Fundamentals of Magnetism

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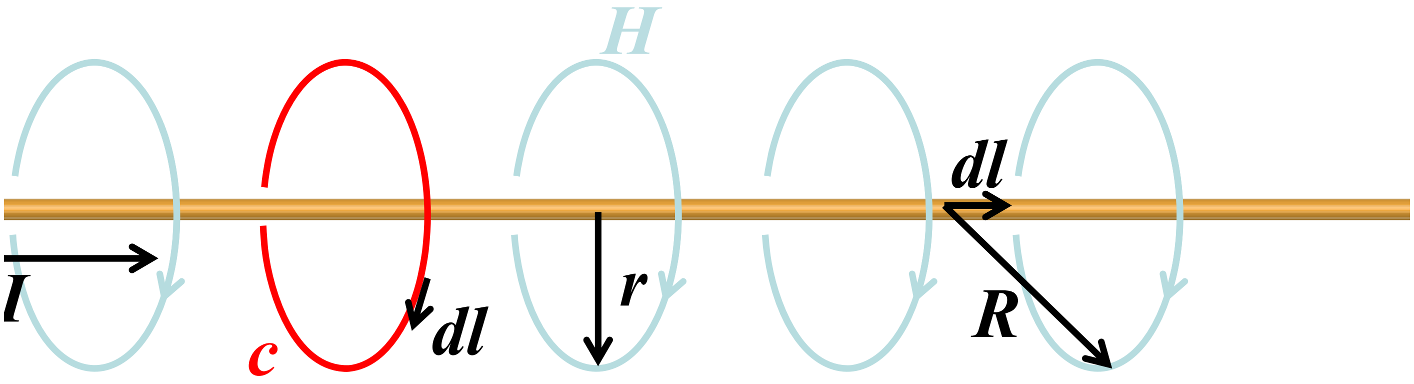
Part I: H,M,B,χ,μ

Part II: M(H)

Magnetic Field from Current in a Wire



Ampere's Circuital Law:

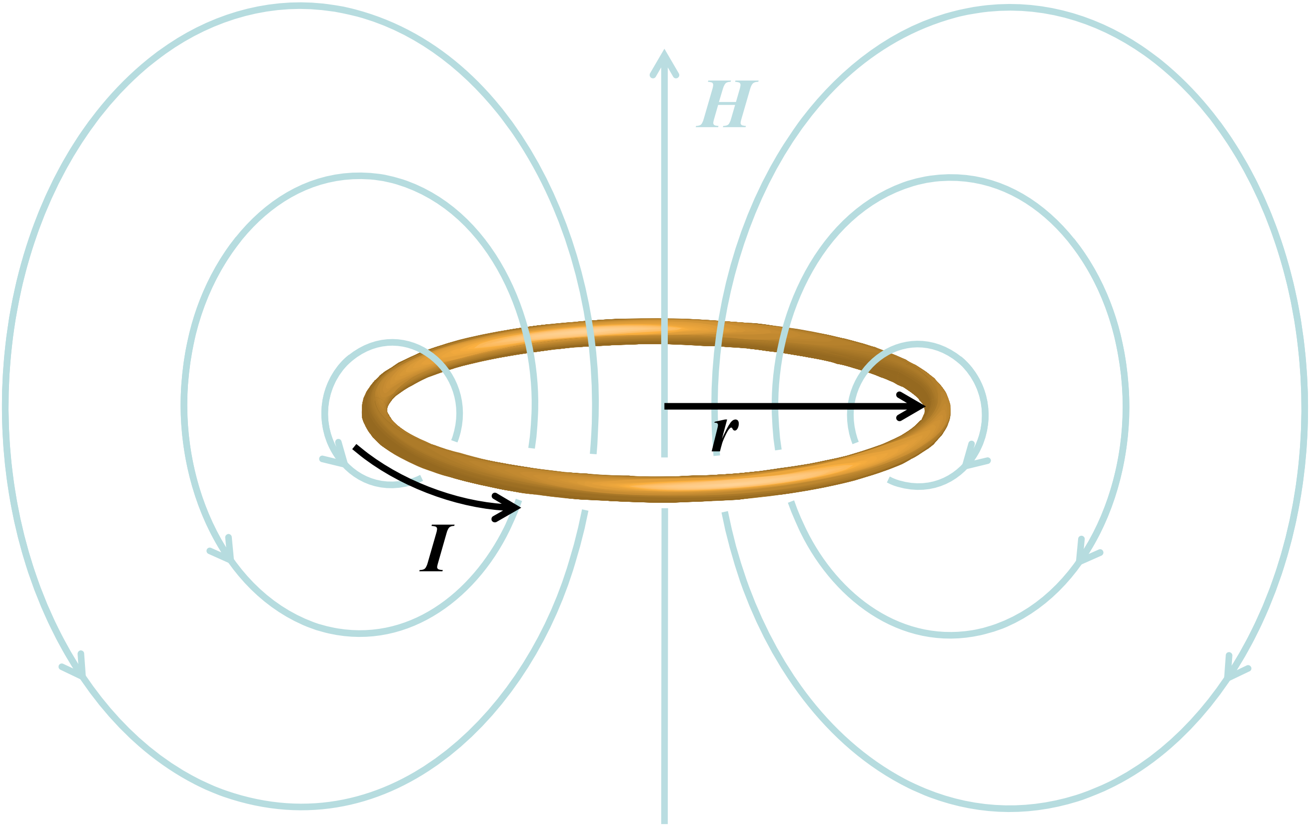
$$\oint_{c} \vec{H}\cdot d\vec{l} = I$$

Biot-Savart Law:

$$d\vec{H} = \frac{I d\vec{l}\times\hat{R}}{4\pi\left|\vec{R}\right|^{2}}$$

$$H = \frac{I}{2\pi r}$$ [A/m]

Magnetic Field from Current Loop



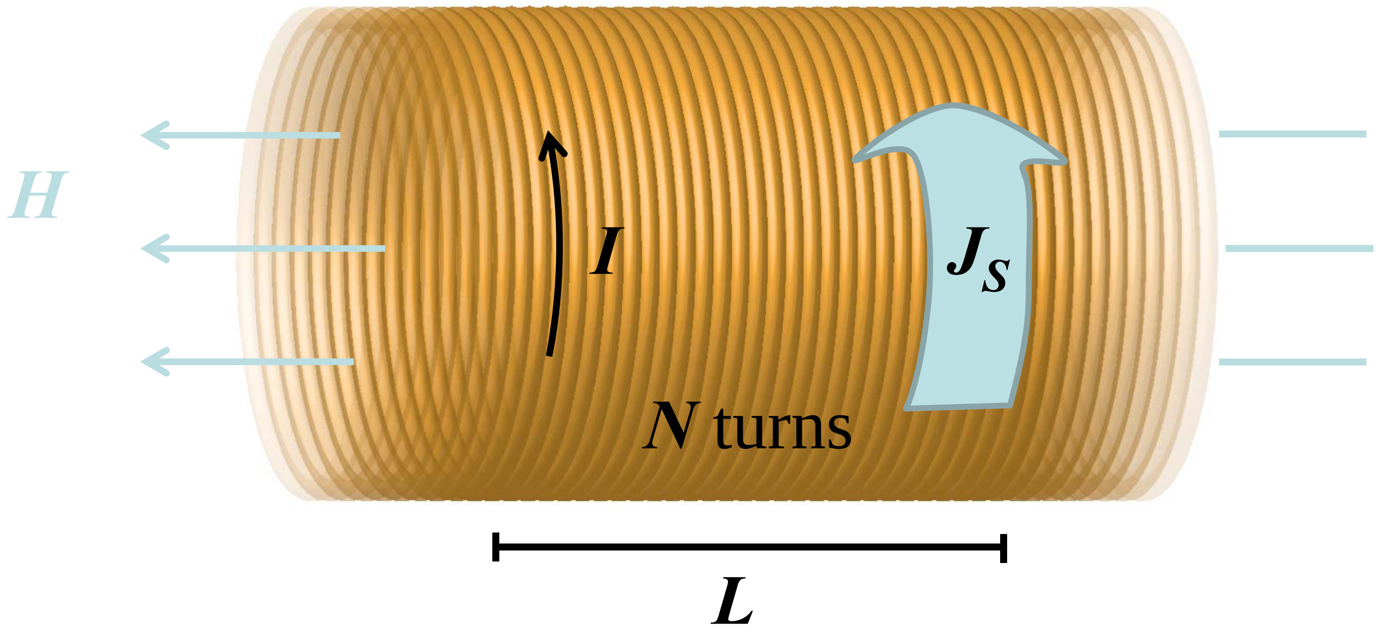
Use Biot-Savart Law:

$$d\vec{H} = \frac{I d\vec{l} \times \hat{R}}{4\pi \left|\vec{R}\right|^2}$$

In the center:

$$H = \frac{I}{2r} \text{ [A/m]}$$

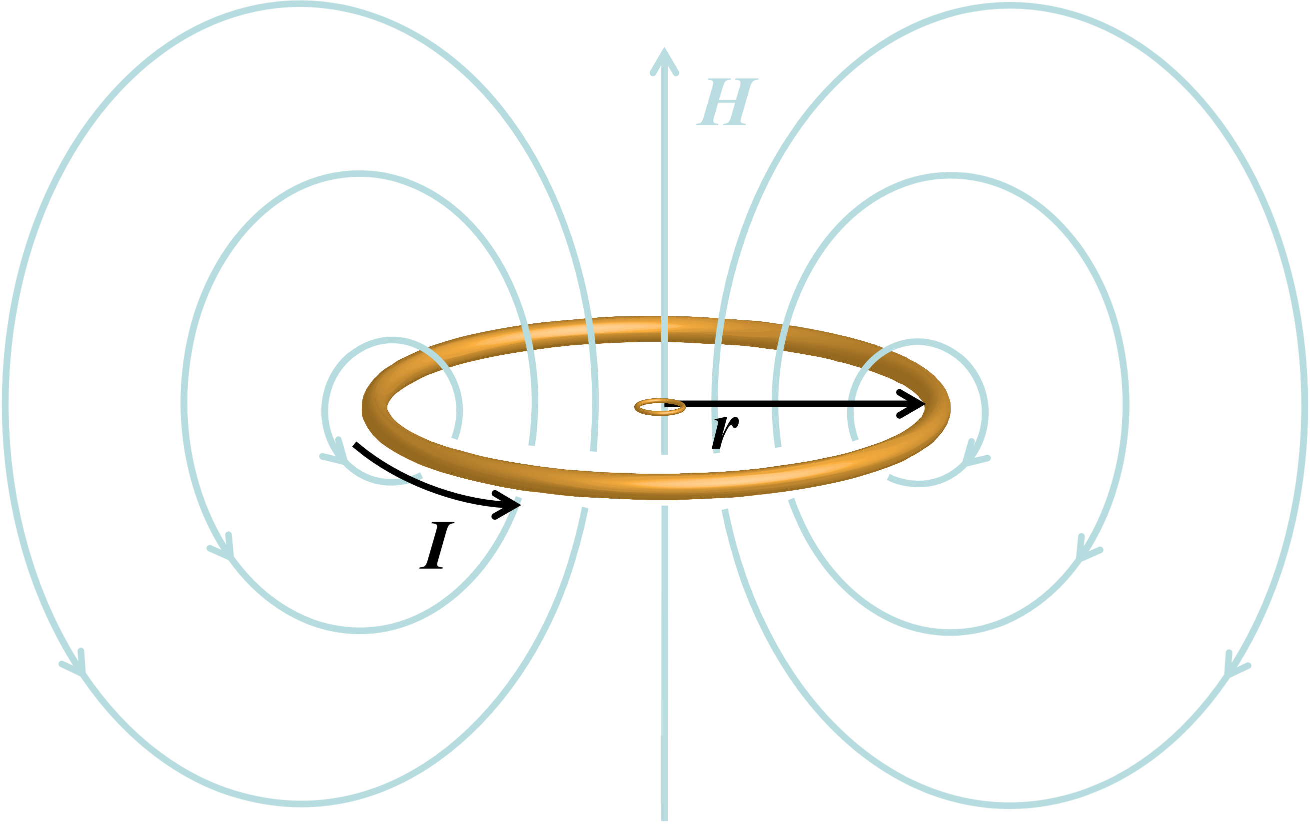
Magnetic Field in a Long Solenoid



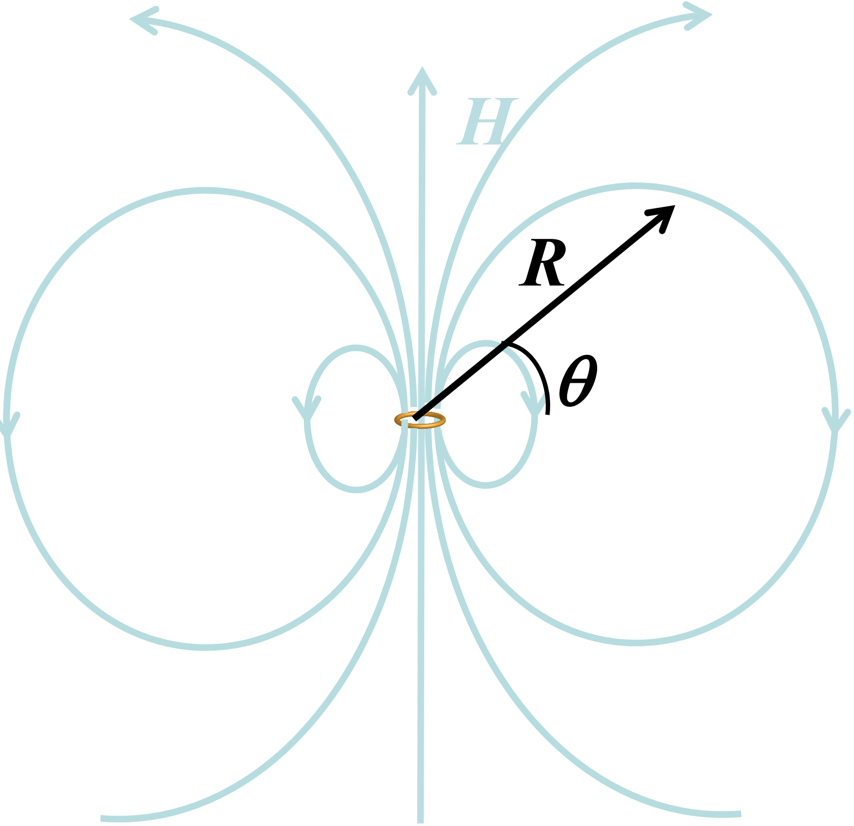
Inside, field is uniform with:

$$H = \frac{NI}{L} \quad \text{[A/m]} \qquad\qquad H = J_S \quad \text{[A/m]}$$

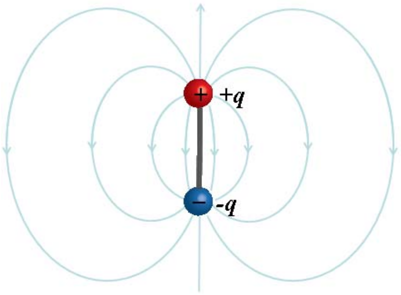
Magnetic Field from Small Loop (Dipole)



Magnetic Field from Small Loop (Dipole)



Compare to Electric dipole:

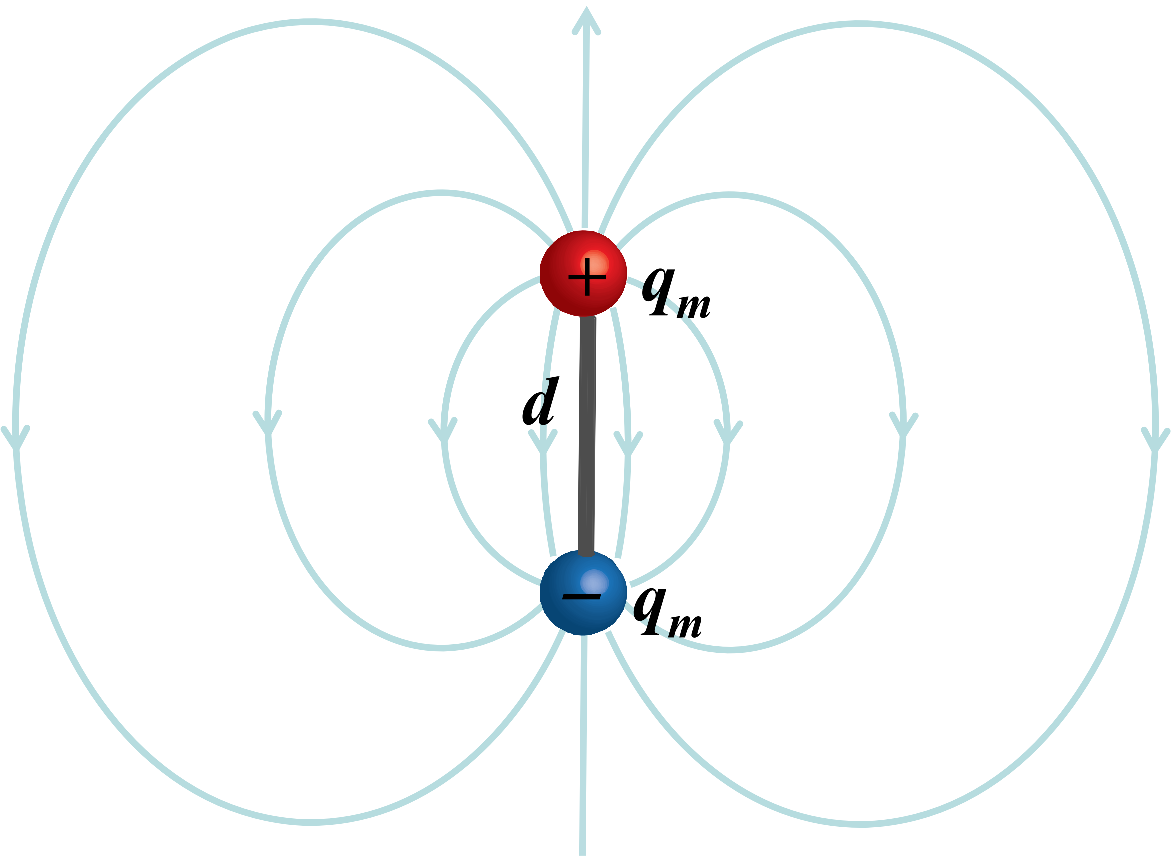


For ***R*** >> ***r***

$$H = \frac{IA}{4\pi R^3}\left[2\hat{r}\cos(\theta) + \hat{\theta}\sin(\theta)\ \right] \quad \text{[A/m]}$$

Magnetic (dipole) moment: $\vec{m} = I\vec{A}$ [Am^2]

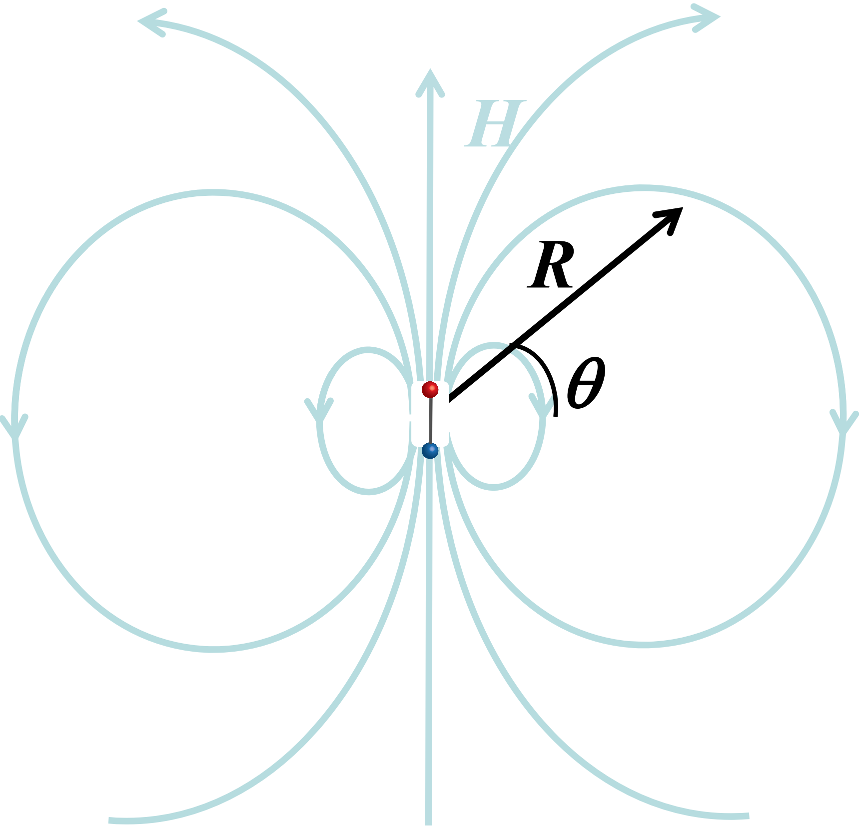
Magnetic Poles and Dipoles



Magnetic field from a “monopole”

$$H = \frac{q_m}{4\pi R^2} \quad \text{[A/m]}$$

Magnetic Field of a Dipole

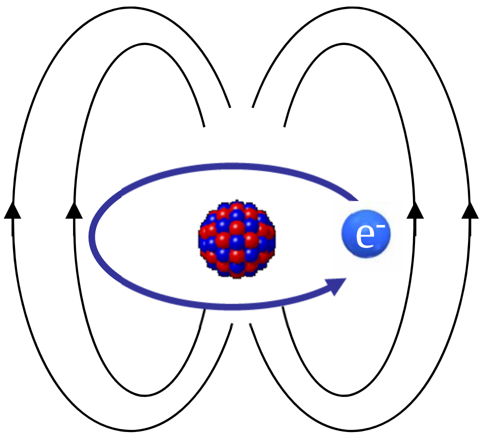


For $\boldsymbol{R} >> \boldsymbol{r}$

$$H = \frac{q_m d}{4\pi R^3}\left[2\hat{r}\cos(\theta) + \hat{\theta}\sin(\theta)\ \right] \quad [\mathrm{A/m}]$$

Magnetic (dipole) moment: $\vec{m} = q_m \vec{d} \quad [\mathrm{Am}^2]$

Orbital Magnetic Moment



•Electrons orbiting a nucleus are like a circulating current producing a magnetic field.

•For an electron with charge q_e orbiting at a radius R with frequency f, the Orbital Magnetic Moment is

$$m = IA = -q_e f \, \pi R^2 \qquad [\mathrm{Am}^2]$$

- It also has an Orbital Angular Momentum

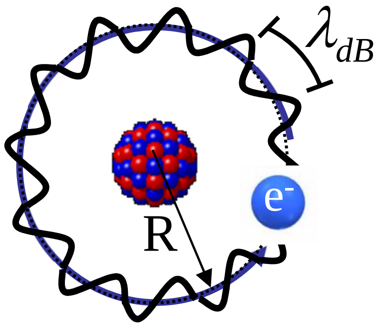
$$L = m_e v R = m_e 2\pi R f \, R$$

- Note: $\dfrac{m}{L} = \dfrac{-q_e}{2m_e}$ m and L are in opposite directions!

Orbital Moment is Quantized

Niels Bohr
(1885-1962)

Bohr model of the atom



•DeBroglie wavelength is:

$$\lambda_{dB} = \frac{h}{m_e v}$$

•Bohr Model: orbit must be integer number of wavelengths

$$2\pi R = N\lambda_{dB} = N\frac{h}{m_e v} = N\frac{h}{m_e 2\pi R f}$$

- Thus the orbital magnetic moment is quantized:

$$m = -q_e f\ \pi R^2 = N\frac{h}{2\pi}\frac{q_e}{2m_e} = N\frac{\hbar q_e}{2m_e}$$

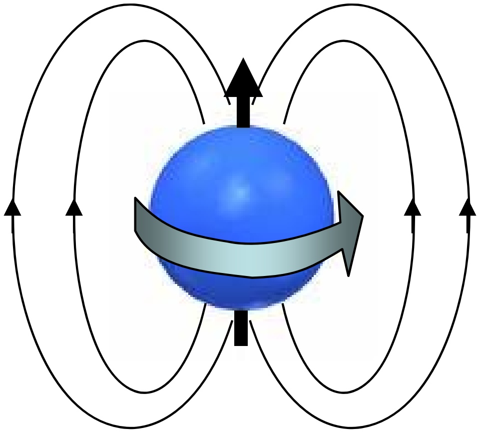
- Magnetic moment restricted to multiples of

<u>Bohr Magneton</u>

$$\mu_B = \frac{\hbar q_e}{2m_e} = 9.2742\times10^{-24}[Am^2]$$

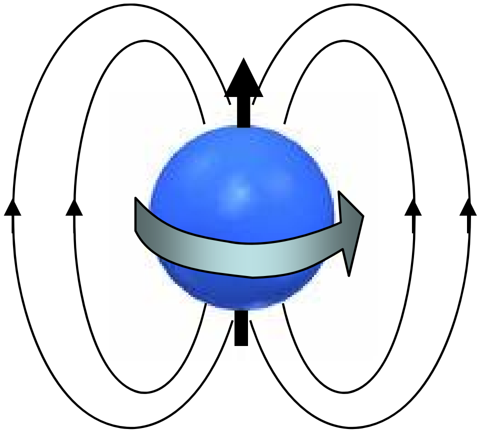
Louis de Broglie
(1892-1987)

Spin



- Spin is a property of subatomic particles (just like charge or mass.)
- A particle with spin has a magnetic dipole moment and angular momentum.
- Spin may be thought of ***conceptually*** as arising from a spinning sphere of charge. (However, note that neutrons also have spin but no charge!)

Pauli and Bohr contemplate the “spin” of a tippy-top



Electron Spin

- Electrons, protons and neutrons (fermions) have spin, s = ½
- Photons (boson) have spin 1

- *When measured in a particular direction*, the measured angular momentum of an electron is

$$L_z = s_z \hbar = \pm \frac{\hbar}{2} \qquad s_z = -s, -s+1 \ldots s$$

- *When measured in a particular direction*, the measured magnetic moment of an electron is

$$m_z = s_z \frac{\hbar q_e}{m_e} = \pm \mu_B$$

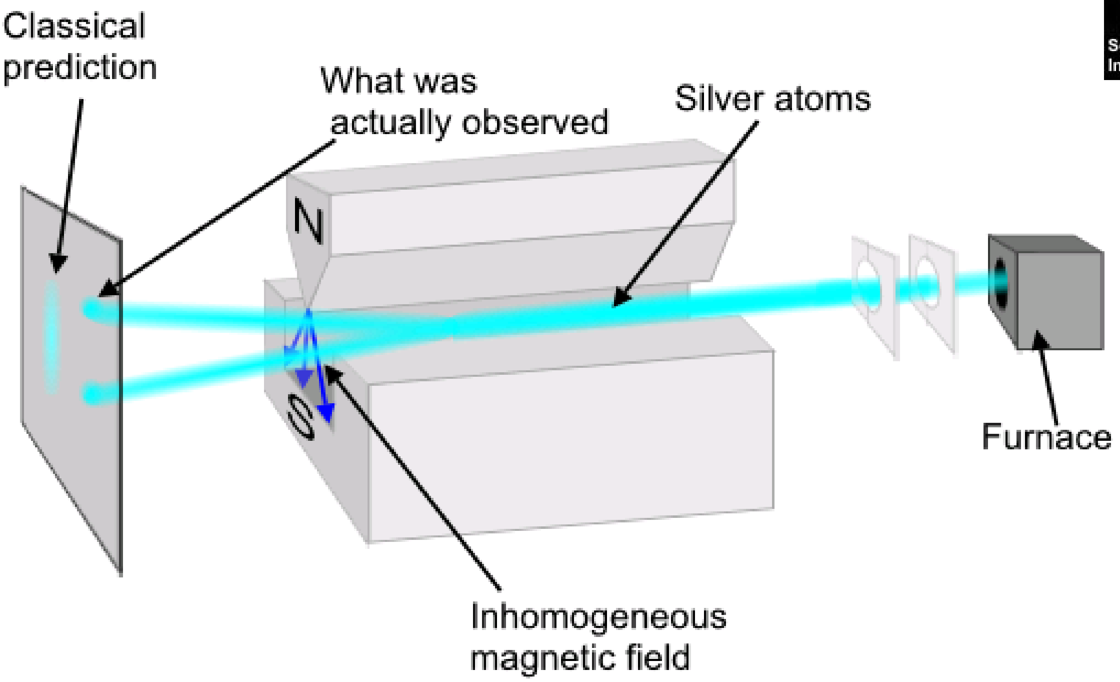
- We say "spin up" and "spin down"
- Note: $\frac{m}{L} = \frac{-q_e}{m_e}$

Compare:

Spin	Orbital
$m_z = \pm \mu_B$	$m_z = N \mu_B$
$\frac{m}{L} = \frac{-q_e}{m_e}$	$\frac{m}{L} = \frac{-q_e}{2m_e}$

Stern-Gerlach Experiment - 1922

Demonstrated that magnetic moment is quantized with ±μ_B

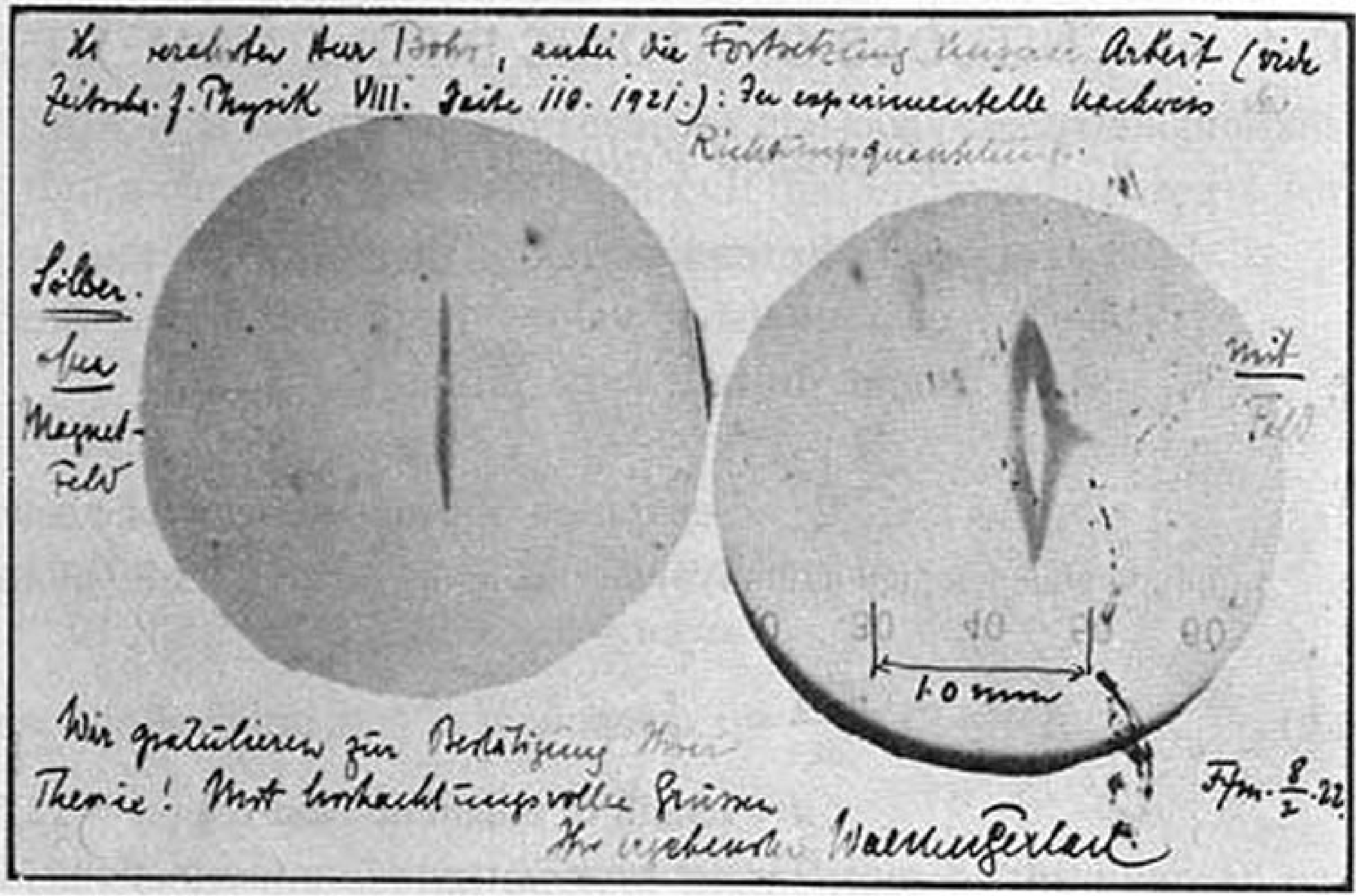


Otto Stern (1888-1969)

Walter Gerlach
(1889-1979)

Walther Gerlach, Otto Stern (1922). "Das magnetische Moment des Silberatoms". *Zeitschrift für Physik A Hadrons and Nuclei* **9**.

Stern-Gerlach Experiment



Sehr verehrter Herr Bohr, anbei die Fortsetzung unserer Arbeit (vide Zeitschr. f. Physik VIII. Seite 110. 1921.): Der experimentelle Nachweis der Richtungsquantelung.

Silber. Ohne Magnet-Feld

Mit Feld

Wir gratulieren zur Bestätigung Ihrer Theorie! Mit hochachtungsvollen Grüssen Ihr ergebenster Walther Gerlach.

Ffm. 8/2. 22.

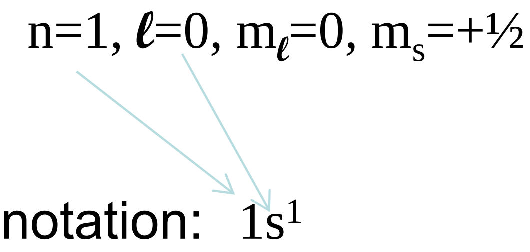
Gerlach's postcard, dated 8 February 1922, to Niels Bohr. It shows a photograph of the beam splitting, with the message, in translation: "Attached [is] the experimental proof of directional quantization. We congratulate [you] on the confirmation of your theory." (Physics Today December 2003)

Quantum Numbers of Electron Orbitals

- Electrons bound to a nucleus move in orbits identified by quantum numbers (from solution of Schrödinger's equation):

n	1…	principal quantum number, identifies the “shell”
ℓ	0..n-1	angular momentum q. #, type of orbital, e.g. s,p,d,f
m_ℓ	$-\ell..\ell$	magnetic quantum number
m_s	+½ or -½	spin quantum number

For example, the electron orbiting the hydrogen nucleus (in the ground state) has:

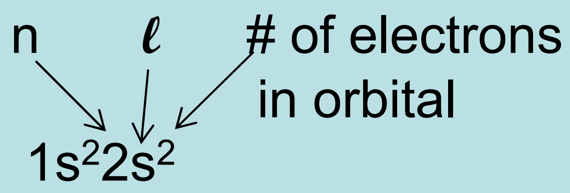


$n=1, \ell=0, m_\ell=0, m_s=+\frac{1}{2}$

In spectroscopic notation: $1s^1$

<u>Spectroscopic notation</u>

$\ell=0$	s
$\ell=1$	p
$\ell=2$	d
$\ell=3$	f



$1s^2 2s^2$

Spin and Orbital Magnetic Moment

- Total orbital magnetic moment (sum over all electron orbitals)

$$m_{tot\_orbital} = \mu_B \sum m_\ell$$

- Total spin magnetic moment

$$m_{tot\_spin} = 2\mu_B \sum m_s$$

full orbitals: no net moment

E.g. Iron:

$1s^2 2s^2 2p^6 3s^2 3p^6 3d^6 4s^2$

How to fill remaining d orbitals (ℓ=2) 6 electrons for 10 spots:

	$m_\ell=-2$	$m_\ell=-1$	$m_\ell=0$	$m_\ell=+1$	$m_\ell=+2$
$m_s=+½$	↑	↑	↑	↑	↑
$m_s=-½$					↓

$$m_{tot\_orbital} = 2\mu_B \qquad m_{tot\_spin} = 4\mu_B$$

Hund's rules:

Maximize Σm_s

Then maximize Σm_ℓ

PERIODIC TABLE

Atomic Properties of the Elements

NIST
National Institute of Standards and Technology
Technology Administration, U.S. Department of Commerce

Physics Laboratory
physics.nist.gov

Standard Reference Data Group
www.nist.gov/srd

Frequently used fundamental physical constants

For the most accurate values of these and other constants, visit physics.nist.gov/constants

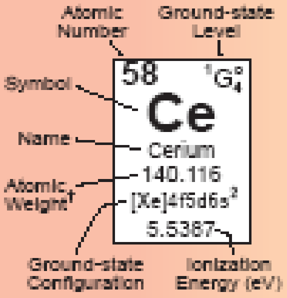
1 second = 9 192 631 770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of ^{133}Cs

speed of light in vacuum	c	299 792 458 m s^{-1}	(exact)
Planck constant	h	6.6261×10^{-34} J s	($\hbar = h/2\pi$)
elementary charge	e	1.6022×10^{-19} C	
electron mass	m_e	9.1094×10^{-31} kg	
	m_ec^2	0.5110 MeV	
proton mass	m_p	1.6726×10^{-27} kg	
fine-structure constant	α	1/137.036	
Rydberg constant	R_∞	10 973 732 m^{-1}	
	$R_\infty c$	$3.289\,842 \times 10^{15}$ Hz	
	$R_\infty hc$	13.6057 eV	
Boltzmann constant	k	1.3807×10^{-23} J K^{-1}	

⬜ Solids
🟦 Liquids
🟥 Gases
🟨 Artificially Prepared

Period	Group 1 IA	2 IIA	3 IIIB	4 IVB	5 VB	6 VIB	7 VIIB	8	9 VIII	10	11 IB	12 IIB	13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA	18 VIIIA
1	🟥 1 $^2S_{1/2}$ **H** Hydrogen 1.00794 $1s$ 13.5984																	🟥 2 $^1S_0$ **He** Helium 4.002602 $1s^2$ 24.5874
2	3 $^2S_{1/2}$ **Li** Lithium 6.941 $1s^22s$ 5.3917	4 $^1S_0$ **Be** Beryllium 9.012182 $1s^22s^2$ 9.3227											5 $^2P^\circ_{1/2}$ **B** Boron 10.811 $1s^22s^22p$ 8.2980	6 $^3P_0$ **C** Carbon 12.0107 $1s^22s^22p^2$ 11.2603	🟥 7 $^4S^\circ_{3/2}$ **N** Nitrogen 14.0067 $1s^22s^22p^3$ 14.5341	🟥 8 $^3P_2$ **O** Oxygen 15.9994 $1s^22s^22p^4$ 13.6181	🟥 9 $^2P^\circ_{3/2}$ **F** Fluorine 18.9984032 $1s^22s^22p^5$ 17.4228	🟥 10 $^1S_0$ **Ne** Neon 20.1797 $1s^22s^22p^6$ 21.5645
3	11 $^2S_{1/2}$ **Na** Sodium 22.989770 $[Ne]3s$ 5.1391	12 $^1S_0$ **Mg** Magnesium 24.3050 $[Ne]3s^2$ 7.6462											13 $^2P^\circ_{1/2}$ **Al** Aluminum 26.981538 $[Ne]3s^23p$ 5.9858	14 $^3P_0$ **Si** Silicon 28.0855 $[Ne]3s^23p^2$ 8.1517	15 $^4S^\circ_{3/2}$ **P** Phosphorus 30.973761 $[Ne]3s^23p^3$ 10.4867	16 $^3P_2$ **S** Sulfur 32.065 $[Ne]3s^23p^4$ 10.3600	🟥 17 $^2P^\circ_{3/2}$ **Cl** Chlorine 35.453 $[Ne]3s^23p^5$ 12.9676	🟥 18 $^1S_0$ **Ar** Argon 39.948 $[Ne]3s^23p^6$ 15.7596
4	19 $^2S_{1/2}$ **K** Potassium 39.0983 $[Ar]4s$ 4.3407	20 $^1S_0$ **Ca** Calcium 40.078 $[Ar]4s^2$ 6.1132	21 $^2D_{3/2}$ **Sc** Scandium 44.955910 $[Ar]3d4s^2$ 6.5615	22 $^3F_2$ **Ti** Titanium 47.867 $[Ar]3d^24s^2$ 6.8281	23 $^4F_{3/2}$ **V** Vanadium 50.9415 $[Ar]3d^34s^2$ 6.7462	24 $^7S_3$ **Cr** Chromium 51.9961 $[Ar]3d^54s$ 6.7665	25 $^6S_{5/2}$ **Mn** Manganese 54.938049 $[Ar]3d^54s^2$ 7.4340	26 $^5D_4$ **Fe** Iron 55.845 $[Ar]3d^64s^2$ 7.9024	27 $^4F_{9/2}$ **Co** Cobalt 58.933200 $[Ar]3d^74s^2$ 7.8810	28 $^3F_4$ **Ni** Nickel 58.6934 $[Ar]3d^84s^2$ 7.6398	29 $^2S_{1/2}$ **Cu** Copper 63.546 $[Ar]3d^{10}4s$ 7.7264	30 $^1S_0$ **Zn** Zinc 65.409 $[Ar]3d^{10}4s^2$ 9.3942	31 $^2P^\circ_{1/2}$ **Ga** Gallium 69.723 $[Ar]3d^{10}4s^24p$ 5.9993	32 $^3P_0$ **Ge** Germanium 72.64 $[Ar]3d^{10}4s^24p^2$ 7.8994	33 $^4S^\circ_{3/2}$ **As** Arsenic 74.92160 $[Ar]3d^{10}4s^24p^3$ 9.7886	34 $^3P_2$ **Se** Selenium 78.96 $[Ar]3d^{10}4s^24p^4$ 9.7524	🟦 35 $^2P^\circ_{3/2}$ **Br** Bromine 79.904 $[Ar]3d^{10}4s^24p^5$ 11.8138	🟥 36 $^1S_0$ **Kr** Krypton 83.798 $[Ar]3d^{10}4s^24p^6$ 13.9996
5	37 $^2S_{1/2}$ **Rb** Rubidium 85.4678 $[Kr]5s$ 4.1771	38 $^1S_0$ **Sr** Strontium 87.62 $[Kr]5s^2$ 5.6949	39 $^2D_{3/2}$ **Y** Yttrium 88.90585 $[Kr]4d5s^2$ 6.2173	40 $^3F_2$ **Zr** Zirconium 91.224 $[Kr]4d^25s^2$ 6.6339	41 $^6D_{1/2}$ **Nb** Niobium 92.90638 $[Kr]4d^45s$ 6.7589	42 $^7S_3$ **Mo** Molybdenum 95.94 $[Kr]4d^55s$ 7.0924	🟨 43 $^6S_{5/2}$ **Tc** Technetium (98) $[Kr]4d^55s^2$ 7.28	44 $^5F_5$ **Ru** Ruthenium 101.07 $[Kr]4d^75s$ 7.3605	45 $^4F_{9/2}$ **Rh** Rhodium 102.90550 $[Kr]4d^85s$ 7.4589	46 $^1S_0$ **Pd** Palladium 106.42 $[Kr]4d^{10}$ 8.3369	47 $^2S_{1/2}$ **Ag** Silver 107.8682 $[Kr]4d^{10}5s$ 7.5762	48 $^1S_0$ **Cd** Cadmium 112.411 $[Kr]4d^{10}5s^2$ 8.9938	49 $^2P^\circ_{1/2}$ **In** Indium 114.818 $[Kr]4d^{10}5s^25p$ 5.7864	50 $^3P_0$ **Sn** Tin 118.710 $[Kr]4d^{10}5s^25p^2$ 7.3439	51 $^4S^\circ_{3/2}$ **Sb** Antimony 121.760 $[Kr]4d^{10}5s^25p^3$ 8.6084	52 $^3P_2$ **Te** Tellurium 127.60 $[Kr]4d^{10}5s^25p^4$ 9.0096	53 $^2P^\circ_{3/2}$ **I** Iodine 126.90447 $[Kr]4d^{10}5s^25p^5$ 10.4513	🟥 54 $^1S_0$ **Xe** Xenon 131.293 $[Kr]4d^{10}5s^25p^6$ 12.1298
6	55 $^2S_{1/2}$ **Cs** Cesium 132.90545 $[Xe]6s$ 3.8939	56 $^1S_0$ **Ba** Barium 137.327 $[Xe]6s^2$ 5.2117		72 $^3F_2$ **Hf** Hafnium 178.49 $[Xe]4f^{14}5d^26s^2$ 6.8251	73 $^4F_{3/2}$ **Ta** Tantalum 180.9479 $[Xe]4f^{14}5d^36s^2$ 7.5496	74 $^5D_0$ **W** Tungsten 183.84 $[Xe]4f^{14}5d^46s^2$ 7.8640	75 $^6S_{5/2}$ **Re** Rhenium 186.207 $[Xe]4f^{14}5d^56s^2$ 7.8335	76 $^5D_4$ **Os** Osmium 190.23 $[Xe]4f^{14}5d^66s^2$ 8.4382	77 $^4F_{9/2}$ **Ir** Iridium 192.217 $[Xe]4f^{14}5d^76s^2$ 8.9670	78 $^3D_3$ **Pt** Platinum 195.078 $[Xe]4f^{14}5d^96s$ 8.9588	79 $^2S_{1/2}$ **Au** Gold 196.96655 $[Xe]4f^{14}5d^{10}6s$ 9.2255	🟦 80 $^1S_0$ **Hg** Mercury 200.59 $[Xe]4f^{14}5d^{10}6s^2$ 10.4375	81 $^2P^\circ_{1/2}$ **Tl** Thallium 204.3833 $[Hg]6p$ 6.1082	82 $^3P_0$ **Pb** Lead 207.2 $[Hg]6p^2$ 7.4167	83 $^4S^\circ_{3/2}$ **Bi** Bismuth 208.98038 $[Hg]6p^3$ 7.2855	84 $^3P_2$ **Po** Polonium (209) $[Hg]6p^4$ 8.414	85 $^2P^\circ_{3/2}$ **At** Astatine (210) $[Hg]6p^5$	🟥 86 $^1S_0$ **Rn** Radon (222) $[Hg]6p^6$ 10.7485
7	87 $^2S_{1/2}$ **Fr** Francium (223) $[Rn]7s$ 4.0727	88 $^1S_0$ **Ra** Radium (226) $[Rn]7s^2$ 5.2784		🟨 104 $^3F_2$? **Rf** Rutherfordium (261) $[Rn]5f^{14}6d^27s^2$? 6.0 ?	🟨 105 **Db** Dubnium (262)	🟨 106 **Sg** Seaborgium (266)	🟨 107 **Bh** Bohrium (264)	🟨 108 **Hs** Hassium (277)	🟨 109 **Mt** Meitnerium (268)	🟨 110 **Uun** Ununnilium (281)	🟨 111 **Uuu** Unununium (272)	🟨 112 **Uub** Ununbium (285)		🟨 114 **Uuq** Ununquadium (289)		🟨 116 **Uuh** Ununhexium (292)		

Lanthanides	57 $^2D_{3/2}$ **La** Lanthanum 138.9055 $[Xe]5d6s^2$ 5.5769	58 $^1G^\circ_4$ **Ce** Cerium 140.116 $[Xe]4f5d6s^2$ 5.5387	59 $^4I^\circ_{9/2}$ **Pr** Praseodymium 140.90765 $[Xe]4f^36s^2$ 5.473	60 $^5I_4$ **Nd** Neodymium 144.24 $[Xe]4f^46s^2$ 5.5250	🟨 61 $^6H^\circ_{5/2}$ **Pm** Promethium (145) $[Xe]4f^56s^2$ 5.582	62 $^7F_0$ **Sm** Samarium 150.36 $[Xe]4f^66s^2$ 5.6437	63 $^8S^\circ_{7/2}$ **Eu** Europium 151.964 $[Xe]4f^76s^2$ 5.6704	64 $^9D^\circ_2$ **Gd** Gadolinium 157.25 $[Xe]4f^75d6s^2$ 6.1498	65 $^6H^\circ_{15/2}$ **Tb** Terbium 158.92534 $[Xe]4f^96s^2$ 5.8638	66 $^5I_8$ **Dy** Dysprosium 162.500 $[Xe]4f^{10}6s^2$ 5.9389	67 $^4I^\circ_{15/2}$ **Ho** Holmium 164.93032 $[Xe]4f^{11}6s^2$ 6.0215	68 $^3H_6$ **Er** Erbium 167.259 $[Xe]4f^{12}6s^2$ 6.1077	69 $^2F^\circ_{7/2}$ **Tm** Thulium 168.93421 $[Xe]4f^{13}6s^2$ 6.1843	70 $^1S_0$ **Yb** Ytterbium 173.04 $[Xe]4f^{14}6s^2$ 6.2542	71 $^2D_{3/2}$ **Lu** Lutetium 174.967 $[Xe]4f^{14}5d6s^2$ 5.4259
Actinides	89 $^2D_{3/2}$ **Ac** Actinium (227) $[Rn]6d7s^2$ 5.17	90 $^3F_2$ **Th** Thorium 232.0381 $[Rn]6d^27s^2$ 6.3067	91 $^4K_{11/2}$ **Pa** Protactinium 231.03588 $[Rn]5f^26d7s^2$ 5.89	92 $^5L^\circ_6$ **U** Uranium 238.02891 $[Rn]5f^36d7s^2$ 6.1941	🟨 93 $^6L_{11/2}$ **Np** Neptunium (237) $[Rn]5f^46d7s^2$ 6.2657	🟨 94 $^7F_0$ **Pu** Plutonium (244) $[Rn]5f^67s^2$ 6.0260	🟨 95 $^8S^\circ_{7/2}$ **Am** Americium (243) $[Rn]5f^77s^2$ 5.9738	🟨 96 $^9D^\circ_2$ **Cm** Curium (247) $[Rn]5f^76d7s^2$ 5.9914	🟨 97 $^6H^\circ_{15/2}$ **Bk** Berkelium (247) $[Rn]5f^97s^2$ 6.1979	🟨 98 $^5I_8$ **Cf** Californium (251) $[Rn]5f^{10}7s^2$ 6.2817	🟨 99 $^4I^\circ_{15/2}$ **Es** Einsteinium (252) $[Rn]5f^{11}7s^2$ 6.42	🟨 100 $^3H_6$ **Fm** Fermium (257) $[Rn]5f^{12}7s^2$ 6.50	🟨 101 $^2F^\circ_{7/2}$ **Md** Mendelevium (258) $[Rn]5f^{13}7s^2$ 6.58	🟨 102 $^1S_0$ **No** Nobelium (259) $[Rn]5f^{14}7s^2$ 6.65	🟨 103 $^2P^\circ_{1/2}$? **Lr** Lawrencium (262) $[Rn]5f^{14}7s^27p$? 4.9 ?



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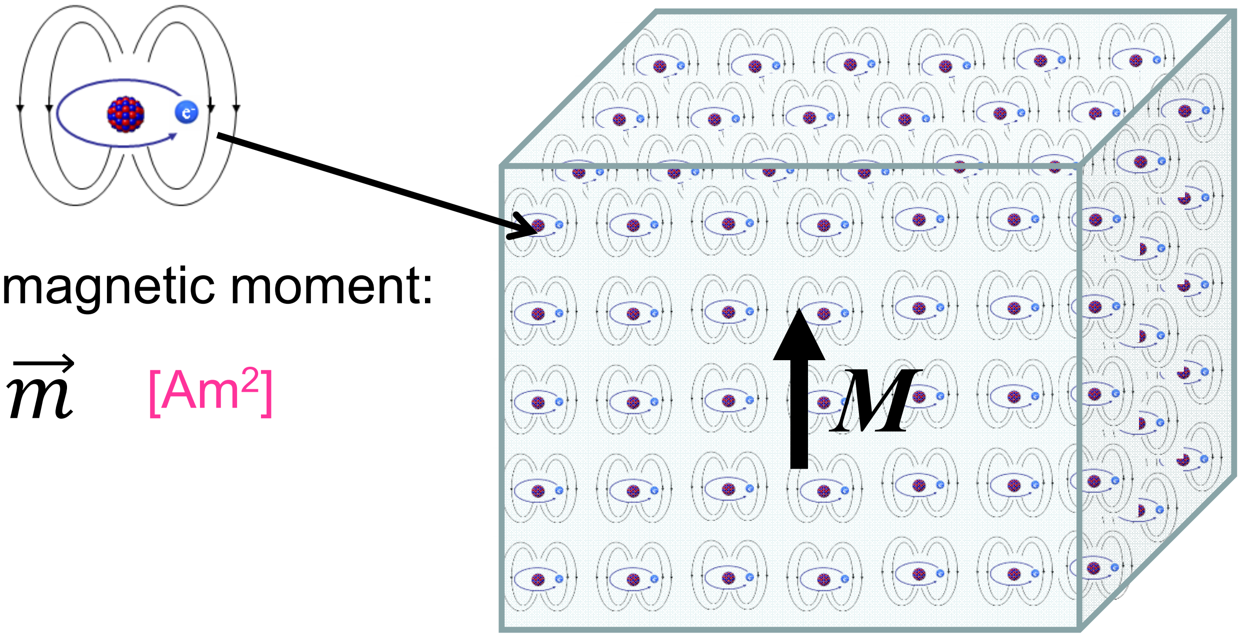
†Based upon ^{12}C. () indicates the mass number of the most stable isotope.

For a description of the data, visit physics.nist.gov/data

NIST SP 966 (September 2003)

Magnetization, $\vec{M}$



Atomic magnetic moment:

$\vec{m}$ [Am^2]

What is H here?

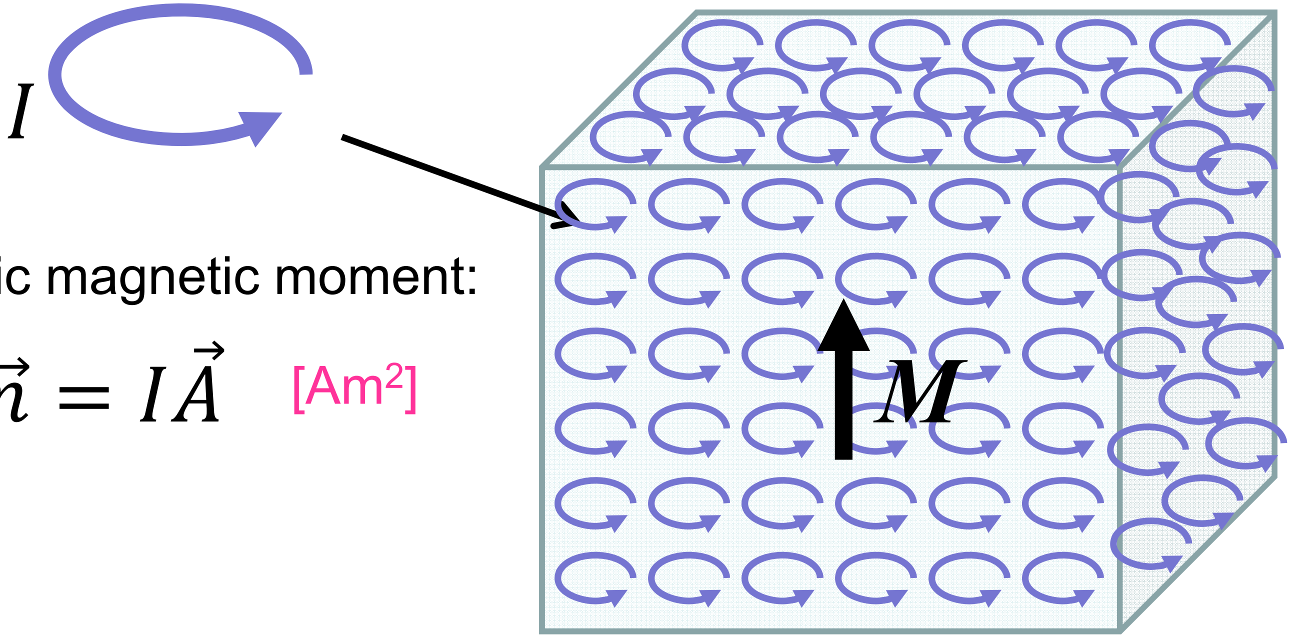
Atomic density: N/vol

[1/m^3]

Magnetic moment per unit volume:

$$\vec{M} \stackrel{\text{def}}{=} \frac{\vec{m}}{vol}$$ [A/m]

Equivalent Surface Current



Atomic magnetic moment:

$$\vec{m} = I\vec{A}$$ [Am^2]

What is H here?

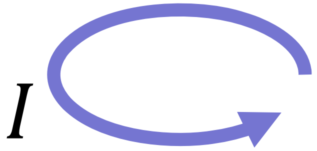
Atomic density: N/vol

[1/m^3]

Magnetic moment per unit volume:

$$\vec{M} \overset{\text{def}}{=} \frac{\vec{m}}{vol}$$ [A/m]

Equivalent Surface Current

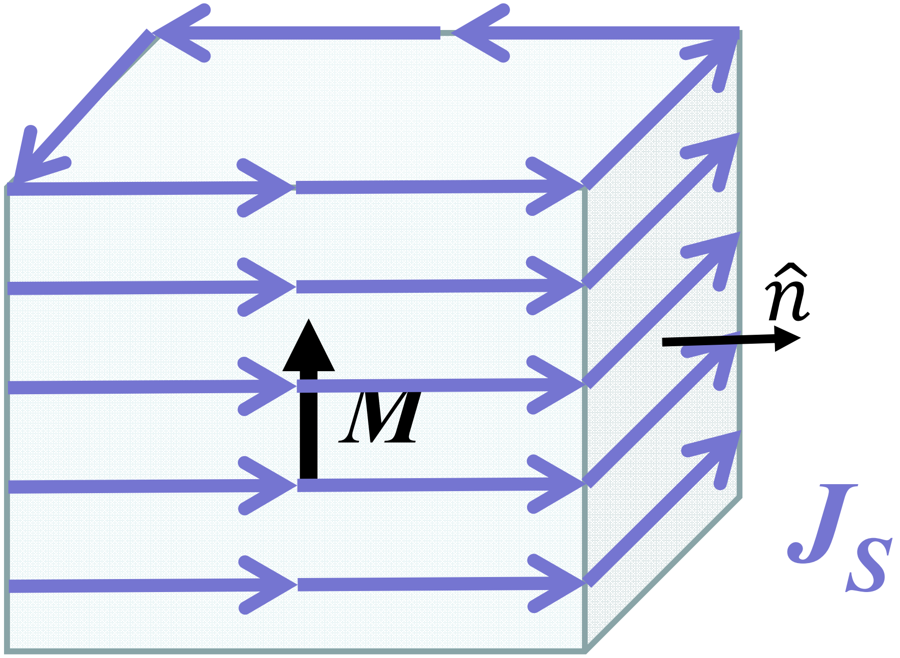


Atomic magnetic moment:

$$\vec{m} = I\vec{A}$$ [Am^2]

Atomic density: N/vol

[1/m^3]



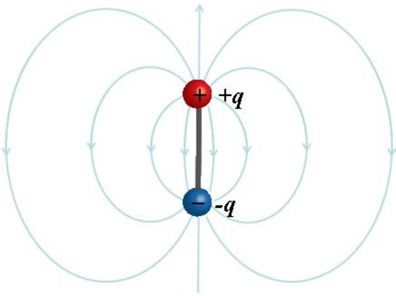
[A/m]

What is H here?

Equivalent surface current:

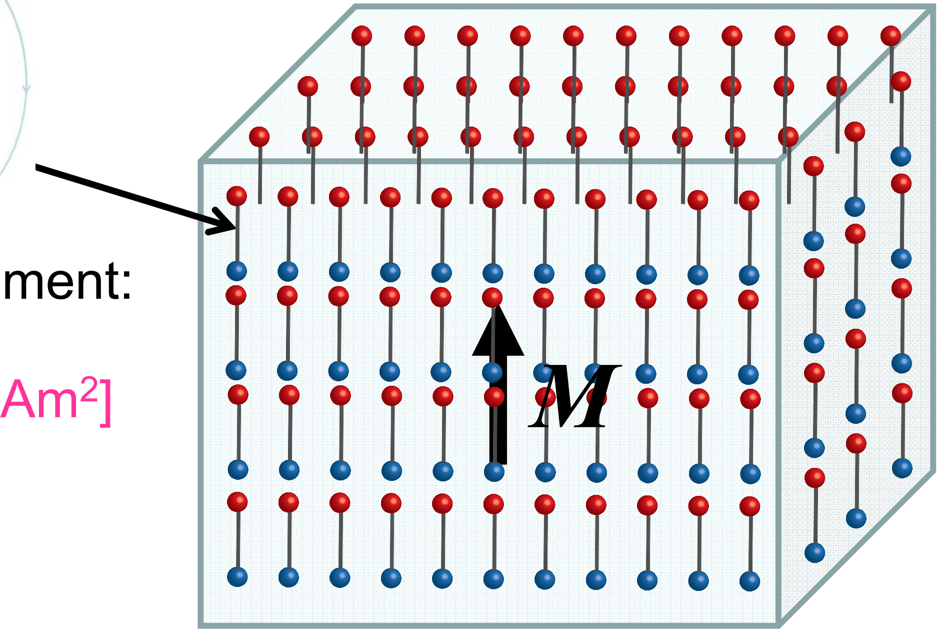
$$\vec{J}_S = \vec{M} \times \hat{n}$$ [A/m]

Magnetic Pole Model



Atomic magnetic moment:

$$\vec{m} = q_m \vec{d}$$ [Am^2]



What is H here?

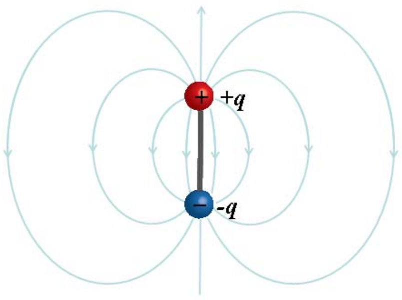
Atomic density: N/vol

[1/m^3]

Magnetic moment per unit volume:

$$\vec{M} \overset{\text{def}}{=} \frac{\vec{m}}{vol}$$ [A/m]

Equivalent Surface Pole Density

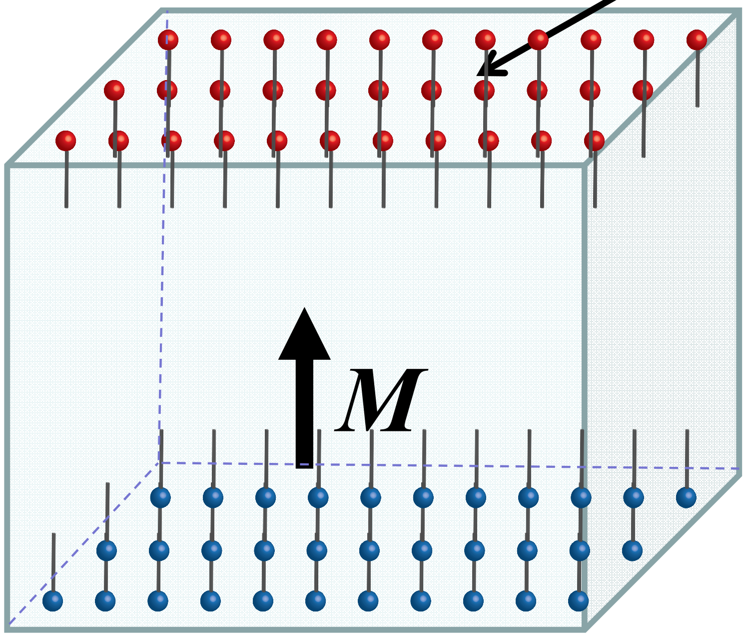


Atomic magnetic moment:

$$\vec{m} = q_m \vec{d} \qquad [\mathrm{Am}^2]$$

Atomic density: N/vol

$[1/\mathrm{m}^3]$



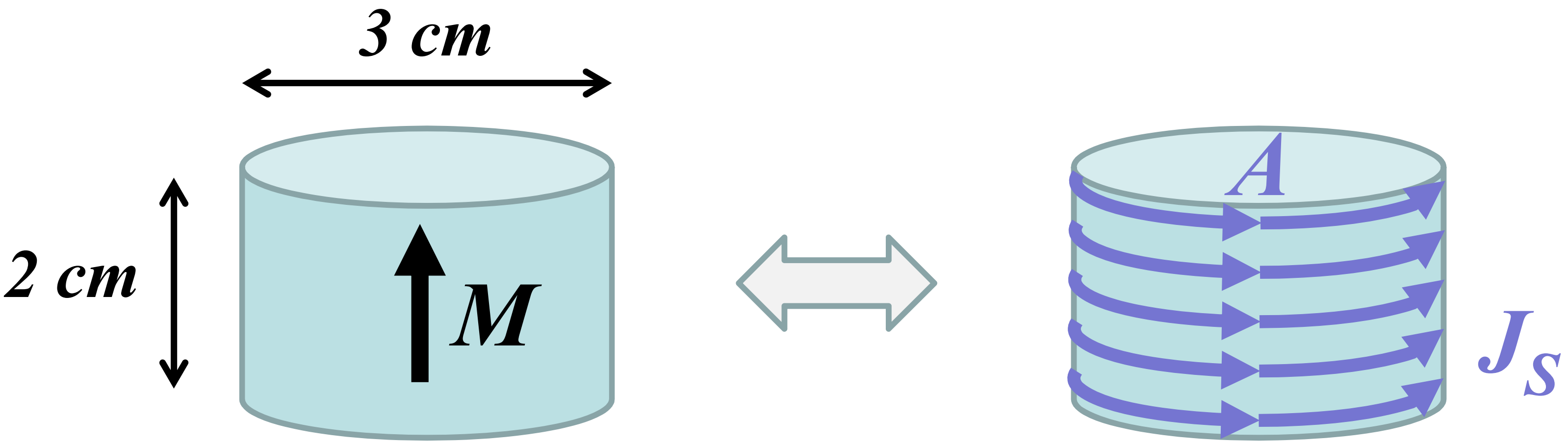
Surface charge density:

$$\rho_{ms} = \frac{q_m}{area} \qquad \frac{[\mathrm{Am}]}{[\mathrm{m}^2]}$$

Magnetic pole density:

$$\rho_{ms} = \vec{M} \cdot \hat{n} \qquad [\mathrm{A/m}]$$

Example: NdFeB Cylinder



$M = 10^6$ [A/m]

$vol = 14 \cdot 10^{-6}$ [m^3]

$m = 14$ [Am^2]

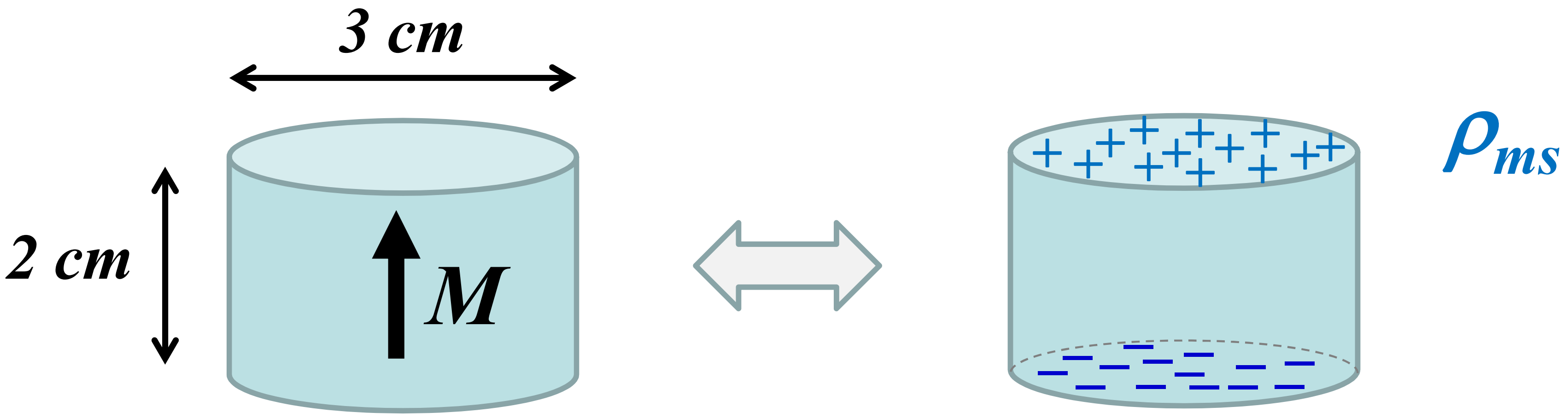
$J_S = 10^6$ [A/m]

$I = 20000$ [A]

$A = 7.07 \cdot 10^{-4}$ [m^2]

$m = IA = 14$ [Am^2]

Example: NdFeB Cylinder



$M = 10^6$ [A/m]

$vol = 14{\cdot}10^{-6}$ [m^3]

$m = 14$ [Am^2]

$\rho_{ms} = 10^6$ [A/m]

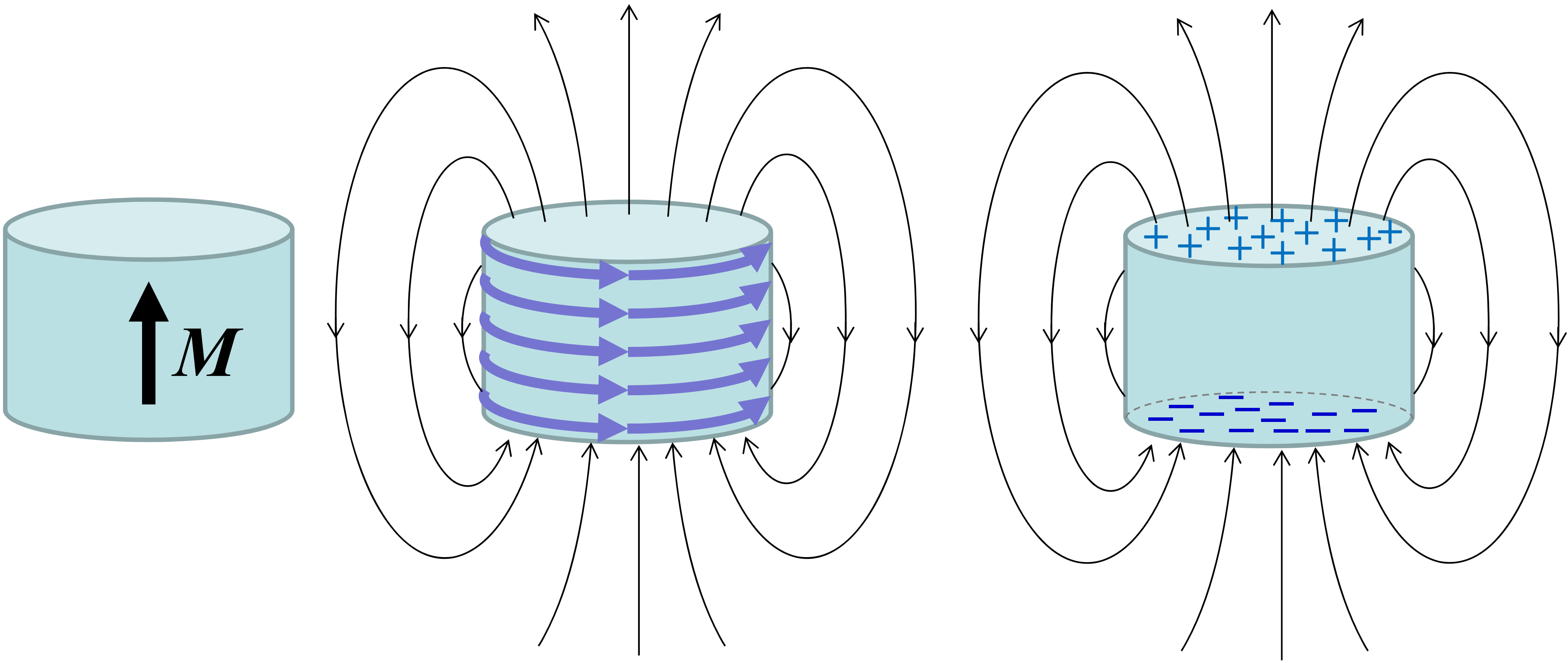
$A = 7{\cdot}10^{-4}$ [m^2]

$q_m = 700$ [Am]

$d = 2{\cdot}10^{-2}$ [m]

$m = q_m d = 14$ [Am^2]

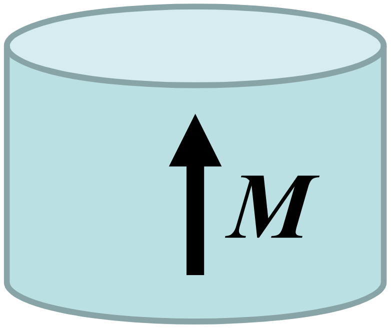
Example: NdFeB Cylinder

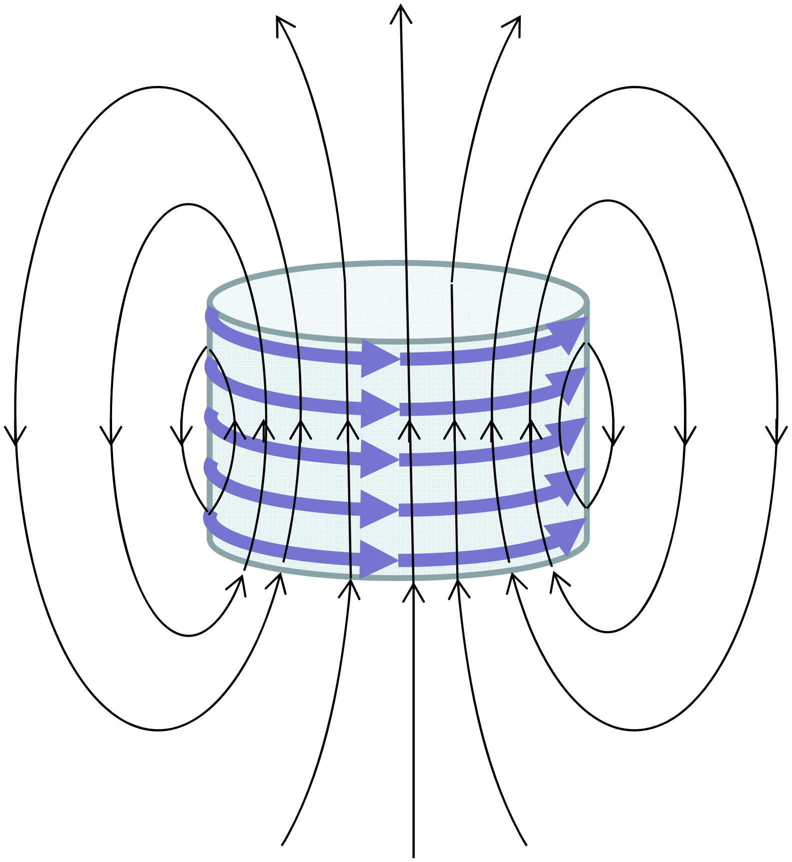


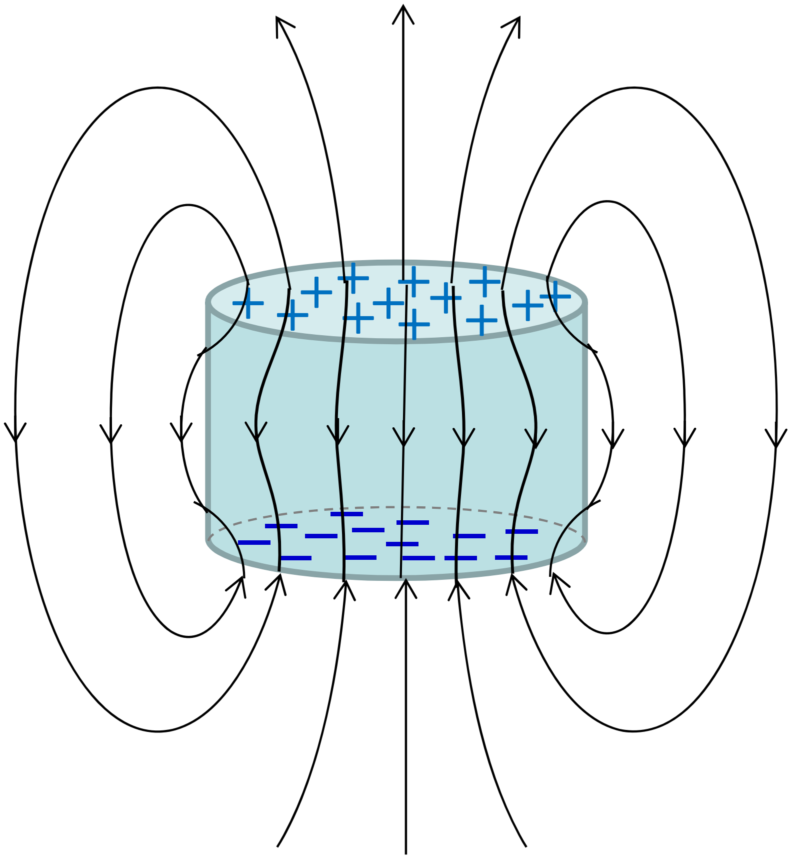
Result is the same external to magnetic material.

Hint: we are far away from all the dipoles!!!

Example: NdFeB Cylinder







Result is <u>different</u> <u>inside</u> the magnetic material.

The Constitutive Relation

$$\boxed{\vec{\boldsymbol{B}} = \boldsymbol{\mu_0}(\vec{\boldsymbol{H}} + \vec{\boldsymbol{M}})}$$

$\boldsymbol{B}$ = Magnetic flux density [Tesla]

$\boldsymbol{H}$ = Magnetic field, “Magnetizing force” [A/m]

$\boldsymbol{M}$ = Magnetization [A/m]

$\boldsymbol{\mu_0}$ = Magnetic constant, “Permeability of free space” [Tesla-m/A] [Henry/m] [N/A^2]

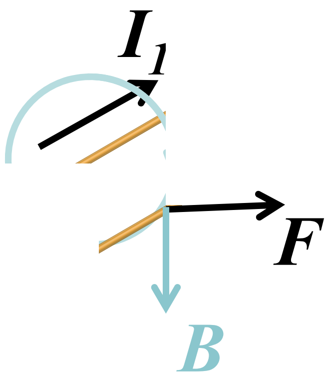
Note, in vacuum, $\boldsymbol{M}$ is zero:

$$\vec{\boldsymbol{B}} = \boldsymbol{\mu_0}\vec{\boldsymbol{H}}$$

What is μ_o ?

μ_0 comes from the SI definition of the Ampere:

> The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per meter of length.



$$B = \mu_0 H = \frac{\mu_0 I_1}{2\pi r}$$

$$\vec{F} = I_2 \vec{L} \times \vec{B}$$

Maxwell's Equations (Magnetostatics)

Differential form

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = \vec{J}$$

Integral form

$$\oiint \vec{B} \cdot d\vec{s} = 0$$

$$\oint \vec{H} \cdot d\vec{l} = I$$

With:

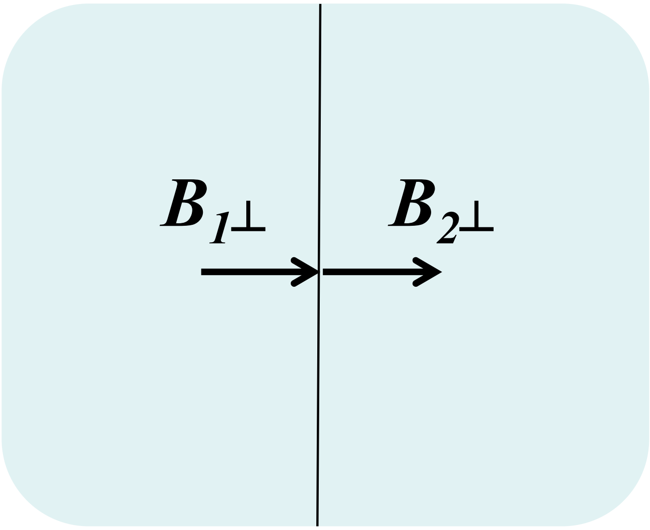
$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

$$\nabla \cdot \vec{B} = \mu_0(\nabla \cdot \vec{H} + \nabla \cdot \vec{M})$$

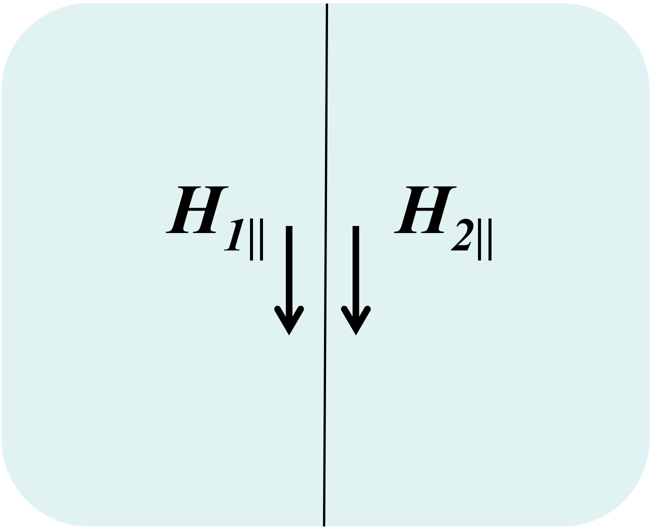
$$\nabla \cdot \vec{H} = -\nabla \cdot \vec{M}) = \rho_m$$ [A/m^2]

Magnetic charge density
i.e. magnetic charge/volume

Boundary Conditions

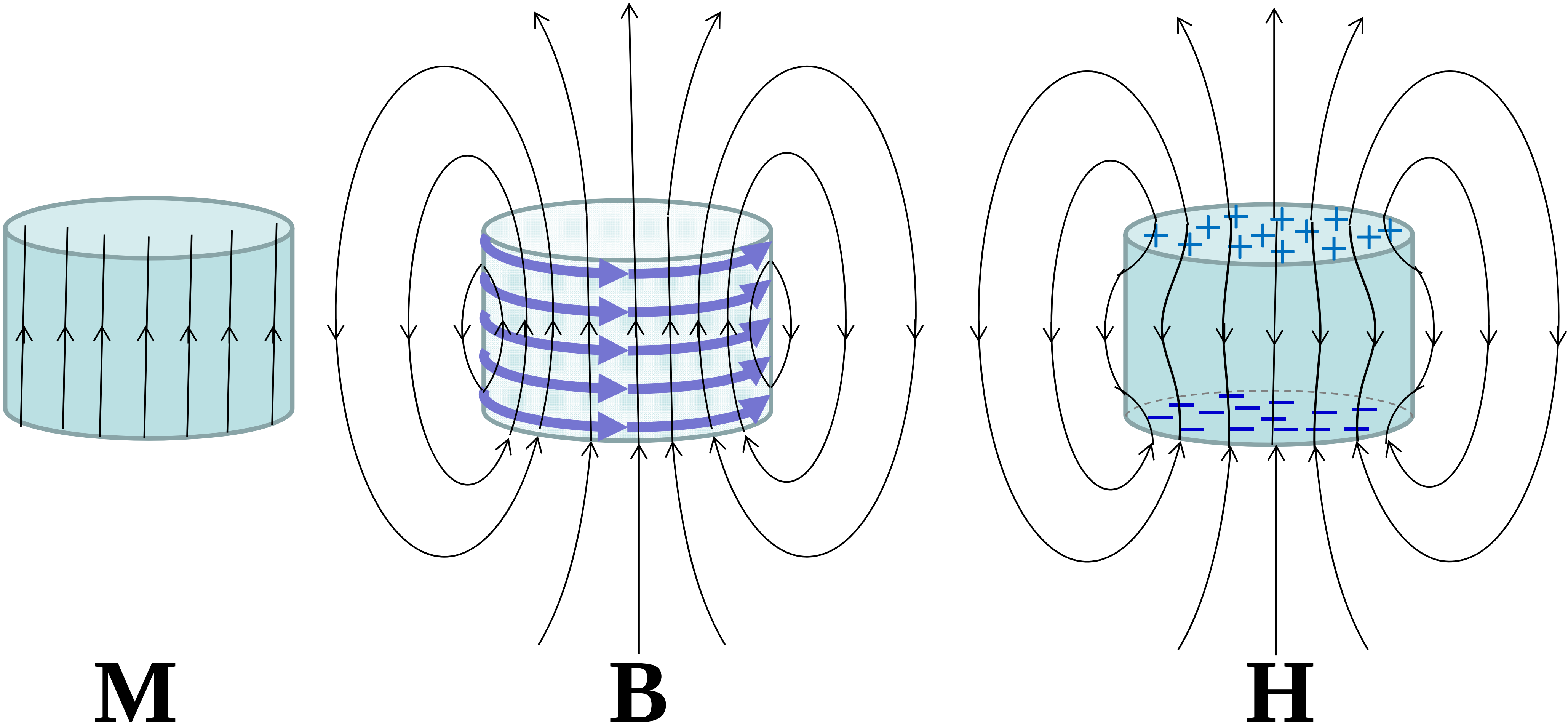


The perpendicular component of B is continuous across a boundary



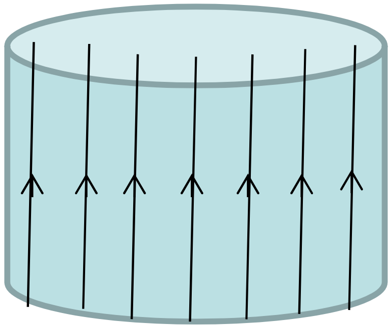
The transverse component of H is continuous across a boundary (unless there is a true current on the boundary)

Example: NdFeB Cylinder

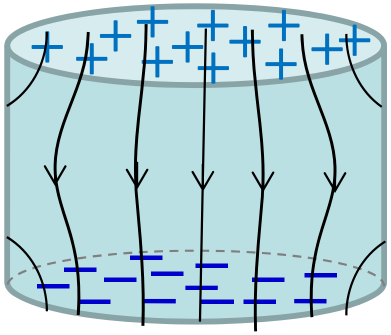


$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

Demagnetizing Fields



$\mathbf{M}$



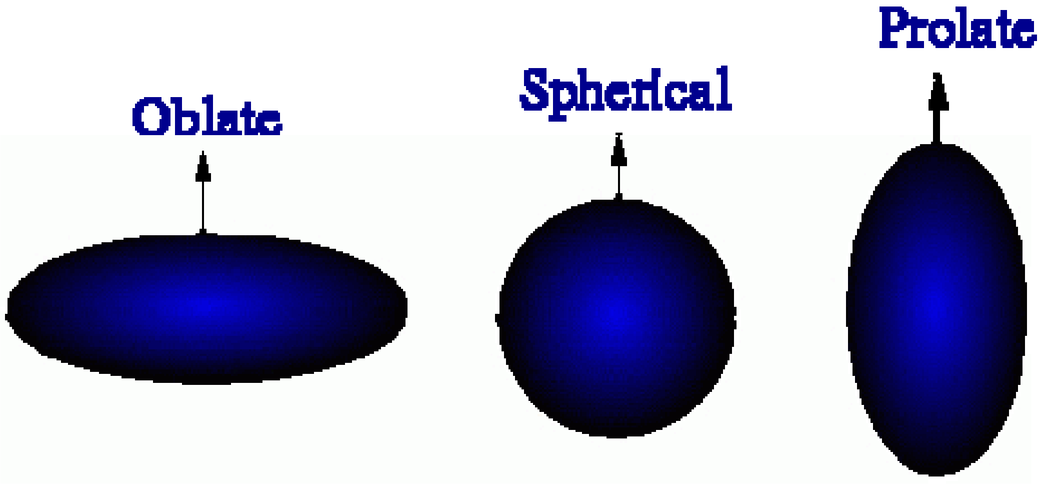
$\mathbf{H_d}$

$$\boldsymbol{H_d} \propto \boldsymbol{M}$$

$$\boldsymbol{H_d} = -\boldsymbol{N}\boldsymbol{M}$$

Demagnetizing Factor

Demagnetizing Factors



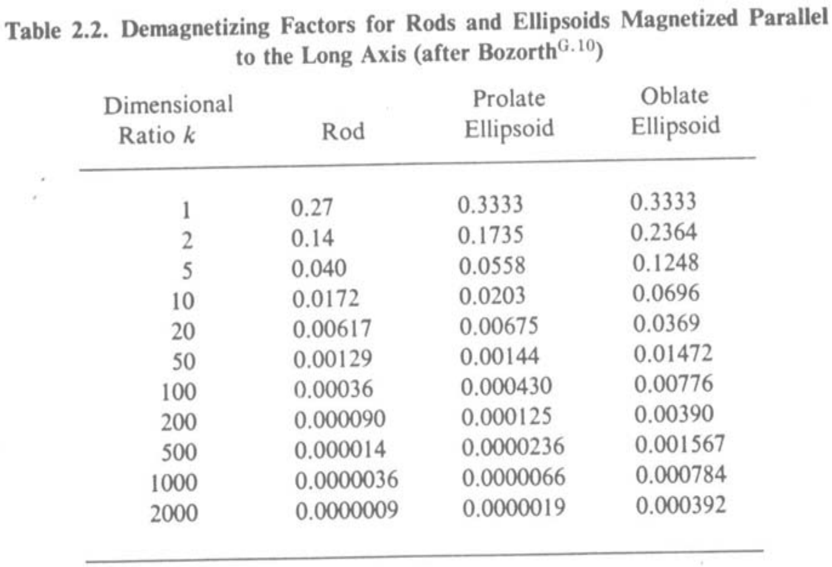


Table 2.2. Demagnetizing Factors for Rods and Ellipsoids Magnetized Parallel to the Long Axis (after Bozorth$^{G.10}$)

Dimensional Ratio *k*	Rod	Prolate Ellipsoid	Oblate Ellipsoid
1	0.27	0.3333	0.3333
2	0.14	0.1735	0.2364
5	0.040	0.0558	0.1248
10	0.0172	0.0203	0.0696
20	0.00617	0.00675	0.0369
50	0.00129	0.00144	0.01472
100	0.00036	0.000430	0.00776
200	0.000090	0.000125	0.00390
500	0.000014	0.0000236	0.001567
1000	0.0000036	0.0000066	0.000784
2000	0.0000009	0.0000019	0.000392

Demag. fields in ellipsoids of revolution are uniform.

$$\boldsymbol{H_d = -NM}$$

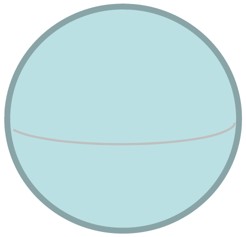
$$\boldsymbol{N_x + N_y + N_z = 1}$$

Demagnetizing Factors: Special Cases

$$H_d = -NM$$

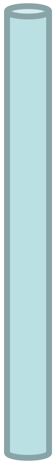
$$N_x + N_y + N_z = 1$$

Sphere



$$N_x = N_y = N_z = \frac{1}{3}$$

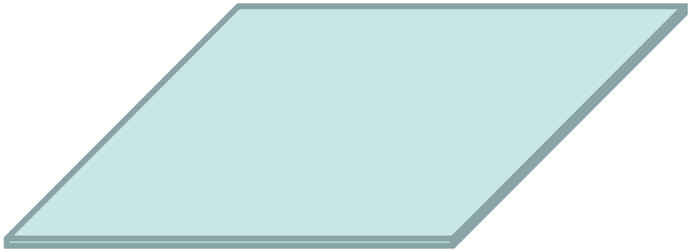
Long Rod



$$N_x = N_y = \frac{1}{2}$$

$$N_z = 0$$

Thin Sheet



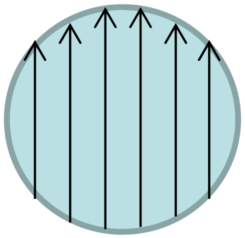
$$N_x = N_y = 0$$

$$N_z = 1$$

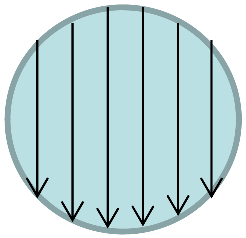
Example: Ba-Ferrite Sphere

Barrium Ferrite:

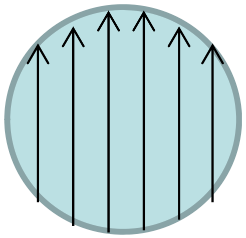
$$M \sim 150 \text{ [kA/m]}$$



M



H



B

$$N_x = N_y = N_z = \frac{1}{3}$$

$$H_d = -NM$$

$$H_d = -50 \text{ [kA/m]}$$

$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

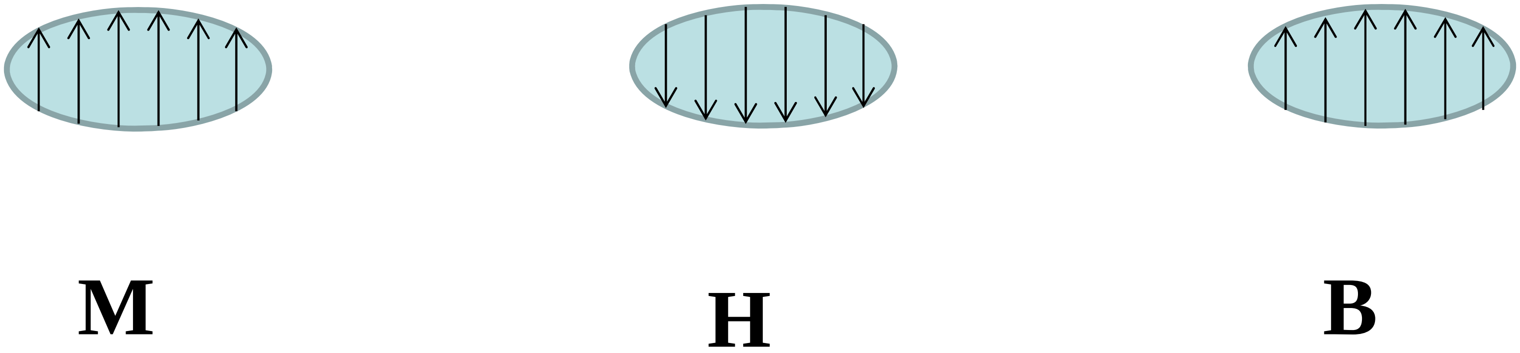
$$\vec{B} = \mu_0(100 \text{ [kA/m]})$$

$$\vec{B} = 0.126 \text{ [Tesla]}$$

Example: Ba-Ferrite Ellipsoid

Barrium Ferrite:

$\boldsymbol{M \sim 150}$ [kA/m]



$$\boldsymbol{N_x = N_y = 0.45}$$

$$\boldsymbol{H_d = -NM}$$

$\boldsymbol{H_d = -67.5}$ [kA/m]

$$\boldsymbol{\vec{B} = \mu_0(\vec{H} + \vec{M})}$$

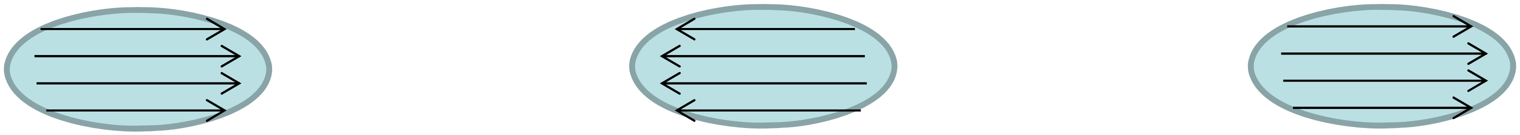
$\boldsymbol{\vec{B} = \mu_0(82.5}$ [kA/m] $\boldsymbol{)}$

$\boldsymbol{\vec{B} = 0.104}$ [Tesla]

Example: Ba-Ferrite Ellipsoid

Barrium Ferrite:

$\boldsymbol{M \sim 150}$ [kA/m]



M

H

$$\boldsymbol{N_z = 0.1}$$

$$\boldsymbol{H_d = -NM}$$

$$\boldsymbol{H_d = -15}$$ [kA/m]

B

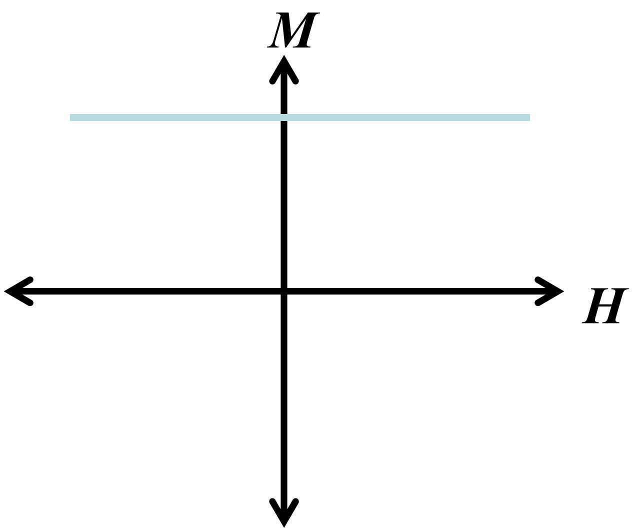
$$\boldsymbol{\vec{B} = \mu_0(\vec{H} + \vec{M})}$$

$$\boldsymbol{\vec{B} = \mu_0(135 \text{ [kA/m]})}$$

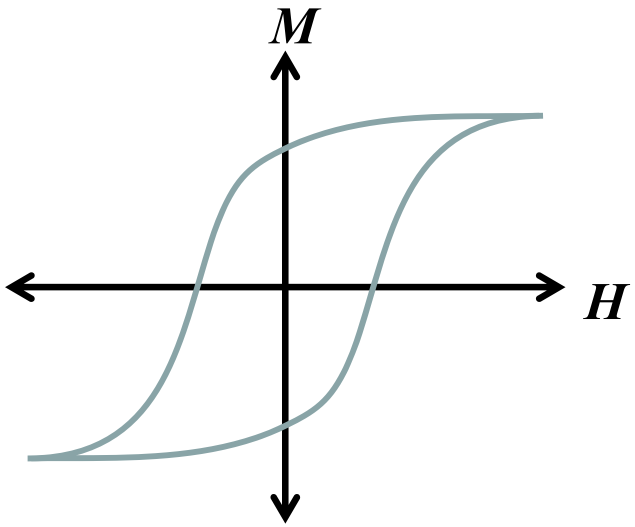
$$\boldsymbol{\vec{B} = 0.17}$$ [Tesla]

Susceptibility and Permeability

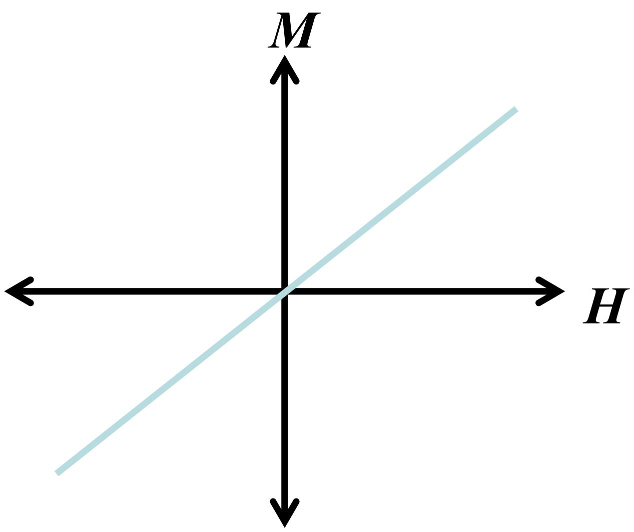
Ideal permanent magnet



Real, useful magnetic material



Ideal, linear soft magnetic material



$$\boldsymbol{M} = \boldsymbol{\chi H}$$

Susceptibility [unitless]

$$\vec{\boldsymbol{B}} = \boldsymbol{\mu_0}(\vec{\boldsymbol{H}} + \boldsymbol{\chi}\vec{\boldsymbol{H}})$$

$$\vec{\boldsymbol{B}} = \boldsymbol{\mu_0}(\boldsymbol{1} + \boldsymbol{\chi})\vec{\boldsymbol{H}}$$

$$\vec{\boldsymbol{B}} = \boldsymbol{\mu_0 \mu_r}\vec{\boldsymbol{H}}$$

Rel. Permeability [unitless]

Example, Long Rod in a Long Solenoid

I* =*1 A

N/L* = *1000/m

$\mu_r = 1000$ $\chi = 999$

$H = \frac{NI}{L}$ = 1 [kA/m]

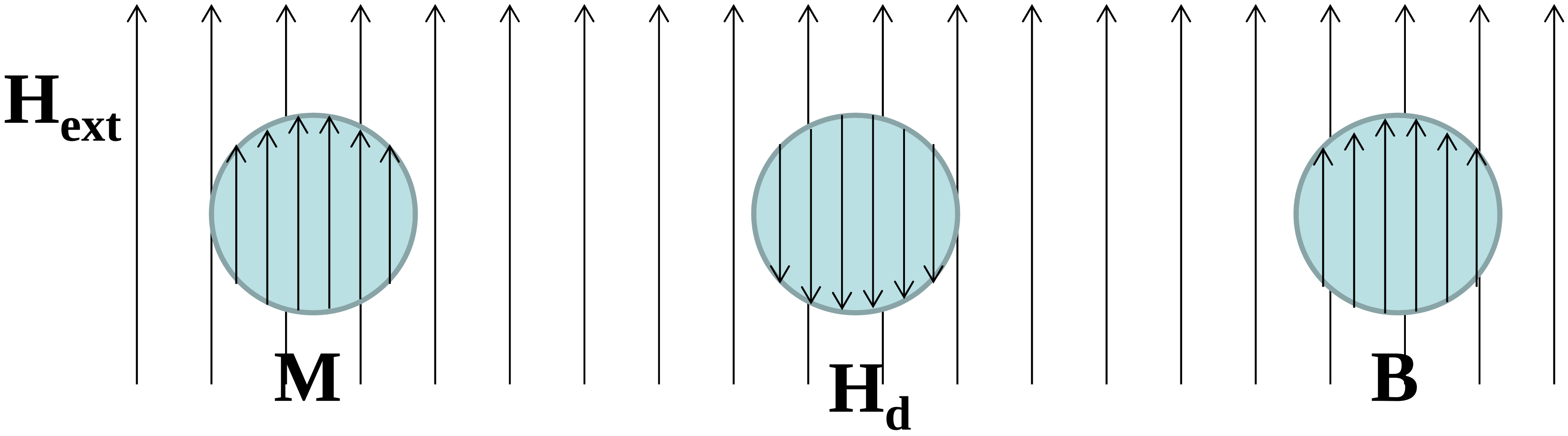
$M = \chi H = \chi \frac{NI}{L} =$ 999 [kA/m]

$B = \mu_0(H + M) = \mu_0$(1000kA/m)= 1.25 [Tesla]

or:

$B = \mu_0 \mu_r H$= 1.25 [Tesla]

Example, Soft Magnetic Ball in a Field

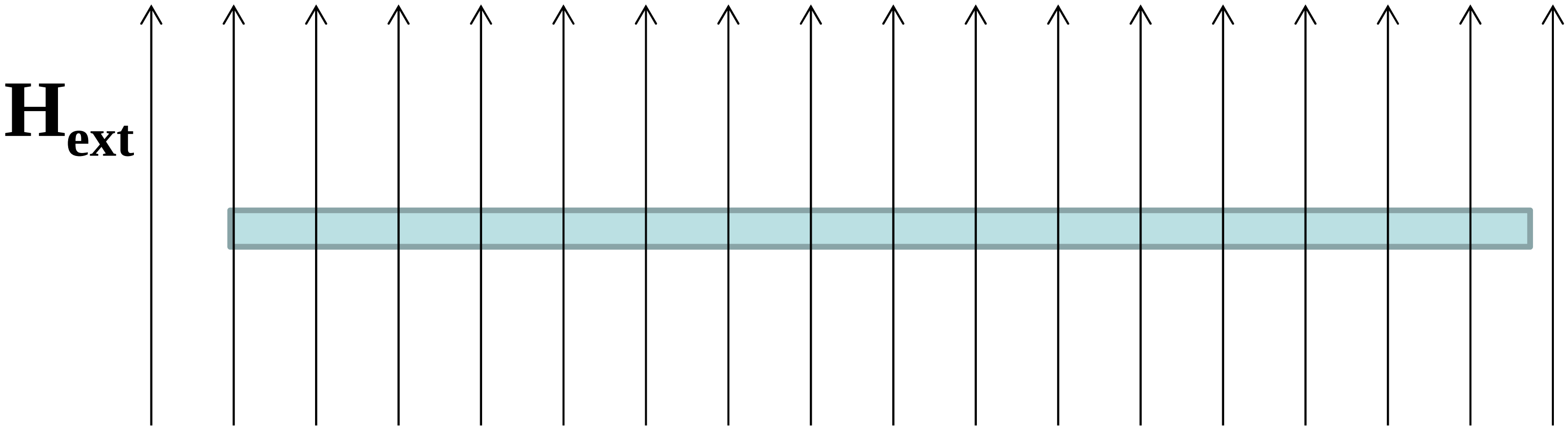


$$M = \chi H = \chi(H_{ext} + H_d) = \chi(H_{ext} - NM)$$

$$M = \frac{\chi}{1 + N\chi} H_{ext}$$

$$\chi_{eff} = \frac{\chi}{1 + N\chi} < 3!!!$$

Example, Thin Film in a Field



$$M = \chi H_{inside} = \chi(H_{ext} + H_d) = \chi(H_{ext} - M)$$

$$M = \frac{\chi}{1+\chi} H_{ext}$$

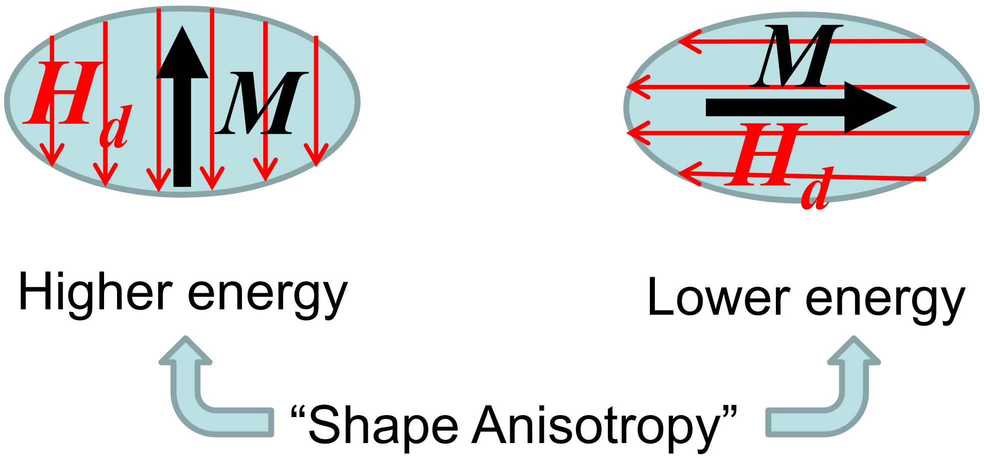
$$H_{inside} = \frac{1}{1+\chi} H_{ext}$$

$$B_{inside} = B_{ext}$$

Why?

Magnetic Energy and Shape Anisotropy

$$E = \iiint_{vol} \mu_0 \vec{M} \cdot \vec{H} dv$$

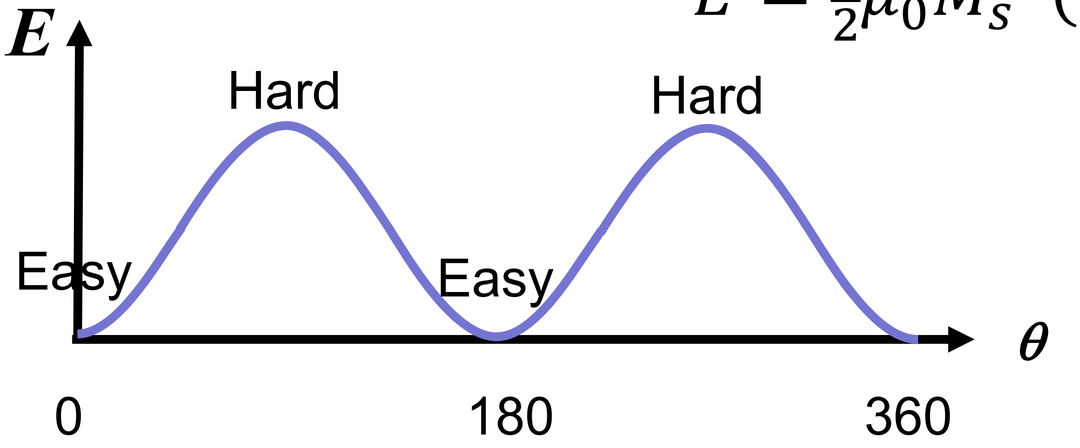


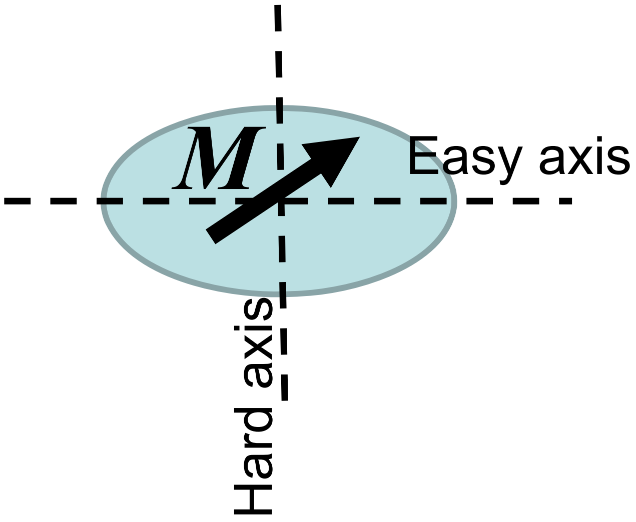
Higher energy

Lower energy

“Shape Anisotropy”

$$E = \frac{1}{2}\mu_0 {M_s}^2 (N_x - N_Y) sin^2(\theta)$$





Coffee Break!