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The Hall Effects Edwin Hall Never imagined

Xiaofeng Jin

Physics Department, Fudan University, Shanghai, China

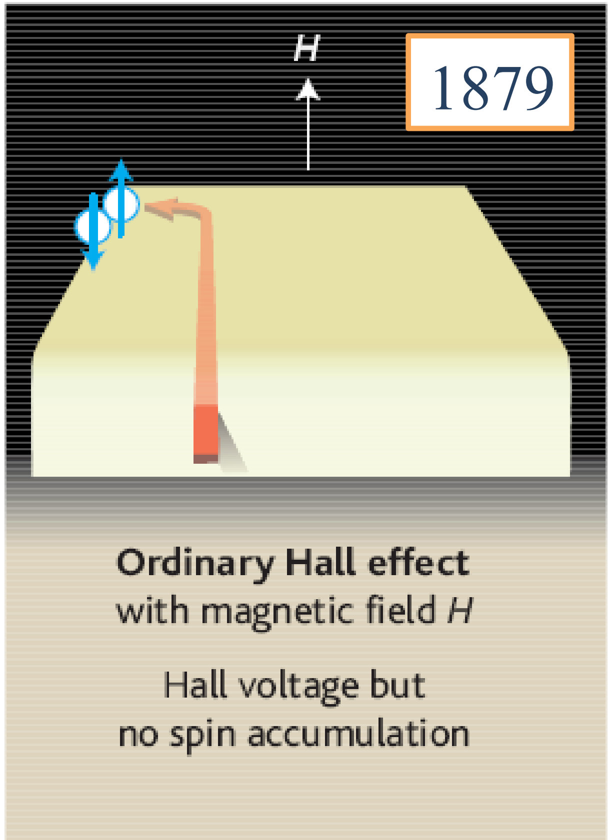
1. Introduction
2. Our approach
3. Conclusions

Santander, June 22, 2017

1. Introduction

On a New Action of the Magnet on Electric Currents.

By E. H. Hall, *Fellow of the Johns Hopkins University.*



Lorentz Force

THE

LONDON, EDINBURGH, AND DUBLIN

PHILOSOPHICAL MAGAZINE

AND

JOURNAL OF SCIENCE.

[FIFTH SERIES.]

NOVEMBER 1880.

XXXVIII. *On the new Action of Magnetism on a permanent Electric Current. By* E. H. Hall, *Assistant in Physics at the Johns Hopkins University*.*

THE

LONDON, EDINBURGH, AND DUBLIN

PHILOSOPHICAL MAGAZINE

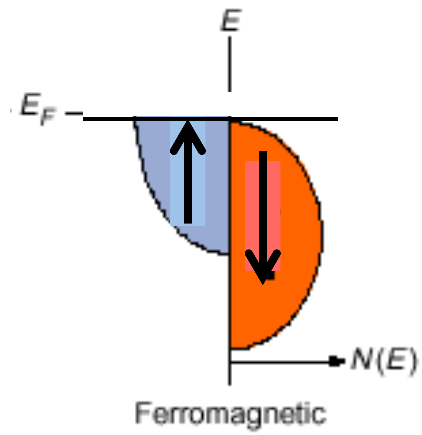
AND

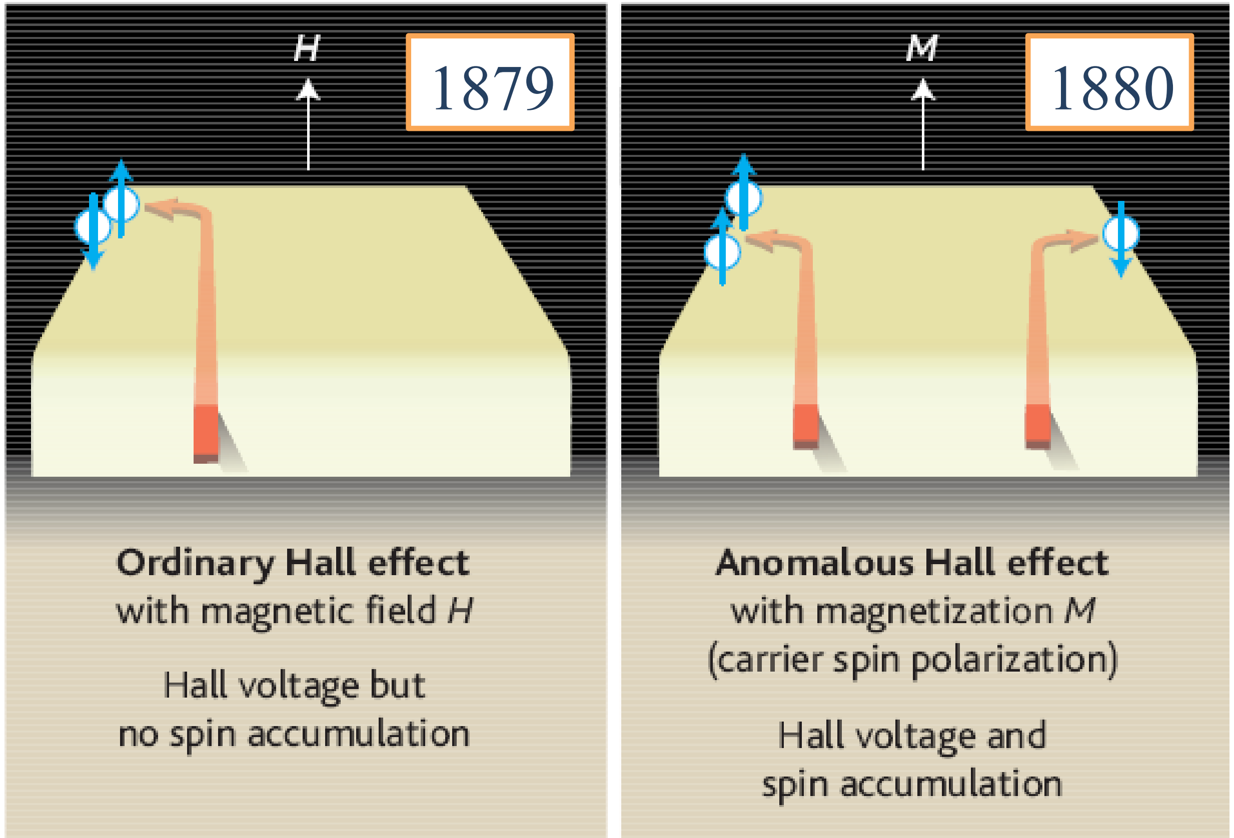
JOURNAL OF SCIENCE.

[FIFTH SERIES.]

SEPTEMBER 1881.

XVIII. *On the "Rotational Coefficient" in Nickel and Cobalt. By* E. H. Hall, *Ph.D., late Assistant in Physics in the Johns Hopkins University, Baltimore*.*

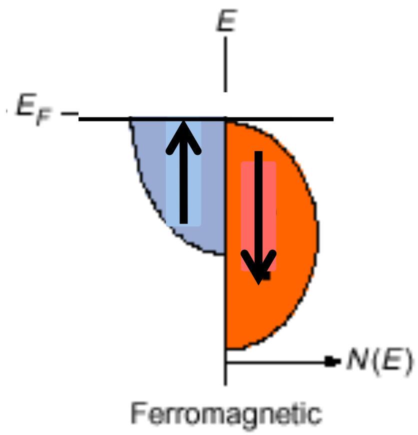


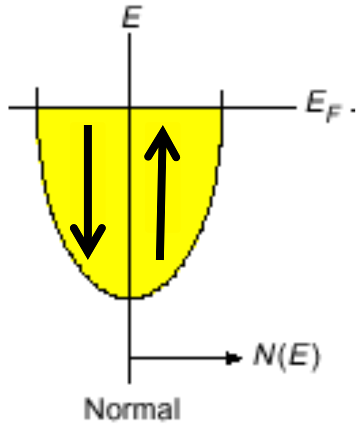


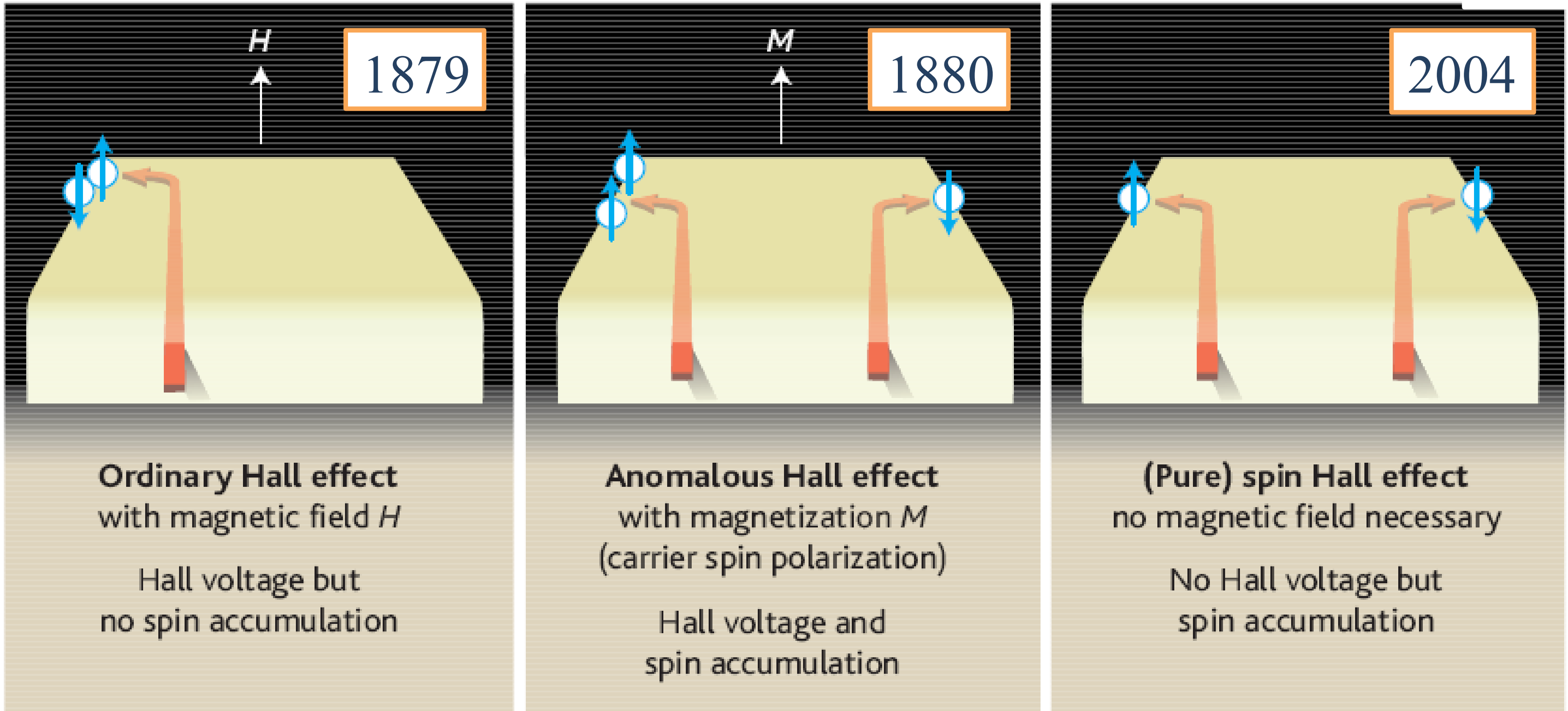
Lorentz Force

Spin-Orbit Coupling

J. Inoue and H. Ohno, Science, 309, 2004 (2005)



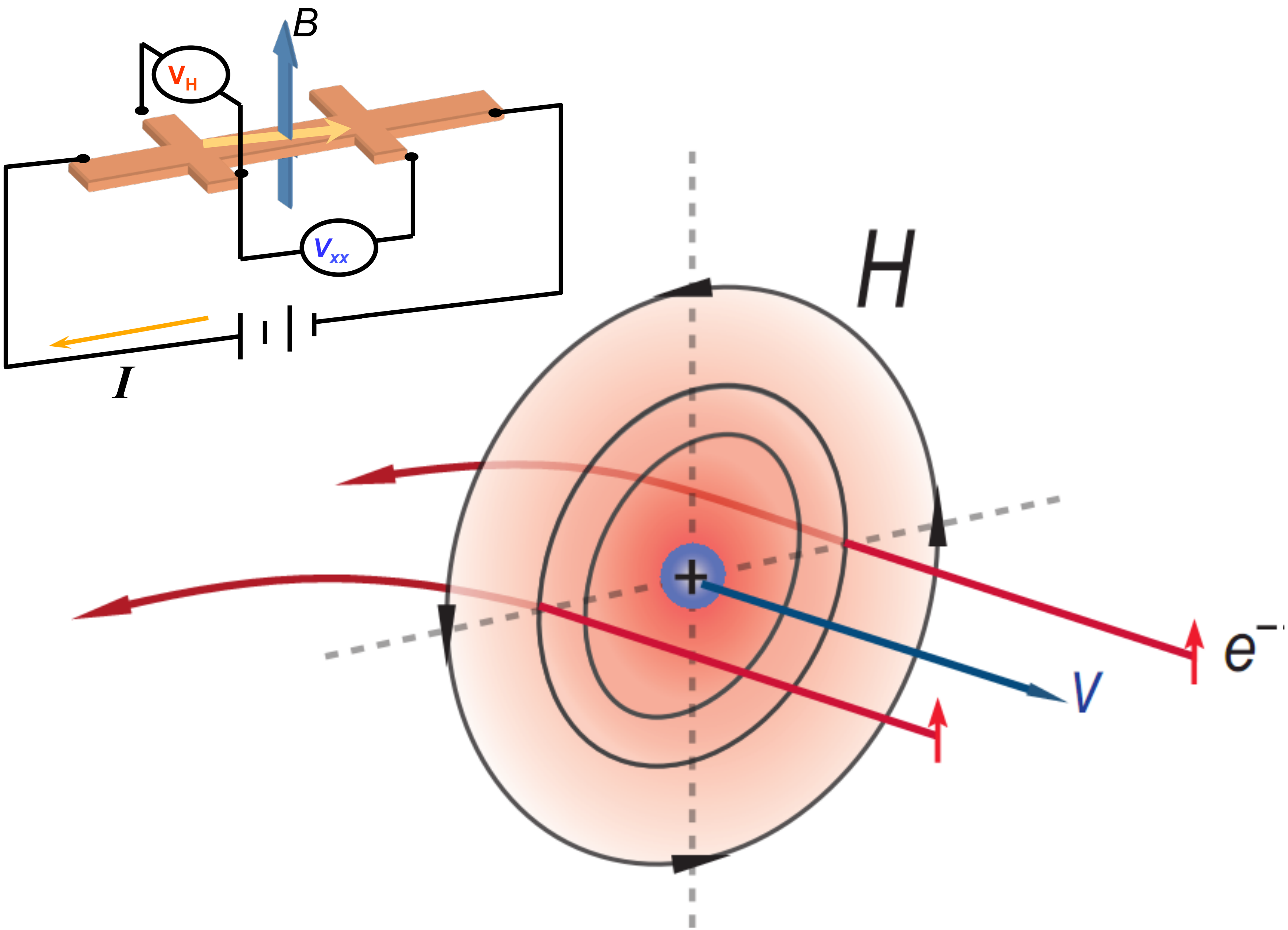




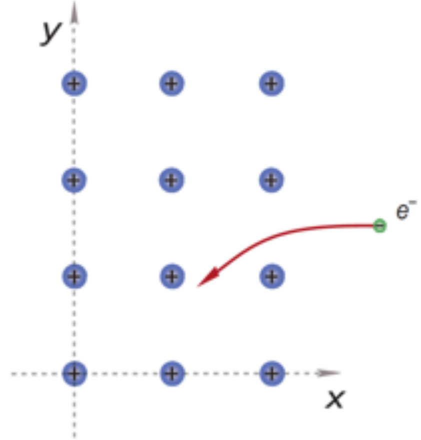
Lorentz Force

Spin-Orbit Coupling

Spin-orbit coupling



(1) Karplus-Luttinger Intrinsic (1954)



Anomalous velocity

$$\frac{d\vec{r}(t)}{dt} = \frac{\partial \varepsilon_n(\vec{k})}{\partial \vec{k}} - \vec{\Omega}_n(\vec{k}) \times \frac{d\vec{k}(t)}{dt}$$

k- space curvature

$$\frac{d\vec{k}(t)}{dt} = \frac{\partial V(\vec{r})}{\partial \vec{r}} - \vec{B}(\vec{r}) \times \frac{d\vec{r}(t)}{dt}$$

r- space curvature

G. Sundaram and Q. Niu, Phys. Rev. B 59 (1999) 14915.

Jungwirth, Niu, MacDonald (2002), Onoda & Nagaosa (2002)

$$\sigma_{xy} = -\frac{e^2}{h}\int d^3\mathbf{k} \sum_n f(\varepsilon_n(\mathbf{k}))\boxed{\Omega_n^z(\mathbf{k})}$$

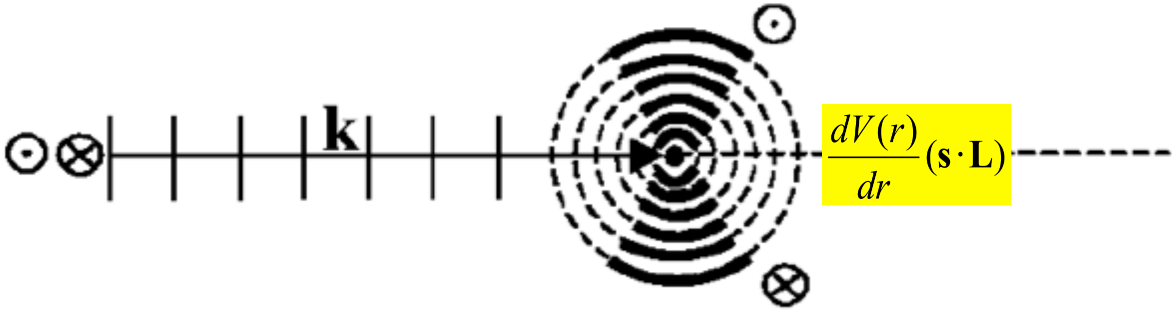
Berry curvature

$$\Omega_n^z(\mathbf{k}) = -\sum_{n' \neq n} \frac{2\,\mathrm{Im}\langle \mathbf{k}n | v_x | \mathbf{k}n' \rangle \langle \mathbf{k}n' | v_y | \mathbf{k}n \rangle}{(\omega_{\mathbf{k}n'} - \omega_{\mathbf{k}n})^2}$$

$\sigma_{\mathrm{int}} = constant$ $\longrightarrow$ $\rho_{\mathrm{int}} = \sigma_{\mathrm{int}}\rho_{xx}^2$

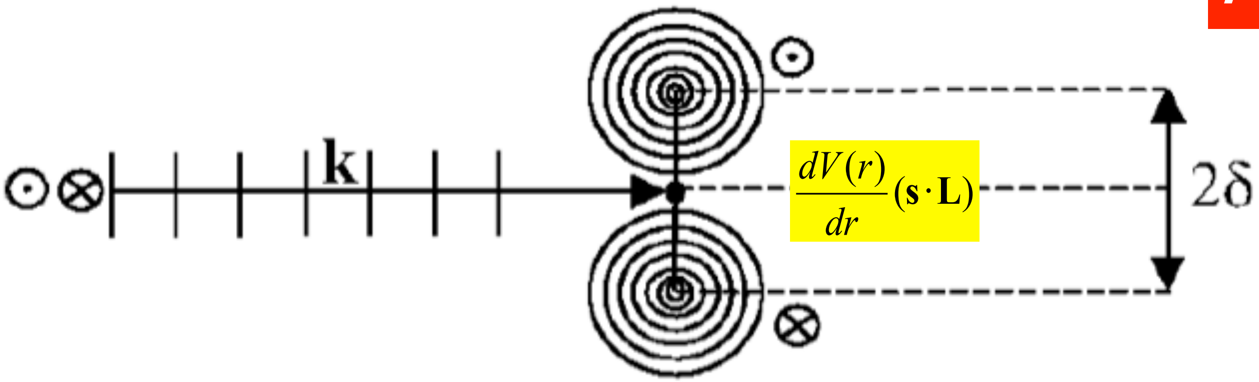
(2) Skew-scattering (Smit, 1955)

$$\rho_{ah} = \alpha \rho_{xx}$$



(3) Side-jump (Berger, 1970)

$$\rho_{ah} = \beta \rho_{xx}^2$$



$$\rho_{ah} = c_1 \rho_{xx} + c_2 \rho_{xx}^2$$

Intrinsic or Extrinsic ?!

$$\rho_{ah} = c_1 \rho_{xx} + c_2 \rho_{xx}^2$$

VOLUME 83, NUMBER 9 PHYSICAL REVIEW LETTERS 30 AUGUST 1999

Spin Hall Effect

J. E. Hirsch
Department of Physics, University of California, San Diego, La Jolla, California 92093-0319
(Received 24 February 1999)

a variety of mechanisms have been proposed to explain the origin of the coefficient R_s. These include skew scattering by impurities and phonons, and the "side jump" mechanism [1]. In early work it was also proposed that the effect will arise in the absence of periodicity-breaking perturbations [2], but this is generally believed not to be correct [1].

Anomalous Hall effect There is a term in the Hall resistivity of a ferromagnet when the field is applied in the z-direction, perpendicular to the plane of the film, in addition to the normal Hall effect (3.53). This is the anomalous Hall effect, which varies with the magnitude of the magnetization M:

$$\varrho_{xy} = \mu_0(R_h H' + R_s M). \tag{5.83}$$

The anomalous Hall effect is yet another consequence of spin-orbit coupling. The symmetry of the radial component of the Lorentz force $\boldsymbol{j} \times \boldsymbol{B}$ which produces the normal Hall effect is the same as the symmetry of the spin-orbit interaction $\boldsymbol{L} \cdot \boldsymbol{S}$ since $\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$, $\boldsymbol{p} \propto \boldsymbol{j}$, $\boldsymbol{S} \propto \mu_0 \boldsymbol{M}$.

In a ferromagnet the anomalous Hall effect varies as the macroscopic average magnetization. General

$$\rho_{ah} = c_1\rho_{xx} + c_2\rho_{xx}^2$$

vary as ϱ_{xx} and as ϱ_{xx}^2, which are associated w … isms. Deviation of the electron trajectories due to spin-orbit interaction is known as skew scattering.

Writing $\varrho_m = \mu_0 R H'$, the Hall angle ϕ_H is defined as ϱ_m/ϱ_{xx}. Thus $\phi_H = \alpha + \beta\varrho_{xx}$; α is the skew scattering angle. The second term is often larger. It is associated with the side-jump mechanism due to impurity scattering. If $\delta \approx 0.1$ nm is the side jump, the Hall angle here is δ/λ, which is proportional to ϱ_{xx}. Here λ is the mean free path.

Anomalous Hall effect

$$\rho_{ah} = c_1 \rho_{xx} + c_2 \rho_{xx}^2$$

Naoto Nagaosa

Department of Applied Physics, University of Tokyo, Tokyo 113-8656, Japan and Cross-Correlated Materials Research Group (CMRG), and Correlated Electron Research Group (CERG), ASI, RIKEN, Wako, 351-0198 Saitama, Japan

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Department of Physics, Texas A&M University, College Station, Texas 77843-4242, USA and Institute of Physics ASCR, Cukrovarnická 10, 162 53 Praha 6, Czech Republic

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A. H. MacDonald

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Department of Physics, Princeton University, Princeton, New Jersey 08544, USA

(Published 13 May 2010)

On the theoretical front, the adoption of the Berry-phase concepts has established a link between the AHE and the topological nature of the Hall currents. On the experimental front, new experimental studies of the AHE in transition metals, transition-metal oxides, spinels, pyrochlores, and metallic dilute magnetic semiconductors have established systematic trends. These two developments, in concert with first-principles electronic structure calculations, strongly favor the dominance of an intrinsic Berry-phase-related AHE mechanism in metallic ferromagnets with moderate conductivity. The intrinsic AHE can be expressed in terms of the Berry-phase curvatures and it is therefore an intrinsic quantum-mechanical property of a perfect crystal.

Berry phase effects on electronic properties

$$\rho_{ah} = c_1 \rho_{xx} + c_2 \rho_{xx}^2$$

Di Xiao

Materials Science and Technology Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA

Ming-Che Chang

Department of Physics, National Taiwan Normal University, Taipei 11677, Taiwan

Qian Niu

Department of Physics, The University of Texas at Austin, Austin, Texas 78712, USA

(Published 6 July 2010)

Because the Berry phase contribution σ_H^{in} is independent of scattering, it can be readily evaluated using first-principles methods or effective Hamiltonians. Excellent agreement with experiments has been demonstrated in ferromagnetic transition metals and semiconductors (Jungwirth *et al.*, 2002; Fang *et al.*, 2003; Yao *et al.*, 2004, 2007; Xiao, Yao, *et al.*, 2006), which leaves little room for the side-jump contribution.

Does the side jump effect exist?

O.P. Sushkov,[1] A. I. Milstein,[2] M. Mori,[3,4] and S. Maekawa[3,4]
[1] *School of Physics, University of New South Wales, Sydney 2052, Australia*
[2] *Budker Institute of Nuclear Physics, 630090 Novosibirsk, Russia*
[3] *The Advanced Science Research Center, Japan Atomic Energy Agency, Tokai 319-1195, Japan*
[4] *CREST, Japan Science and Technology Agency, Sanbancho 102-0075, Japan*

(Dated: November 13, 2012)

The side-jump effect is a manifestation of the spin orbit interaction in electron scattering from an atom/ion/impurity. The effect has a broad interest because of its conceptual importance for generic spin-orbital physics, in particular the effect is widely discussed in spintronics. We reexamine the effect accounting for the exact nonperturbative electron wave function inside the atomic core. We find that value of the effect is much smaller than estimates accepted in literature. The reduction factor is $1/Z^2$, where Z is the nucleus charge of the atom/impurity. This implies that the side-jump effect is practically irrelevant for spintronics, the skew scattering and/or the intrinsic mechanism always dominate the anomalous Hall and spin Hall effects.

Relativistic effects in scattering of polarized electrons

O. P. Sushkov[1], A. I. Milstein[2], M. Mori[3,4] and S. Maekawa[3,4]

[1] *School of Physics, University of New South Wales - Sydney 2052, Australia*
[2] *Budker Institute of Nuclear Physics - 630090 Novosibirsk, Russia*
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Euro. Phys. Lett. 103 (2013) 47003

PACS 72.25.-b – Spin polarized transport
PACS 31.15.aj – Relativistic corrections, spin-orbit effects, fine structure; hyperfine structure
PACS 34.80.Nz – Spin dependence of cross sections; polarized beam experiments

Abstract – The right-left asymmetry (skew scattering) and the side jump effect are manifestations of the spin-orbit interaction in scattering of polarized electrons. While the side jump effect is less known than the right-left asymmetry, the effect is of conceptual importance for generic spin-orbital physics, and the effect is widely discussed in spintronics. We reexamine the side jump effect accounting for the exact nonperturbative electron wave function inside the atom/impurity/host atomic core. We find that the size of the effect is much smaller than estimates accepted in the literature. The reduction factor is $1/Z^2$, where Z is the nuclear charge. This implies that the side jump effect is practically irrelevant, the skew scattering and/or the intrinsic mechanism always dominate the transverse deflection of the electron beam and hence dominate the anomalous Hall and spin Hall effects.

Theoretically:

(1) Karplus-Luttinger Intrinsic (1954) $\rho_{intrinsic} = b\rho_{xx}^2$

(2) Skew-scattering (Smit, 1955) $\rho_{skew} = \alpha\rho_{xx}$

(3) Side-jump (Berger, 1970) $\rho_{side-jump} = \beta\rho_{xx}^2$

All based on single type of scatters !

Theoretically:

(1) Karplus-Luttinger Intrinsic (1954) $\rho_{intrinsic} = b\rho_{xx}^2$

(2) Skew-scattering (Smit, 1955) $\rho_{skew} = \alpha\rho_{xx}$

(3) Side-jump (Berger, 1970) $\rho_{side-jump} = \beta\rho_{xx}^2$

In real materials: $\rho_{xx} = \rho_{xx0} + \rho_{xxT}$

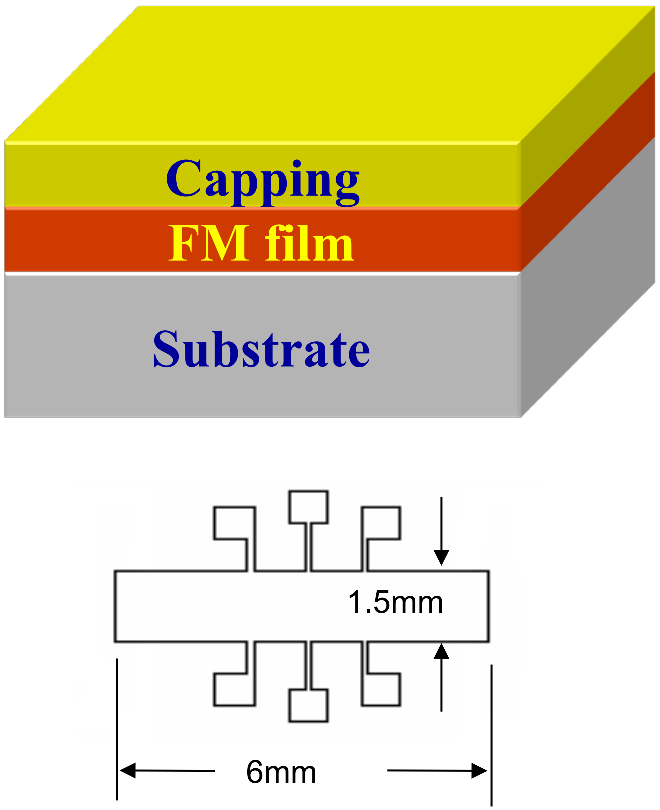
(Matthiessen's

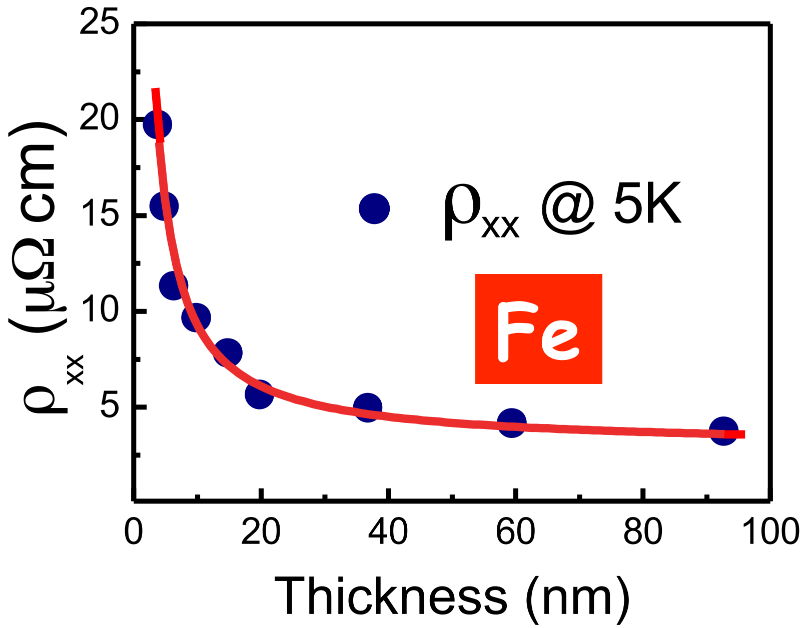
Key Issue: ρ_{ah} **should scale with** ρ_{xx} **or** ρ_{xx0} **or** ρ_{xxT} **?**

2. Our approach

$$\rho_{ah} = f(\rho_{xx0}, \rho_{xxT}, \rho_{xx})$$

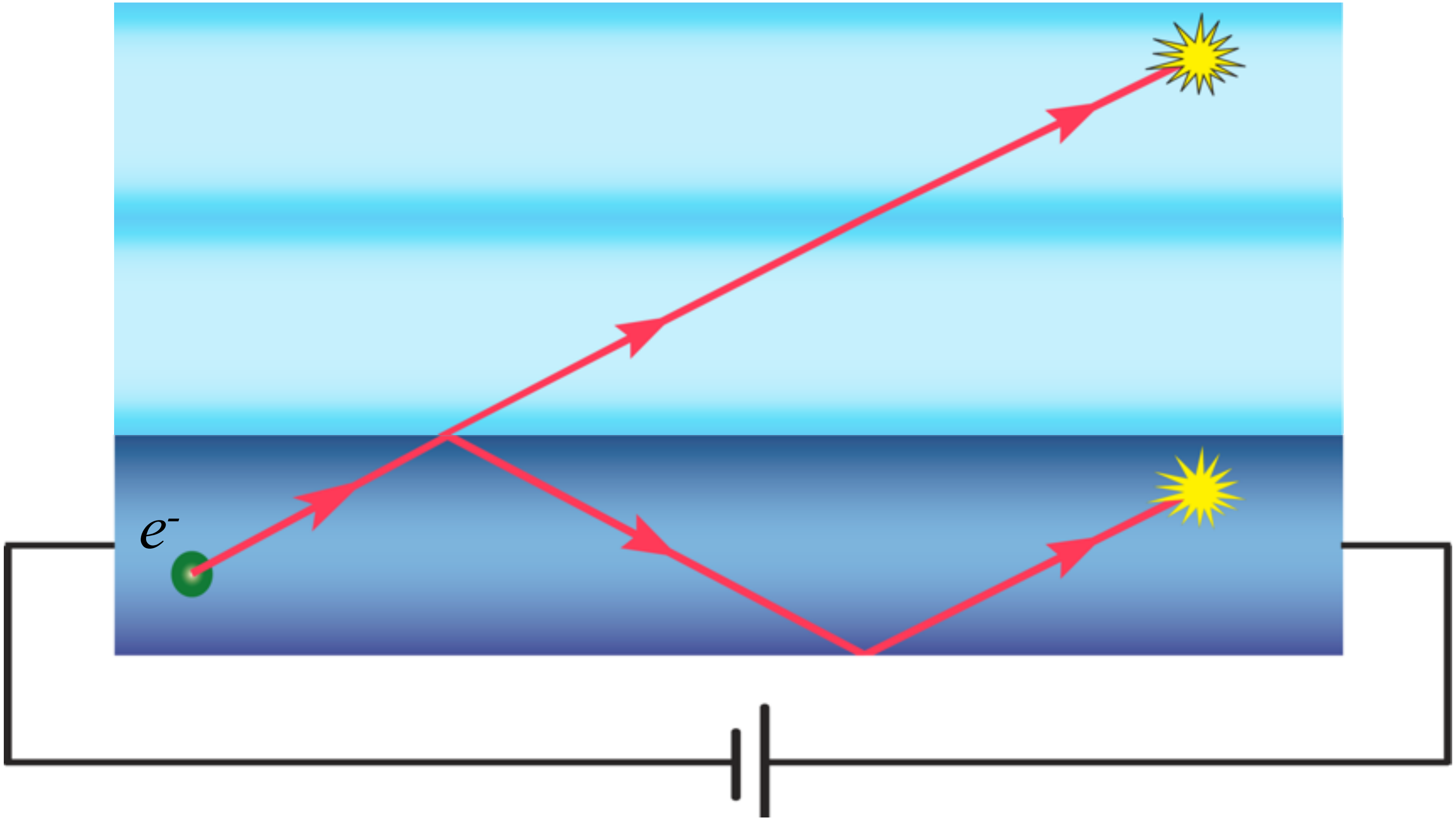
Tuning with thickness and temperature





D.Z. Hou et al., J. Phys. Cond. Matt., 24 (2012) 482001

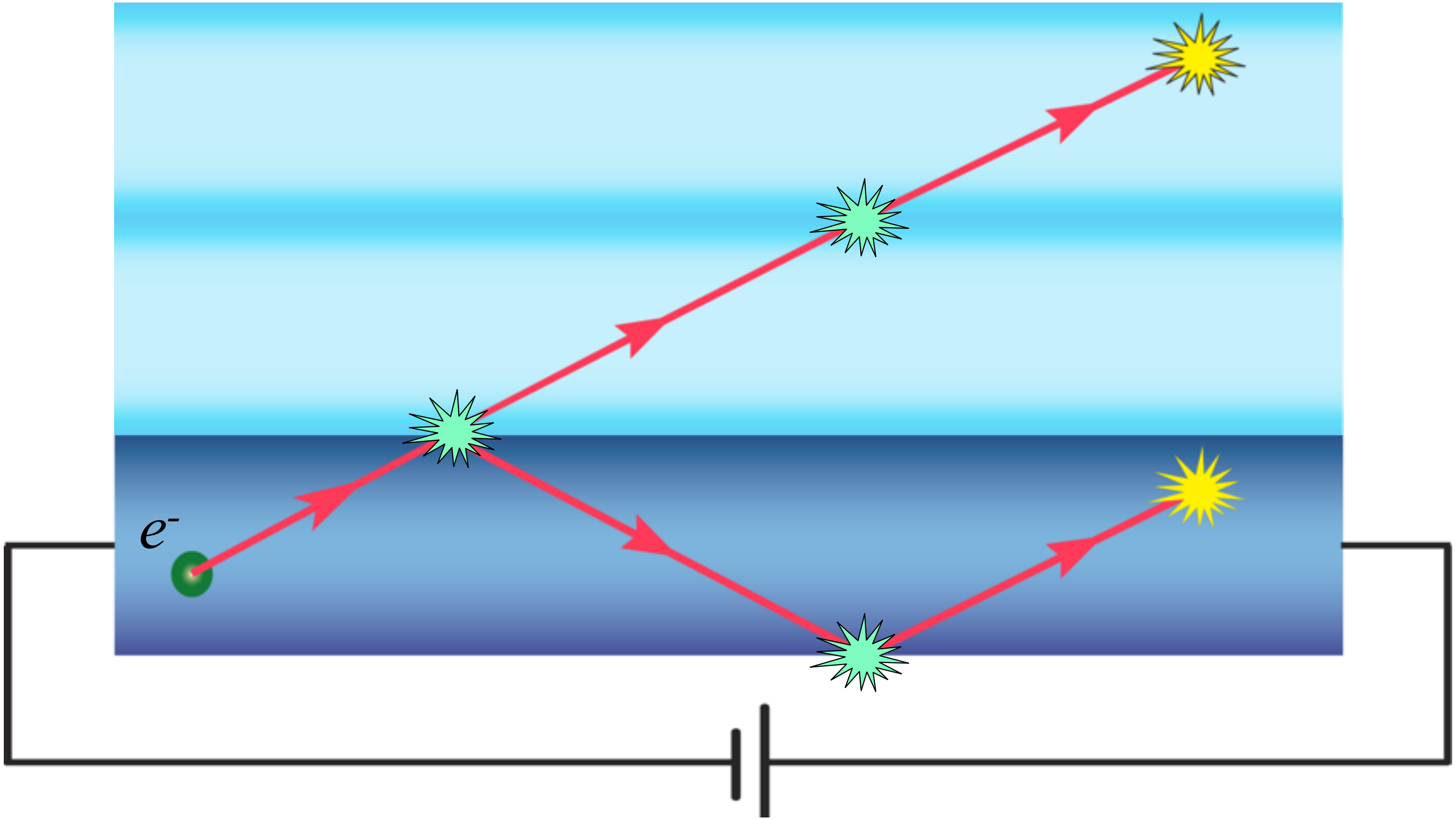
A. Cottey, Thin solid films 1, (1967) 297-397



R=1 at interface

D.Z. Hou et al., J. Phys. Cond. Matt., 24 (2012) 482001

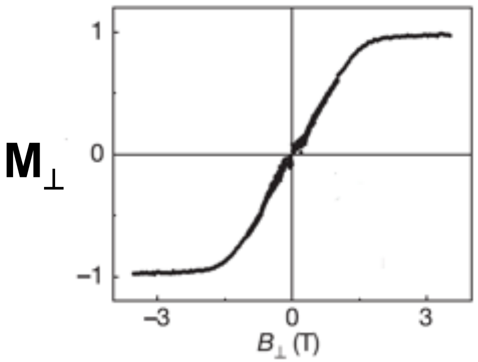
A. Cottey, Thin solid films 1, (1967) 297-397



R<1 at interface

$$\rho_{xy} = R_0 B + \overset{\rho_{ah}}{\boxed{R_{ah} M_z}}$$

$$\rho_{ah} = f(\rho_{xx})$$



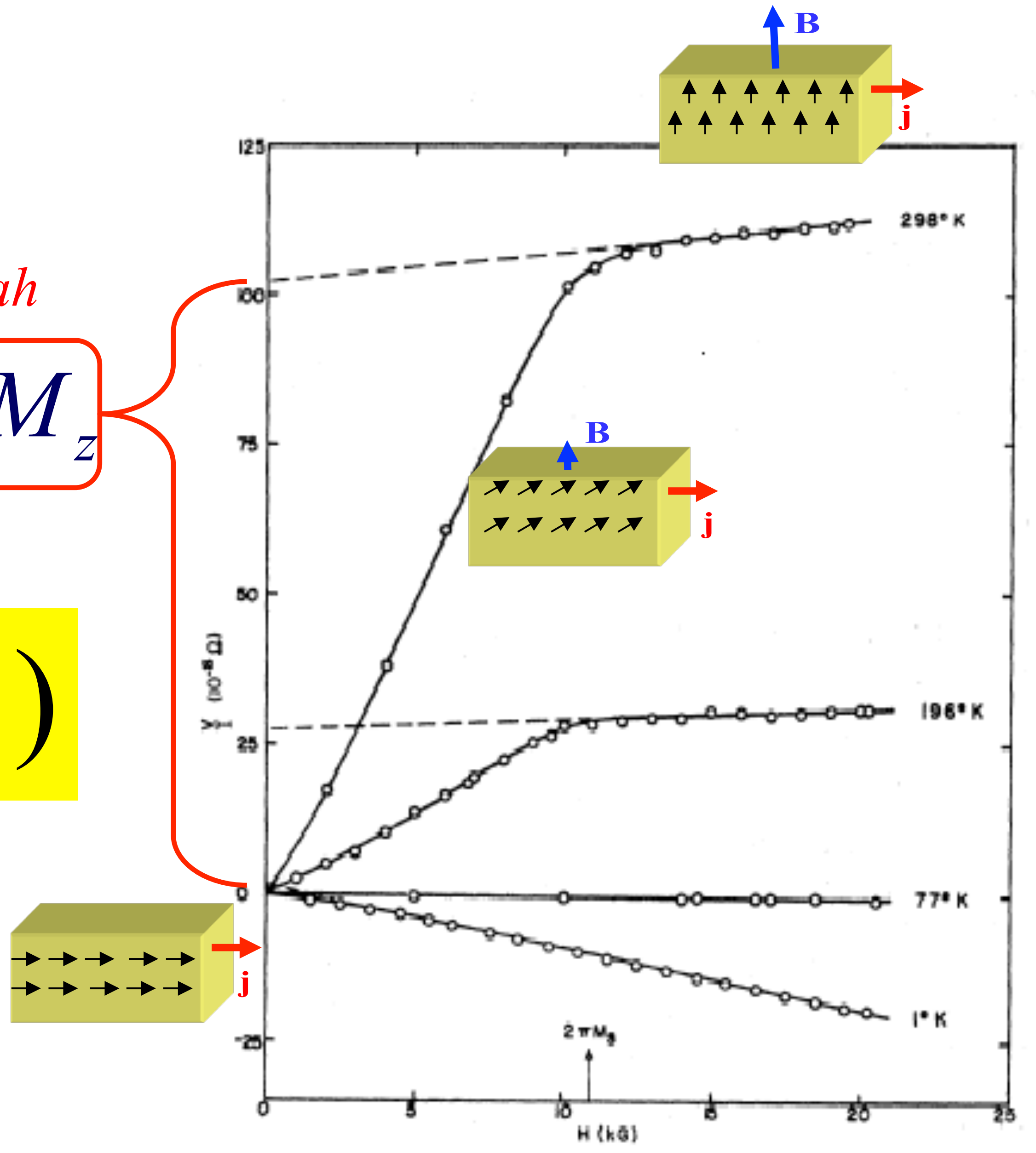
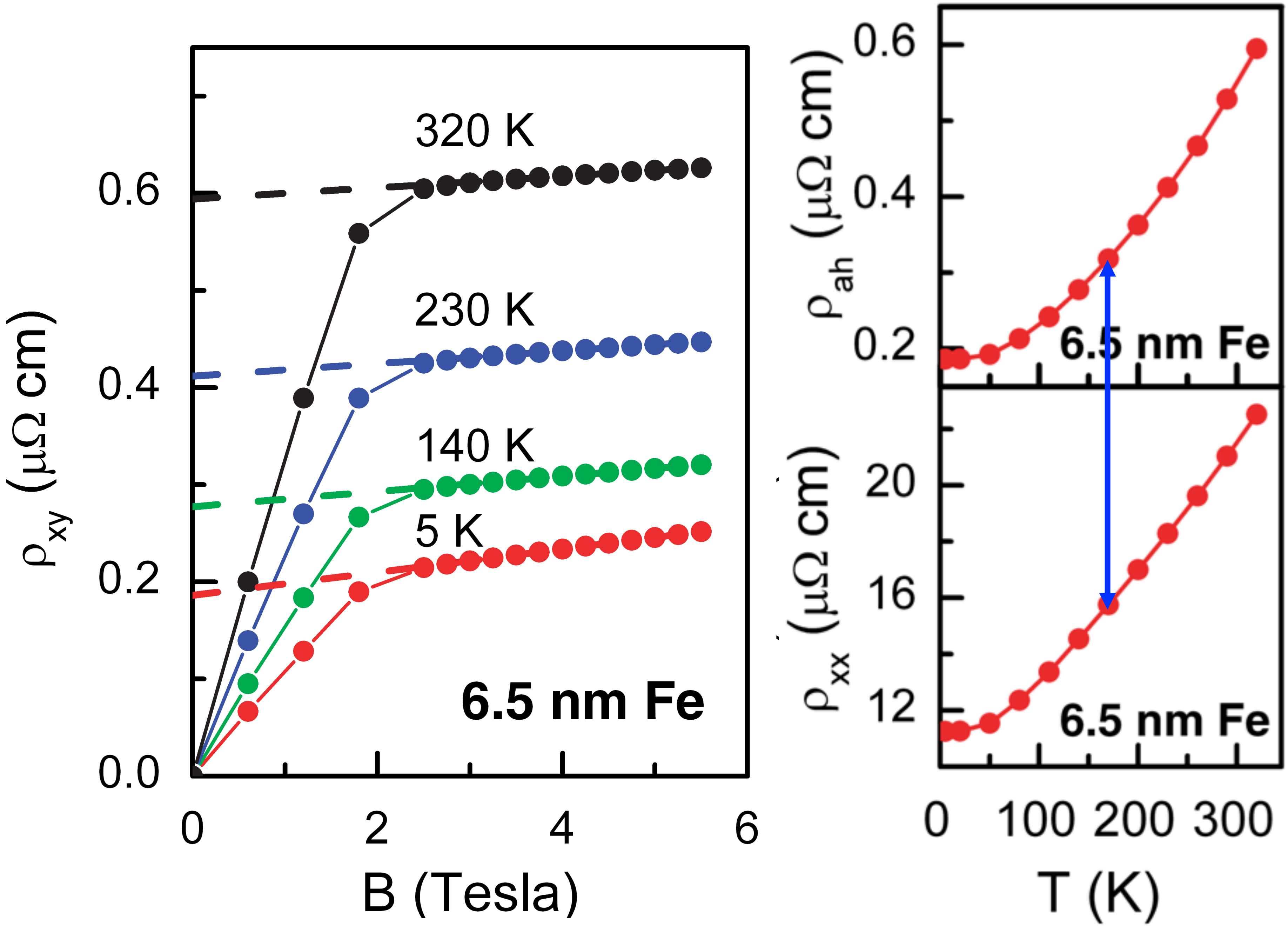
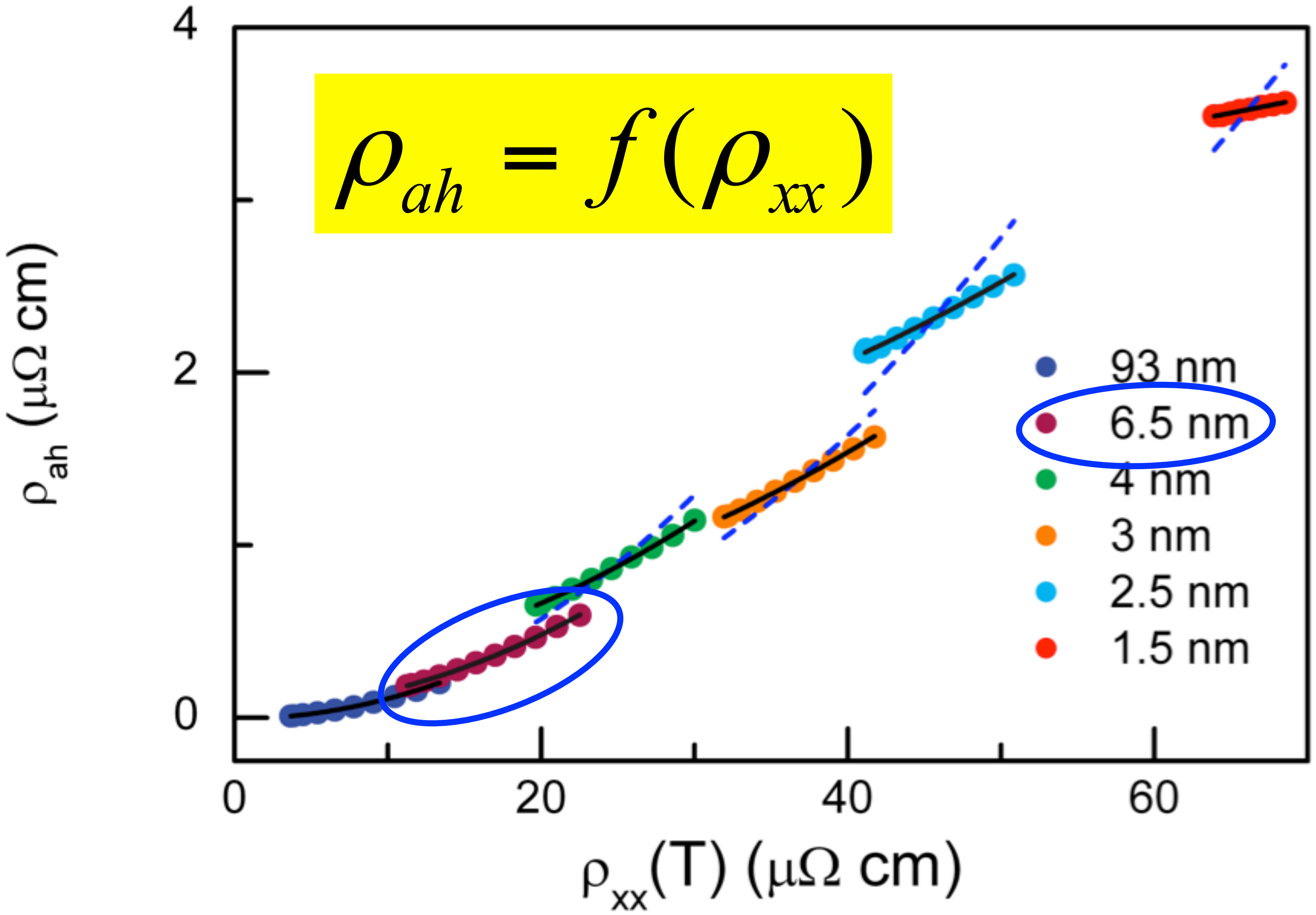


FIG. 1. Variation of the Hall resistance with applied magnetic field for whisker Fe

P.N. Dheer Phys Rev (1967)



320 K
230 K
140 K
5 K
6.5 nm Fe
0.0
0.2
0.4
0.6
ρ_{xy} ($\mu\Omega$ cm)
0
2
4
6
B (Tesla)
ρ_{ah} ($\mu\Omega$ cm)
0.2
0.4
0.6
6.5 nm Fe
ρ_{xx} ($\mu\Omega$ cm)
12
16
20
6.5 nm Fe
0
100
200
300
T (K)



$\rho_{ah} = f(\rho_{xx})$
93 nm
6.5 nm
4 nm
3 nm
2.5 nm
1.5 nm
ρ_{ah} ($\mu\Omega$ cm)
$\rho_{xx}(T)$ ($\mu\Omega$ cm)
0
2
4
0
20
40
60

Matthiessen's rule :

$$\rho_{xx} = \rho_{xx0} + \rho_{xxT}$$

impurity --> skew !

Phonon --> no skew !

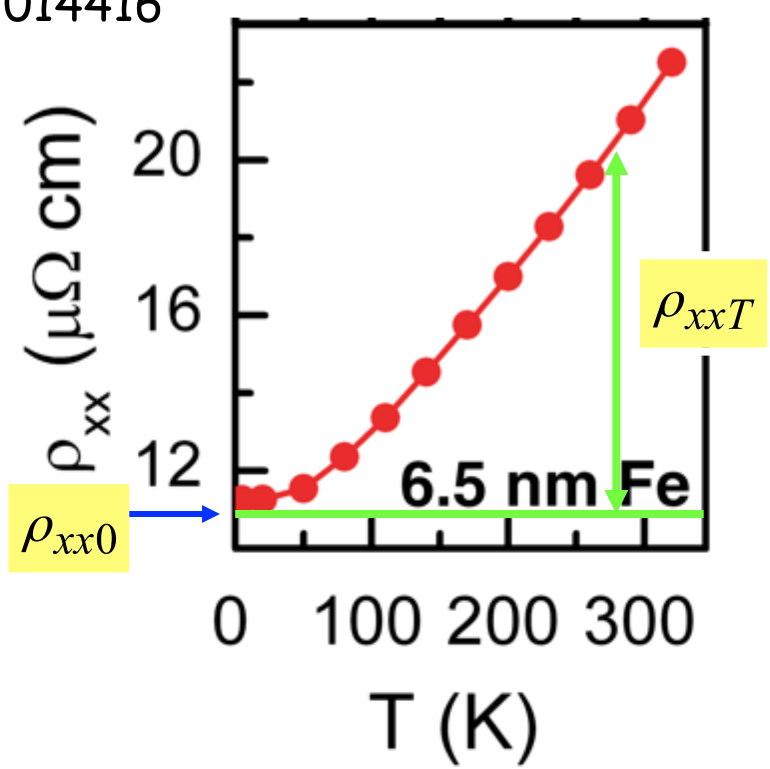
A. Crepieux et al., PRB, 64 (2001) 014416

Contradictory to the traditionally used scaling:

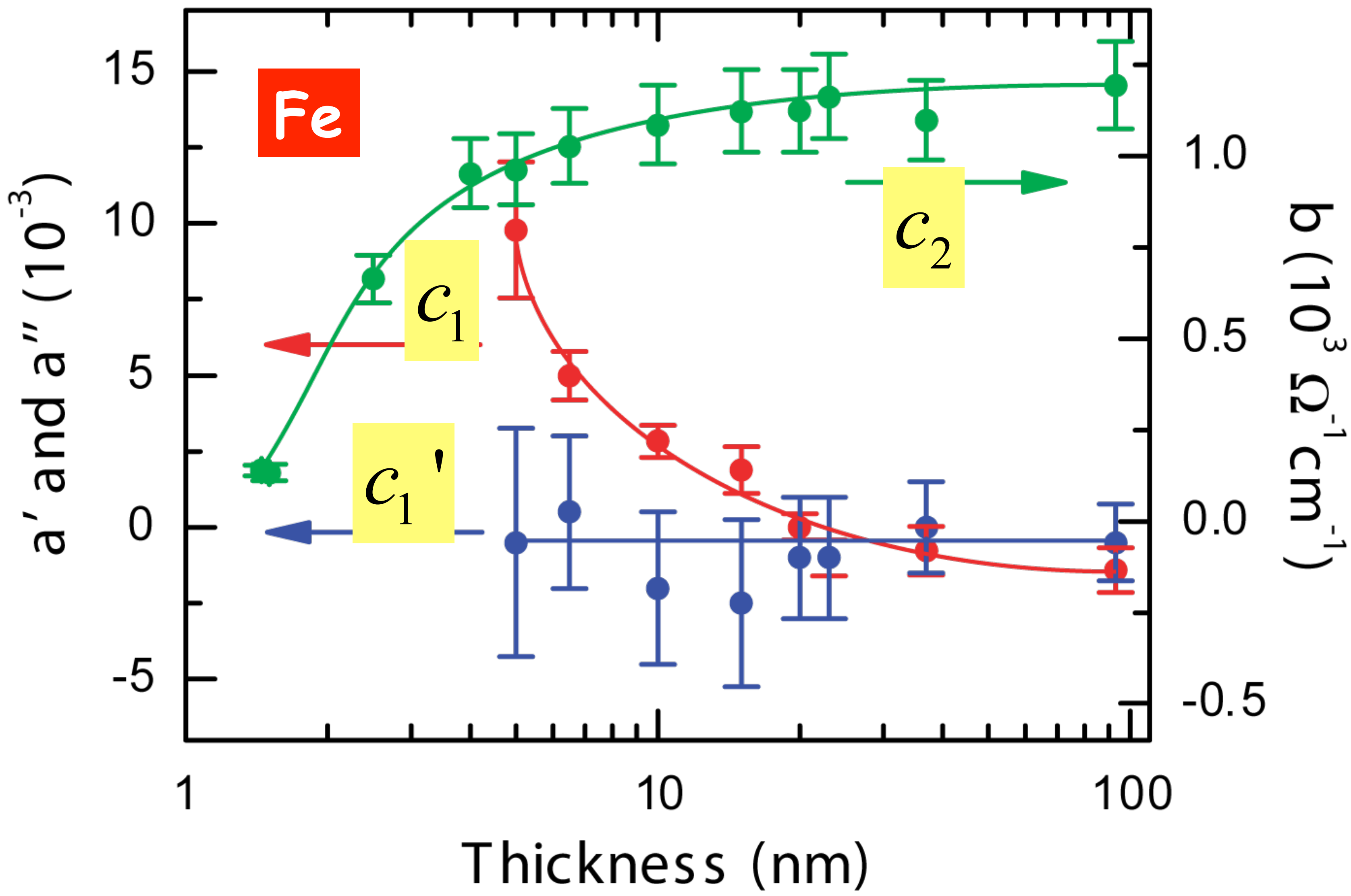
$$\rho_{ah} = c_1\rho_{xx} + c_2\rho_{xx}^2$$

Experimental verification !

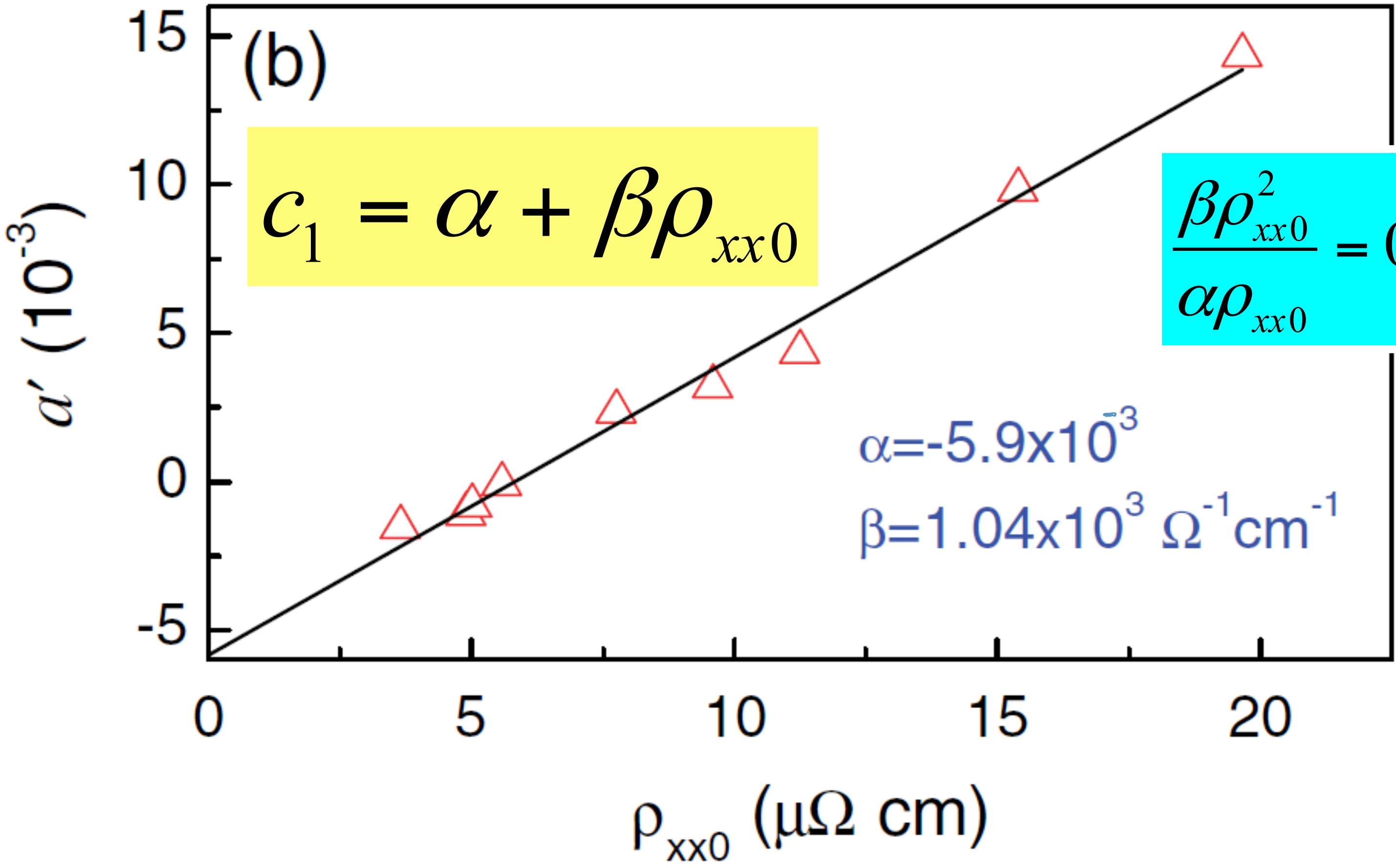
$$\rho_{ah} = c_1\rho_{xx0} + c_1'\rho_{xxT} + c_2\rho_{xx}^2$$



$$\rho_{ah} = c_1\rho_{xx0} + c_2\rho_{xx}^2$$



$$\rho_{ah} = c_1\rho_{xx0} + c_2\rho_{xx}^2$$



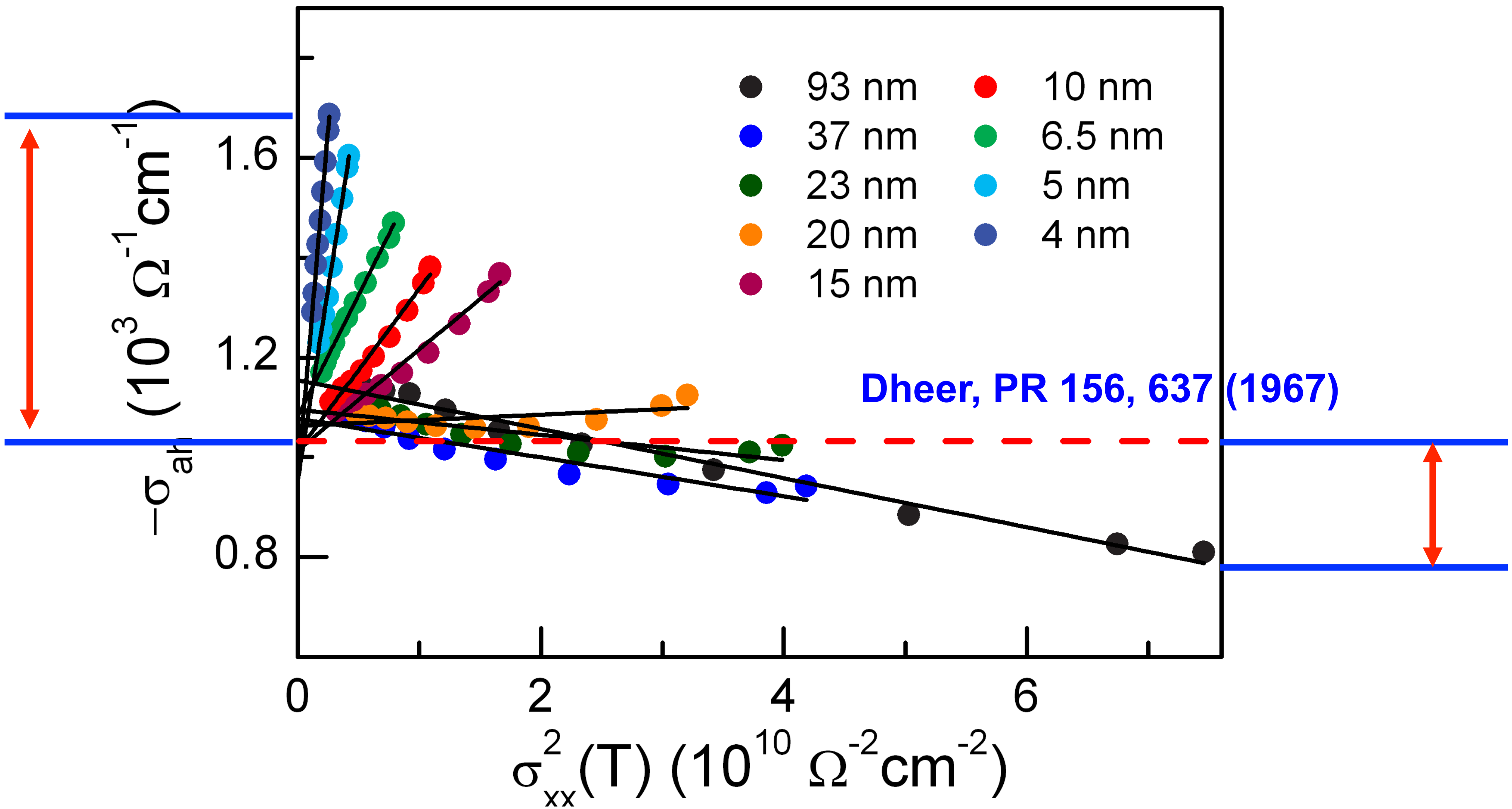
$$c_1 = \alpha + \beta\rho_{xx0}$$

$$\frac{\beta\rho_{xx0}^2}{\alpha\rho_{xx0}} = 0.7 \rightarrow 3.7$$

$$\rho_{ah} = \alpha\rho_{xx0} + \beta\rho_{xx0}^2 + c_2\rho_{xx}^2$$

$$\rho_{ah} = \alpha\rho_{xx0} + \beta\rho_{xx0}^2 + c_2\rho_{xx}^2$$

$$\sigma_{ah} = -(\alpha\sigma_{xx0}^{-1} + \beta\sigma_{xx0}^{-2})\sigma_{xx}^2 - c_2$$



$$c_2 = b = \sigma_{\text{int}\,rinsic} \approx 1.1\times10^3 \quad \Omega^{-1}cm^{-1}$$

Proper Scaling of the Anomalous Hall Effect

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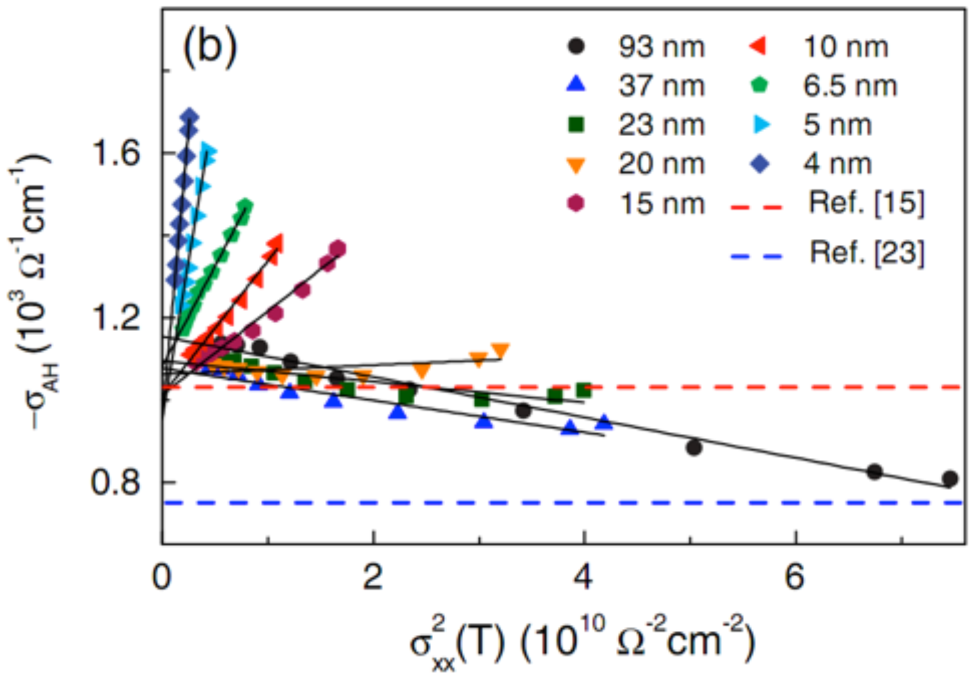
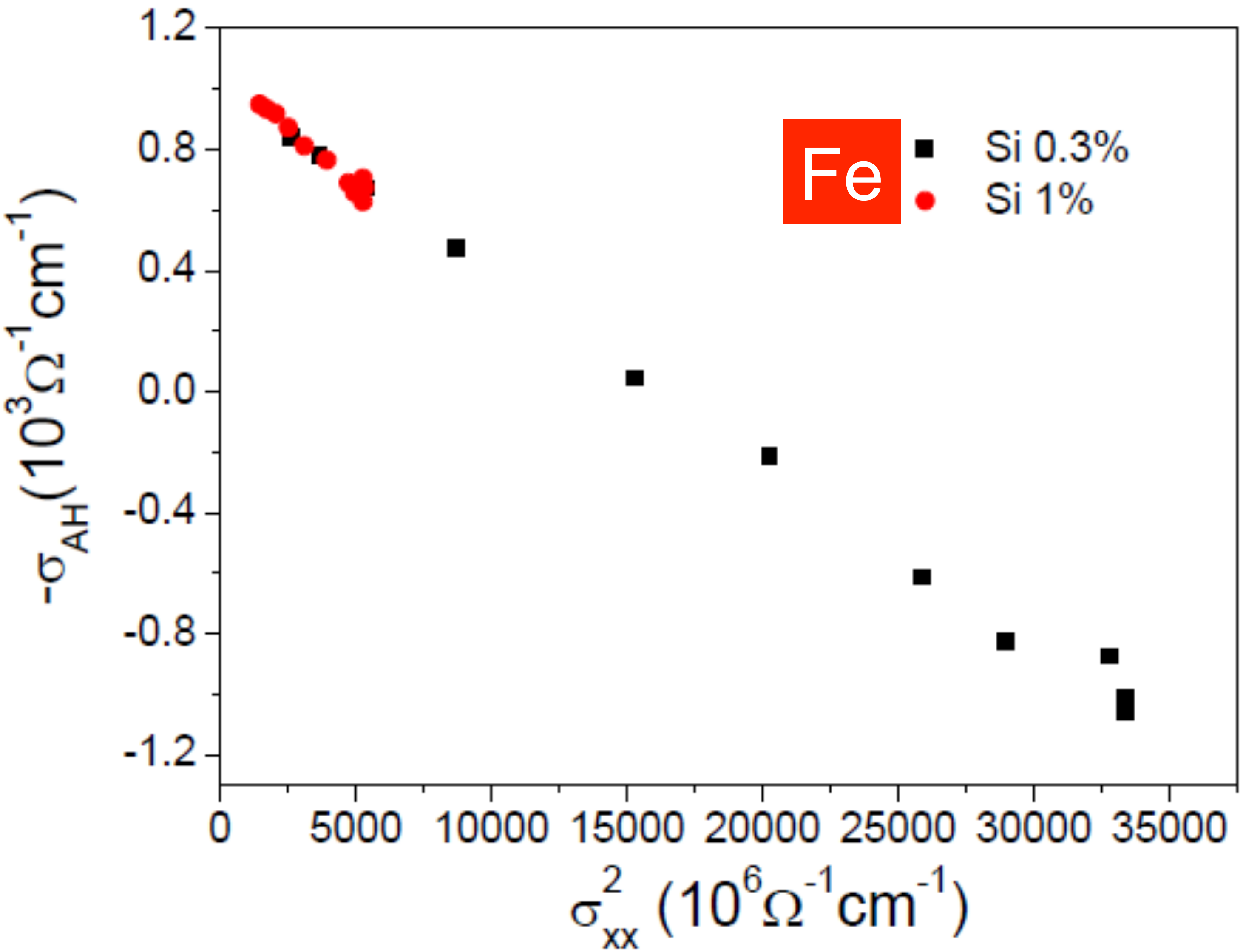
(Received 30 March 2009; published 21 August 2009)

Working with epitaxial films of Fe, we succeeded in independent control of different scattering processes in the anomalous Hall effect. The result clearly exposed the fundamental flaws of the conventional scaling $\rho_{AH} = f(\rho_{xx})$ between the anomalous Hall resistivity and longitudinal resistivity. A new scaling $\rho_{AH} = f(\rho_{xx0}, \rho_{xx})$ that also involves the residual resistivity has been established which helps identify the intrinsic and extrinsic mechanisms of the anomalous Hall effect.



Ni,Co♪

Intercept: 0.3%: $0.99{\times}10^3\ \Omega^{-1}cm^{-1}$, 1%: $1.07{\times}10^3\ \Omega^{-1}cm^{-1}$



Y. Shiomi, Y. Onose and Y. Tokura, PRB 79, 100404(R) 2009

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(Dated: November 13, 2012)

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Euro. Phys. Lett. 103 (2013) 47003

PACS 72.25.-b – Spin polarized transport
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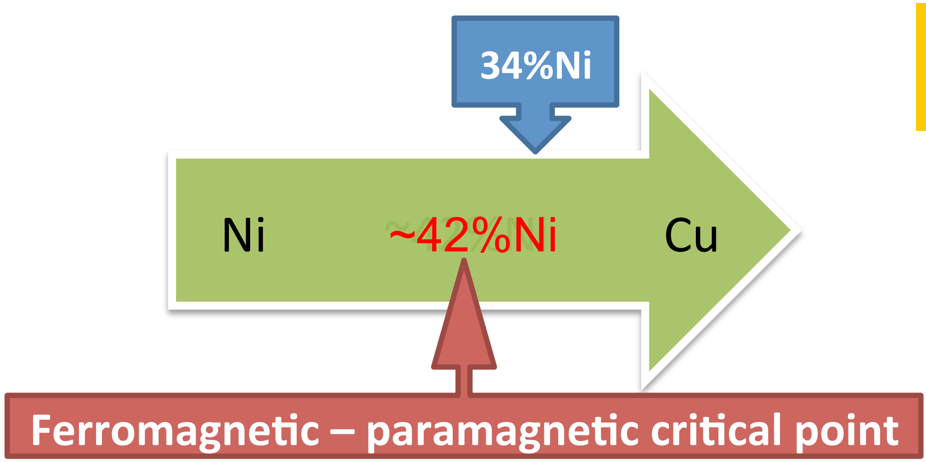
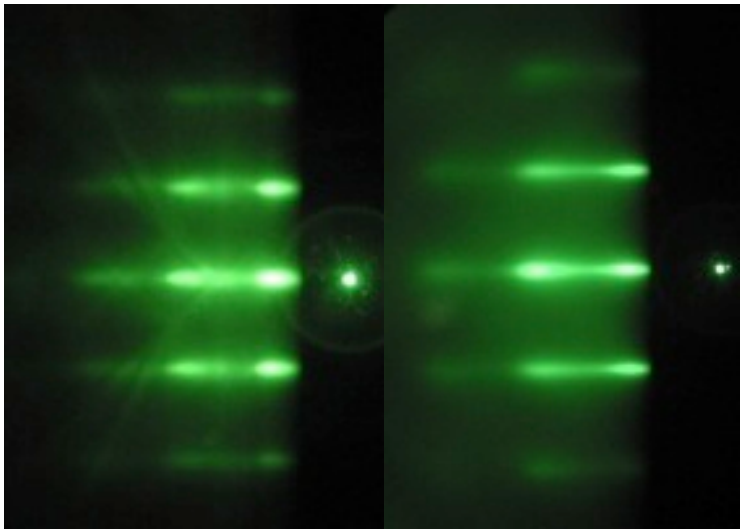
A System without Intrinsic AHE

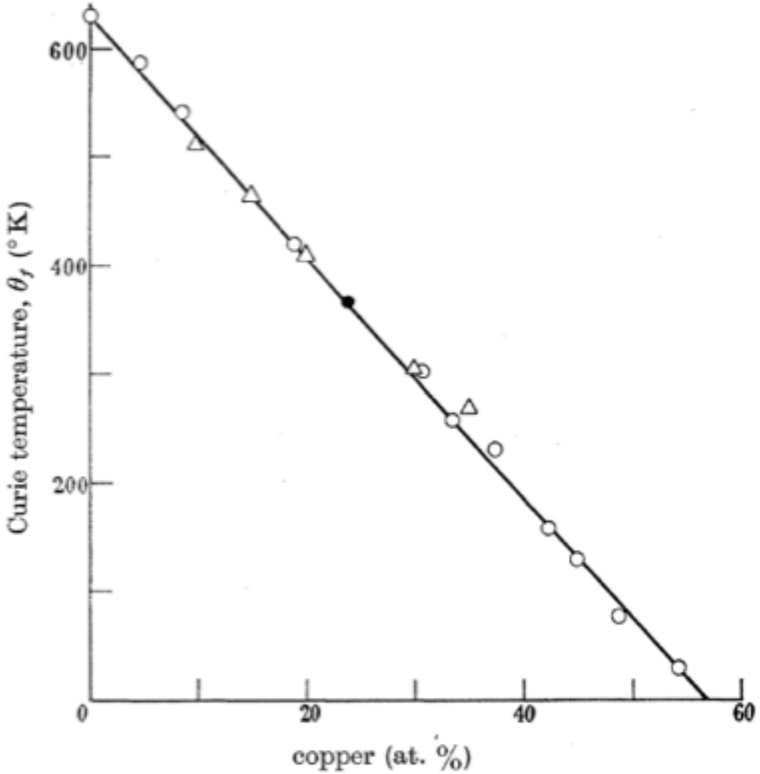
$$\rho_{ah} = \alpha\rho_{xx0} + \beta\rho_{xx0}^2 + b\rho_{xx}^2$$





MgO anealed @500°C

NiCu as grown



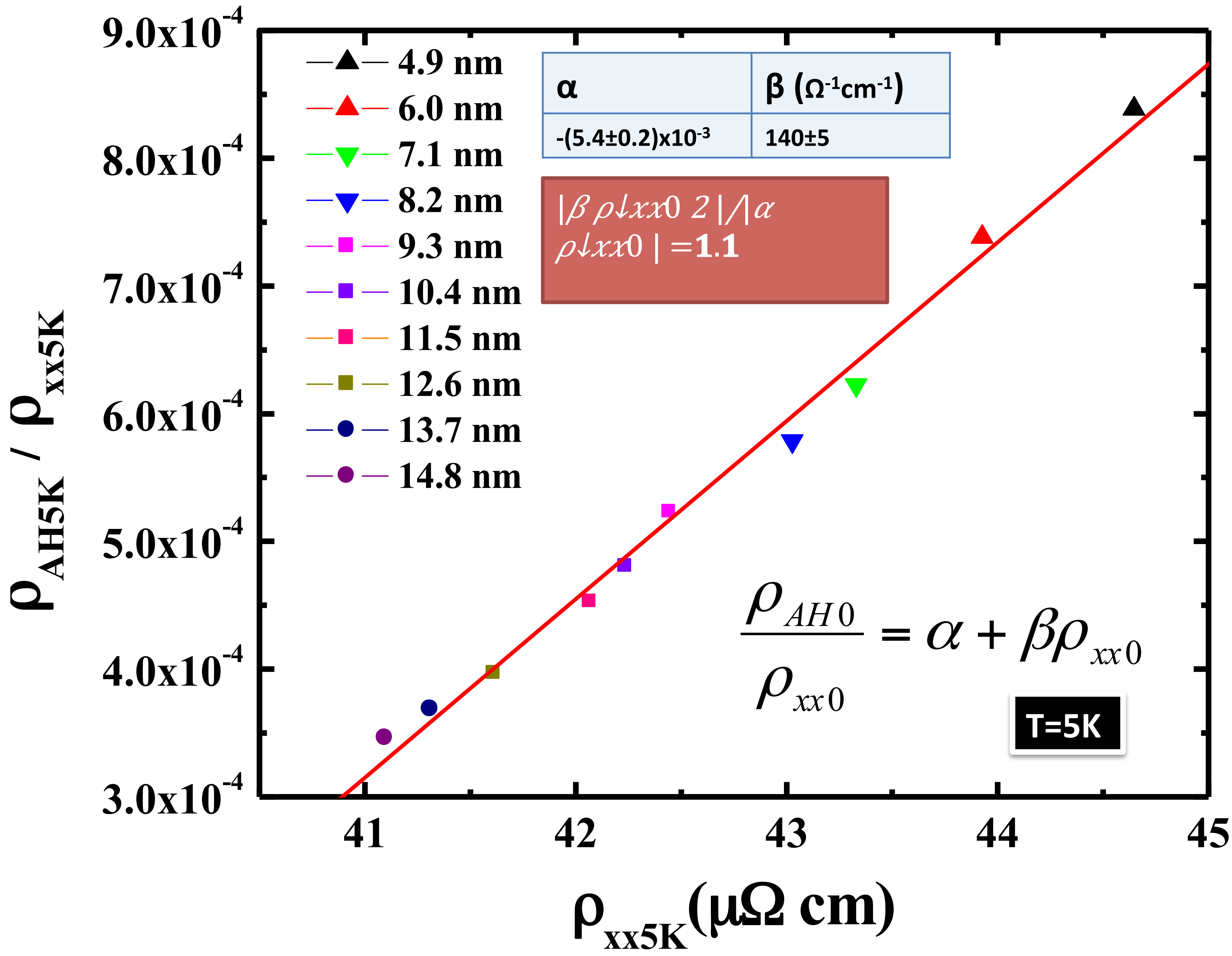
S.A.Aherm et al., Proc. Royal Soc. London, 248 A, (1958) 145

$$\rho_{ah} = \alpha\rho_{xx0} + \beta\rho_{xx0}^2 + b\rho_{xx}^2$$

$$\rho_{AH} = \alpha\rho_{xx0} + \beta\rho_{xx0}^2$$

At 5 K

$$\frac{\rho_{AH0}}{\rho_{xx0}} = \alpha + \beta\rho_{xx0}$$



4.9 nm
6.0 nm
7.1 nm
8.2 nm
9.3 nm
10.4 nm
11.5 nm
12.6 nm
13.7 nm
14.8 nm
α
β (Ω-1cm-1)
-(5.4±0.2)x10-3
140±5
|β ρ↓xx0 2|/|α ρ↓xx0 | =1.1
$\frac{\rho_{AH0}}{\rho_{xx0}} = \alpha + \beta\rho_{xx0}$
T=5K
9.0x10-4
8.0x10-4
7.0x10-4
6.0x10-4
5.0x10-4
4.0x10-4
3.0x10-4
41
42
43
44
45
ρAH5K / ρxx5K
ρxx5K(μΩ cm)

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Anomalous Hall Effect in Ferromagnetic Metals: Role of Phonons at Finite Temperature

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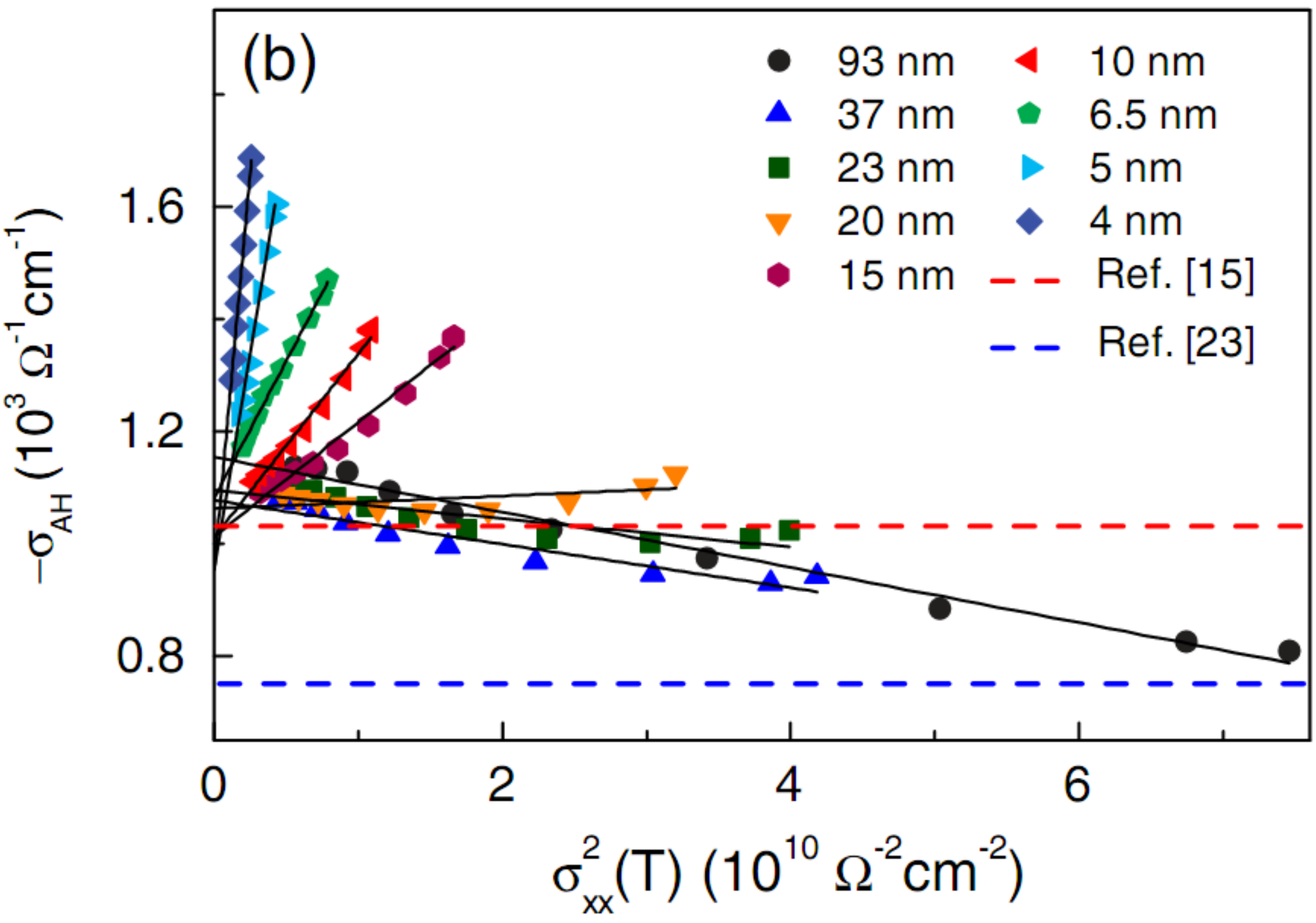
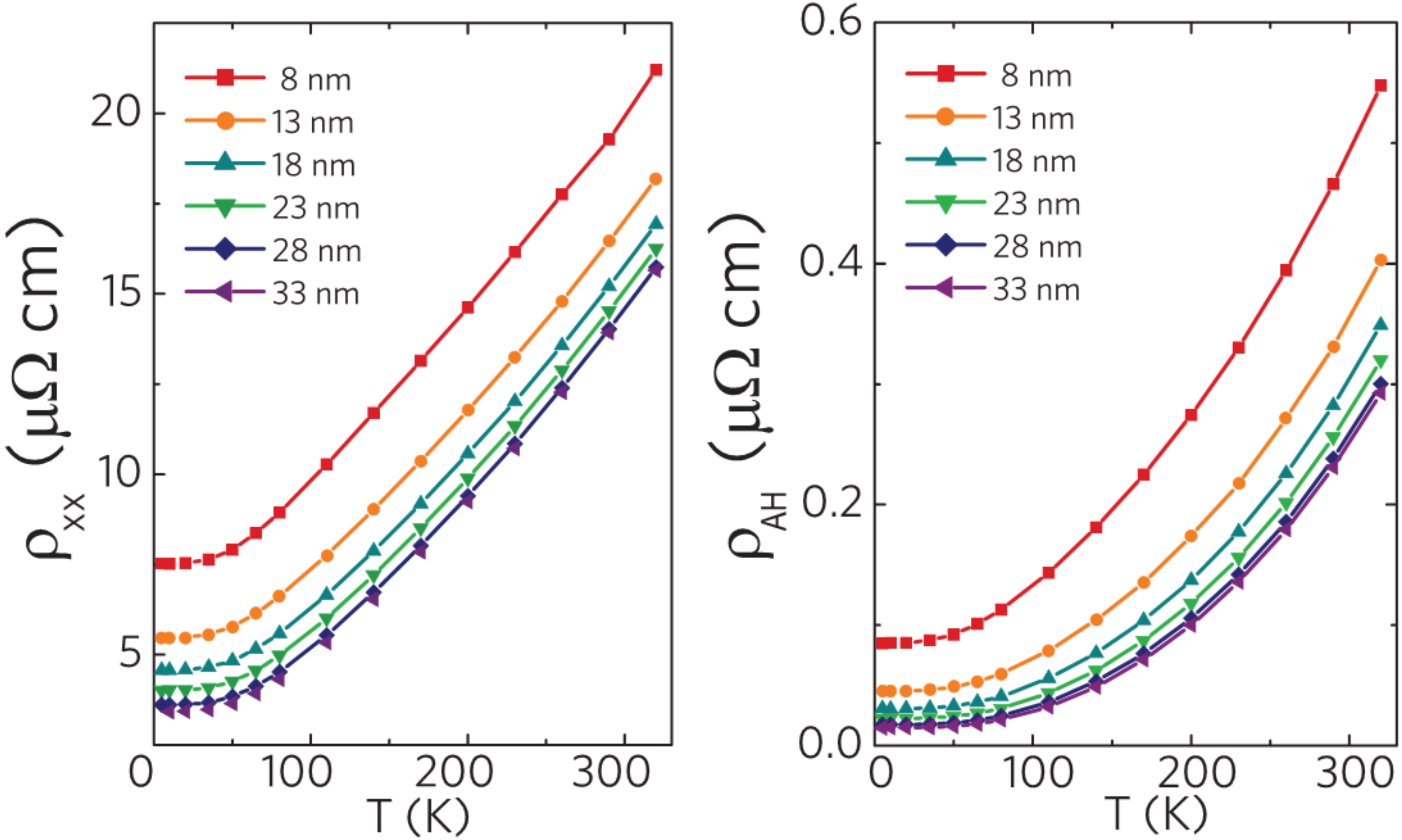
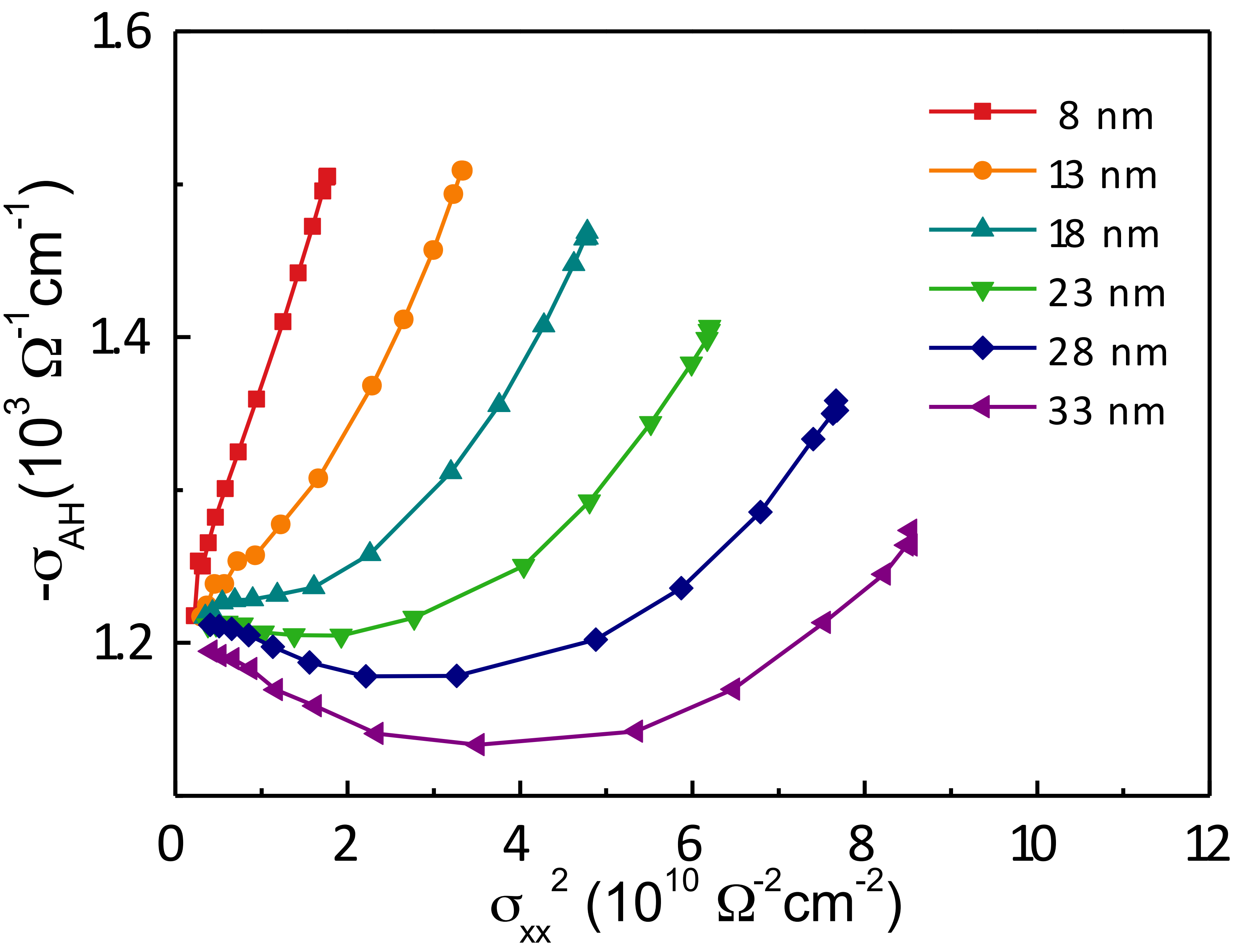
The anomalous Hall effect in a multiband tight-binding model is numerically studied taking into account both elastic scattering by disorder and inelastic scattering by the electron–phonon interaction. The Hall conductivity is obtained as a function of temperature T, inelastic scattering rate γ, chemical potential μ, and impurity concentration x_{imp}. We find that the new scaling law holds over a wide range of these parameters; $-\sigma_{xy} = (\alpha\sigma_{xx0}^{-1} + \beta\sigma_{xx0}^{-2})\sigma_{xx}^{2} + b$, with $\sigma_{\mu\nu}$ ($\sigma_{\mu\nu0}$) being the conductivity tensor (with only elastic scattering), which corresponds to the recent experimental observation [Phys. Rev. Lett. **103** (2009) 087206]. The condition of this scaling is examined. Also, it is found that the intrinsic mechanism depends on temperature under a resonance condition.

KEYWORDS: anomalous Hall effect, inelastic scattering, phonon

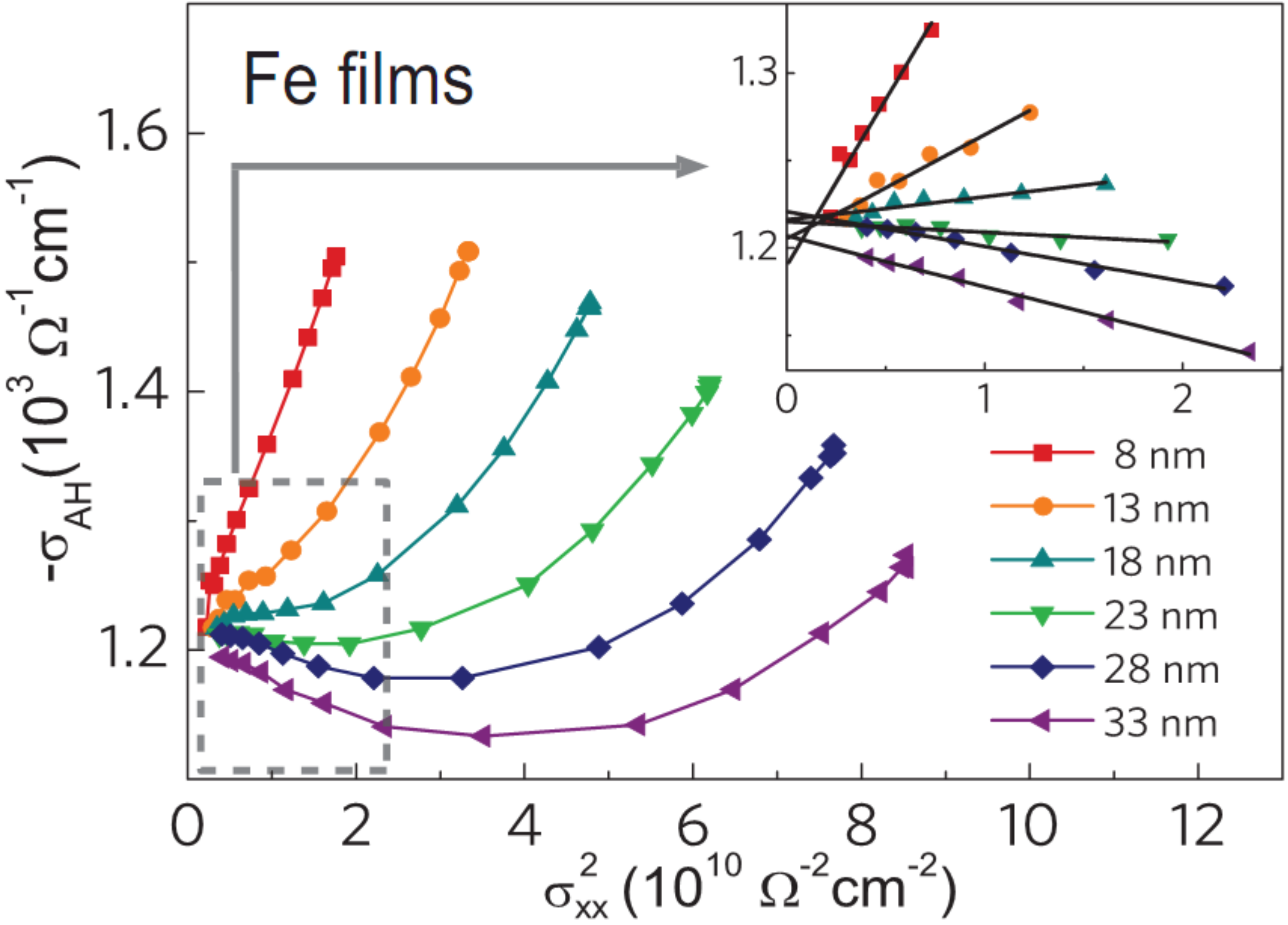

(b)
93 nm
37 nm
23 nm
20 nm
15 nm
10 nm
6.5 nm
5 nm
4 nm
Ref. [15]
Ref. [23]
1.6
1.2
0.8
0
2
4
6
$-\sigma_{AH}$ (10^3 Ω^{-1}cm^{-1})
σ_{xx}^2(T) (10^{10} Ω^{-2}cm^{-2})

Fe/MgO(001) films

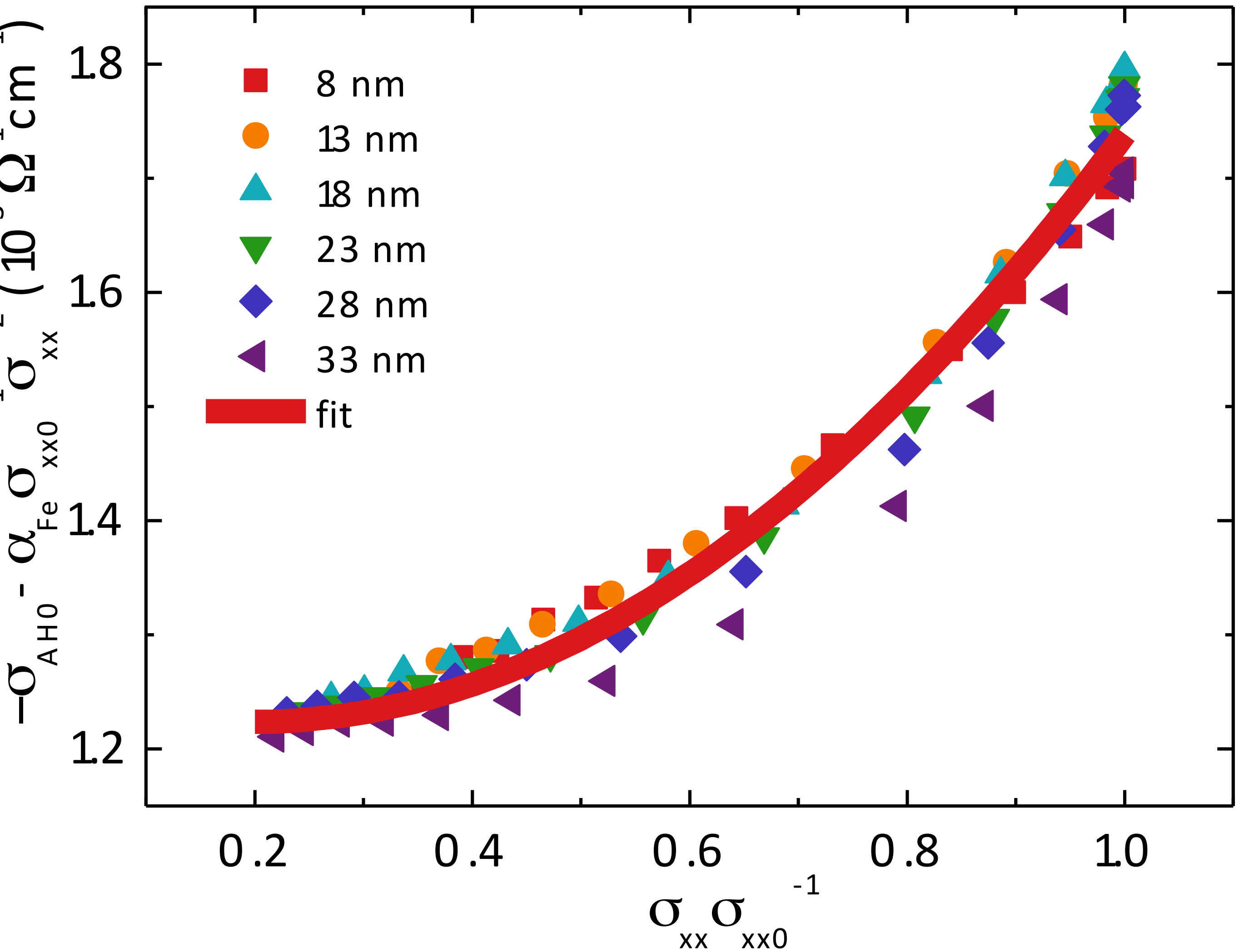




1.6
1.4
1.2
0
2
4
6
8
10
12
-σAH(10³ Ω⁻¹cm⁻¹)
σxx² (10¹⁰ Ω⁻²cm⁻²)
8 nm
13 nm
18 nm
23 nm
28 nm
33 nm



Fe films
1.6
1.4
1.2
$-\sigma_{AH}(10^3\ \Omega^{-1}cm^{-1})$
0
2
4
6
8
10
12
$\sigma_{xx}^2\ (10^{10}\ \Omega^{-2}cm^{-2})$
1.3
1.2
0
1
2
8 nm
13 nm
18 nm
23 nm
28 nm
33 nm



8 nm
13 nm
18 nm
23 nm
28 nm
33 nm
fit
1.2
1.4
1.6
1.8
0.2
0.4
0.6
0.8
1.0
$-\sigma_{AH0} - \alpha_{Fe}\sigma_{xx0}^{-1}\sigma_{xx}^{2}$ ($10^{3}\,\Omega^{-1}cm^{-1}$)
$\sigma_{xx}\sigma_{xx0}^{-1}$

Theory with multiple competing

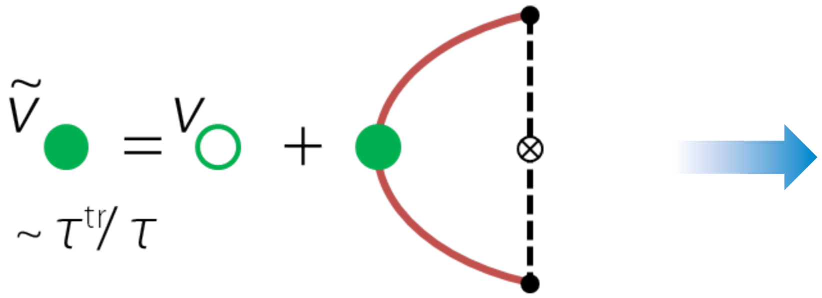
AH conductivity in Kubo-Streda formula

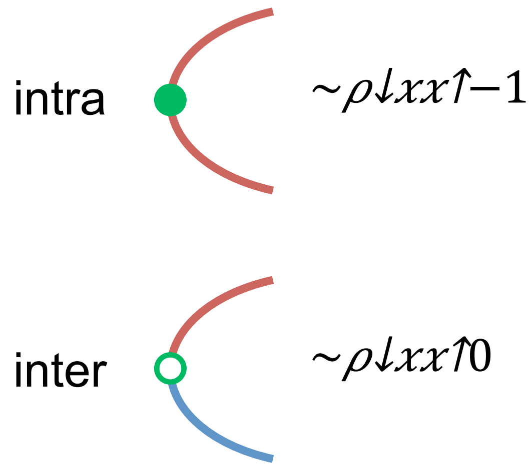
scattering effects in the Fermi surface term

$$\sigma_{AH} = \sigma_{AH}^{I} + \sigma_{AH}^{II}$$

$$\sigma_{AH}^{I} = \frac{e^2}{2\pi A}\langle v_x G^R(\varepsilon_F) v_y G^A(\varepsilon_F)\rangle$$

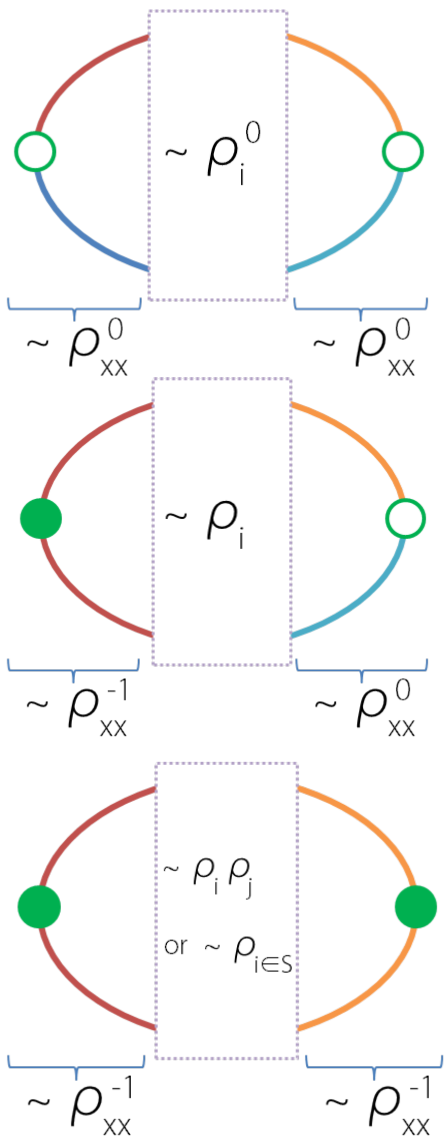
velocity vertex: intraband vertex needs to be dressed





note that $1/\rho_{xx} \cong \sigma_{xx} \propto \tau^{tr}$

Three groups of diagrams



$+\sigma_{AH}^{I}$ → c intrinsic Berry curvature

→ $\sum_i c_i \rho_i / \rho_{xx}$

→ $(\sum_{ij} c_{ij} \rho_i \rho_j + \sum_{i\in S} \alpha_i \rho_i)/\rho_{xx}^2$

$$-\sigma_{AH} = c + \sum_i c_i \rho_i / \rho_{xx} + \left(\sum_{ij} c_{ij} \rho_i \rho_j + \sum_{i \in S} \alpha_i \rho_i\right) / \rho_{xx}^2$$

$$\boxed{\rho_{AH} \cong -\sigma_{AH} \rho_{xx}^2 = c\rho_{xx} + \sum_i c_i \rho_i \rho_{xx} + \sum_{ij} c_{ij} \rho_i \rho_j + \sum_{i \in S} \alpha_i \rho_i}$$

- partial resistivities ρ_i as scaling variables
- coefficients c_i, c_{ij}, and α_i not depend on disorder concentratior

for two scattering sources: one static (impurity) and one dynamic (phonon)

$$\boxed{\rho_{AH} = \alpha \rho_{xx0} + \beta_0 \rho_{xx0}^2 + \gamma \rho_{xx0} \rho_{xxT} + \beta_1 \rho_{xxT}^2}$$

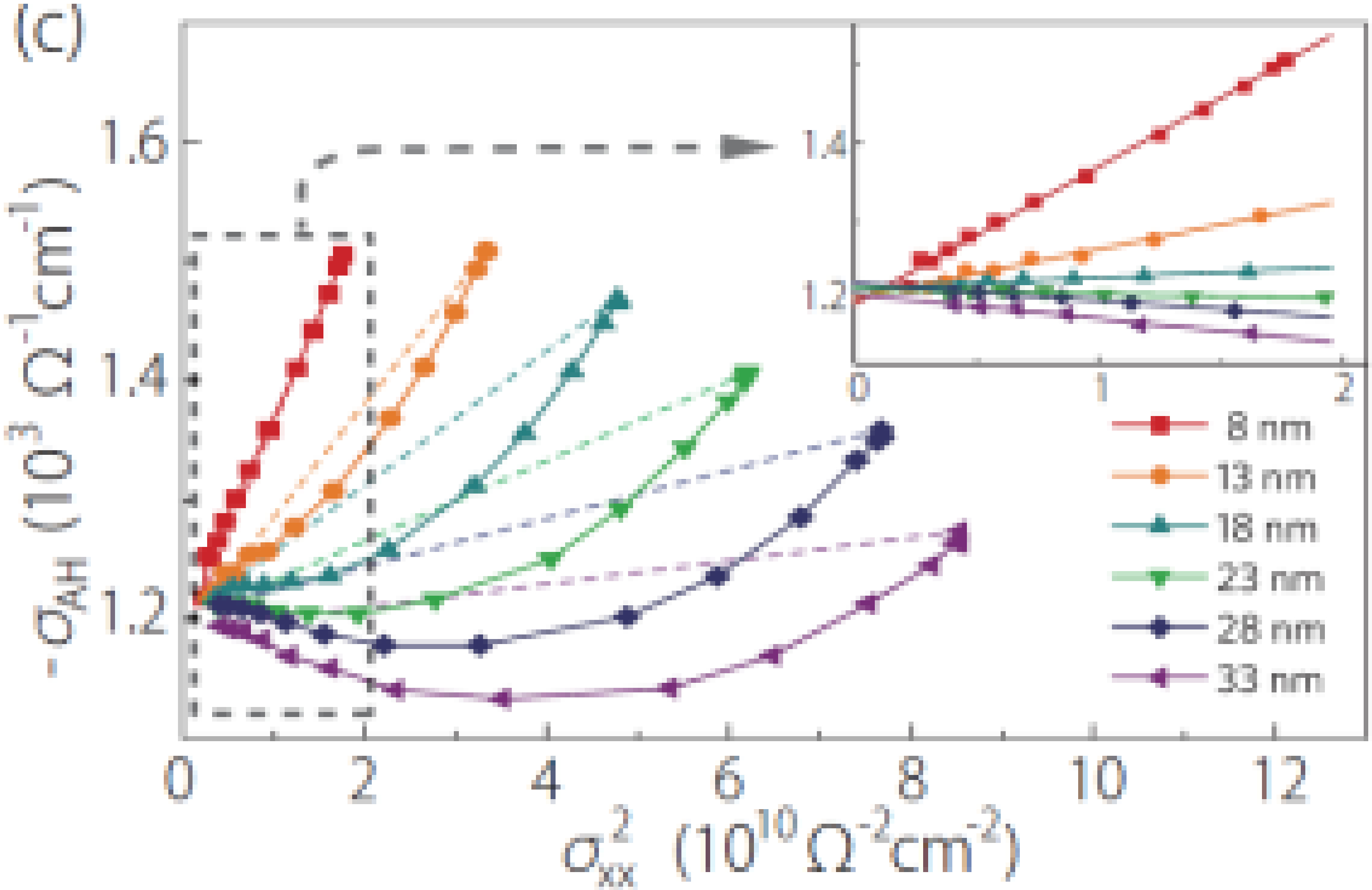
a quadratic surface passing through origin in (ρ_{xx0}, ρ_{xxT}, ρ_{AH}) space

$$\beta_0 = c + c_0 + c_{00}$$
$$\gamma = 2c + c_0 + c_1 + c_{01}$$
$$\beta_1 = c + c_1 + c_{11}$$

$$\rho_{ah} = \alpha \rho_{xx0} + \beta \rho_{xx0}^2 + d_0 \rho_{xx}^2 + d_1 \rho_{xx0} \rho_{xx}$$

$$\sigma_{ah} = -(\alpha \sigma_{xx0}^{-1} + \beta \sigma_{xx0}^{-2}) \sigma_{xx}^2 - d_0 - d_1 \sigma_{xx0}^{-1} \sigma_{xx}$$

$$\sigma_{ah} = -(\alpha\sigma_{xx0}^{-1} + \beta\sigma_{xx0}^{-2})\sigma_{xx}^{2} - d_0 - d_1\sigma_{xx0}^{-1}\sigma_{xx}$$



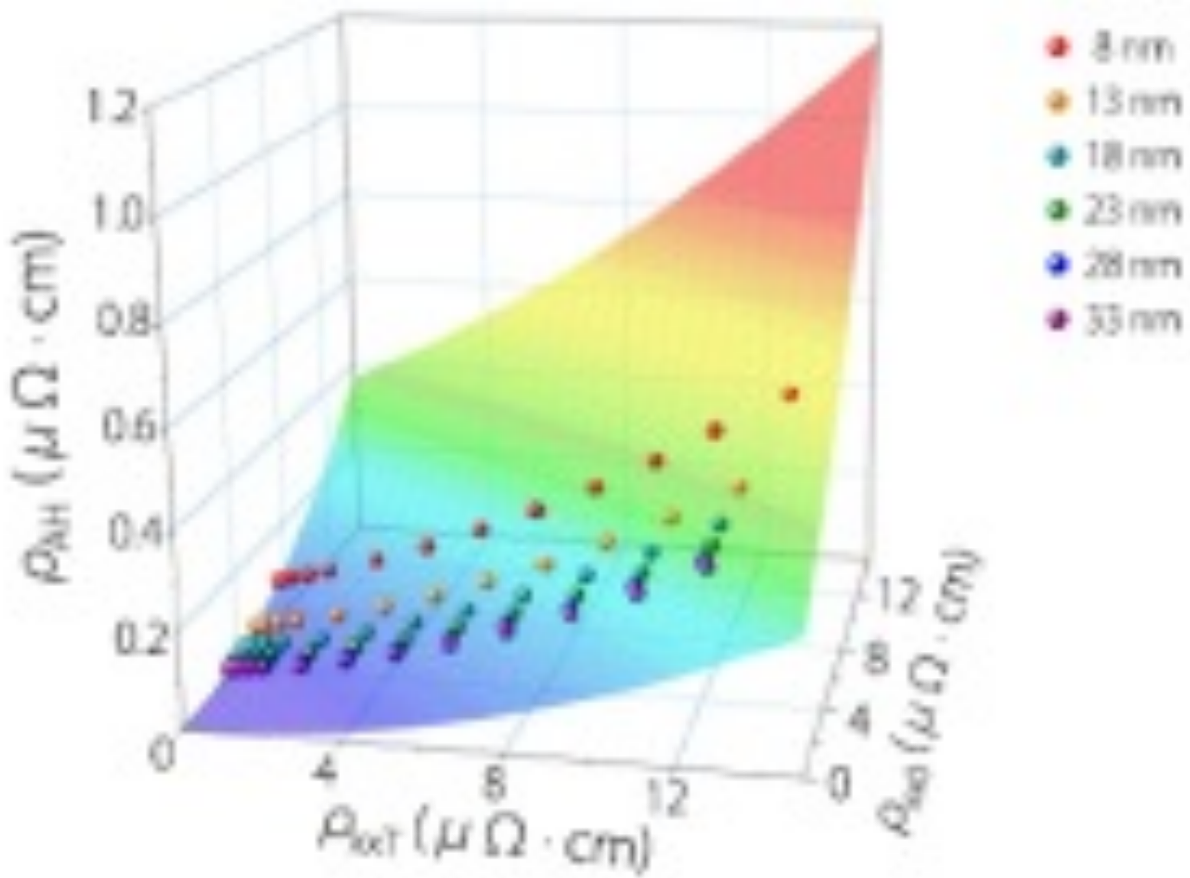


FIG. 3 (color online). The quadratic surface of $\rho_{AH}(\rho_{xx0}, \rho_{xxT})$ of Eq. (3) with the values of four parameters determined in Fig. 2. The deviation of each raw data point from the surface is within the size of the dots.

Multivariable Scaling for the Anomalous Hall Effect

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(Received 11 March 2015; published 29 May 2015)

We derive a general scaling relation for the anomalous Hall effect in ferromagnetic metals involving multiple competing scattering mechanisms, described by a quadratic hypersurface in the space spanned by the partial resistivities. We also present experimental findings, which show strong deviation from previously found scaling forms when different scattering mechanisms compete in strength but can be nicely explained by our theory.

DOI: 10.1103/PhysRevLett.114.217203 PACS numbers: 75.47.Np, 72.15.Eb, 73.50.Jt

$$\rho_{ah} = \alpha\rho_{xx0} + \beta\rho_{xx0}^2 + c_2\rho_{xx}^2$$

Au

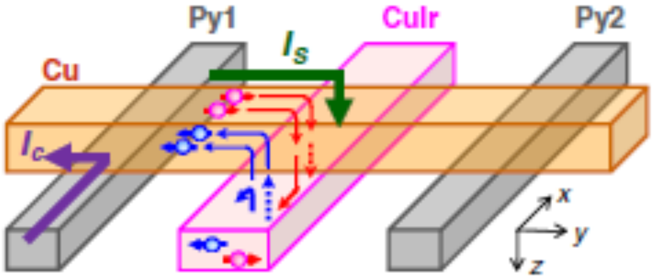
T (K)	σ^c_{xx} $\left(\frac{10^6}{\Omega \mathrm{m}}\right)$	$\sigma^{s_z}_{xy}$ $\left(\frac{\hbar}{e}\frac{10^3}{\Omega \mathrm{m}}\right)$	Θ_{SH} (%)	**Ref.**
4.5	48.3	< 1110	< 2.3	[97]
293	25.2	88 ± 8	$\boxed{0.35 \pm 0.03}$	[15]
293	5.3	84 ± 5	1.6 ± 0.1	[98]
293	7.0	23.4 ± 0.4	0.335 ± 0.006	[98]
293	20	50 ± 10	0.25 ± 0.05	[99]
295	37	≈ 4200	$\boxed{\approx 11}$	[100]
295	25.7	< 694	< 2.7	[97]

Spin Hall Effect♪

A. Hoffmann, IEEE transactions on magnetics 49, 5172 (2013)

Method 1: non-local spin valve

Spin injection with FM/Cu/NM bridge



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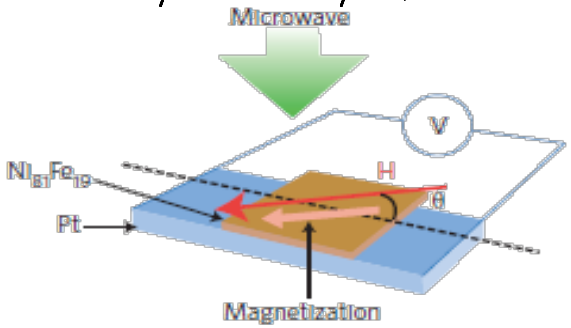
Y. Niimi, et al, Phys. Rev. Lett. 106, 126601 (2011)

Method 2: spin pumping

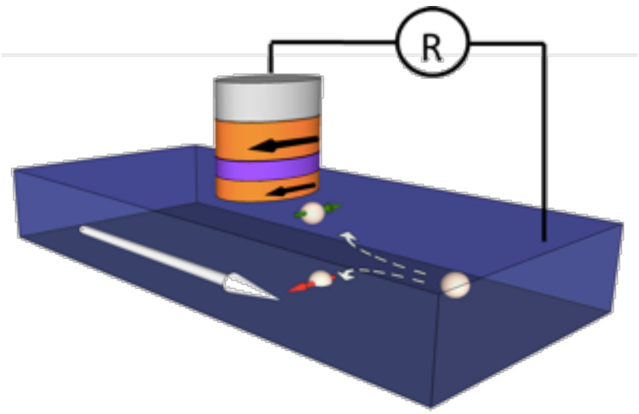
FM/NM bilayer driven by Brf



E. Saitoh, et al . Appl. Phys. Lett 88, 182509 (2006).

Method 3: spin transfer torque

NM/FM bilayer structure



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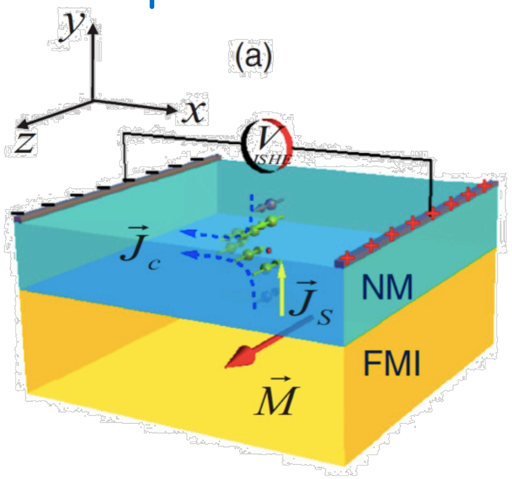
Method 4: spin Seebeck effect



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Generation

Detection

Shunting

Interface

Au

T (K)	σ_{xx}^{c} $\left(\frac{10^6}{\Omega\text{m}}\right)$	$\sigma_{xy}^{s_z}$ $\left(\frac{\hbar}{e}\frac{10^3}{\Omega\text{m}}\right)$	Θ_{SH} (%)	**Ref.**
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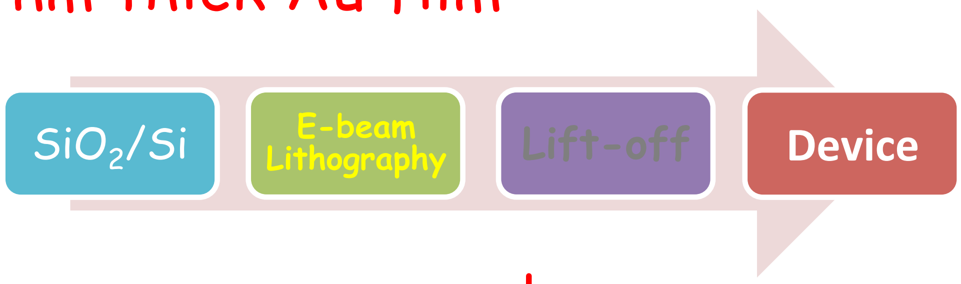
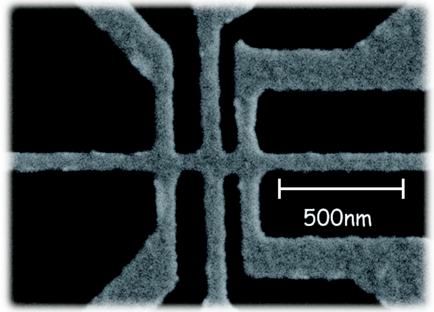
295	37	≈ 4200	$\boxed{\approx 11}$	[100]
295	25.7	< 694	< 2.7	[97]

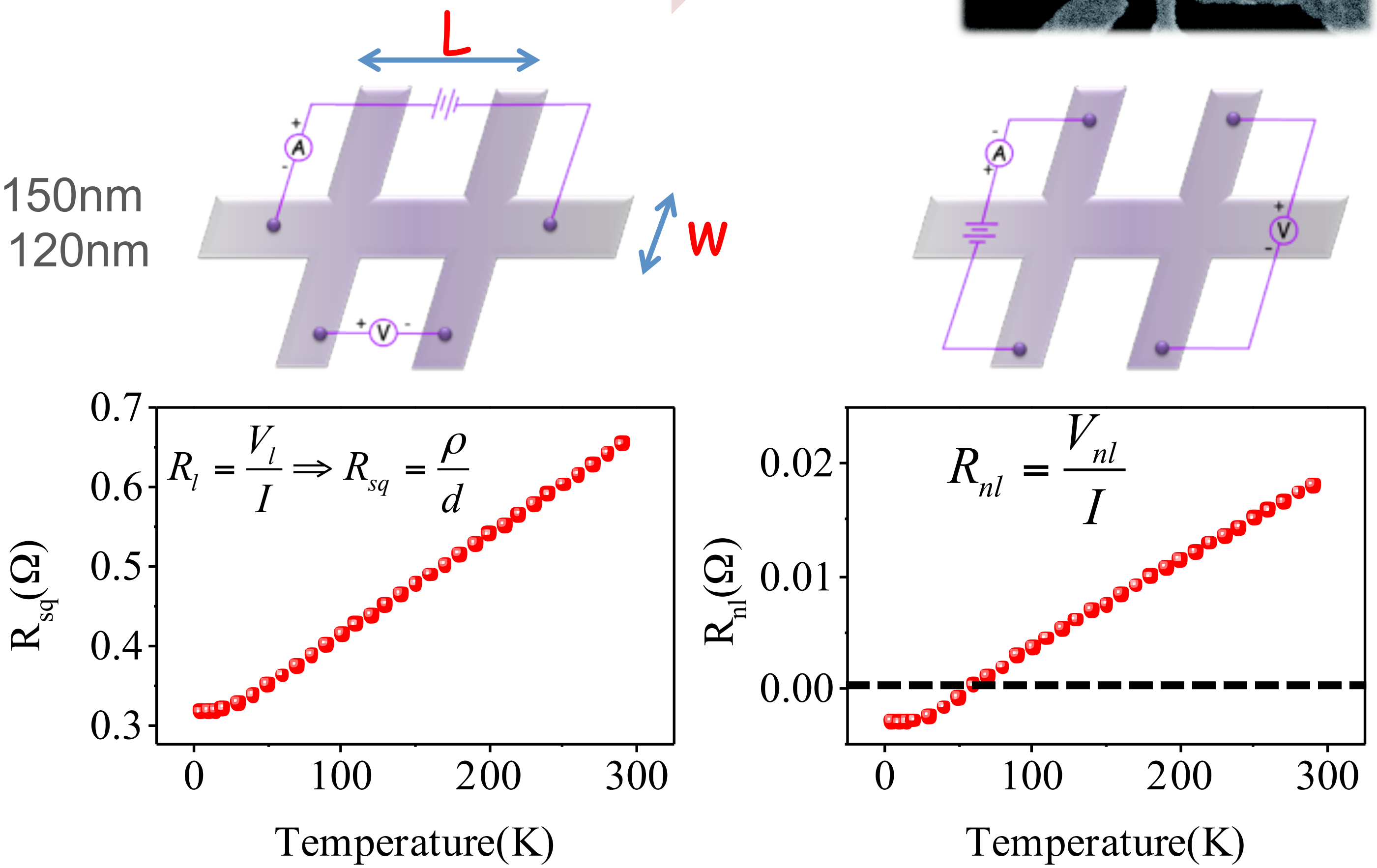
A. Hoffmann, IEEE transactions on magnetics 49, 5172 (2013)

60 nm thick Au film





L = 150nm
W= 120nm

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Dr. G. Su


Dr. J.L. Xu


Dr. L. Wu

Caigan Chen

Dai Tian

3. Conclusions

$$-\sigma_{ah} = \underbrace{(\alpha\sigma_{xx0}^{-1} + \beta\sigma_{xx0}^{-2})\sigma_{xx}^2 + d_1\sigma_{xx0}^{-1}\sigma_{xx}}_{\text{Extrinsic}} + \underbrace{d_0}_{\text{"Intrinsic"}}$$

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THANK YOU