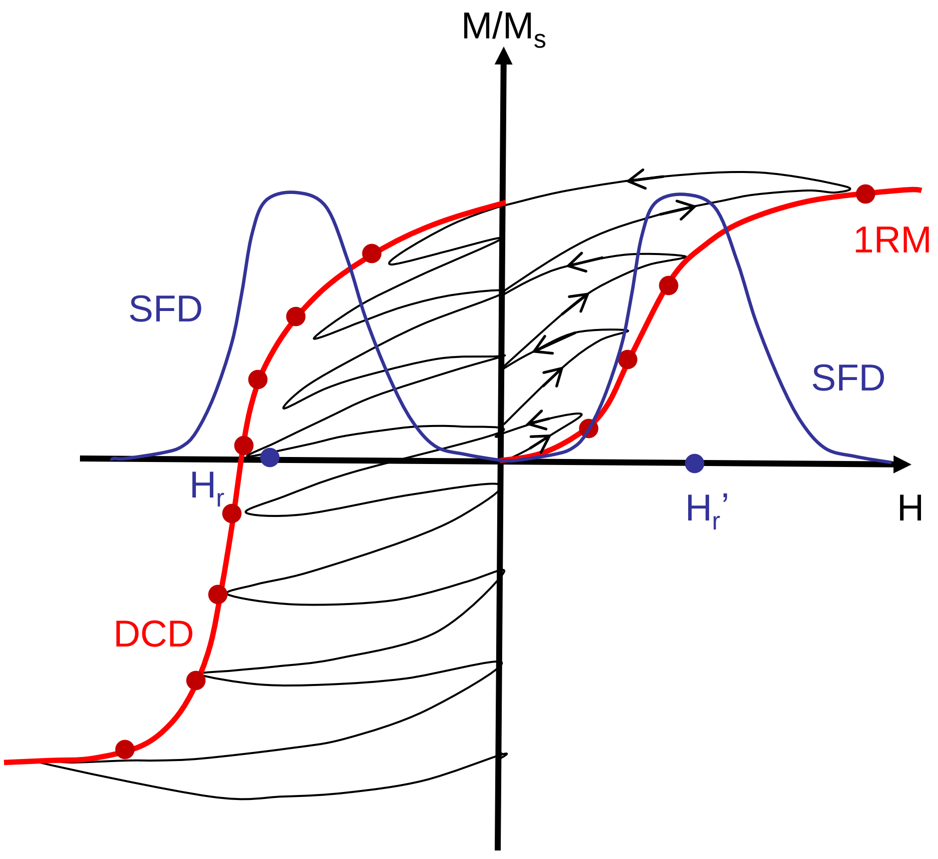
- Distribution of orientation

- Dipole-Dipole exchange interactions



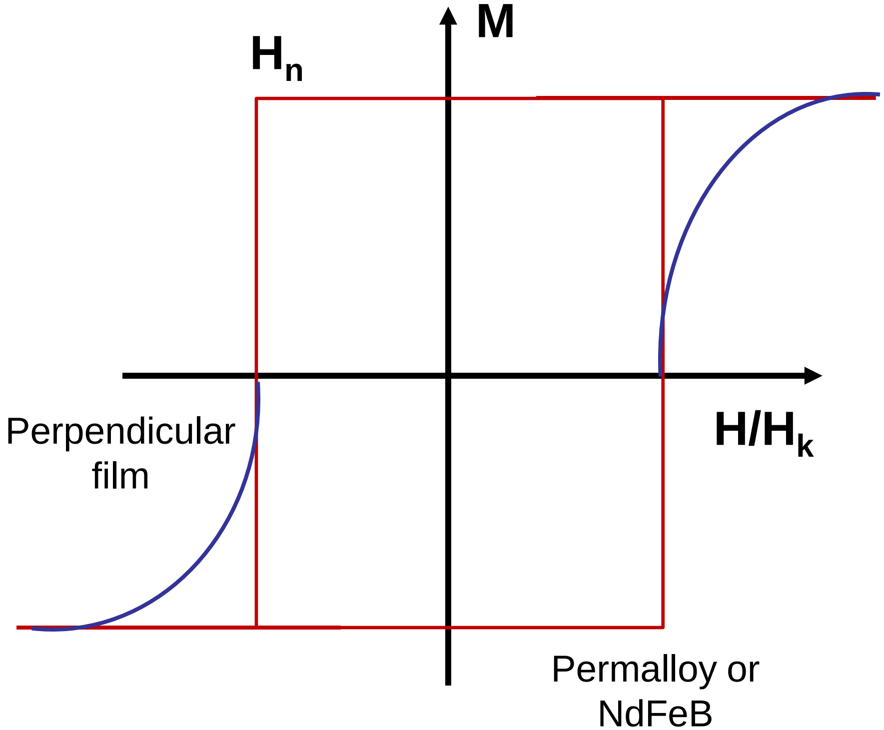
# Measurement of SFD

- We can measure f(ΔE) from f($T_B$)
- To get the SFD it is best to measure the remenance curve.
- This measures the irreversible switching only
- The differential gives the SFD for granular or domain wall pinned systems.



# Nucleation and Propagation

- When a material reverses a reverse domain must nucleate.
- If $H_n > H_{dw}$ then the domain sweeps through the sample giving a square loop.
- If $H_n < H_{dw}$ the loop will be round as $H_{dw}$ is overcome gradually
- In perpendicular films the loop can be round as $H_d$ reduces



# Summary

- Magnetisation produces effective surface and volume charges. Demagnetisation field and energy depends on sample shape.
- Magnetic anisotropy determines preferred orientations of the magnetisation. Source of anisotropy can be shape, crystalline, stress, exchange, …
- Magnetisation can form domains to minimize magnetostatic energy. Domain wall width is determined by minimising exchange and anisotropy energy.
- Quasistatic magnetisation reversal in small particles can be described by coherent reversal model. Larger systems reverse incoherently.

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