Part II - Mechanisms and examples

- Magnetization switching
- Ferromagnetic resonance
- Spin-transfer-driven magnetization dynamics
- Non-uniform magnetization configurations
- Thermal fluctuations

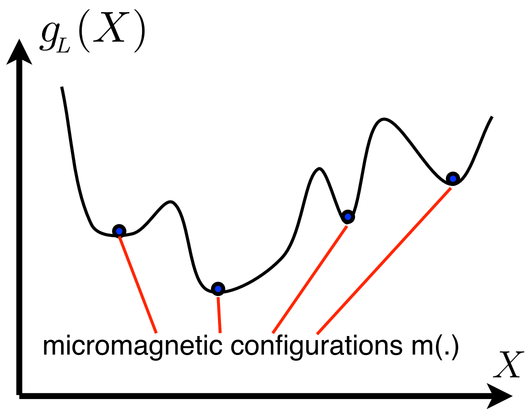
Disorder versus dynamical nonlinearities

$$g_L(\mathbf{m}(.);\mathbf{h}_a,T)=\int_V\left(l_{EX}^2\frac{(\nabla\mathbf{m})^2}{2}-\kappa\frac{(\mathbf{m}\cdot\mathbf{e}_{AN})^2}{2}-\frac{1}{2}\mathbf{h}_M\cdot\mathbf{m}-\mathbf{h}_a\cdot\mathbf{m}\right)\frac{dV}{V}$$

$$\frac{\partial\mathbf{m}}{\partial t}-\alpha\,\mathbf{m}\times\frac{\partial\mathbf{m}}{\partial t}=-\mathbf{m}\times\mathbf{h}_{\mathrm{eff}}$$

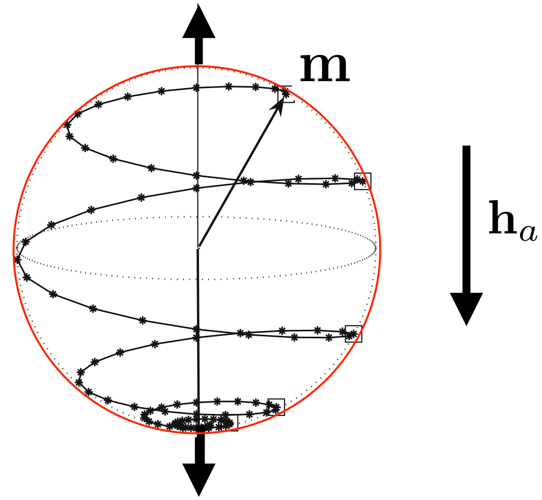
magnetization processes dominated by disorder

- domain nucleation
- domain wall motion
- rate-independent hysteresis
- ...



magnetization processes dominated by dynamical nonlinearities

- magnetization switching
- magnetization precession
- spin-transfer-driven dynamics
- ...

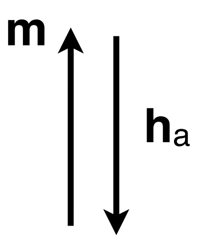


Magnetization switching

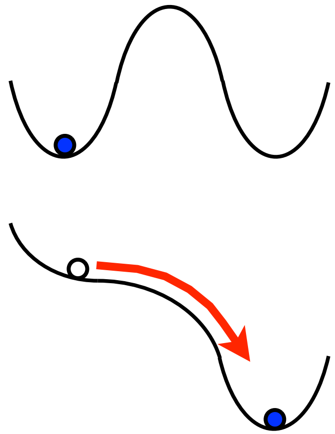
- In the micromagnetic approach, the central notion is that of stable equilibrium state: when stability is lost, it is tacitly understood that the system will "jump" to some other stable state.
- What occurs to the system when stability is lost is a dynamical aspect that must be studied by considering the equations for the magnetization dynamics (LLG equation).
- By the term "switching" one means situations where there are only two states involved in the process: the initial state which becomes unstable and a second stable state (typically with reversed magnetization) which the system reaches (switching mechanism) after a certain characteristic time (switching time).
- The key problem is to predict under what field conditions and on what time scale switching will take place.
- A number of questions of interest to applications can be made: Which are the conditions that will make the reversal particularly fast? How important is the role of damping? Are there cases where the magnetization remains spatially uniform during the reversal? Is the initial presence or lack of spatial uniformity a feature that significantly affects the conditions for magnetization reversal?

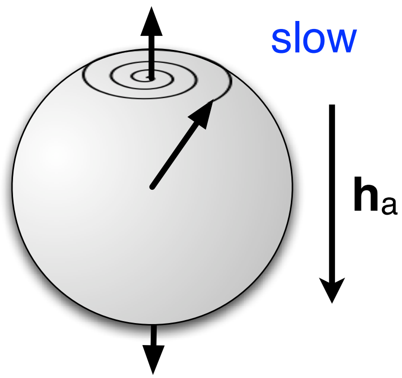
Magnetization switching strategies

- Going over energy barriers: damping-dominated switching



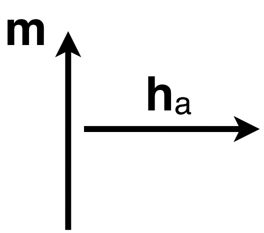
- small initial torque
- long switching time
- switching is guaranteed if the field is applied for a sufficiently long time
- damping plays a dominant role



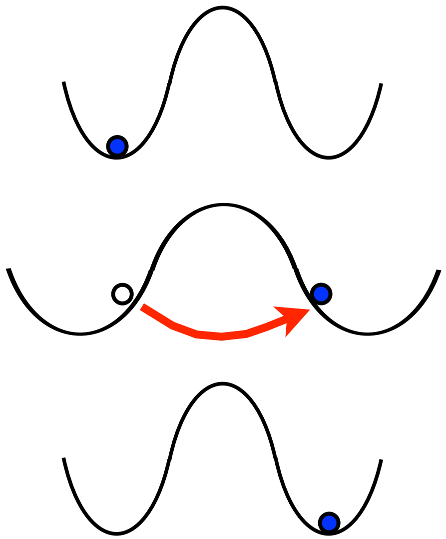


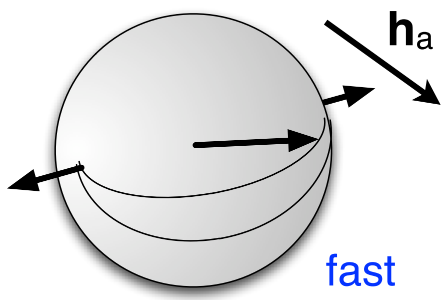
$$\boxed{\frac{dg_L}{dt} = -\alpha \left|\mathbf{m} \times \mathbf{h}_{\mathrm{eff}}\right|^2}$$

- Going around energy barriers: precessional switching



- large initial torque
- short switching time
- switching occurs only if a field pulse of suitable duration is applied
- damping plays no role (quasi-conservative dynamics)





$$\boxed{\frac{d\mathbf{m}}{dt} = -\mathbf{m} \times \mathbf{h}_{\mathrm{eff}}}$$

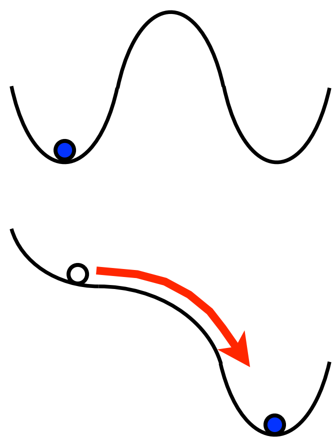
Damping-dominated switching

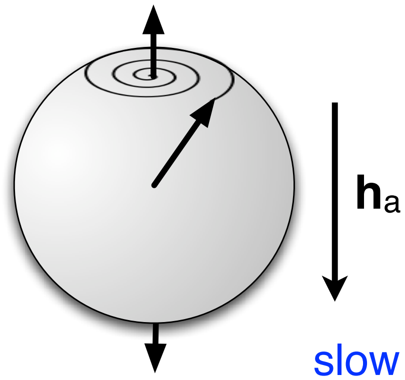
$(D_x,\ D_y,\ D_z)$ describe shape + crystal anisotropy

$$g_L\left(\mathbf{m};\mathbf{h}_\mathrm{a}\right)=\frac{1}{2}\left(D_x m_x^2+D_y m_y^2+D_z m_z^2\right)-\mathbf{h}_\mathrm{a}\cdot\mathbf{m}$$

external field

- Spheroidal particle, i.e., $D_x = D_y \equiv D_\perp$
- Easy axis is along z axis, i.e., $D_z < D_\perp$
- Initially, $\mathbf{m} = \mathbf{e}_z$, field $\mathbf{h}_\mathrm{a}$ is applied along -z direction

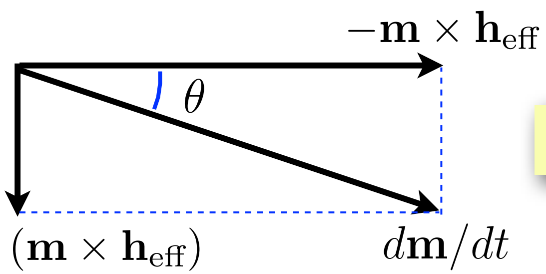




$$\frac{dg_L}{dt}=-\alpha\left|\mathbf{m}\times\mathbf{h}_\mathrm{eff}\right|^2$$

$$\frac{1}{\kappa_\mathrm{eff}}\frac{dm_z}{dt}=\alpha\left(m_z+\frac{h_a}{\kappa_\mathrm{eff}}\right)\left(1-m_z^2\right)$$

$\kappa_\mathrm{eff}=D_\perp - D_z \qquad h_a < -\kappa_\mathrm{eff}$

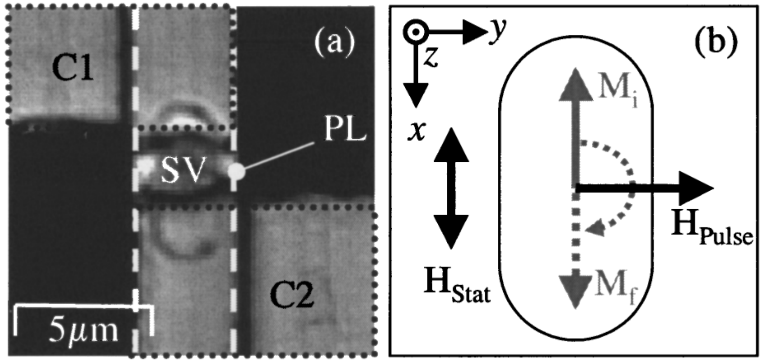


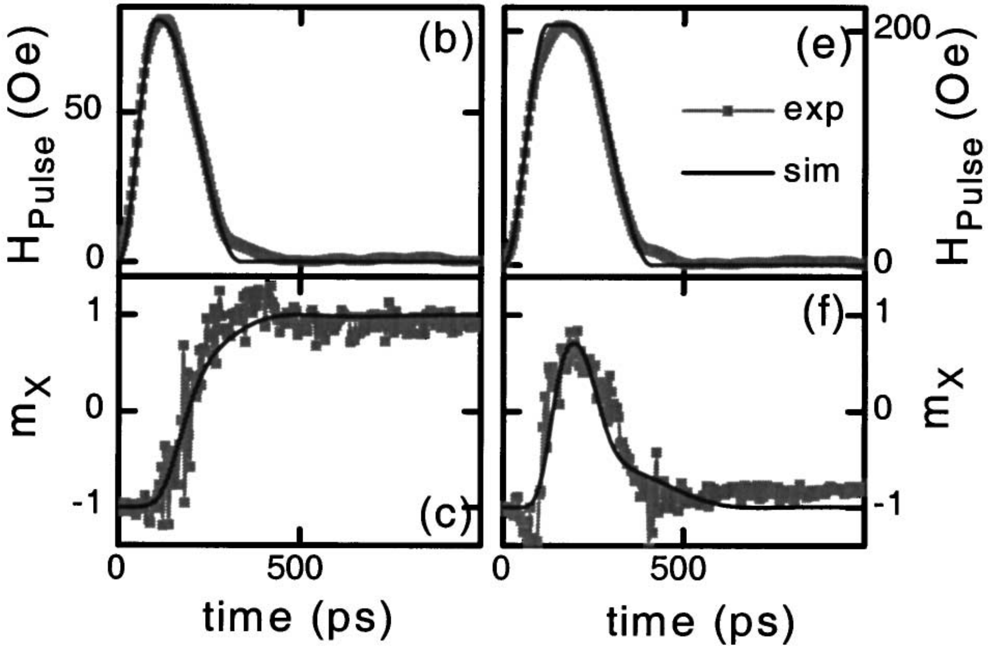
$$\tan\theta=\alpha$$

- When the initial state becomes unstable, the system starts to precess around the anisotropy axis and gradually approaches the reversed state. This approach is controlled by the damping constant α.
- Trajectories traversed under nonzero damping form a fixed angle with respect to the trajectories of the conservative dynamics.
- The magnetization motion during switching is the same for all systems if one uses $\kappa_\mathrm{eff}\,t$ as normalized time coordinate and h_a/κ_eff as normalized field intensity.

Precessional magnetization switching

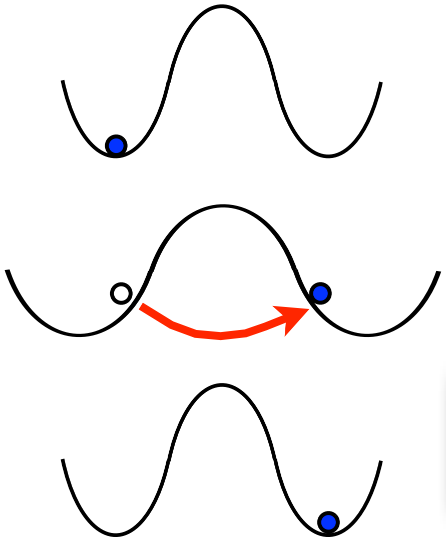
H. W. Schumacher et al.,
Phys. Rev. Lett. **90**, 017204 (2003)

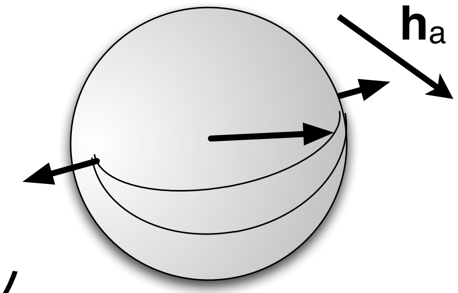




precessional magnetization dynamics is exploited, relaxation effects play a minor role

system is in initial state, field is switched on → precession under field → field is switched off, system is in reversed state

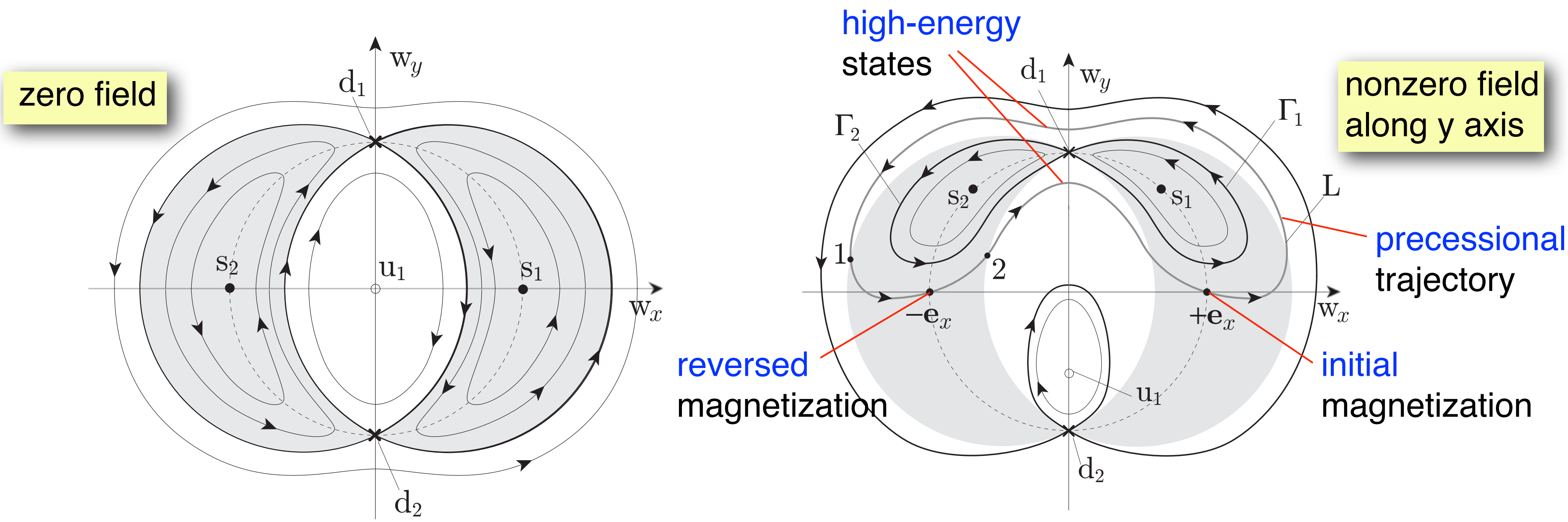




fast

$$\frac{d\mathbf{m}}{dt} = -\mathbf{m} \times \mathbf{h}_{\text{eff}}$$

Precessional magnetization switching



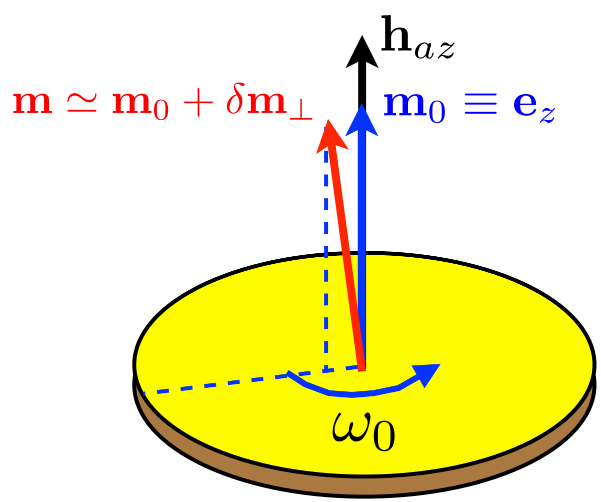
- Precessional switching is fast, so it is a good approximation to neglect the effect of damping and assume that the magnetization motion is conservative during the switching process.
- Precessional switching is possible when there exists a conservative trajectory under nonzero field visiting both of the states (initial and reversed magnetization) that are stable under zero field.
- Since the field induces a persistent precessional motion, precessional switching occurs only if the field is switched off at the right time.
- Therefore, there exists a regular sequence of time windows inside which the field should be switched off in order to produce switching.
- After the field is switched off, the magnetization reaches the final stable state of reversed magnetization by oscillations of decreasing amplitude controlled by damping ("ringing" effect).

Ferromagnetic resonance

- Magnetization tends to precess around the effective field.
- Because of stiffness brought about by exchange interactions, magnetization precession in ferromagnets tends to be coherent in space.
- There are situations, typically uniformly magnetized particles with convenient shape, subject to some external dc magnetic field, in which uniform and coherent precession of the magnetization around its equilibrium orientation is one of the natural oscillation modes for the system ("Kittel" mode).
- The associated precessional angular frequency ω_0 depends on the dc magnetic field as well as on the geometrical shape of the particle.
- Resonance occurs when a small rf magnetic field is applied transversally to the equilibrium magnetization and its frequency is equal to ω_0.
- When the rf field amplitude is increased, nonlinear effects appear in the form of distortions in the dynamic susceptibility curve ("foldover" effects).

Spatially uniform precession (Kittel mode)

- Ellipsoidal particle with principal axes along x, y, z; crystal anisotropy is neglected
- Demagnetizing factors are: D_x , D_y , D_z
- Demagnetizing field is spatially uniform if particle is uniformly magnetized
- There exist exact spatially uniform solutions to the LLG equation
- External dc magnetic field is applied along z axis
- $\mathbf{m}_0 \equiv \mathbf{e}_z$ is a stable equilibrium orientation for **m**



$$\boxed{\frac{d\mathbf{m}}{dt} = -\mathbf{m} \times \mathbf{h}_{\text{eff}} + (\text{damping})}$$

- we consider small motions $\delta\mathbf{m}_\perp$ around the equilibrium orientation $\mathbf{m}_0 \equiv \mathbf{e}_z$
- $\delta\mathbf{m}_\perp$ is a solution of the equation obtained by linearizing the LLG equation around $\mathbf{m}_0$
- $\delta\mathbf{m}_\perp$ lies in x-y plane because the magnitude of **m** must be preserved
- we neglect damping effects in the computation of $\delta\mathbf{m}_\perp$

$$\mathbf{h}_{\text{eff}} = (h_{az} - D_z m_z)\,\mathbf{e}_z - D_x m_x \mathbf{e}_x - D_y m_y \mathbf{e}_y$$

$$\mathbf{m} \simeq \mathbf{e}_z + \delta m_x\,\mathbf{e}_x + \delta m_y\,\mathbf{e}_y$$

$$\mathbf{h}_{\text{eff}} \simeq (h_{az} - D_z)\,\mathbf{e}_z - D_x \delta m_x\,\mathbf{e}_x - D_y \delta m_y\,\mathbf{e}_y$$

$$\frac{d}{dt}\begin{pmatrix} \delta m_x \\ \delta m_y \end{pmatrix} = \begin{pmatrix} 0 & -(h_{az} + D_y - D_z) \\ (h_{az} + D_x - D_z) & 0 \end{pmatrix} \begin{pmatrix} \delta m_x \\ \delta m_y \end{pmatrix}$$

$$\boxed{\frac{d^2 x}{dt^2} + \omega_0^2\, x = 0}$$

$$\boxed{\omega_0^2 = (h_{az} + D_x - D_z)\,(h_{az} + D_y - D_z)}$$

Kittel frequency

Kittel frequency

$$\omega_0^2 = (h_{az} + D_x - D_z)\,(h_{az} + D_y - D_z)$$

Kittel frequency depends on the geometrical shape of the particle, but it is independent of its absolute linear dimensions

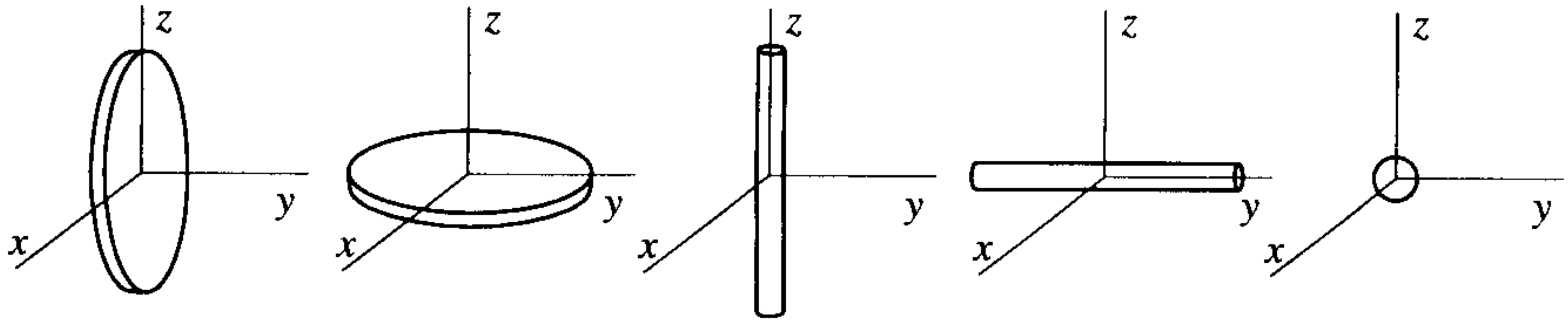
Examples of Kittel frequency for a soft material with no crystal anisotropy

$$D_x + D_y + D_z = 1$$

$$\omega_0 = h_{az} - 1$$

$$\omega_0 = h_{az} + 1/2$$

$$\omega_0 = h_{az}$$

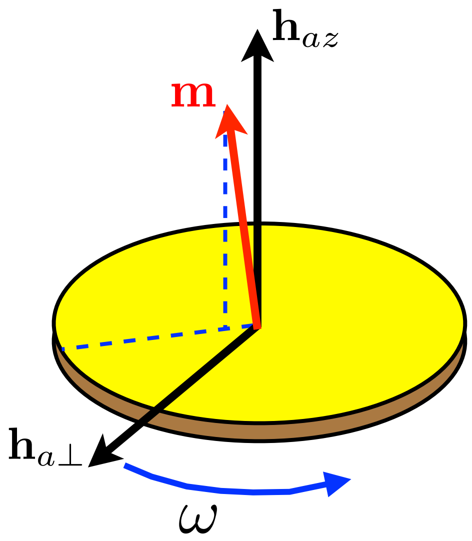


$$\omega_0 = \sqrt{h_{az}(h_{az} + 1)}$$

$$\omega_0 = \sqrt{h_{az}(h_{az} - 1/2)}$$

Ferromagnetic resonance curves

- spheroidal symmetry: $D_x = D_y \equiv D_\perp$
- circularly polarized rf magnetic field $\mathbf{h}_{a\perp}(t)$ is applied in addition to the dc field
- magnetization response to the rf field is computed from the linearized equation for the forced motion under $\mathbf{h}_{a\perp}(t)$
- damping must be included in the computation



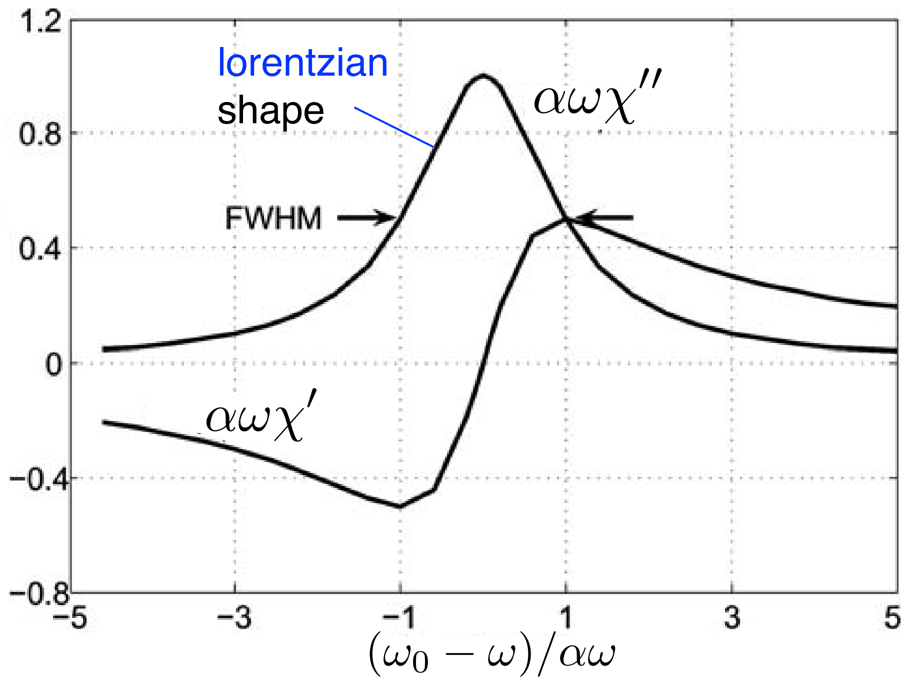
$$\boxed{\frac{d\mathbf{m}}{dt} - \alpha\,\mathbf{m} \times \frac{d\mathbf{m}}{dt} = -\mathbf{m} \times \mathbf{h}_{\text{eff}}}$$

$$\mathbf{h}_{\text{eff}} = (h_{az} - D_z m_z)\,\mathbf{e}_z - D_\perp \mathbf{m}_\perp + \mathbf{h}_{a\perp}(t)$$

$$\mathbf{h}_{a\perp}(t) = h_{a\perp}\,(\mathbf{e}_x\,\cos\omega t + \mathbf{e}_y\,\sin\omega t)$$

$$\omega_0 = h_{az} + D_\perp - D_z$$

$$\boxed{\text{absorbed power} = \frac{\alpha\omega^2 h_{a\perp}^2}{(\omega - \omega_0)^2 + \alpha^2\omega^2}}$$

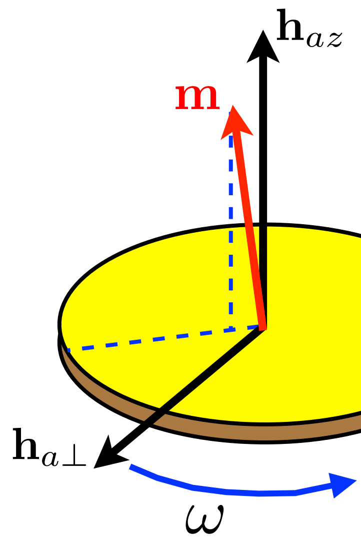


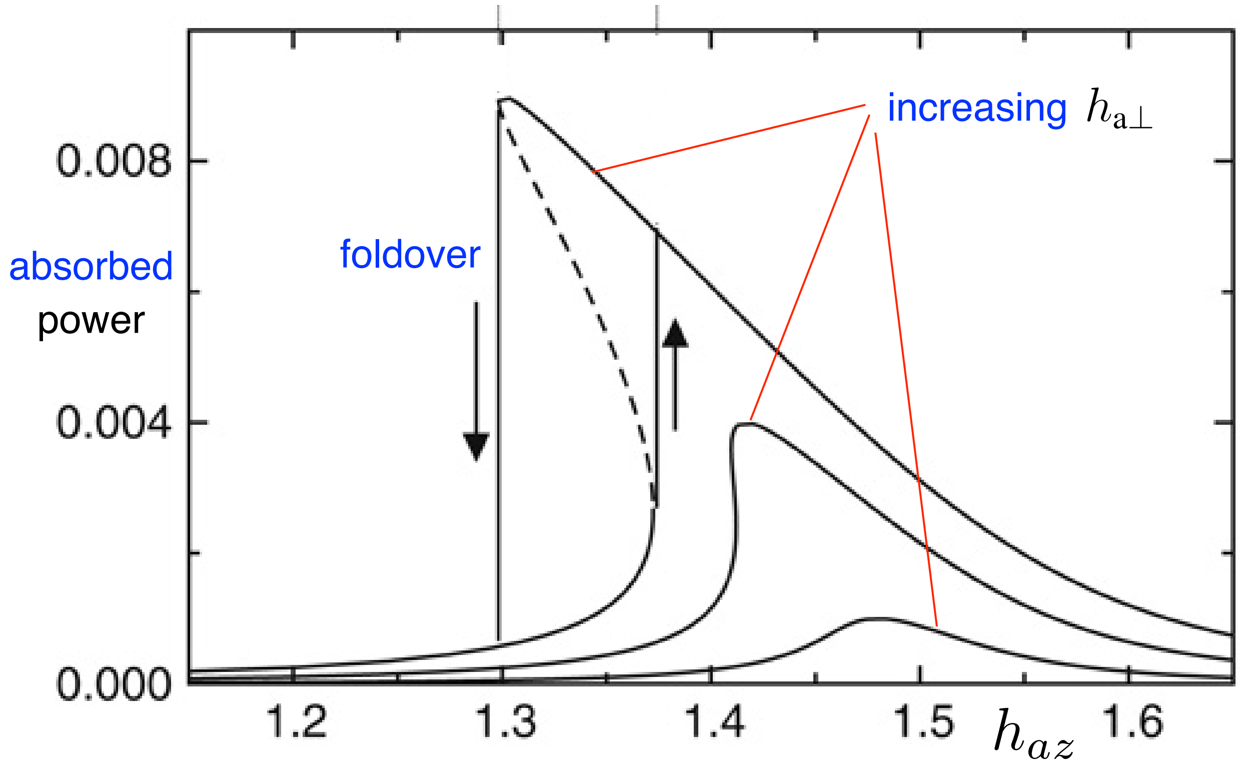
Nonlinear effects (foldover)

$$\frac{d\mathbf{m}}{dt} - \alpha\,\mathbf{m} \times \frac{d\mathbf{m}}{dt} = -\mathbf{m} \times \mathbf{h}_{\text{eff}}$$

$$\boxed{\frac{d\mathbf{m}}{dt} = \omega_0\,(m_z)\,\mathbf{e}_z \times \mathbf{m} - \mathbf{m} \times \left[\mathbf{h}_{a\perp}(t) - \alpha\frac{d\mathbf{m}}{dt}\right]}$$

$$\omega_0\,(m_z) = [h_{az} + (D_\perp - D_z)\,m_z]$$





$\omega = 0.5$

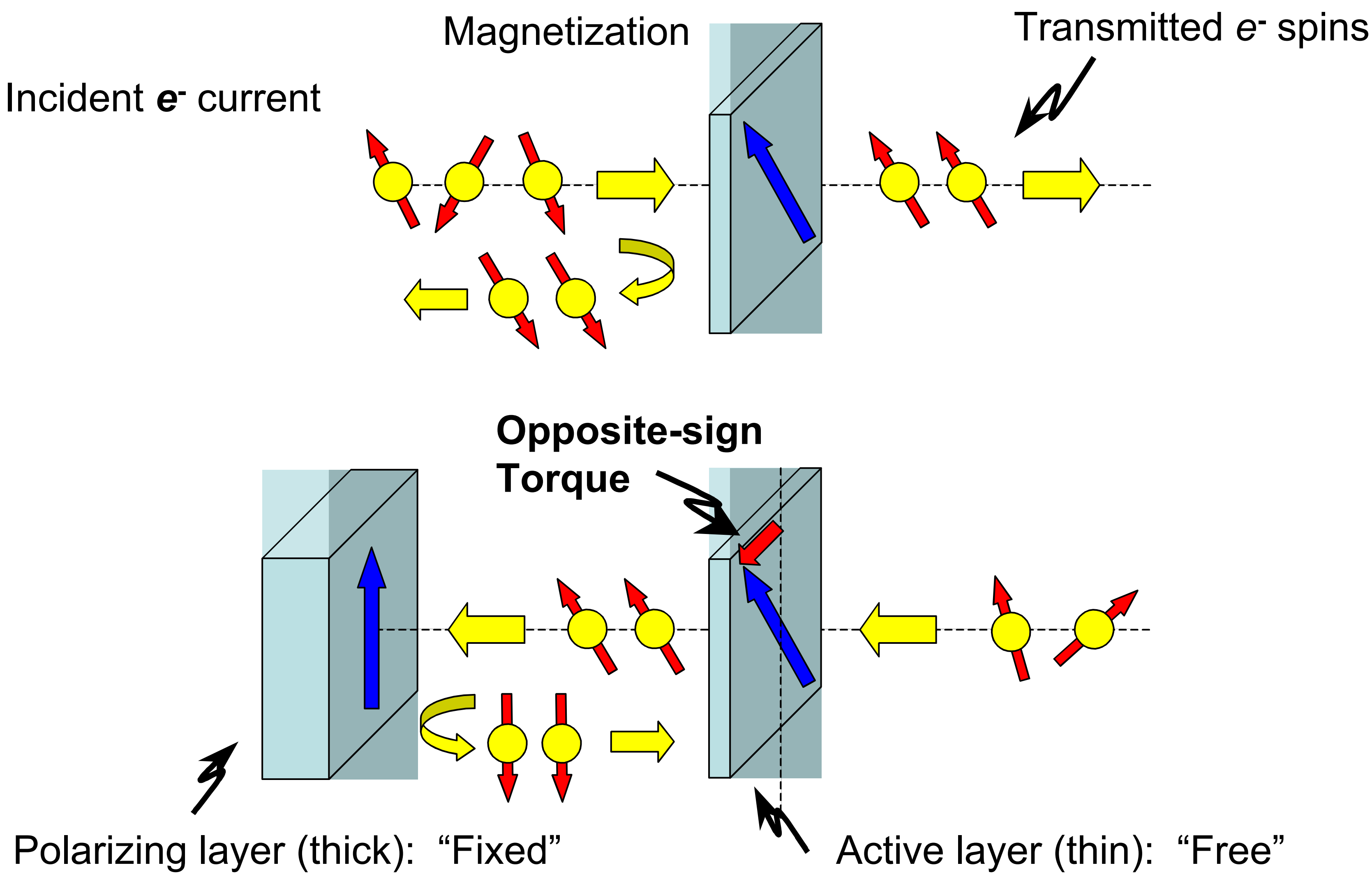
$D_z = 1$

$D_\perp = 0$

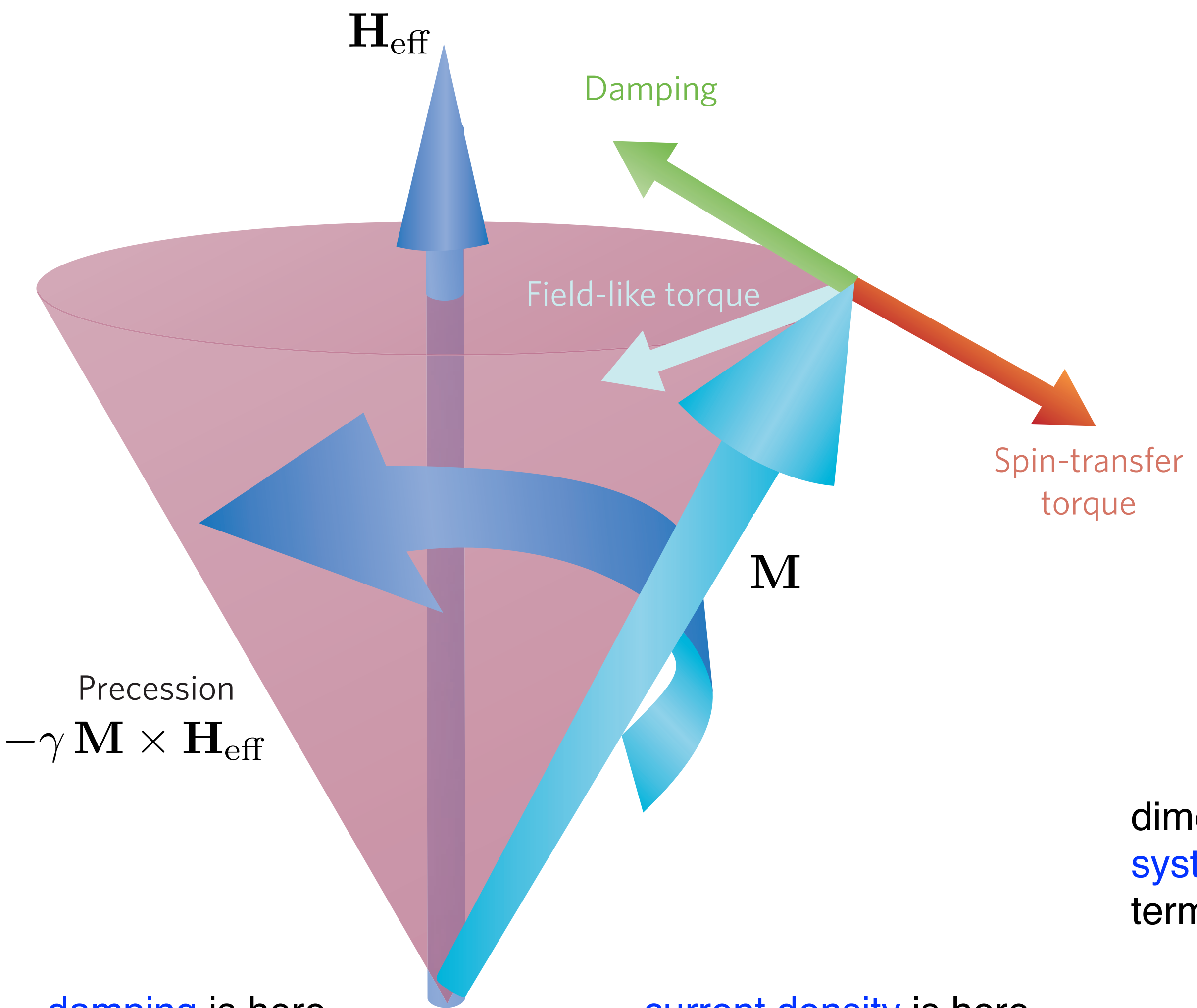
$\alpha = 0.1$

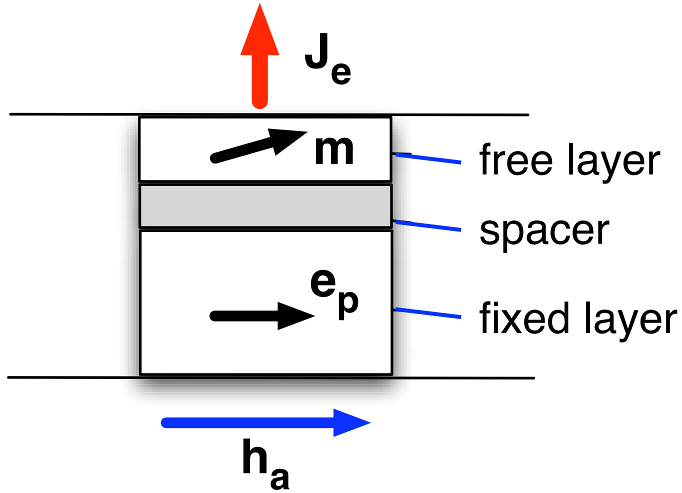
Spin transfer: angular momentum absorption

J. C. Slonczewski,
J. Magn. Magn. Mater. **159**, L1 (1996)



Spin-transfer-driven magnetization dynamics





damping is here

current density is here

$$\boxed{\frac{d\mathbf{m}}{dt} - \alpha\,\mathbf{m}\times\frac{d\mathbf{m}}{dt} = -\mathbf{m}\times\mathbf{h}_{\text{eff}} + \beta\,\mathbf{m}\times(\mathbf{m}\times\mathbf{e}_p)}$$

spin-transfer torque

dimensionless
system-dependent
term ~ 1

current density

$$\beta = b_p\,\frac{J_e}{J_p}$$

free-layer
thickness

$$J_p = \mu_0 M_s^2\,\frac{|e|d}{\hbar}$$

$$J_p \simeq 10^9\,\mathrm{A\,cm^{-2}}$$

Spin-transfer-induced phenomena

spin transfer is strong enough to reverse the free-layer magnetization

→

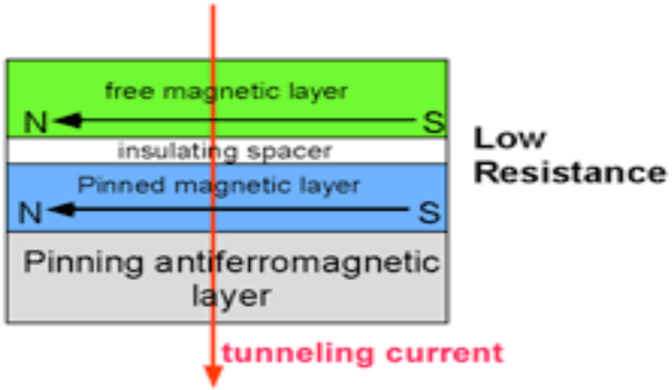
current-induced magnetization switching

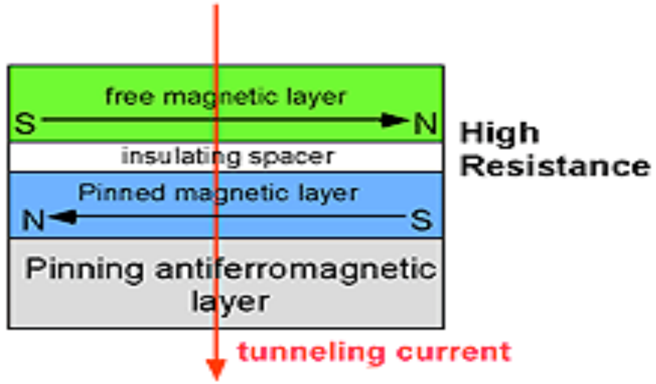
spin transfer is of the right strength so as to compensate for the effect of damping

→

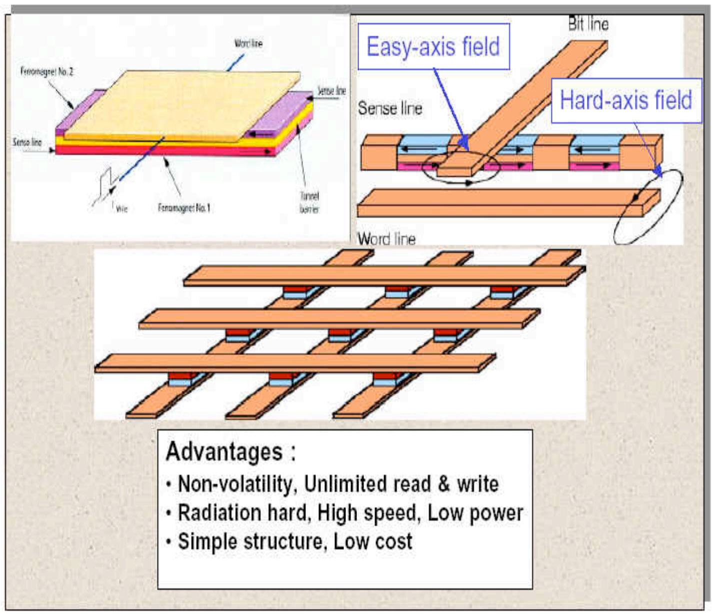
current-induced magnetization microwave oscillations

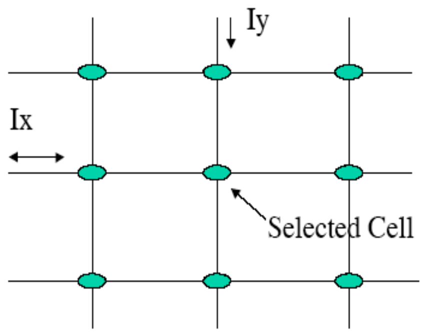
Magnetic tunnel junctions and MRAMs

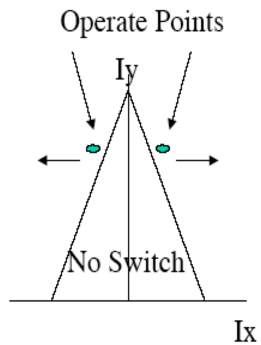




MRAM Cell Architecture

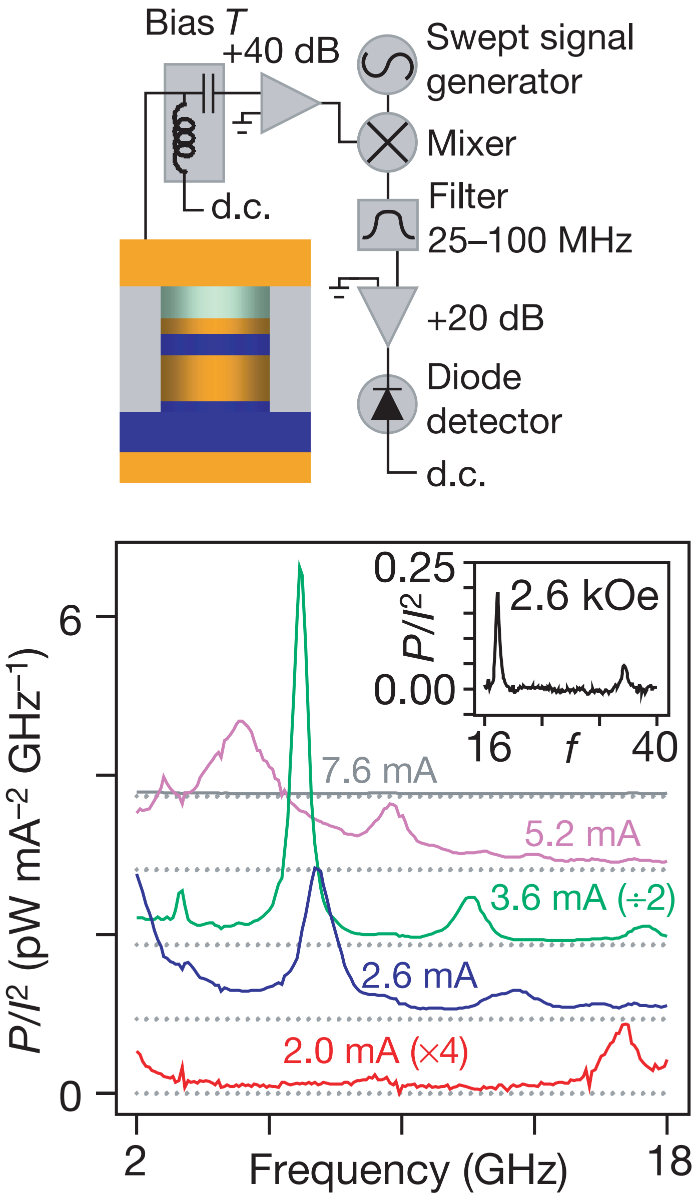






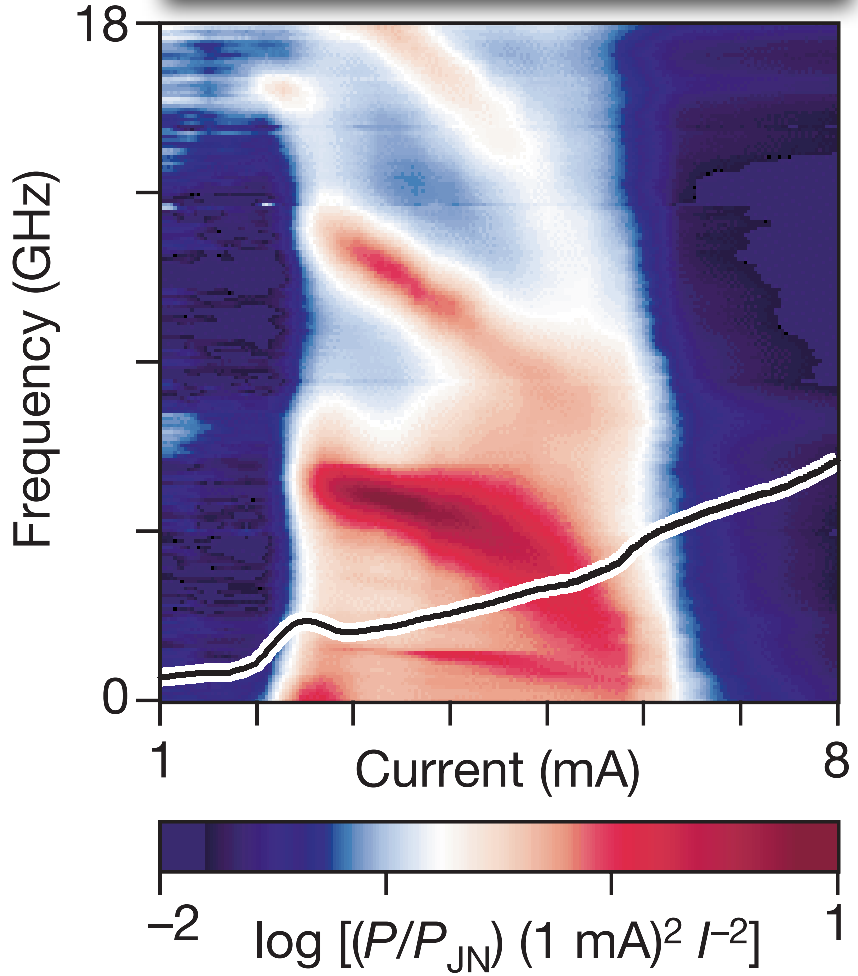
Microwave oscillations in nanopillars

S. I. Kiselev et al., Nature **425**, 380 (2003)



70 x 130 nm^2 elliptic nanopillar

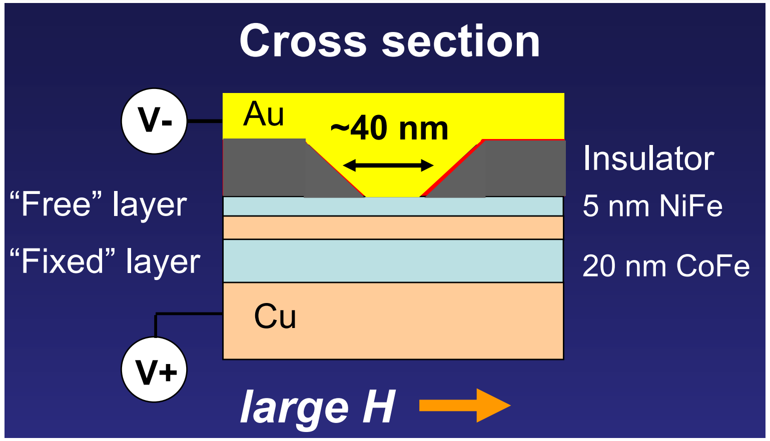
40 nm Co / 10 nm Cu / 3 nm Co
room temperature

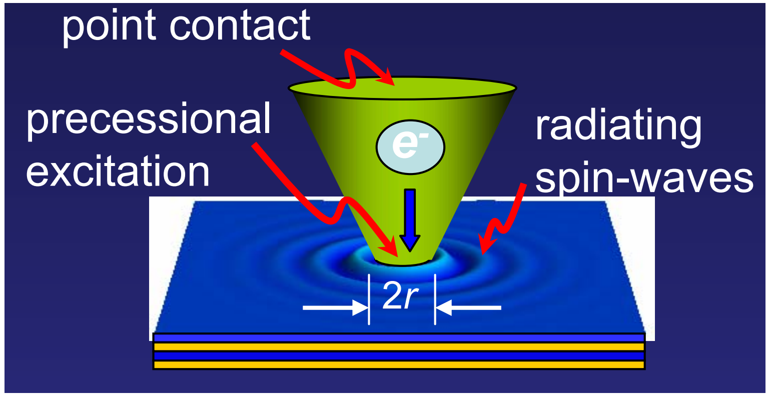


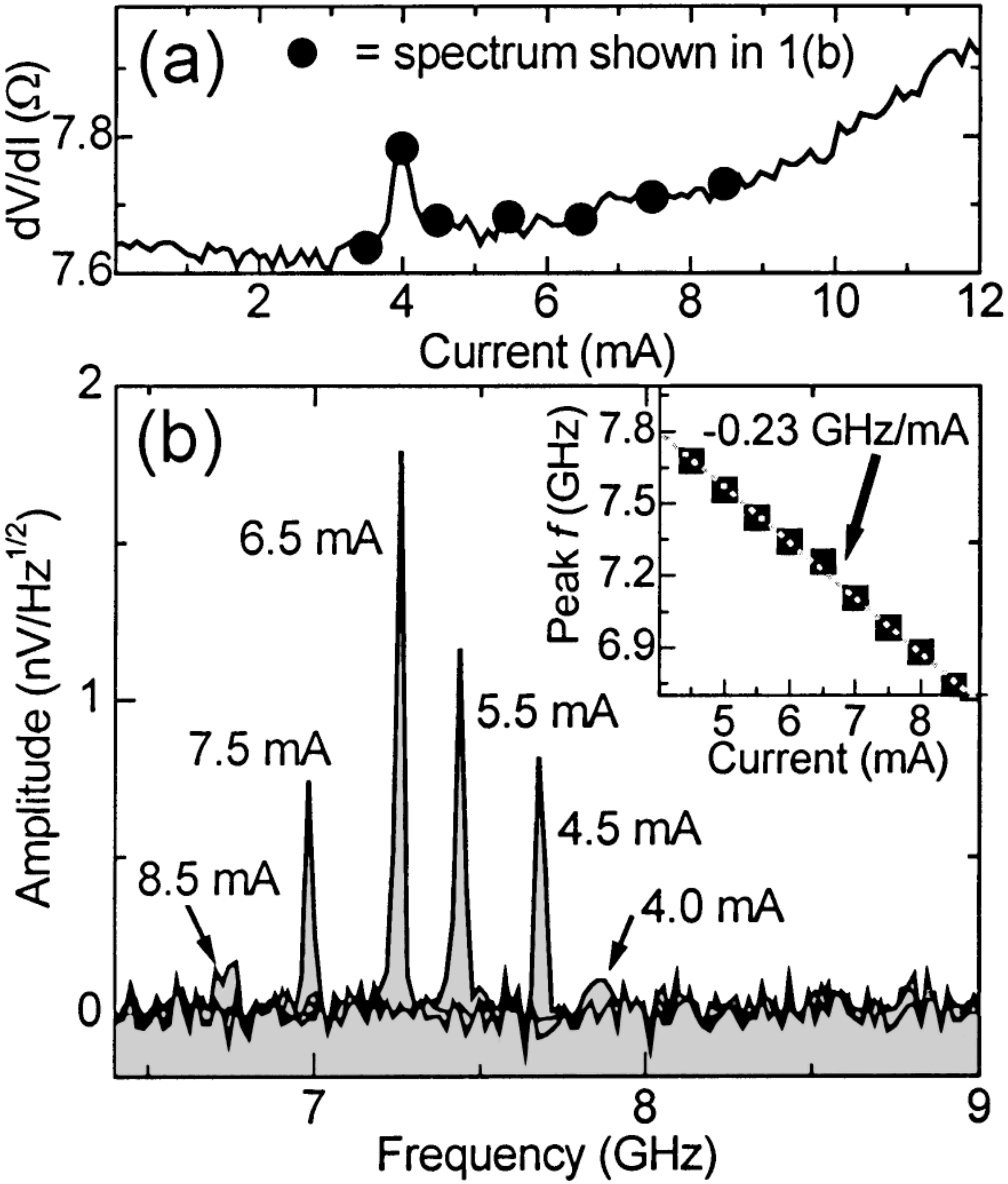
Microwave oscillations in nanocontacts

W. H. Rippard et al.,
Phys. Rev. Lett. **92**, 027201 (2004)

20 nm $Co_{90}Fe_{10}$ / 5 nm Cu / 5 nm $Ni_{80}Fe_{20}$
room temperature







Competition between damping and current

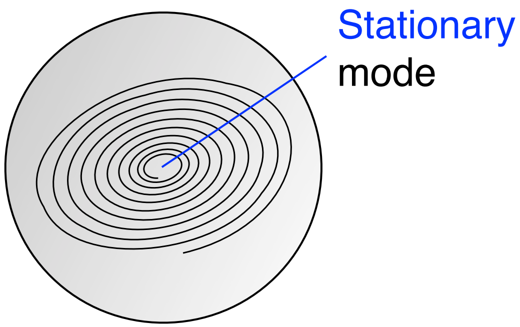
$$\frac{dg_L}{dt} = -\alpha \left| \frac{d\mathbf{m}}{dt} \right|^2 - \beta \left(\mathbf{m} \times \mathbf{e}_p \right) \cdot \frac{d\mathbf{m}}{dt}$$

Energy balance equation governing deviations from conservative trajectories

- Under zero current, the energy can only decrease, so the only admissible process is relaxation toward local energy minima and the only possible persistent states are stationary modes.
- Under nonzero current, the rate of change of energy has no longer a definite sign and stable limit cycles (i.e., persistent magnetization oscillations) become possible.

$\alpha \neq 0$

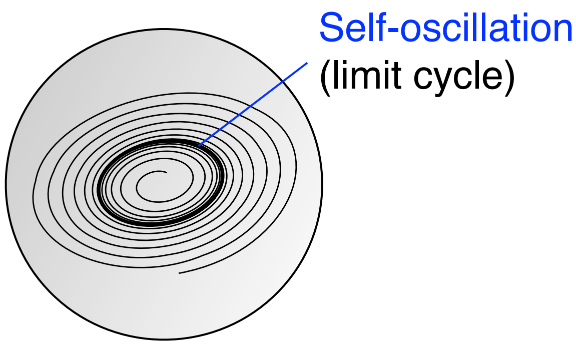
$\beta = 0$



$$\frac{dg_L}{dt} \leq 0$$

$\alpha \neq 0$

$\beta \neq 0$



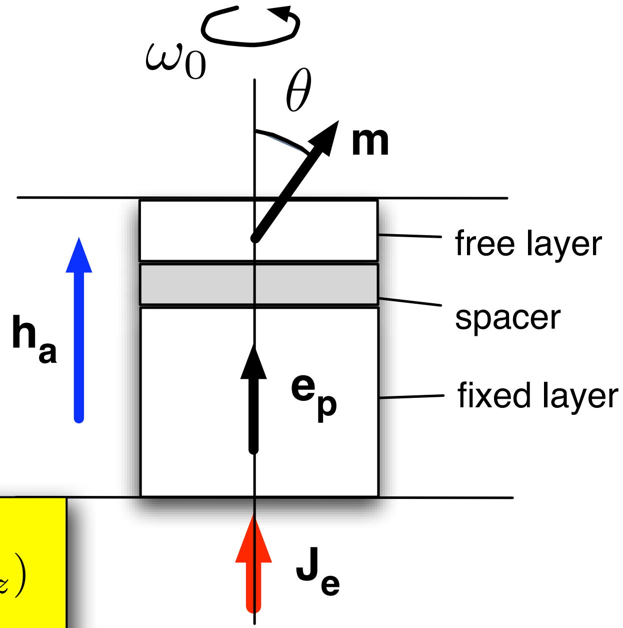
$$\frac{dg_L}{dt} = ?$$

Current-driven magnetization precession

$$D_x = D_y = D_\perp$$

$$\kappa_{\rm eff} = D_\perp - D_z$$

- spatially uniform magnetization dynamics
- uniaxial system, i.e., rotational invariance around *z* axis



dc field

shape + crystal anisotropy

$$\frac{d\mathbf{m}}{dt} - \alpha\,\mathbf{m}\times\frac{d\mathbf{m}}{dt} = -\left(h_{az} + \kappa_{\rm eff}\,m_z\right)\mathbf{m}\times\mathbf{e}_z + \beta\,\mathbf{m}\times(\mathbf{m}\times\mathbf{e}_z)$$

damping is here

current density is here

$$\frac{d\mathbf{m}}{dt} = -\omega_0\,\mathbf{m}\times\mathbf{e}_z \quad \Rightarrow \quad \Big[\omega_0 - \left(h_{az} + \kappa_{\rm eff}\,m_z\right)\Big]\mathbf{m}\times\mathbf{e}_z - \alpha\big(\omega_0 - \beta/\alpha\big)\,\mathbf{m}\times(\mathbf{m}\times\mathbf{e}_z) = 0$$

precession around z axis is an exact solution

the frequency of the current-induced motion coincides with that of free precession

$$\omega_0 = h_{az} + \kappa_{\rm eff}\,\cos\theta$$

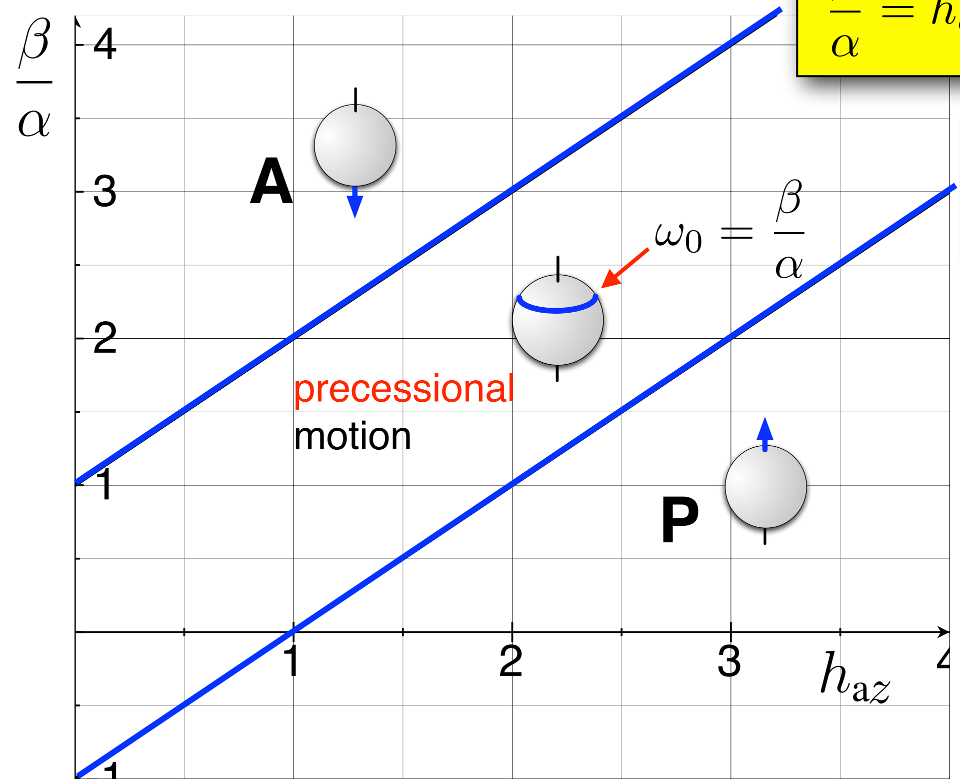
$$\omega_0 = \frac{\beta}{\alpha}$$

$$\frac{\beta}{\alpha} = h_{az} + \kappa_{\rm eff}\,\cos\theta$$

$$-1 \le \frac{\beta/\alpha - h_{az}}{\kappa_{\rm eff}} \le 1$$

Stability diagram

$$\kappa_{\text{eff}} = -1$$



$$\frac{\beta}{\alpha} = h_{az} - \kappa_{\text{eff}}$$

$$\frac{\beta}{\alpha} = h_{az} + \kappa_{\text{eff}}$$

- The magnetization dynamics excited by the current is indistinguishable from the free precessional motion taking place under no current and negligible damping.

- The frequency of current-induced precession coincides with the frequency of free precession of the same amplitude. The current does not contribute to the field torque that determines the frequency of the precession.

- The effect of the current is to select a particular precessional amplitude and to elevate it to stable current-induced precession.

Non-uniform magnetization configurations

- Rigid configurations moving by preserving their integrity: domain walls, vortex structures
- Parametrized descriptions by collective variables
- Wave-like propagating disturbances: spin waves
- Magnetostatic waves and exchange waves
- Nonuniform modes in confined structures

Non-uniform magnetization configurations

J. K. Ha et al., Phys. Rev. B **67**, 064418 (2003)

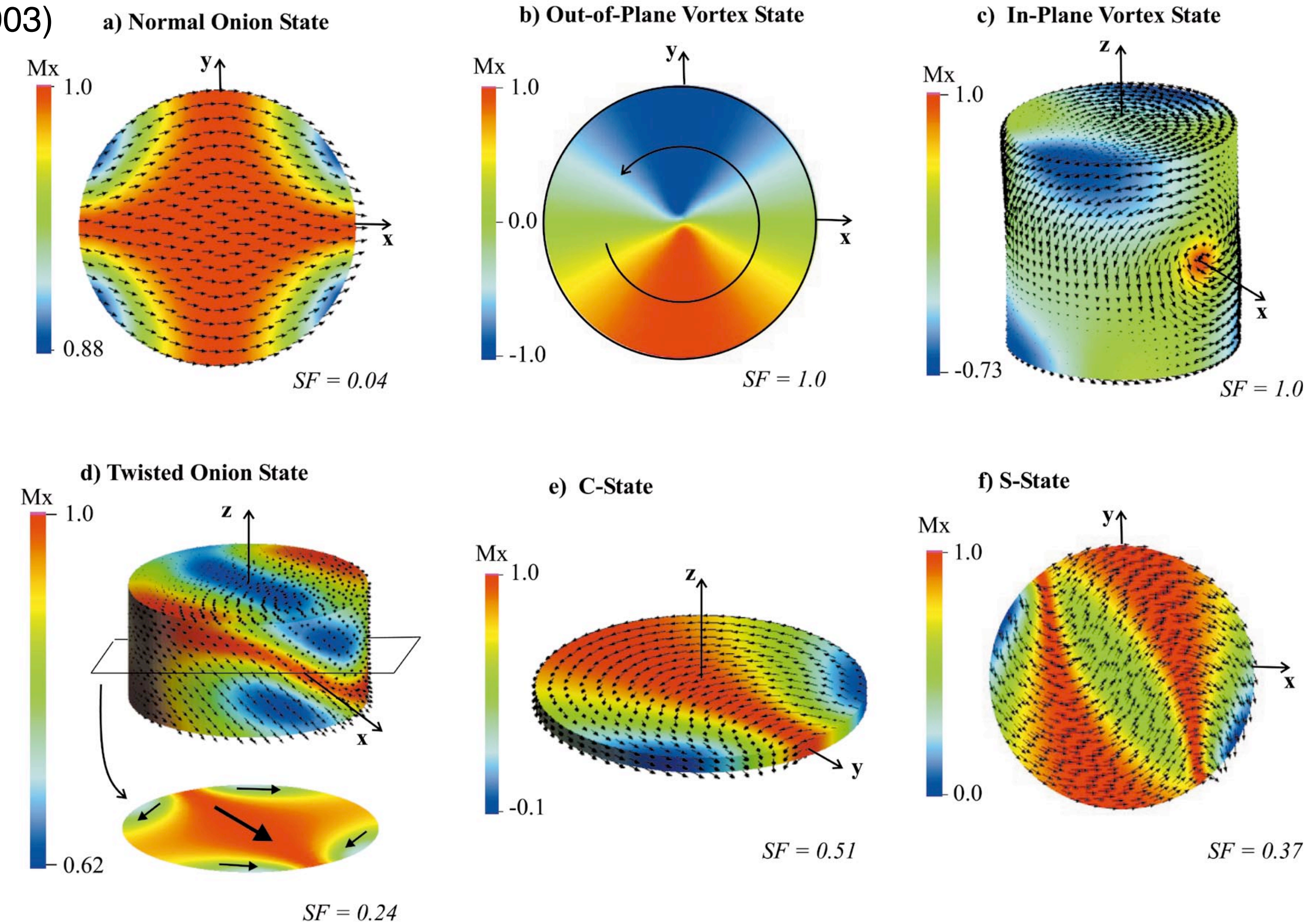


FIG. 1. (Color) Magnetic configurations observed in submicron permalloy disks. The notation such as D200R20 is used to specify the disk size. For example, D200R20 means that the disk diameter is 200 nm and its diameter-to-thickness ratio d/t is 20. On the bottom right-hand corner of each configuration is the corresponding spread function SF value. (a) Onion state seen in D200R20 at 10 mT, (b) out-of-plane vortex state in D400R10 at zero field, (c) in-plane vortex state in D100R1 at zero field, (d) twisted onion state in D200R2 at 162 mT, (e) *C* state in D200R20 at zero field, and (f) *S* state in D500R30 at 4 mT field.

Non-uniform magnetization configurations

PRL **98,** 117201 (2007) PHYSICAL REVIEW LETTERS week ending 16 MARCH 2007

Ultrafast Nanomagnetic Toggle Switching of Vortex Cores

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We present an ultrafast route for a controlled, toggle switching of magnetic vortex cores with ultrashort unipolar magnetic field pulses. The switching process is found to be largely insensitive to extrinsic parameters, like sample size and shape, and it is faster than any field-driven magnetization reversal process previously known from micromagnetic theory. Micromagnetic simulations demonstrate that the vortex core reversal is mediated by a rapid sequence of vortex-antivortex pair creation and annihilation subprocesses. Specific combinations of field-pulse strength and duration are required to obtain a controlled vortex core reversal. The operational range of this reversal mechanism is summarized in a switching diagram for a 200 nm Permalloy disk.

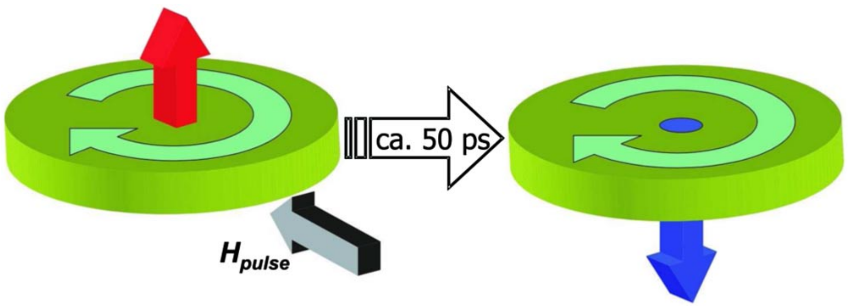


FIG. 1 (color online). Schematics of a field-pulse driven vortex core switching. The vortex core magnetization can be switched by a short magnetic field pulse applied in the film plane. This switching process requires only 40–50 ps.

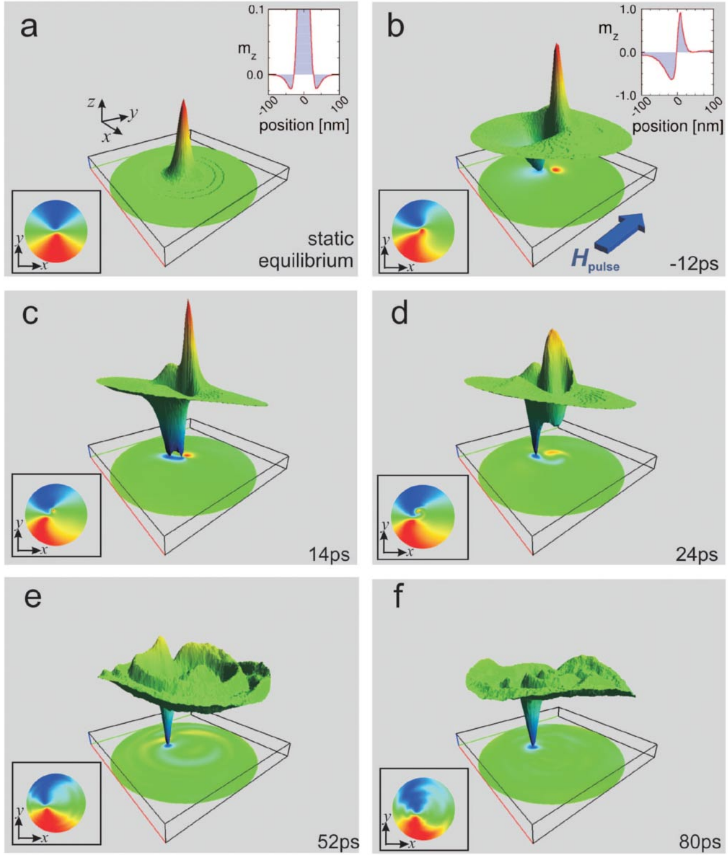


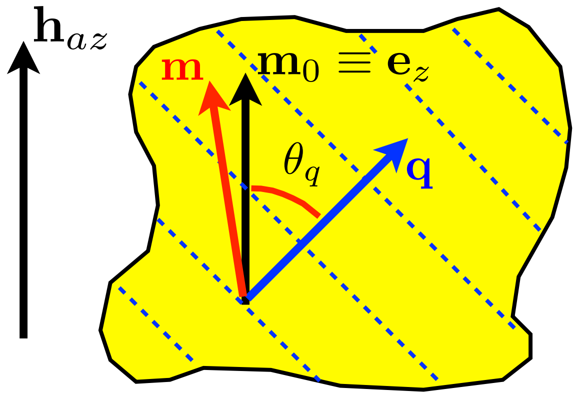
FIG. 3 (color). Topography of m_z during the core switching process. (a) Static equilibrium. The cutline on the upper right shows the small negative dip in m_z near the core. (b) Formation of a large negative dip of m_z. The cutline goes along the diameter through the vortex core and the adjacent dip. (c) Pair creation leads to two separate points with $m_z = -1$. (d) The original vortex core annihilates with the new antivortex. (e) Spin waves are emitted after the annihilation. Finally, a vortex core with opposite polarization remains (f). The insets on the lower left show the x component of the magnetization (red: $m_x = 1$, blue: $m_x = -1$). The times are relative to the pulse maximum.

Spin waves

- Given the static, spatially uniform, saturated magnetization state, it is natural to consider the nature and properties of small-amplitude perturbations of the saturated state.
- These perturbations will be solutions of an equation that is obtained by linearizing the LLG equation around the saturation state.
- The Kittel mode is just one of these solutions, with the property of being spatially uniform; spatially non-uniform solutions analogous to the Kittel mode, in the sense that they too depend on the geometrical shape of the particle but are independent of absolute dimensions, are termed magnetostatic modes.
- There also exist wave-like solutions to the linearized equations, characterized by a wave-vector and a well-defined dispersion relation: these solutions are termed spin waves.
- The simplest situation occurs for bulk spin waves in extended media, where boundary conditions play a negligible role.
- Substantial deviations from this behavior occur in confined media (e.g., thin films, multilayers, etc.) due to the dominant role played by boundary conditions.
- When the wavelength is long enough with respect to the exchange length, one can neglect exchange interaction: the result are so-called magnetostatic waves.
- When exchange interaction becomes important, one has exchange waves.

Spin waves

- Spheroidal object with polar axis along the z direction; crystal anisotropy is neglected
- Demagnetizing factors are: $D_x = D_y \equiv D_\perp\ ,\ D_z$
- External dc magnetic field is applied along z axis
- $\mathbf{m}_0 \equiv \mathbf{e}_z$ is a stable equilibrium orientation for the magnetization



$$\boxed{\frac{\partial \mathbf{m}}{\partial t} = -\mathbf{m} \times \mathbf{h}_{\text{eff}} + (\text{damping})}$$

- We consider plane-wave perturbations with wave-vector **q**: $\mathbf{m} \simeq \mathbf{e}_z + \delta\mathbf{m}_\perp \exp\left(i\mathbf{q}\cdot\mathbf{r}\right)$
- Boundary effects are neglected (this is an acceptable approximation if 1/q << *L*, where *L* is typical linear dimension of the object)
- Damping effects are neglected
- We assume that **q** lies in the x-z plane (this is no restriction since problem is invariant with respect to rotations around z axis)

$$\mathbf{h}_{\text{eff}} = \nabla^2\mathbf{m} + h_{az}\mathbf{e}_z + \mathbf{h}_M(\mathbf{m})$$

$$\mathbf{m} \simeq \mathbf{e}_z + \delta\mathbf{m}_\perp \exp\left(i\mathbf{q}\cdot\mathbf{r}\right)$$

this a matrix that operates on the vector $\delta\mathbf{m}_\perp$

$$\mathbf{h}_{\text{eff}} \simeq \left(h_{az} - D_z\right)\mathbf{e}_z - \left(q^2\mathbf{I} + \frac{\mathbf{q}\otimes\mathbf{q}}{q^2}\right)\cdot\delta\mathbf{m}_\perp \exp(i\mathbf{q}\cdot\mathbf{r})$$

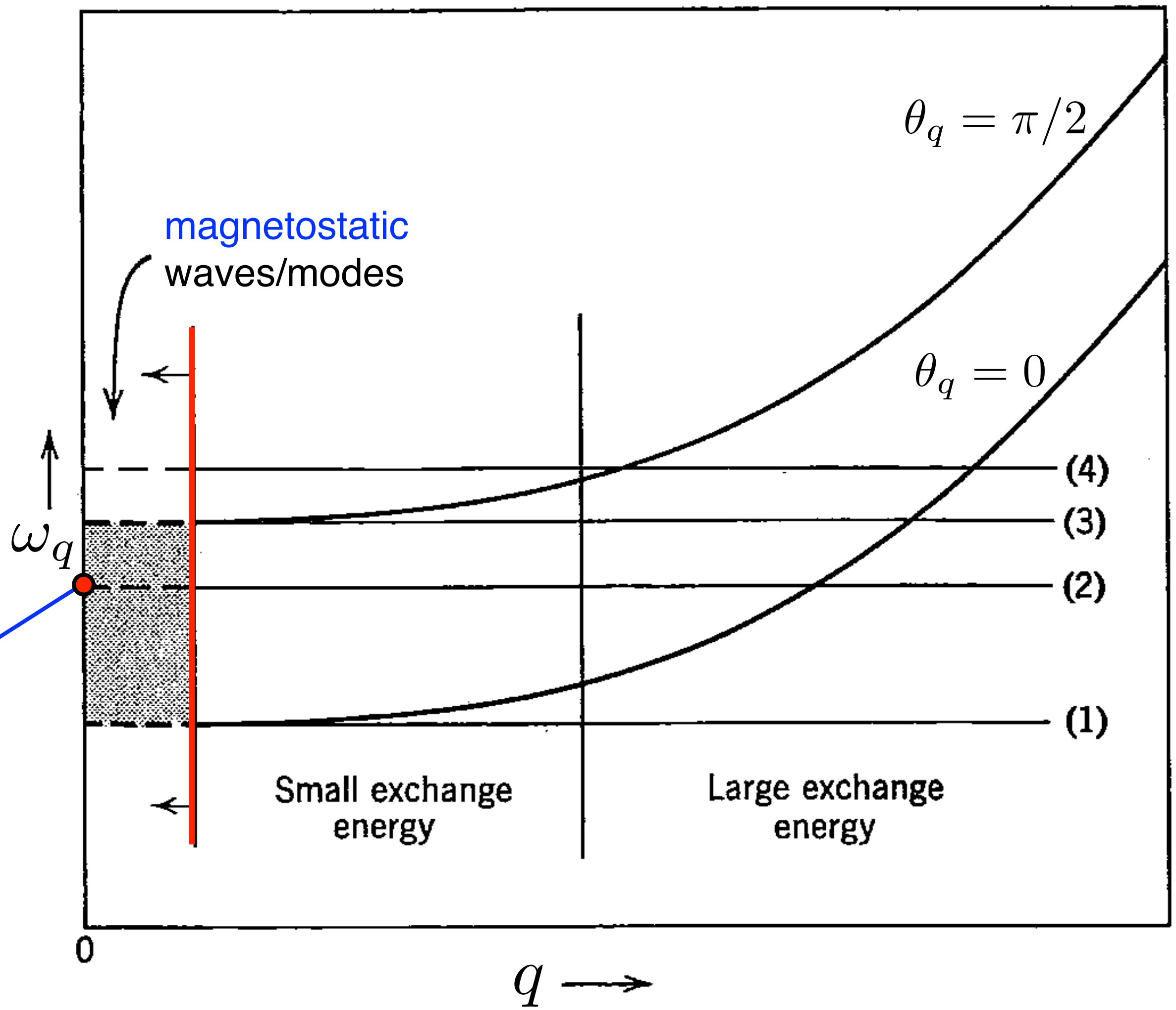
$$\frac{d}{dt}\begin{pmatrix}\delta m_x \\ \delta m_y\end{pmatrix} = \begin{pmatrix} 0 & -\left(h_{az} - D_z + q^2\right) \\ \left(h_{az} - D_z + q^2 + \sin^2\theta_q\right) & 0 \end{pmatrix}\begin{pmatrix}\delta m_x \\ \delta m_y\end{pmatrix}$$

$$\boxed{\frac{d^2x}{dt^2} + \omega_0^2\, x = 0}$$

$$\boxed{\omega_q^2 = \left(h_{az} - D_z + q^2\right)\left(h_{az} - D_z + q^2 + \sin^2\theta_q\right)}$$

spin-wave dispersion relation

Spin-wave manifold

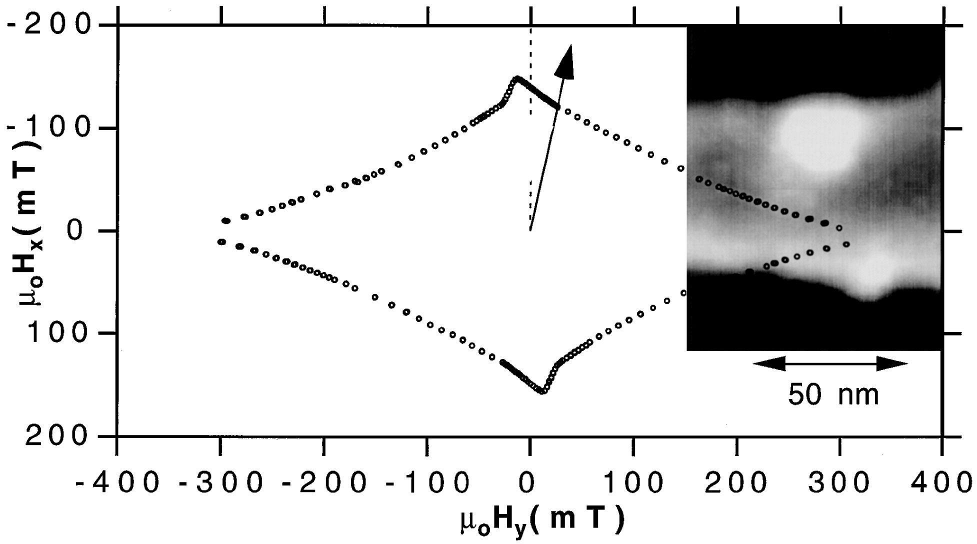


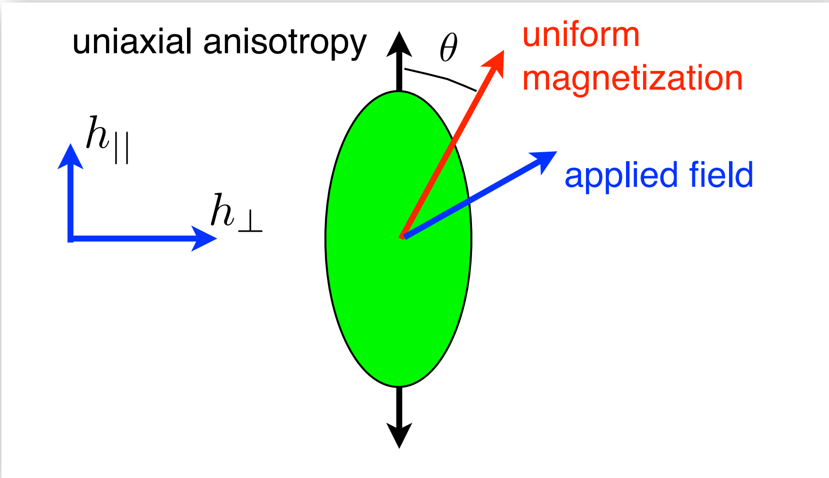
$$\omega_0 = h_{az} + D_\perp - D_z$$

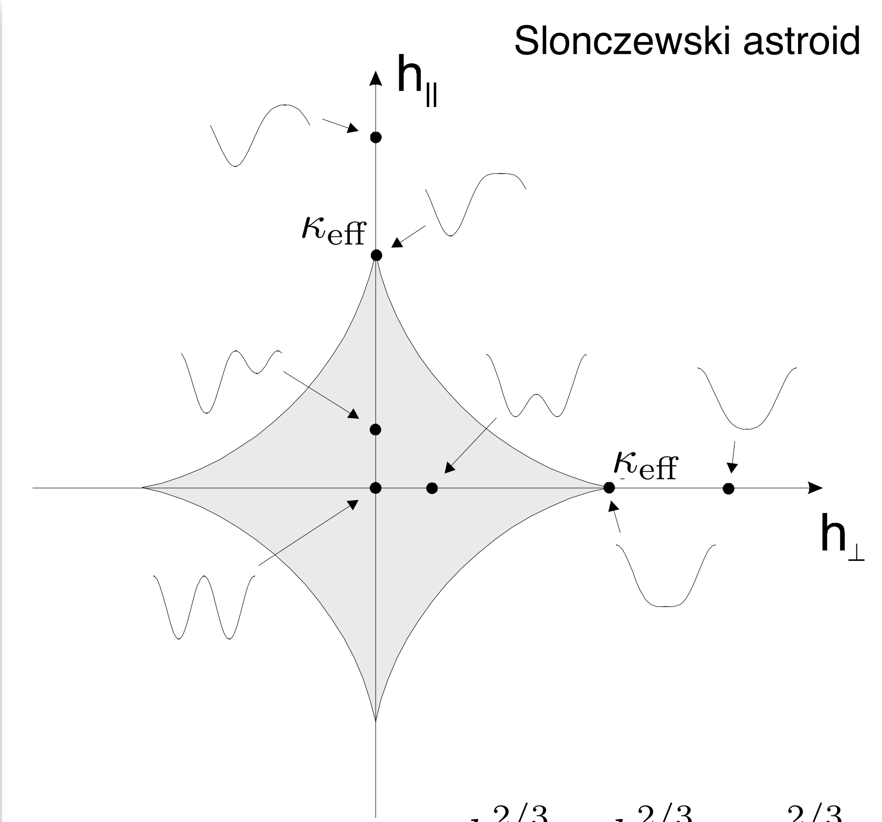
$$\omega_q = \sqrt{\left(h_{az} - D_z + q^2\right)\left(h_{az} - D_z + q^2 + \sin^2\theta_q\right)}$$

Thermal fluctuations in single-domain nanoparticles

W. Wernsdorfer et al.,
Phys. Rev. Lett. **78**, 1791 (1997)







$$h_{\perp}^{2/3} + h_{||}^{2/3} = \kappa_{\text{eff}}^{2/3}$$

$$g_L(\theta; \mathbf{h}_{\text{a}}) = \frac{\kappa_{\text{eff}}}{2} \sin^2\theta - h_{\perp} \sin\theta - h_{||} \cos\theta$$

Astroid behavior is fingerprint of single-domain character of magnetization in the nanoparticle

Thermal fluctuations in single-domain nanoparticles

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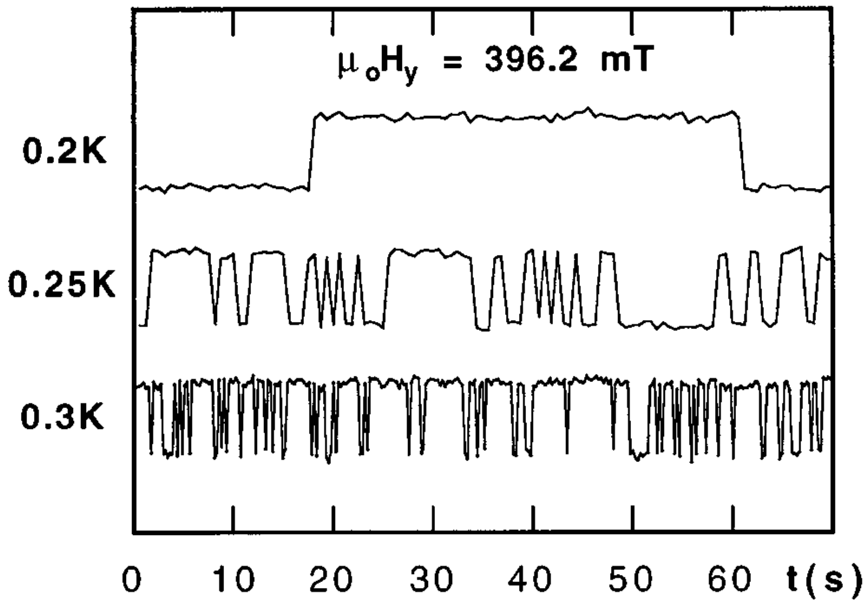

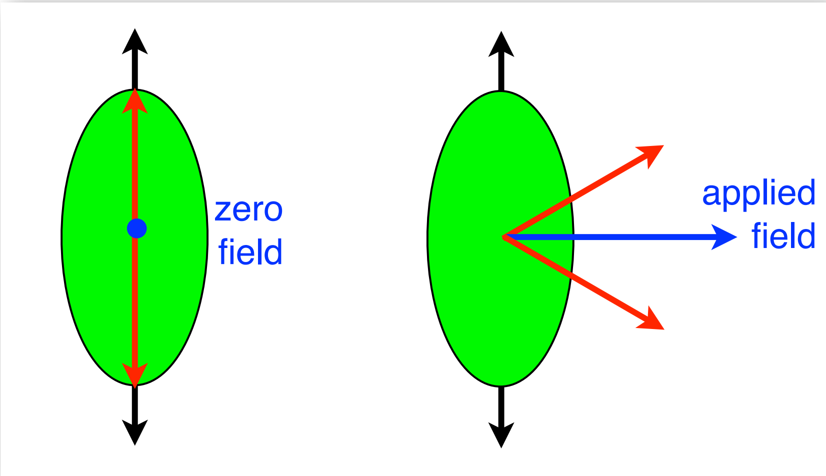

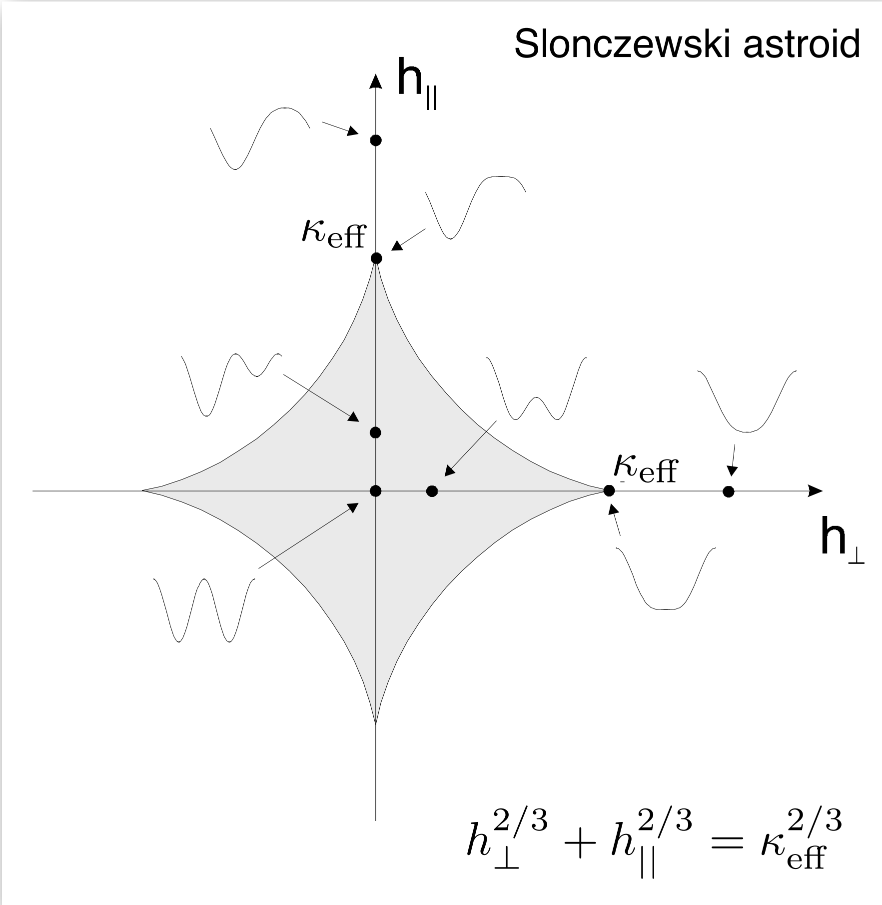

$$g_L(\theta; \mathbf{h}_{\mathrm{a}}) = \frac{\kappa_{\mathrm{eff}}}{2} \sin^2\theta - h_\perp \sin\theta - h_{||} \cos\theta$$

Random telegraph signal is fingerprint of coherent thermally activated jumps over energy barrier

THE END

GOOD LUCK !