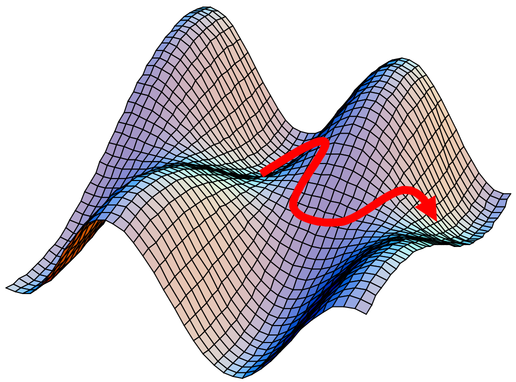
Dynamics Modelling & Simulation



Robert Stamps received BS and MS degrees from the University of Colorado, and a PhD in Physics from Colorado State University. He was with the University of Western Australia until 2010, and is currently Professor of Solid State Physics at the University of Glasgow in Scotland. He was an IEEE Magnetics Society Distinguished Lecturer in 2008 (including visits to CBPF and elsewhere in Brazil), and he was the IEEE/IOP Wohlfarth Lecturer in 2004. He is chair of the IRUK IEEE Magnetics Society Chapter, was chair of the 2007 MML Symposium, and will co-chair the Joint European Magnetics Symposia in 2016. This is the fourth time he has lectured at an IEEE Magnetics School.

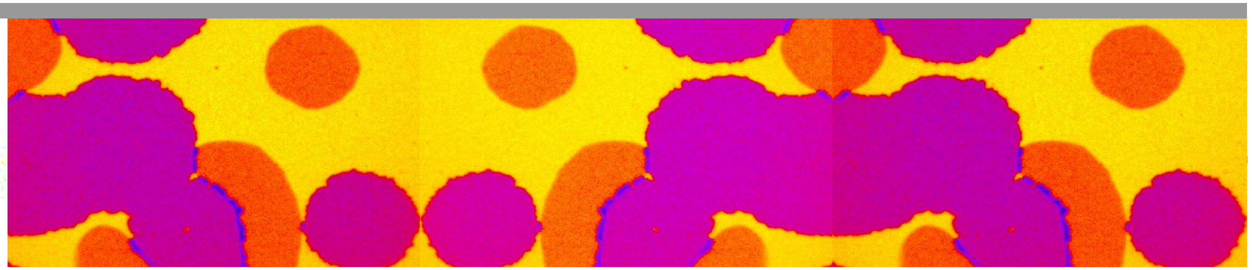
University of Glasgow

Dynamics: Modelling and Simulation

Robert Stamps

IEEE Magnetics Society School 2017

IEEE MAGNETICS



Aim of lectures:

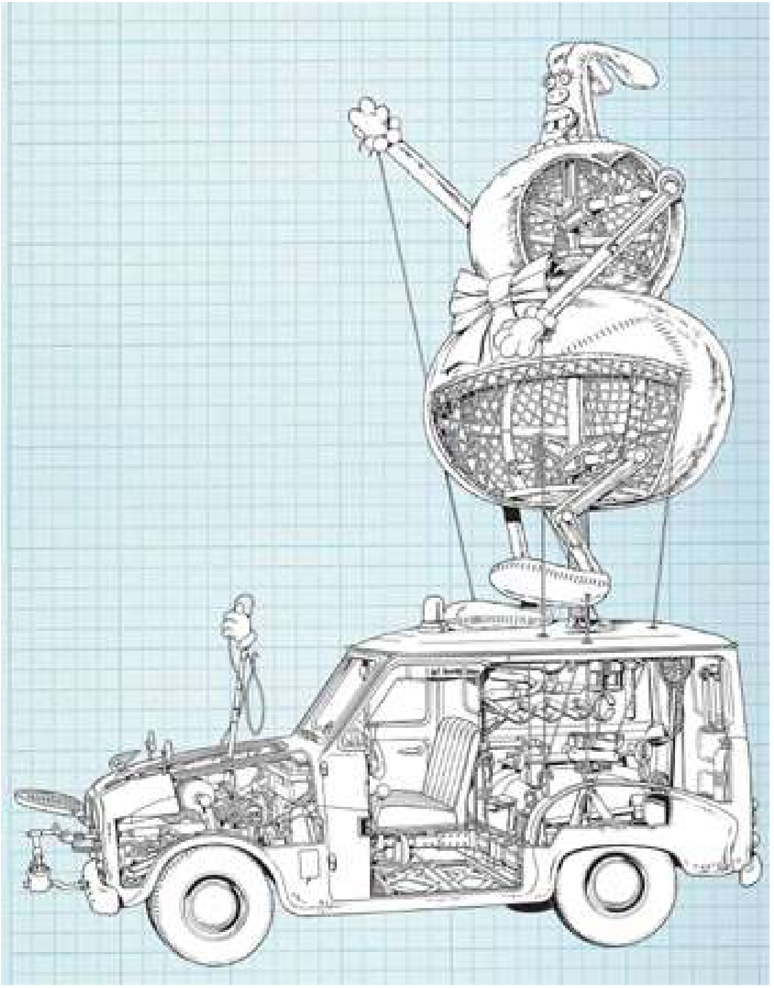
To provide an introduction to the philosophy and art of modelling of the essential physics at play in dynamic magnetic systems.

Examples will be given of how simple models can be constructed and applied to understand and interpret observable phenomena, ranging from magnetisation processes to high frequency spin wave dynamics.

Along the way, an introduction to some general tools will be provided, including Monte Carlo models and micromagnetics.

Outline

- **Modelling Dynamics: where to start?**
  – Starting points
  – Phenomenology
- **Some generic tools:**
  – Micromagnetics
  – Mean field theory & Monte Carlo
- **Spin dynamics**
  – Torque equations
  – Spinwaves & resonances
- **Domains and domain walls**
  – Stoner-Wohlfarth models
  – Magnetic domains and domain walls



With examples from PhD works!

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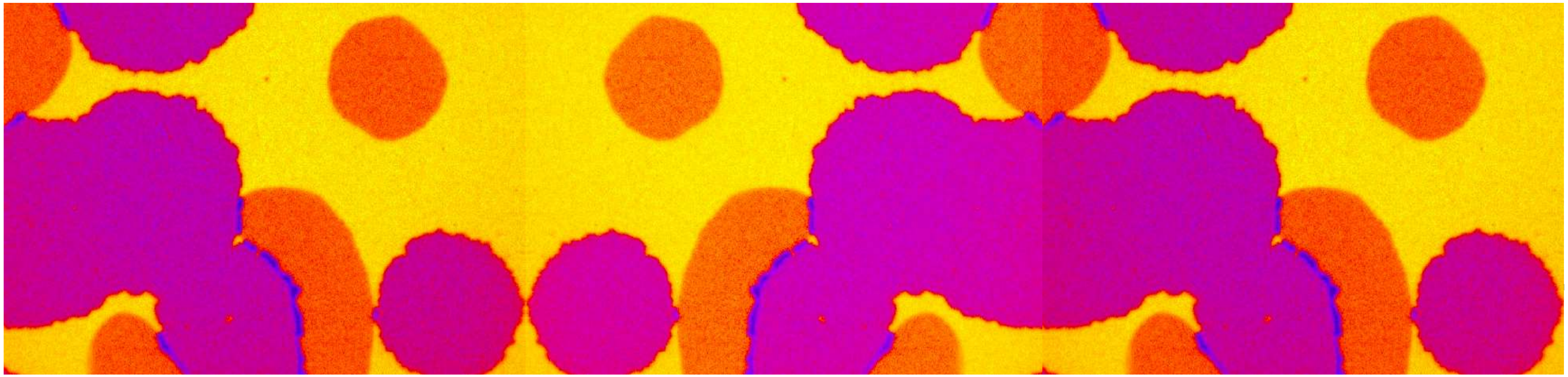
Modelling: where to start?



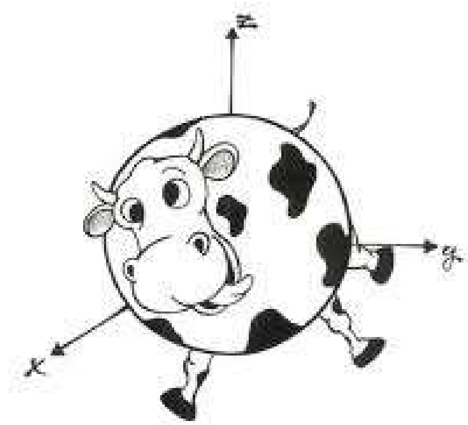
Models for **research & development**: magnetic ordering, dynamics, transport ...

Some starting points for model makers

Tools

1) **Simulations**
do not by themselves provide interpretations or insights

2) **Analytic/conceptutal models**
often go where simulations cannot



The dark arts of simplification:

Energies, symmetry and phenomenology

Energies

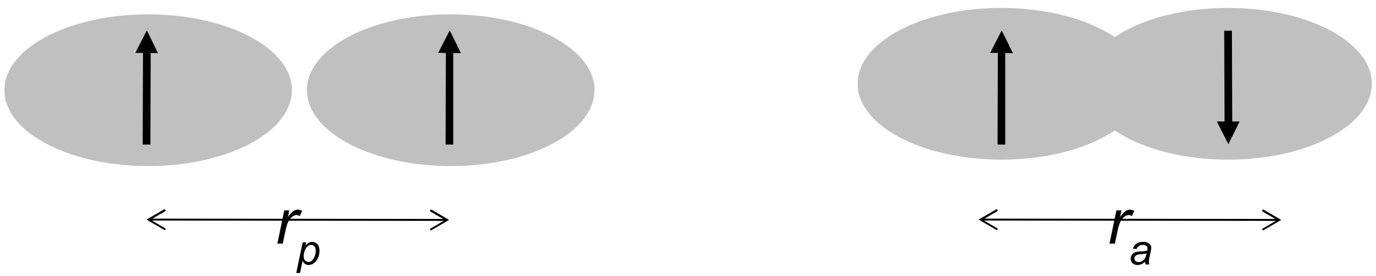
Relevant energy scales (P. W. Anderson, 1953):

1 – 10 eV	Atomic Coulomb integrals Hund's rule exchange energy Electronic band widths Energy/state at ε_f
0.1 – 1.0 eV	Crystal field splitting
10^{-2} – 10^{-1} eV	Spin-orbit coupling $k_B T_C$ or $k_B T_N$
10^{-4} eV	Magnetic spin-spin coupling Interaction of a spin with 10 kG field
10^{-6} – 10^{-5} eV	Hyperfine electron-neuclear coupling

magnon region (bracketing 10^{-4} eV and 10^{-6} – 10^{-5} eV)

Concept: Exchange Energy

Pauli exclusion **separates** like spins:



Can be **energetically favourable**: *suppose* alignment determines average separation. Then *if*:

$$\langle r_a \rangle \sim 0.3\,nm \Rightarrow \frac{e^2}{r_a} \sim 4.8\,eV$$

$$\langle r_p \rangle \sim 0.31\,nm \Rightarrow \frac{e^2}{r_p} \sim 4.75\,eV$$

$$\left.\right\} E_{\uparrow\uparrow} - E_{\uparrow\downarrow} = 0.05eV\left(580K\right)$$

… equivalent field: $\frac{E_{\uparrow\uparrow} - E_{\uparrow\downarrow}}{\mu_B} = 870T$

Exchange Interactions

Exchange: electrostatic repulsion + quantum mechanics.

Hamiltonian as **spin functions:** (Dirac & Heisenberg)

Pauli spin matrices

$$H = -J_{1,2}\boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2}$$

Generalised for multi-electron orbitals (**van Vleck**):

$$H_{ex} = -\sum J(r_a - r_b)S(r_a)\cdot S(r_b)$$

*total spin at sites **r***

Using Symmetry: Exchange

Measurable moment **density** (not an operator):

$$m(r) = Tr\left(\rho \hat{M}(r)\right)$$

density matrix

Exchange still in Heisenberg form:

$$E_{ex} = \sum J m(r_j) \cdot m(r_{j+\delta})$$

neighbours

Atomic to continuum: Expand ***m*** field about ***r_j***

$$m(r_{j+\delta}) = m(r_j) + \left[(\delta \cdot \nabla) m(r_{j+\delta})\right]_{j=\delta} + \frac{1}{2}\left[(\delta \cdot \nabla)^2 m(r_{j+\delta})\right]_{j=\delta} + \ldots$$

Using Symmetry: Exchange

When **lattice symmetry** allows:

$$\delta_x \frac{\partial m}{\partial x} + (-\delta_x)\frac{\partial m}{\partial x} = 0$$

Example: **isotropic medium**

$$E_{ex} = m_x\left(\nabla^2 m_x\right) + m_y\left(\nabla^2 m_y\right) + m_z\left(\nabla^2 m_z\right)$$

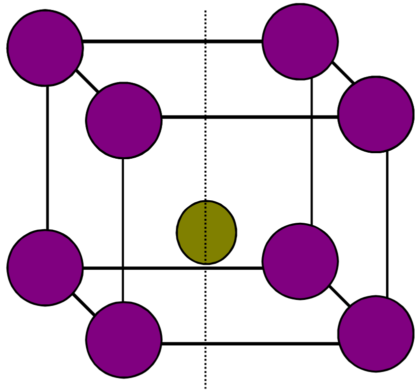
Exchange energy must be compatible with symmetry of the crystal

$$E_{ex} = \sum C_{kl} \frac{\partial m_\alpha(r)}{\partial r_k}\frac{\partial m_\alpha(r)}{\partial r_l}$$

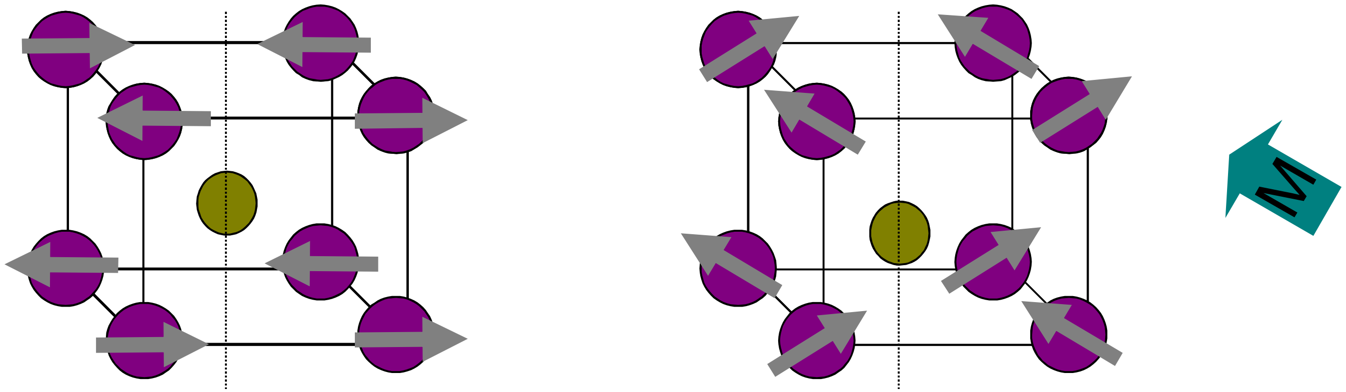
Dzyaloshinski-Moriya Interaction

Asymmetric interaction possible when **inversion symmetry** is absent:

$$H = \sum \left[J S_i \cdot S_j - D \cdot \left(S_i \times S_j \right) \right]$$



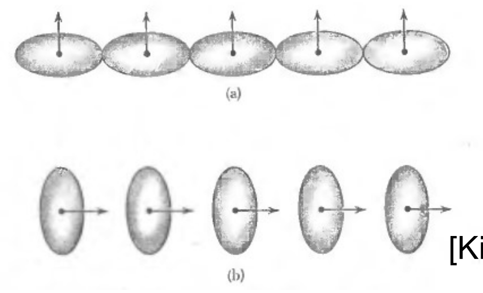
Describes weak ferromagnetism of canted antiferromagnets:



D = 0 *D* $\neq$ *0*

Using Symmetry: Anisotropy

Local **atomic environment** affects spin orientation:



[Kittel, Introduction to Solid State]

Spin orbit interaction and crystal field effects

Anisotropies & **symmetries**: (**u** = **m**/M_s)

- Uniaxial: $E_{ani}(u_z) = E_{ani}(-u_z)$

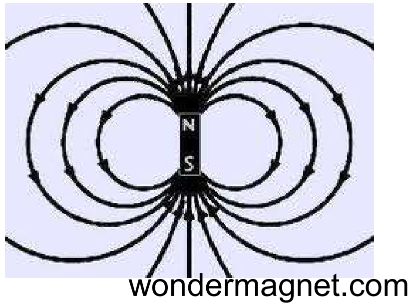
$$E_{ani} = -K_u^{(1)} u_z^2 - K_u^{(2)} u_z^4 + \dots$$

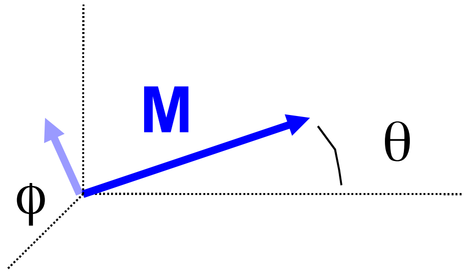
- Cubic: $E_{ani}(u_x, u_y, u_z) = E_{ani}(-u_x, u_y, u_z)$, etc.

$$E_{ani} = K_4 (u_x^2 u_y^2 + u_x^2 u_z^2 + u_y^2 u_z^2) + \dots$$

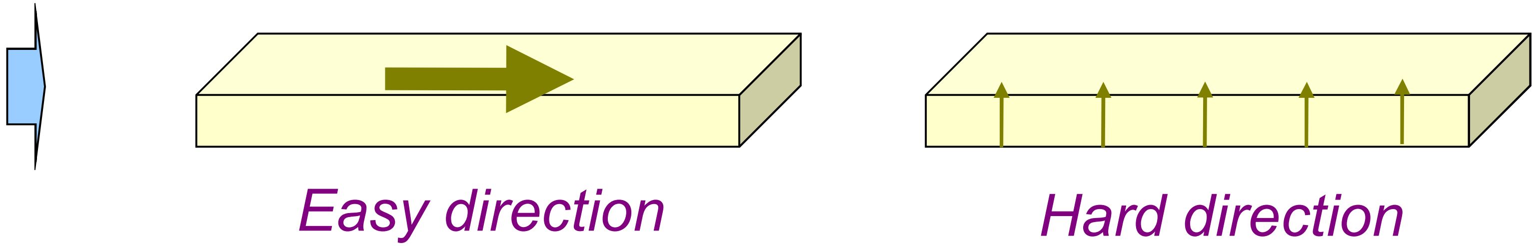
Using Symmetry: Dipolar Fields

All moments interact throughout a sample via dipolar fields. Sample **shape** creates an **effective** anisotropy:





$$E_{ani} = \frac{M^2 V}{2\mu_o}\left(N_x \sin^2\theta\cos^2\phi + N_y \sin^2\theta\sin^2\phi + N_z\cos^2\theta\right)$$



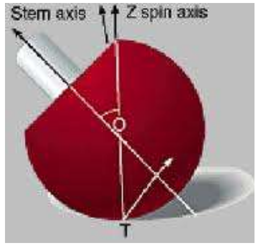
It's only Angular Momentum

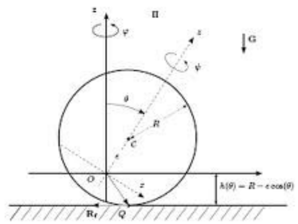
Everything (nearly) important for magnetic dynamics can be understood from a toy…

Precession
Dissipation
Instabilities

It's Only Angular Momentum

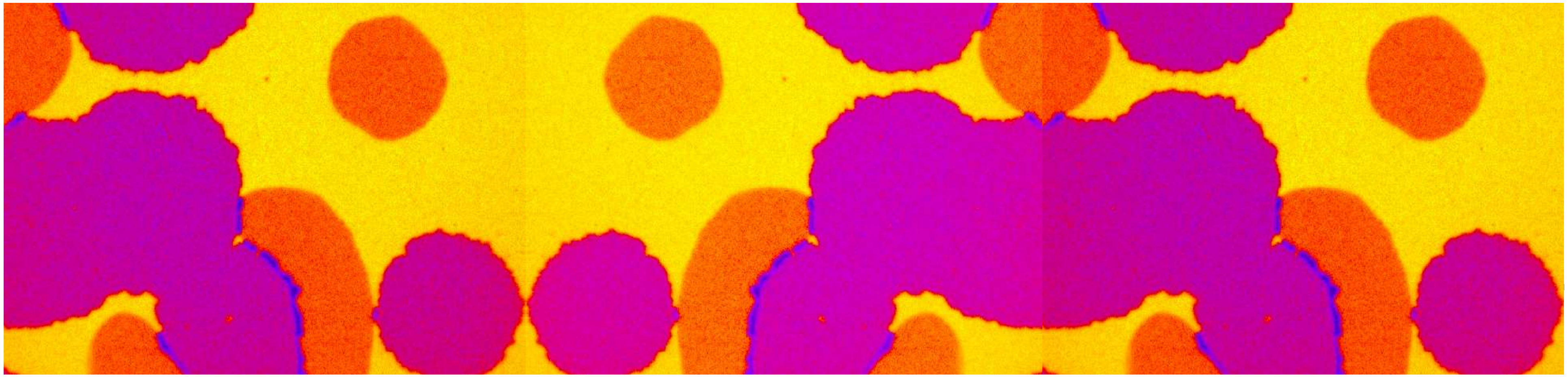
Bohr and Pauli Study
Angular Momentum





Interesting video at:
https://www.youtube.com/watch?v=58sryfWQOa0

Some generic tools



Tools

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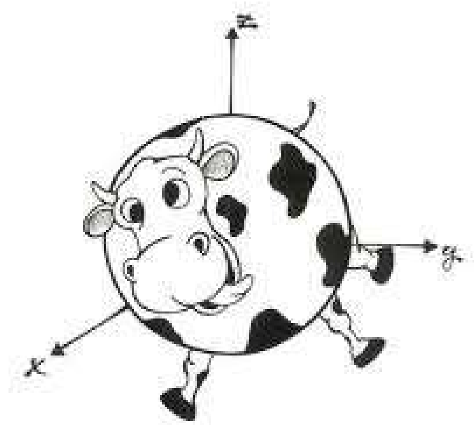
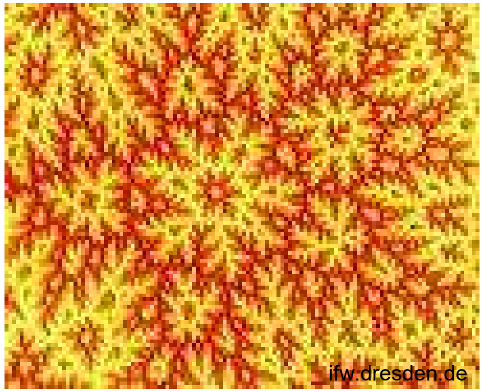
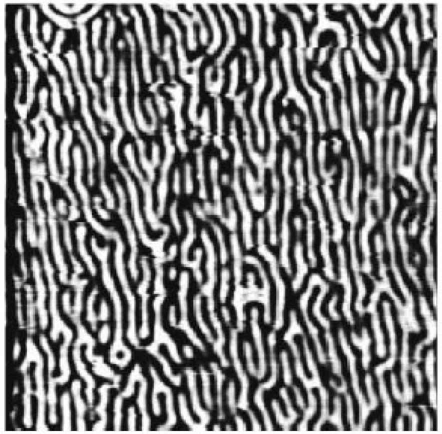
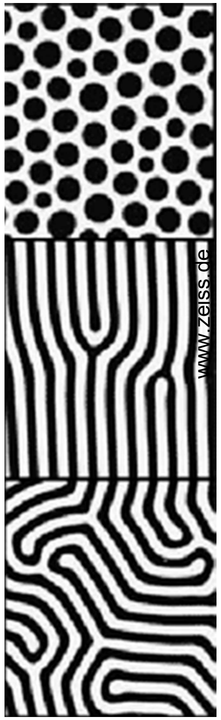


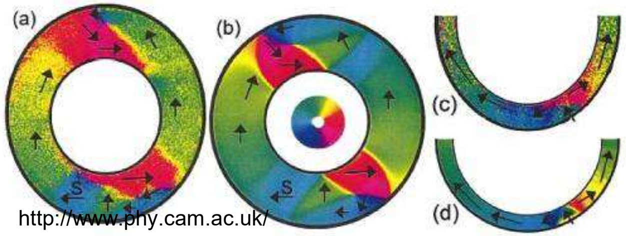
Domain Patterns

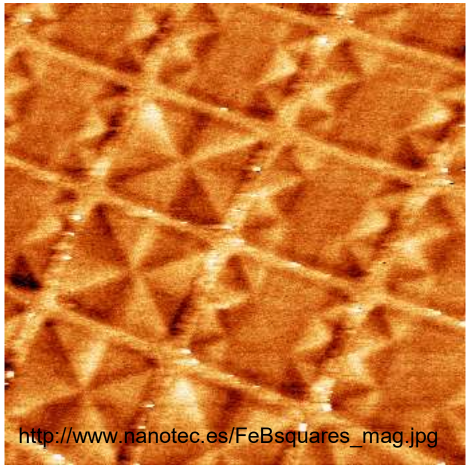
Pattern detail depends on competition between dipolar interactions, exchange and anisotropy.





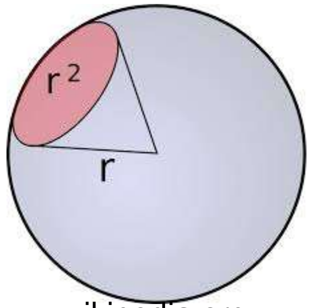






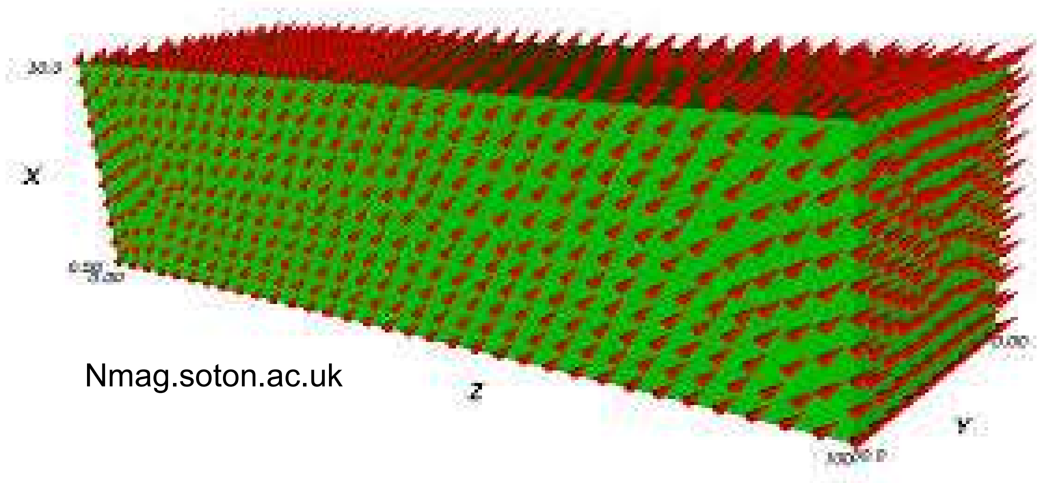
The Problem of Dipolar Interactions

Magnetic fields decrease slowly
with distance-- sample shape matters



wikipedia.org

Magnetisation is generally <u>not</u> uniform:



Nmag.soton.ac.uk

Tools:
Micromagnetics

Minimising the Energy

Goal: find stable (and metastable) configurations that define minima of the total energy *E*

$$E(\vec{u}) = \int \left[A(\nabla \vec{u})^2 - K_n(\hat{n} \cdot \vec{u})^2 - \mu_o M_s \left(\vec{u} \cdot \vec{H}_a + \vec{u} \cdot \vec{h}_d \right) \right] dV$$

exchange *anisotropy* *applied field* *magnetostatic*

$$\vec{u} = \frac{\vec{M}}{M_s}$$

reduced M

Minimisation = vanishing **torques**:

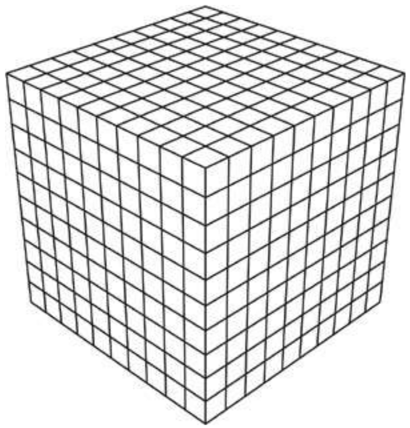
$$\delta E = 0 \Rightarrow \vec{u} \times \left(-\frac{\partial E}{\partial \vec{u}} \right) = 0$$

A Numerical Method: Finite Differences

Convert differential equations to difference equations:

$$u_\beta(x+\Delta x) = u_\beta(x) + \Delta x \frac{\partial}{\partial x} u_\beta(x) + \frac{1}{2}(\Delta)^2 \frac{\partial^2}{\partial x^2} u_\beta(x)$$

$$u_\beta(x-\Delta x) = u_\beta(x) - \Delta x \frac{\partial}{\partial x} u_\beta(x) + \frac{1}{2}(\Delta)^2 \frac{\partial^2}{\partial x^2} u_\beta(x)$$



Divide **magnetisation** into blocks, replace differentials, construct torque equations for **each** block

Magnetostatic Terms

Maxwell equations can define a magnetostatic potential (if we are not worried about an electric field during dynamics)

$$\vec{B} = \mu_o\left(\vec{H} + \vec{M}\right)$$

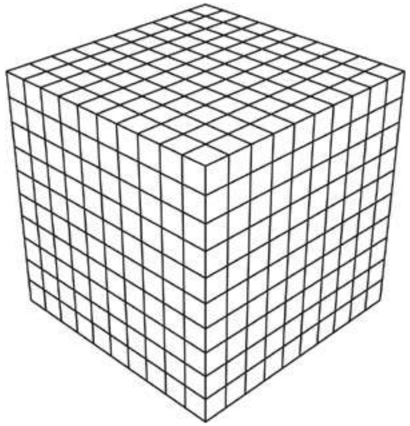
$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \approx 0$$

$$\vec{H} = -\nabla \Phi$$

$$\nabla^2 \Phi = -\nabla \vec{M}$$

Blocks are ***sources*** *of H field*



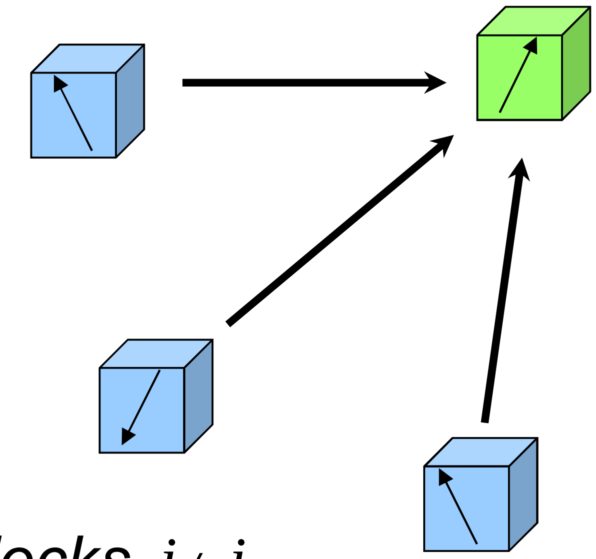
The magnetostatic terms link **all blocks** throughout the sample

Note: Micromagnetics and GPU's

The magnetostatic calculation involves convolution over all blocks:

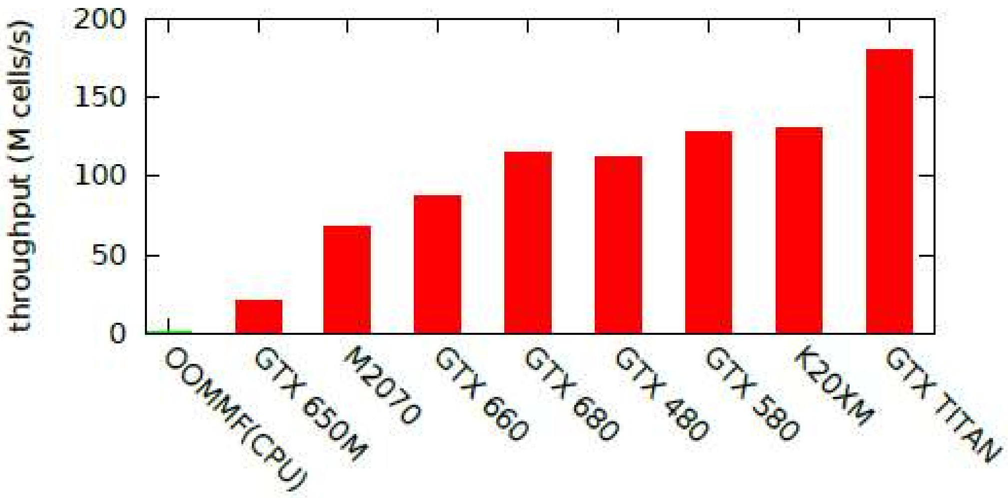
$$\vec{H}(i) = \hat{K}(i,j) * \vec{M}(j)$$

block i



blocks $j \neq i$

Accelerate calculations using ***Graphical Processing Units***



[Vansteenkiste, et al. arXiv:1406.7635]

Example: Mumax3

```
// Standard Problem #4

SetGridsize(256, 64, 1)
SetCellsize(500e-9/256, 125e-9/64, 3e-9)

Msat  = 800e3
Aex   = 13e-12
alpha = 0.02

m = uniform(1, .1, 0)
relax()
save(m)    // relaxed state

autosave(m, 200e-12)
tableautosave(10e-12)

B_ext = vector(-24.6E-3, 4.3E-3, 0)
run(1e-9)
```

define grid and sizes (m)

parameters (SI)

initialse M
find zero torque configuration
save configuration

save configurations every 0.2 ns
create table of m(t)

apply magnetic field
time evolution

Information & Download:
http://mumax.github.io/index.html

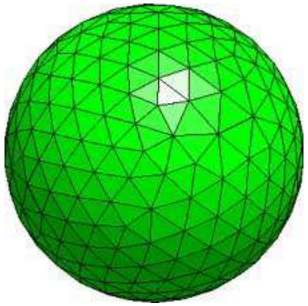
Run Standard Problem 4

https://www.youtube.com/watch?v=DPQFppEbqf4

Approaches (with example codes)

Finite difference: mumax3, OOMMF

Finite element: useful for complex geometries



http://nmag.soton.ac.uk

Nmag
http://nmag.soton.ac.uk/nmag/

MAGPAR
http://magnet.atp.tuwien.ac.at/

Atomistic: model atomic lattice scale variations

VAMPIRE

http://www-users.york.ac.uk/~rfle500/research/vampire/

… and many more !

Limitations!

Lengthscales are limited

Shapes are approximate

Timescales are limited

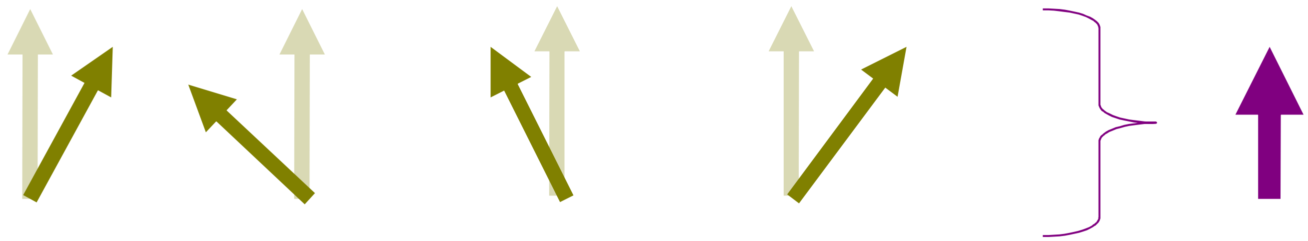
Classical limits: dynamics
& thermodynamics

Questions?

Tools:
Mean field approximation

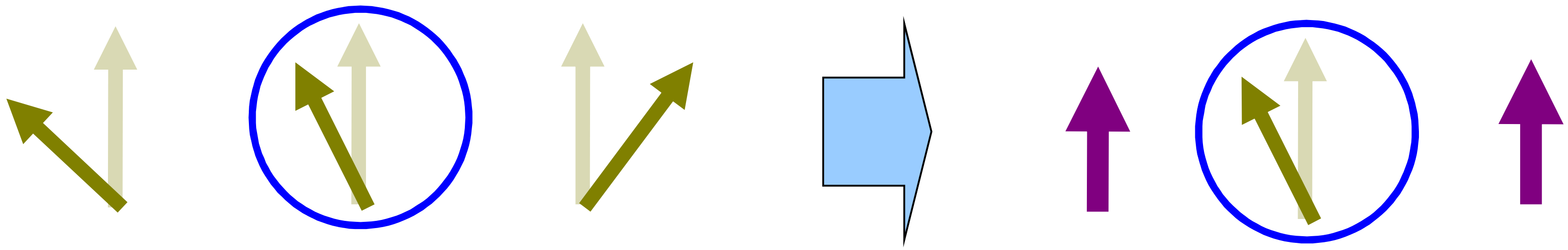
Thermal Fluctuations

Reduction in magnetisation:



$$M(T) \sim S$$

Replace local site field with **averaged effective field**:



Dynamic correlations are replaced by a **static field**:

$$H = -2\sum J_{ex} S_i \cdot S_j \approx -2\sum S_i \cdot B_{ex} \qquad B_{ex} = \frac{2ZJ_{ex}}{Ng\mu_B} S$$

Heisenberg Model and Mean Field

Heisenberg exchange energy:

$$H = -\sum J_{ij} \vec{S}_i \cdot \vec{S}_j$$

Thermal averaged magnetisation (N moments):

$$\vec{M} = Ng\mu_B \vec{S}$$

Fluctuations:

$$\vec{s}_i = \vec{S}_i - \vec{S}$$

$$\Rightarrow \quad H = -\sum J_{ij} \left(\vec{s}_i + \vec{S}\right) \cdot \left(\vec{s}_j + \vec{S}\right)$$

Heisenberg Model and Mean Field

Z near neighbours:

$$H = -J\sum \vec{s}_i \cdot \vec{s}_j - 2ZJ\sum \vec{S}_i \cdot \vec{S} + ZN\left|\vec{S}\right|^2$$

Second term is the **mean field**:

$$\vec{B}_{ex} = -2ZJ\,\vec{S}$$

Mean field approximation: neglect first term **(correlations)**

$$H_{fluctuations} = -J\sum \vec{s}_i \cdot \vec{s}_j$$ ✗

Reminder: Paramagnetism

Probabilities to be antiparallel (down) and parallel (up):

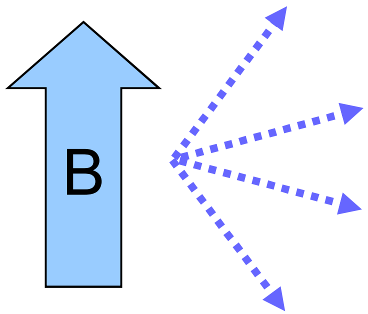
$$\frac{n_\downarrow}{N} \propto \exp\left(\frac{-\mu_B B}{k_B T}\right) \qquad \frac{n_\uparrow}{N} \propto \exp\left(\frac{\mu_B B}{k_B T}\right)$$

Magnetisation = difference:

$$\Rightarrow \quad S = \left(\frac{N_\uparrow - N_\downarrow}{N}\right) = \tanh\left(\frac{\mu_B B}{k_B T}\right)$$

Generalised Paramagnetism

Angular momentum states
(*J* = 1/2, 3/2, 5/2, …):



Brillouin function for any *J*:

$$B_J(x) = \frac{2J+1}{2J}\coth\left(\frac{2J+1}{2J}x\right) - \frac{1}{2J}\coth\left(\frac{1}{2J}x\right)$$

$$x = \frac{gJ\mu_B B}{k_B T}$$

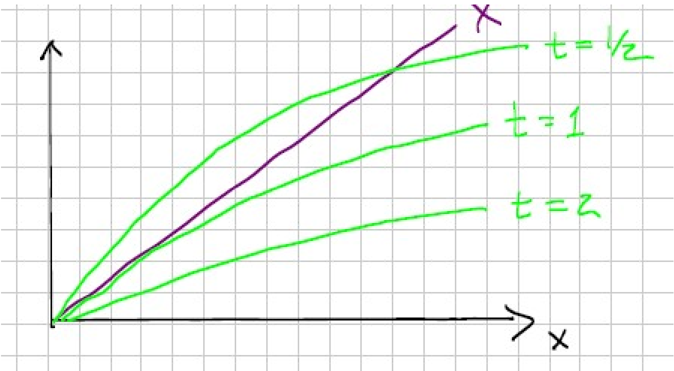
Average magnetisation from: $M \propto S = B_J(x)$

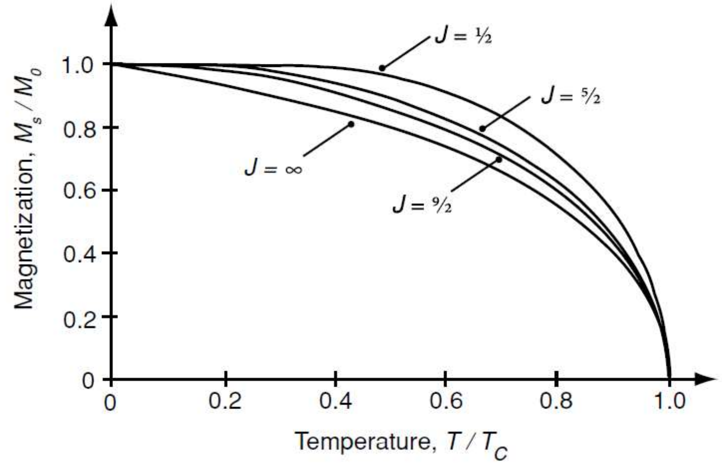
Exchange: Replace B by B_{ex}

Average M with **mean field** B_{ex}:

$$\vec{S} = B_J\left(\frac{g\mu_B Z J_{ex}\vec{S}}{k_B T}\right)$$

Plot left and right hand sides to see graphical solution:





Note: Landau Ginzburg Theory

A **general form** for mean field theory, created by Landau and Ginzburg, begins with an energy that is a function of an **order parameter** ψ :

$$F = \int \left[F_o + \frac{1}{2} a \mid \psi \mid^2 + \frac{1}{4} b \mid \psi \mid^4 + \ldots + \frac{1}{2} \lambda \mid \nabla \psi \mid^2 \ldots \right] d^3 r$$

Allowed terms must be consistent with the **symmetries** of the problem that the order parameter ψ must obey. **The equilibrium value of the order parameter minimises F**. The coefficients represent various contributions to the system's energy. **Temperature** is introduced in the first coefficient:

$$a = \alpha (T - T_c)$$

L-G and the Ferromagnet

Let the order parameter be the ferromagnetic M that is uniformly magnetised over a volume V:

$$\psi = M \quad \Rightarrow \quad \frac{F}{V} = F_o + \frac{1}{2}\alpha(T - T_c)M^2 + \frac{1}{4}bM^4$$

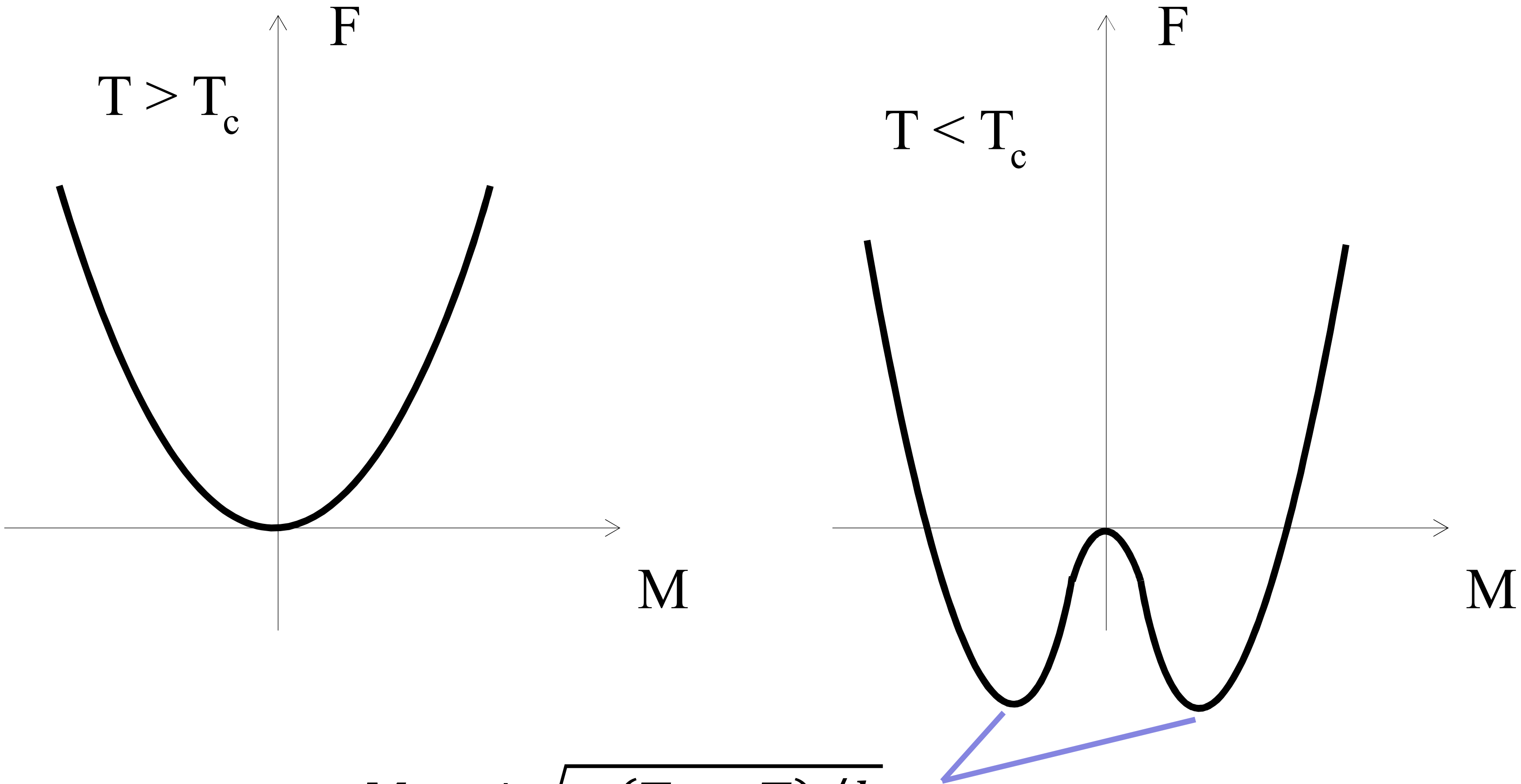
This energy is easily minimised with respect to M:

$$\frac{d}{dM}\left(\frac{F}{V}\right) = 0 \quad \Rightarrow \quad \alpha(T - T_c)M + bM^3 = 0$$

$$\Rightarrow \quad M = \pm\sqrt{\alpha\,(T_c - T)/b}$$

Energy Landscapes

This can be pictured using a plot of the energy landscape for F(M):

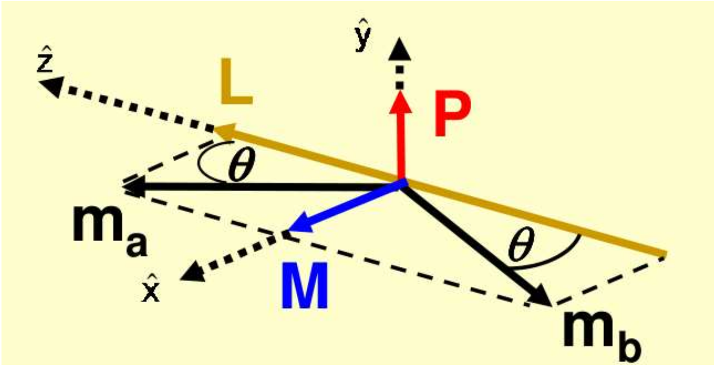


$$M = \pm\sqrt{\alpha\,(T_c - T)/b}$$

Example: Multiferroics

Coupled order parameters: **M** & **P**

(M = sum of canted antiferromagnetic sublattices)

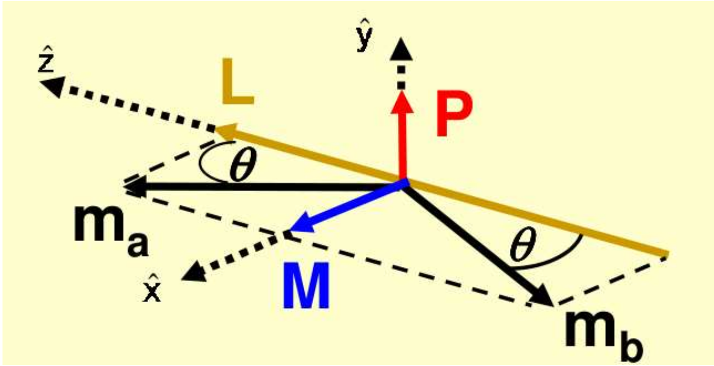


Challenges:

- correlations between spin and charge distributions
- how to describe dynamics?
- how to describe effects of thermal fluctuations?

Example Application: Multiferroics

Coupled order parameters: **M** & **P**



Approach:
(Vincinsius Gunawan PhD 2012)

Mean field approximation for **free energy**:

$$F = F_{FE}(P) - \vec{P}\cdot\vec{E} - \lambda\vec{m}_a\cdot\vec{m}_b - K\left(m_{az}^2 + m_{bz}^2\right) - \vec{m}\cdot\vec{H} + F_{ME}$$

polarization part

magnetization part

magneto-electric coupling

Example Application: Multiferroics

Brillouin function for components of **m**:

$$m_{s,\alpha} = g\mu_B J B_J\left(\vec{m}_s \cdot \vec{B}_s\right)$$

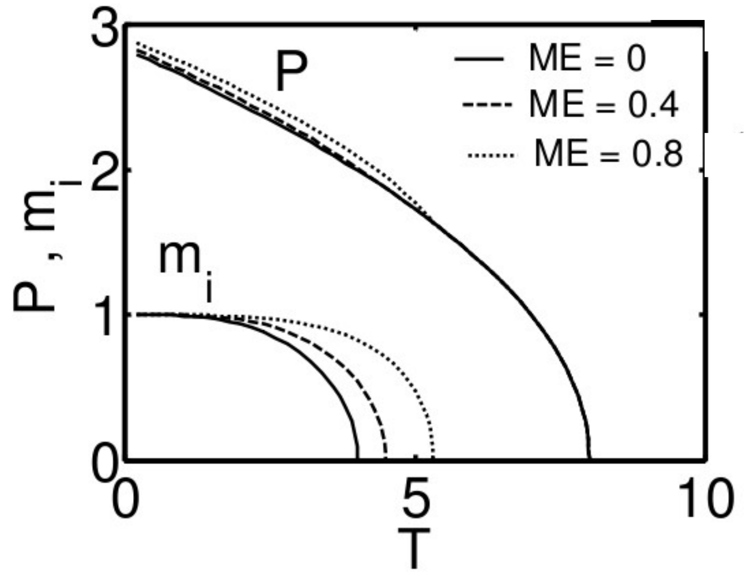
Landau-Ginzburg mean field theory for P:

$$F_{FE}(P) = \alpha_o(T - T_c)P^2 + \beta P^4$$

Minimise free energy for P and θ:

$$\frac{d}{d\theta}F = 0$$

$$\frac{d}{dP}F = 0$$



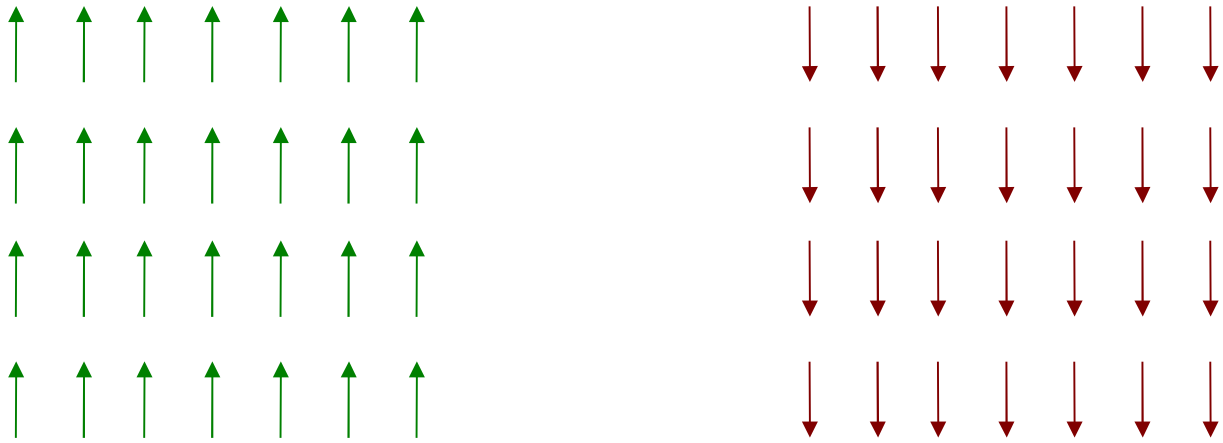
[Gunawan et al., JPCM (2011)]

Break!

Tools:
Monte Carlo methods

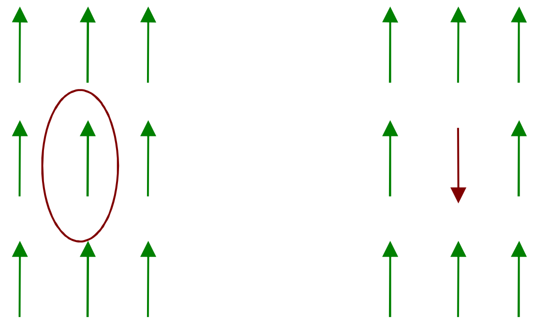
Ising model and Monte Carlo

Suppose two possible states: 'up' and 'down'



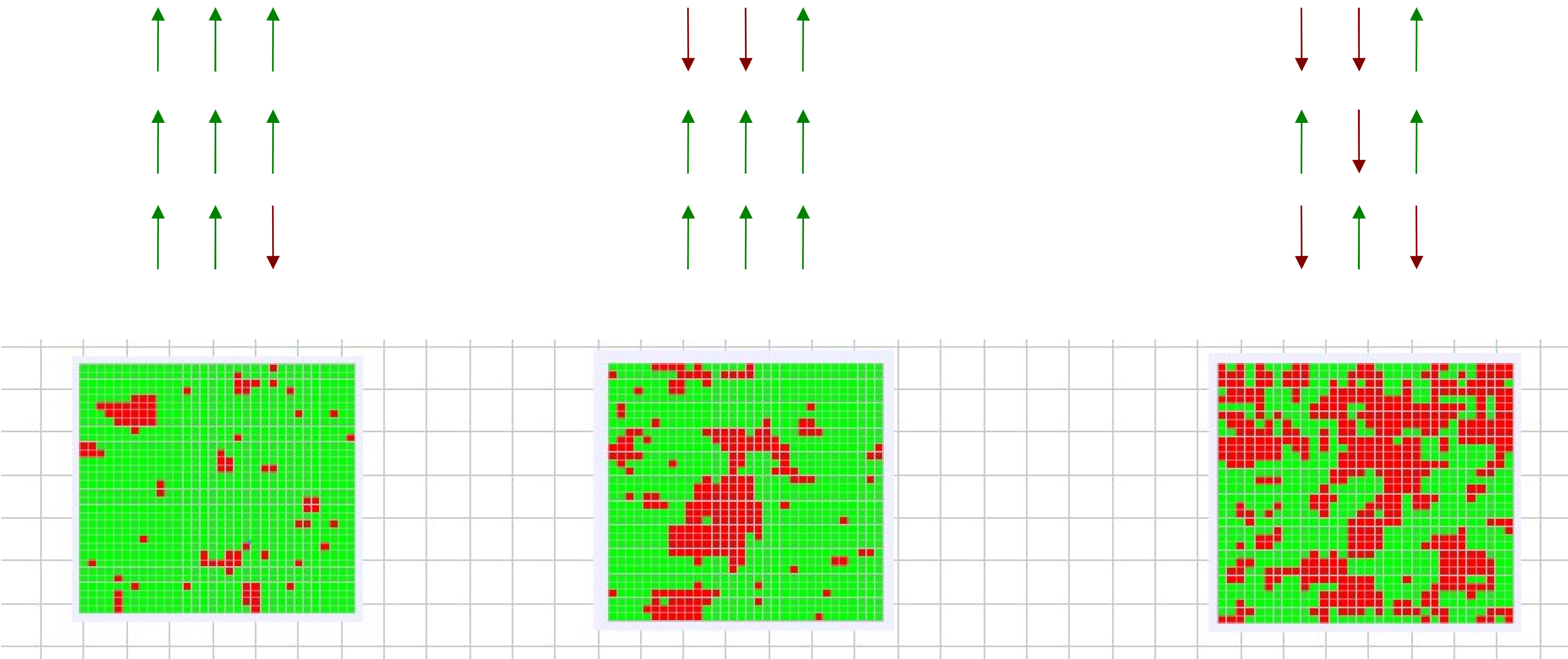
Suppose near neighbour interactions. Probability to flip depends on 4 neighbours:

$$P(-S_i) \sim \exp\left(\frac{-J(\sum S_i)}{k_B T}\right)$$



Sampling Random Fluctuations

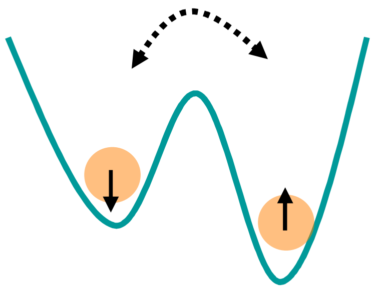
Thermal fluctuations and 2 dimensional Ising model:



Low T — Near T_c — Above T_c

Constructing Averages

Fluctuations drive the system towards thermal **equilibrium**.



$$P_{\uparrow\downarrow} \sim \exp\left(\frac{-\Delta E(\uparrow \Rightarrow \downarrow)}{k_B T}\right)$$

$$P_{\downarrow\uparrow} \sim \exp\left(\frac{-\Delta E(\downarrow \Rightarrow \uparrow)}{k_B T}\right)$$

Sample a **distribution** for averages:

$$A = \sum A(\sigma)\rho(\sigma) \qquad \rho(\sigma) = \frac{1}{Z}\exp\left(-\frac{E(\sigma)}{k_B T}\right)$$

***Key idea**: σ is a configuration from the **ensemble** of **equilibrium** spin configurations*

The Metropolis Algorithm

Sample from {σ}: Start with some ξ, generate a σ' with a single spin flip.

Rules: Calculate Δ*E* = *E*(ξ) - *E*(σ')
1) If Δ*E* < 0, accept σ' as an equilibrium fluctuation
2) If Δ*E* > 0, accept σ' if *P*(Δ*E*) < 1

For **equilibrium** fluctuations, *P*(Δ*E*) must satisfy **detailed balance**:

$$P(\sigma')W(\uparrow\Rightarrow\downarrow) = P(\xi)W(\downarrow\Rightarrow\uparrow)$$

$$\Rightarrow \frac{W(\uparrow\Rightarrow\downarrow)}{W(\downarrow\Rightarrow\uparrow)} = \frac{P(\xi)}{P(\sigma')} = P(\Delta E) = \exp\left(-\frac{E(\xi)-E(\sigma')}{k_B T}\right)$$

Monte Carlo for the Ising Model

OSP Library 2.0 released December 8, 2008

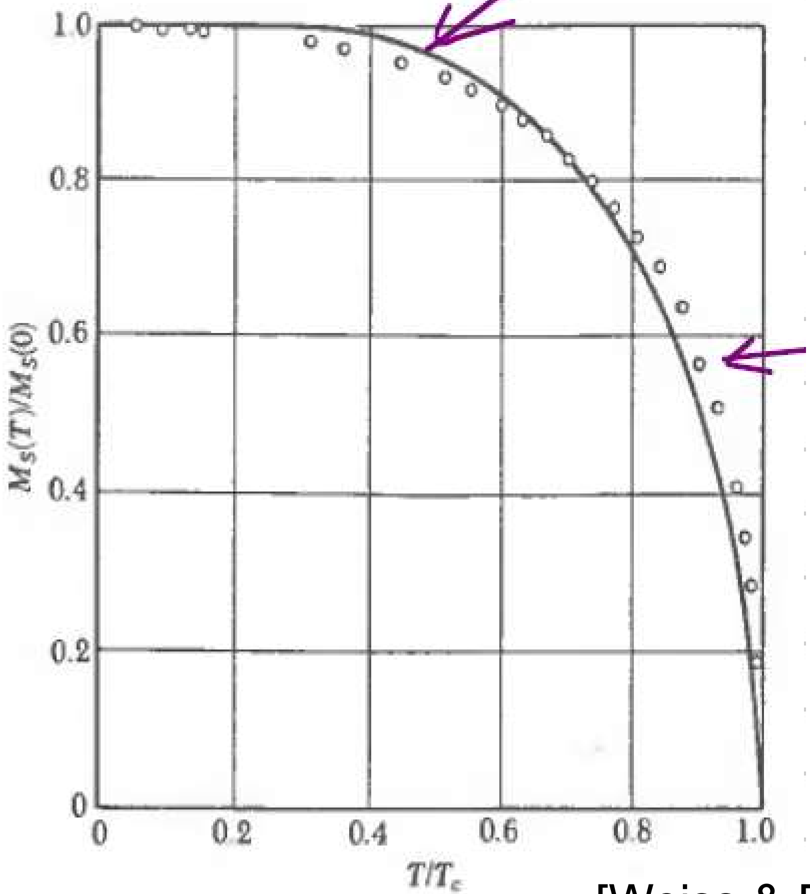
Open Source Physics Project

www.opensourcephysics.org

Note on phase transitions:
Scaling near *critical points*

Schematic of the Transition (2nd order)

Linear spinwaves



Large amplitude fluctuations

[Weiss & Forrer]

Scaling

Mean field theory: $M(T) \sim (T - T_C)^{1/2}$

Reality includes correlations: $M(T) \sim (T - T_C)^{\beta} \quad \beta \approx 0.34$

Note on dimensionality:
- Ultra thin films ~ two dimensional systems
- fluctuations destroy long range order
- nano-thermodynamics for small elements (~ 0 D!)

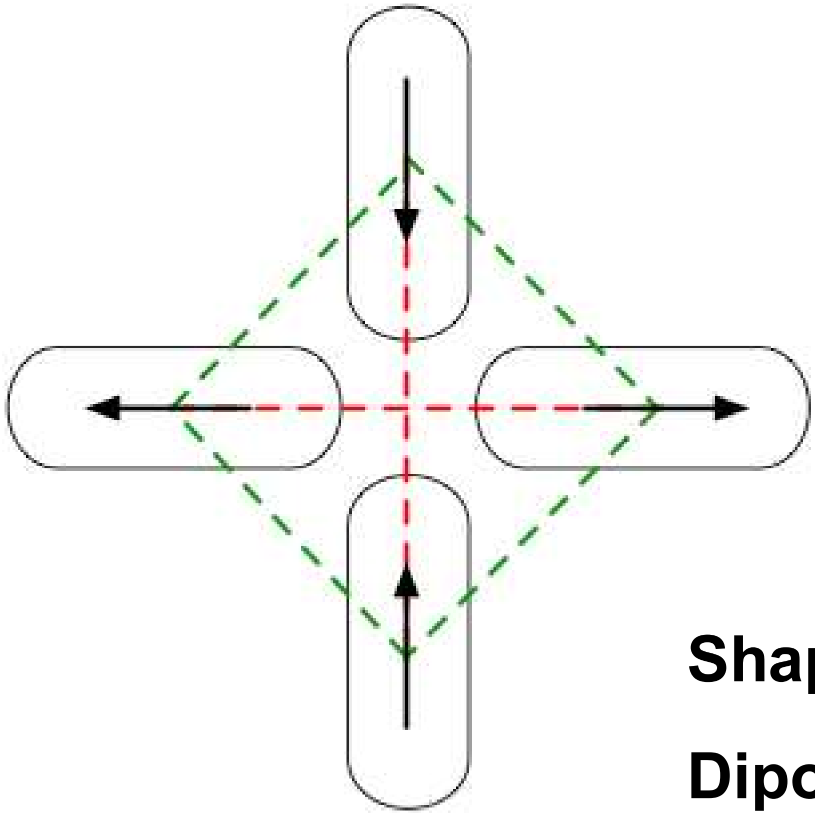
***Remember** this for later when we talk about domain wall creep*

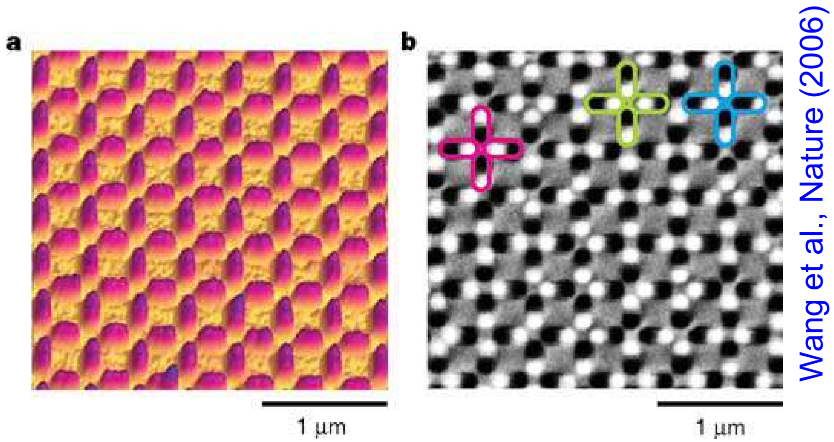
Example: Interacting magnetic particles

Challenges:
- large arrays of submicron elements
- super-paramagnetic
- long range interactions

Approach: ***(Zoe Budrikis, PhD 2012)***
Combine Mean Field & Monte Carlo

An Artificial Antiferromagnet (artificial square spin ice)





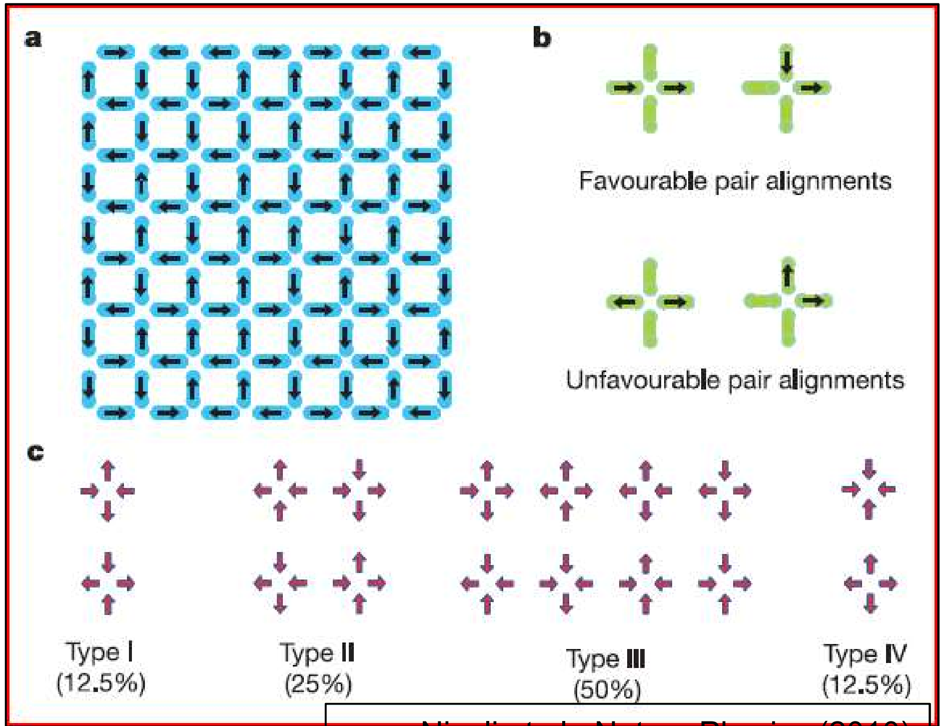
Shape anisotropy: Ising spins

Dipolar interactions

6 interactions but can only **minimise** 4

Configurations

Local spin configurations:



Nisoli et al., Nature Physics (2010)

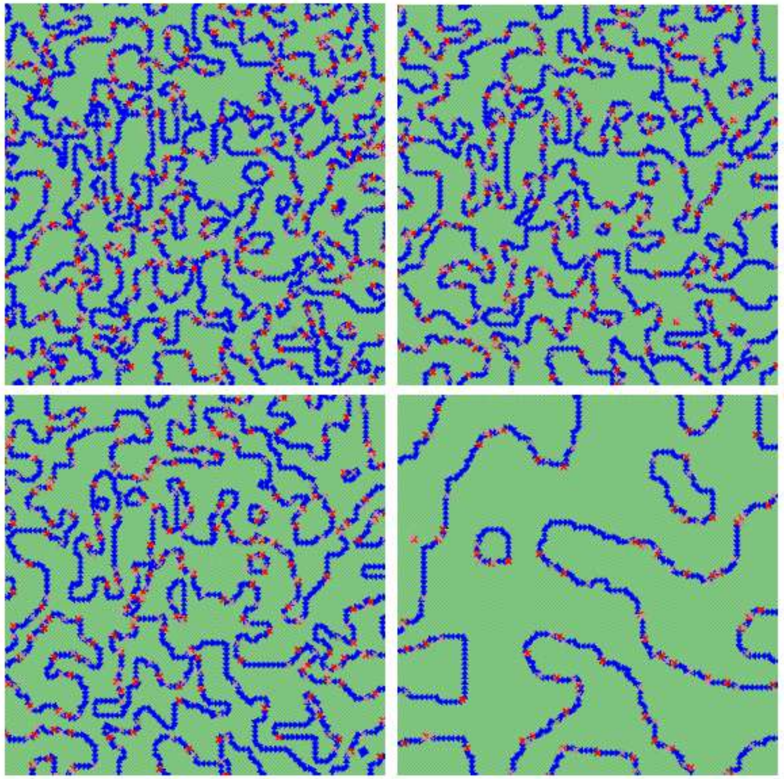
Type I
(ground)

Type II
(wall)

Type III
(defect)

Growth of Domains and Wall Motion

Type I domains separated by Type II walls:



- Type I
- Type II
- Type III

Type III 'charge' production during wall motion

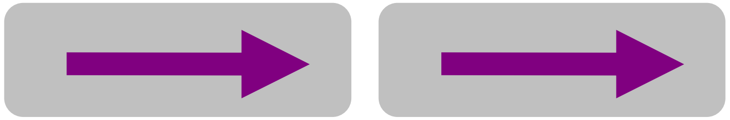
Thermal evolution of domains

Thermal fluctuations on 2 timescales:

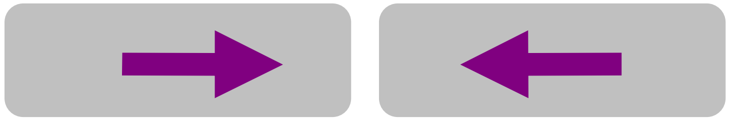
- small volumes (reversal)

$$\frac{KV}{k_B T} \sim 1$$

- thermal reduction of element M



enhancement



suppression

Configuration dependent local M

Mean field model: thermal dynamics

Mean field model for ***element*** magnetisations:

$$m_j = B_{1/2}\left\lfloor \beta m_j \cdot \left(h_c + \sum J_{j,k}\, m_k\right)\right\rfloor \qquad h_c = K m_j$$

Algorithm:

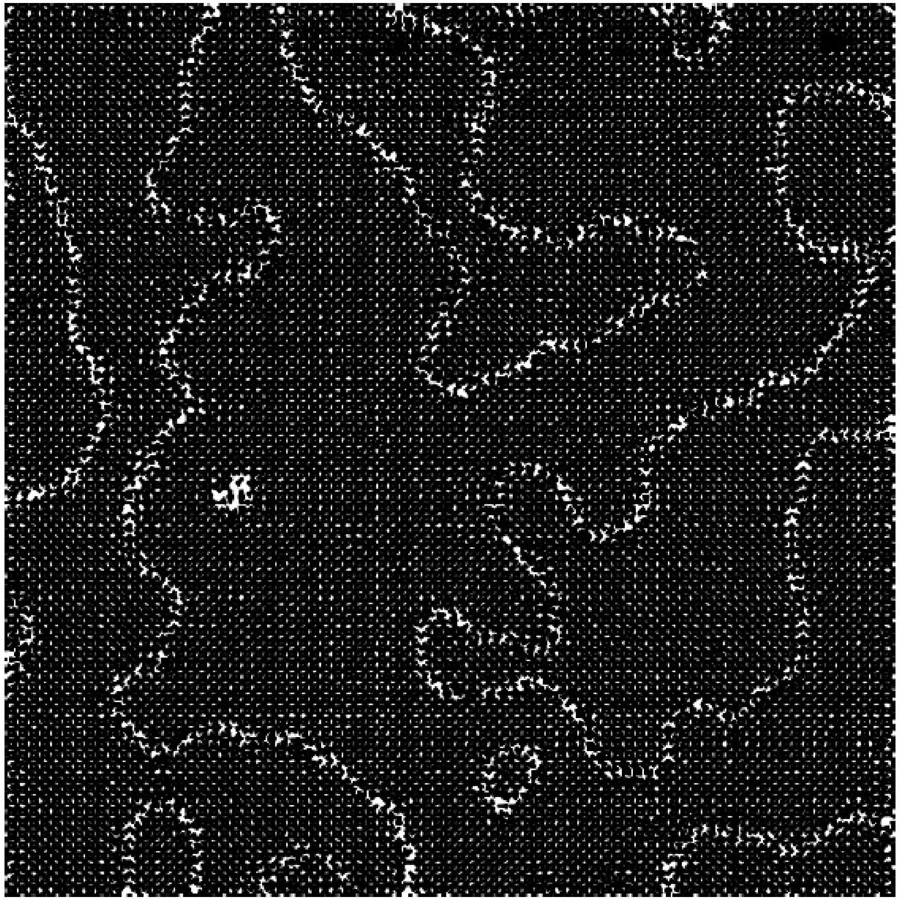
- self consistent iteration for <m_j>
- stochastic reversal (**Monte Carlo**)

Disorder: uniform distribution for K centred on K_o

$$K = K_o\left(1 + \frac{r}{2}\right) \qquad r \in [-\Delta, \Delta]$$

Thermal fluctuations at walls

(Karen Livesey PhD 2010)



- **M = 1**
- **M = 0.1**

*Thermal **fluctuations** largest on domain walls*

Challenge: modelling kinetics in real time with Monte Carlo

Continuous Time Monte Carlo

Probability for **acceptance** of a single flip (out of N spins):

$$Q = \frac{1}{N} \sum n(\Delta E) P(\Delta E)$$

number of spins with ΔE

Probability that a spin **will flip** in time Δ*t*:

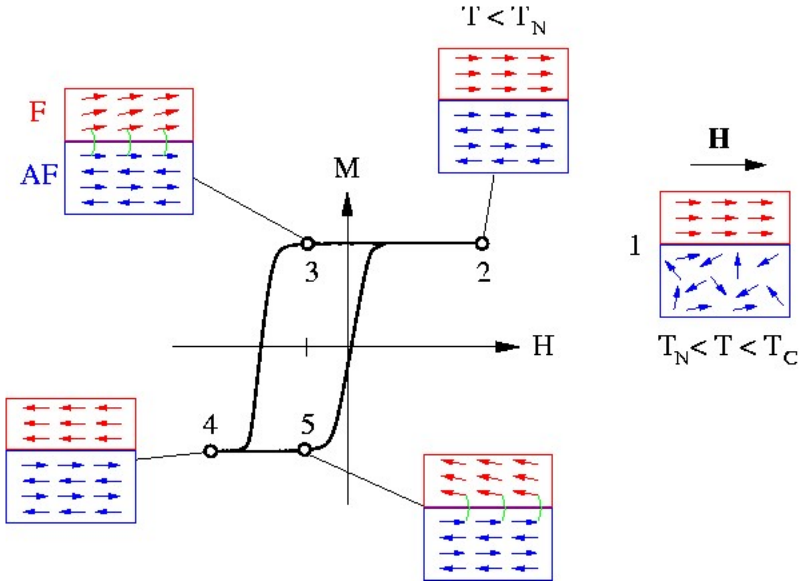
$$P_{flip}(\Delta t) = \exp\left(-\frac{\Delta t}{\tau} Q \right)$$

Rejection free algorithm:
1) track **all** possible transitions
2) accept **one** according to random *R*
3) **time** update determined by *R*

$$\Delta t = -\frac{\tau}{Q} \ln R$$

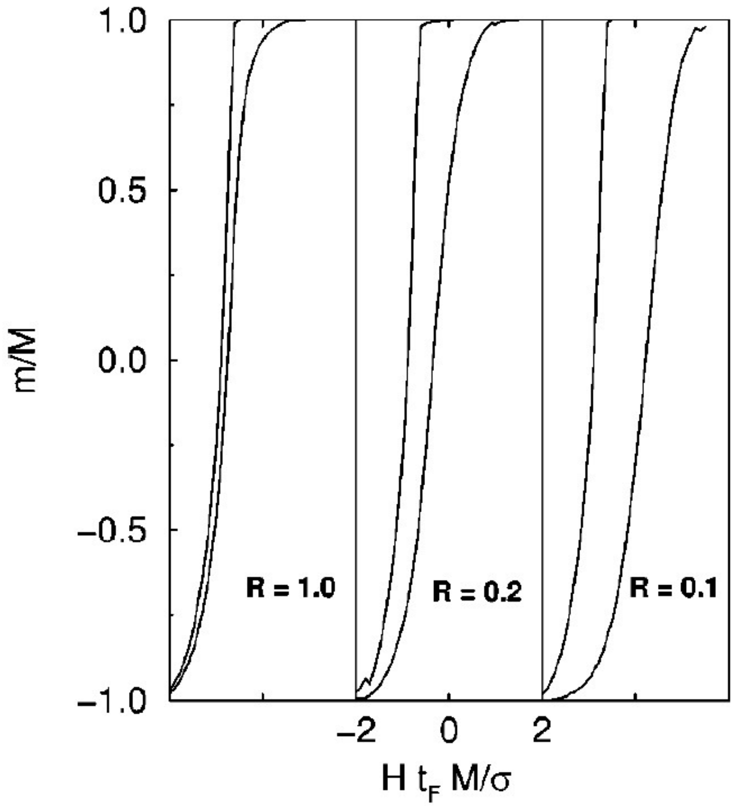
Example: Exchange Bias

Thermal setting of bias:



M Kirschner http://magnet.atp.tuwien.ac.at

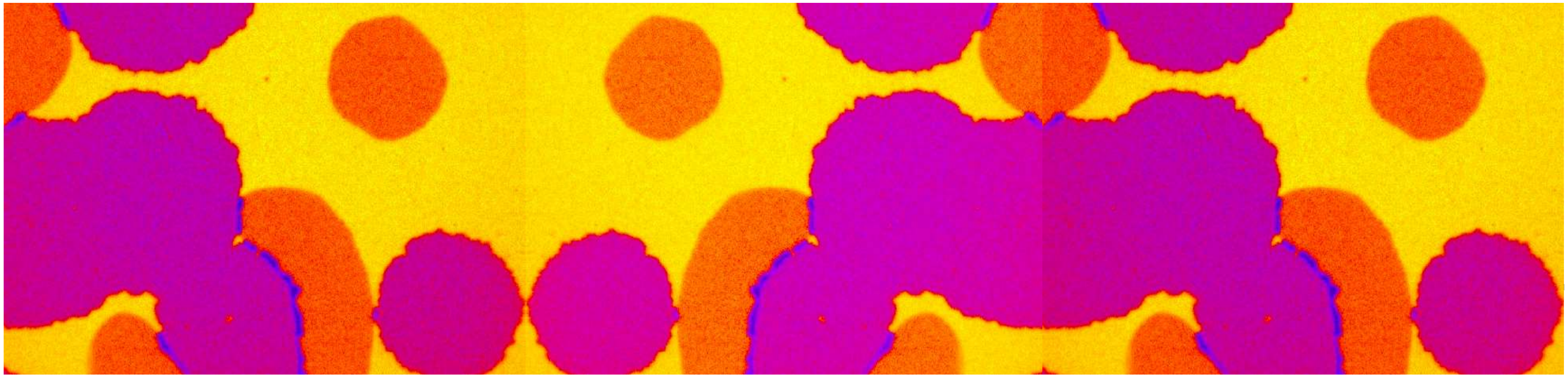
Time dependent coercivity:
Field sweep rates



Stamps, PRB 2000

University of Glasgow

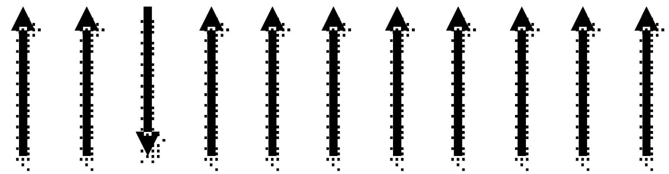
Spin Wave Dynamics



Low Temperature Fluctuations

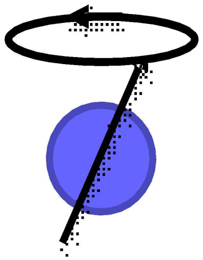
Energy to reverse one spin: *2 J*

$$H = \sum_{ij} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$



Superposition of ways to flip one spin:

$$|\, n = 1\rangle = |\uparrow\downarrow\uparrow\uparrow\uparrow\rangle + |\uparrow\uparrow\downarrow\uparrow\uparrow\rangle + |\uparrow\uparrow\uparrow\downarrow\uparrow\rangle + \ldots$$

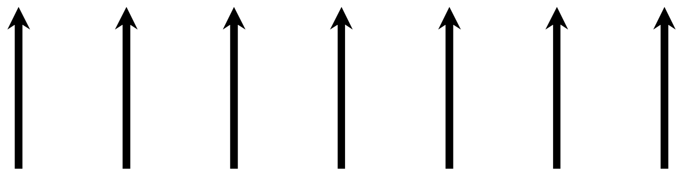


Spinwave excitation

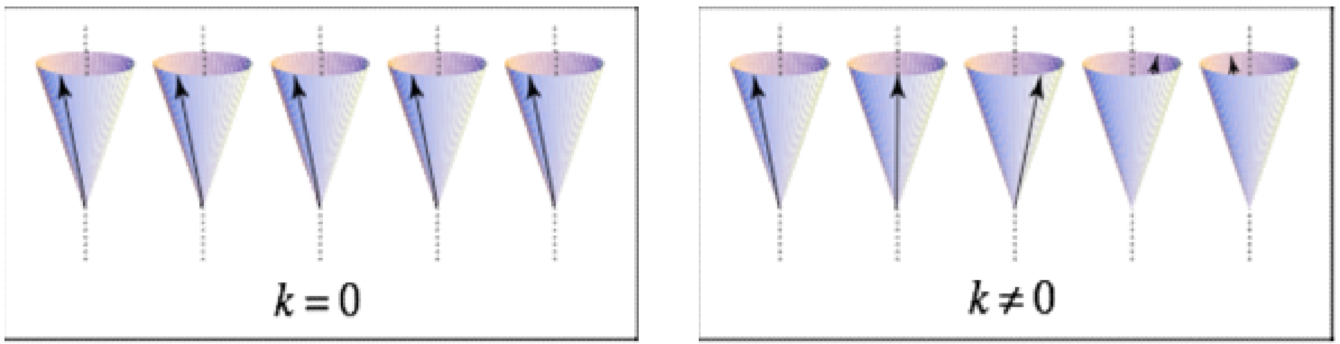
Model: Torque equations

Excitations: Spin Waves

Ground state magnetic orderings:



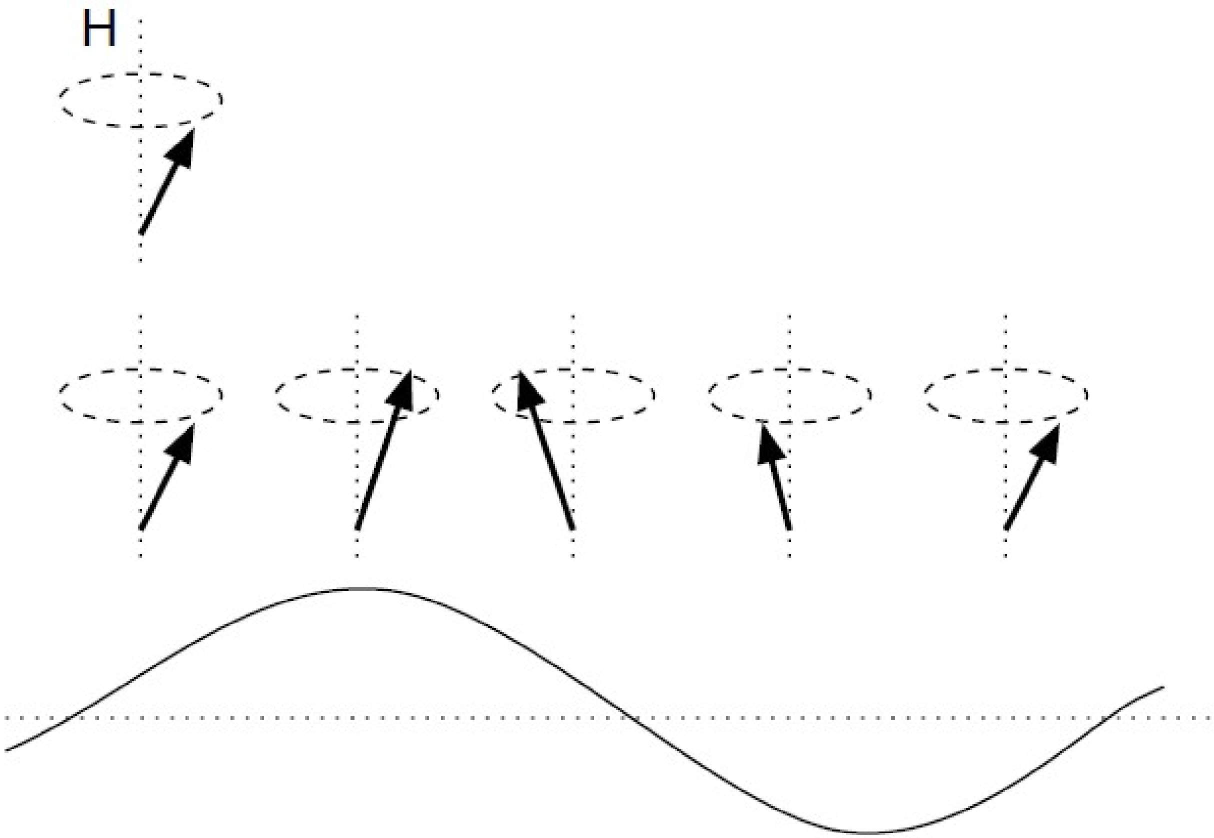
Excitations: Precessional dynamics



slide courtesy J-V Kim

***Note**: The excitations are bosons!*

Classical Precession



Transverse oscillations define wavelength

Equations of Motion

Torque equations:

$$\frac{\partial}{\partial t}\mathbf{m}(\mathbf{r}) = -\gamma\mathbf{m}(\mathbf{r}) \times \mathbf{H}_{eff}$$

$$\mathbf{H}_{eff} = \mathbf{H_a} + A\nabla^2\mathbf{m}(\mathbf{r}) + \mathbf{K}\big(\mathbf{m}(\mathbf{r})\big) - \mathbf{h}_{dip}$$

applied field

exchange

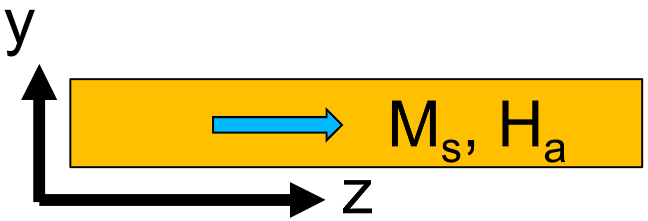
anisotropy

Note: Dissipation adds additional torques

$$\frac{\partial}{\partial t}\mathbf{m}(\mathbf{r}) = -\gamma\mathbf{m}(\mathbf{r}) \times \mathbf{H}_{eff} + \lambda\mathbf{m}(\mathbf{r}) \times \frac{\partial\mathbf{m}(\mathbf{r})}{\partial t}$$

Equations of Motion: FMR

No exchange contribution for uniform precession and dipole field modelled as shape anisotropy ($K \sim M_s$).



$$\mathbf{H}_{eff} = \hat{\mathbf{z}} H_a + \hat{\mathbf{y}} 2K m_y$$

$$\frac{d}{dt} m_x = -\gamma \left[H_a m_y - 2K M_z m_y \right]$$

$$\frac{d}{dt} m_y = \gamma [H_a m_x]$$

$$\frac{d}{dt} M_z = \gamma 2K m_x m_y$$

Linearisation:

$$\frac{d}{dt} M_z \approx 0$$

$$(m_x, m_y) \sim e^{-i\omega t}$$

$$\frac{\omega^2}{\gamma^2} = H_a (H_a - 2K M_s)$$

Anisotropy shifts frequency

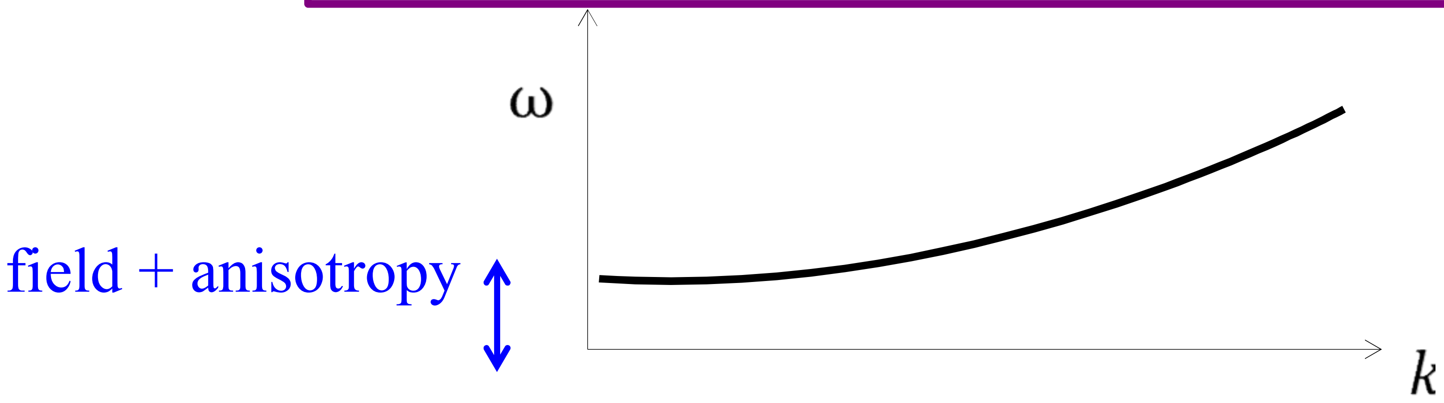
With Exchange: Dispersion

Effect of interactions: exchange

$$(m_x, m_y) \sim \exp[-i(\omega t - \mathbf{k} \cdot \mathbf{r})]$$

$$\mathbf{H}_{ex} \sim A\nabla^2 \mathbf{m}(\mathbf{r}) \quad \Rightarrow \quad \mathbf{H}_{ex} \sim -Ak^2 \mathbf{m}(\mathbf{r})$$

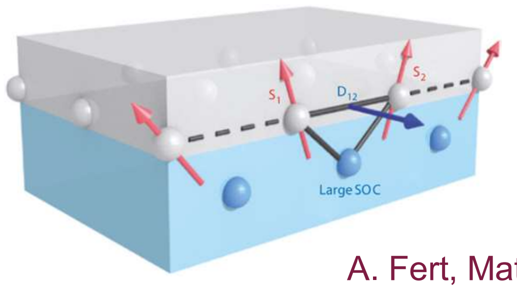
$$\frac{\omega^2}{\gamma^2} = (H_a + Ak^2)(H_a - 2KM_s + Ak^2)$$



Dzyaloshinskii-Moriya Interactions

Interface-driven DMI interaction in ultrathin ferromagnets

$$\mathcal{H}_{\rm DM} = -\vec{D}_{12} \cdot \left(\vec{S}_1 \times \vec{S}_2\right)$$



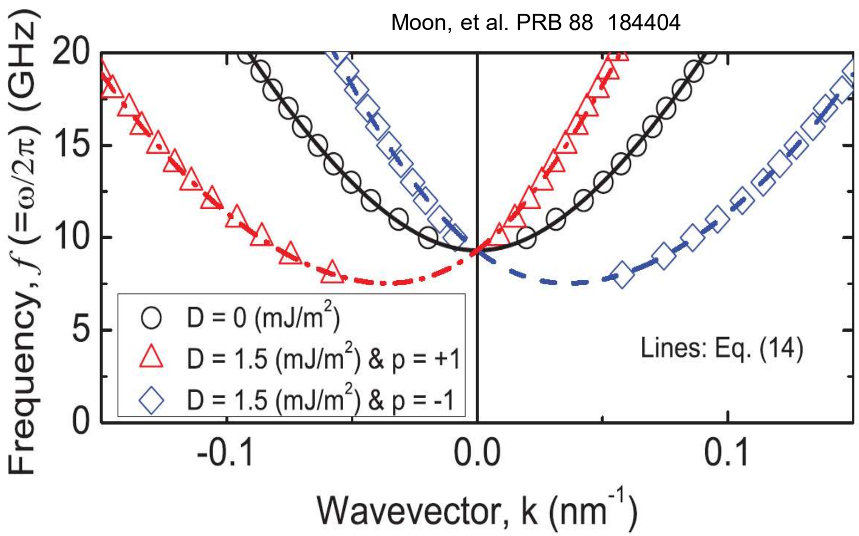
A. Fert, Mat. Sci. Forum (1990)
A. Fert and P. M. Levy, PRL (1980)

Effects of DMI on spin waves:

$$\omega(k) = \Omega(k^2) \pm Dk_{||}$$

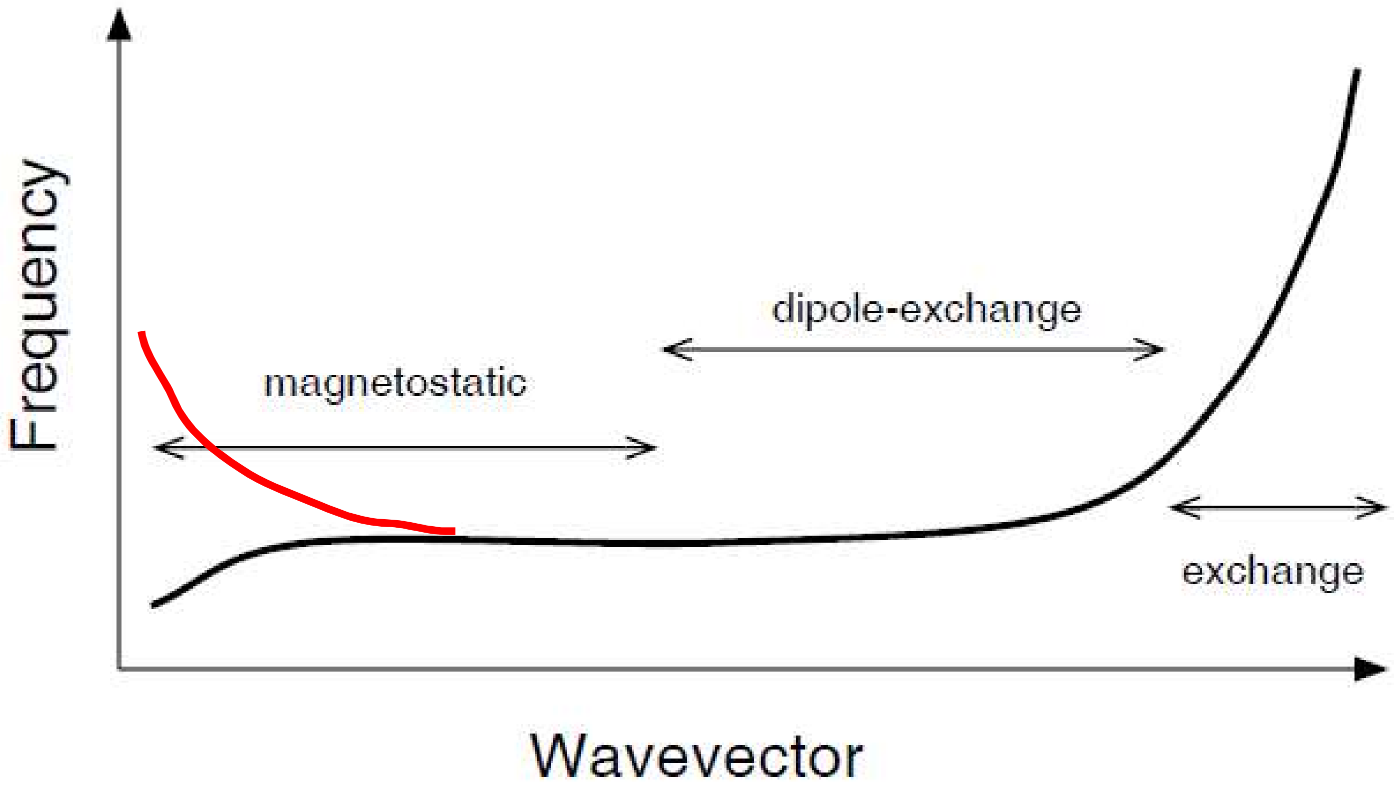
Moon et al. PRB 2013
Iguchi et al., ArXiv 2015

Moon, et al. PRB 88 184404



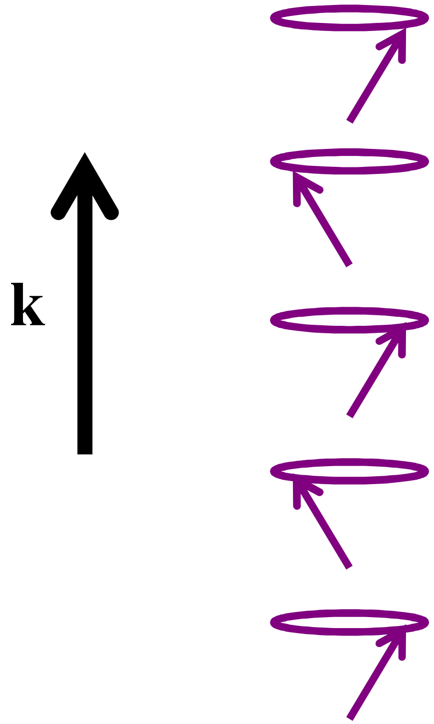
Dispersion: Dipole Exchange Modes

Dipolar: long range interaction terms compete with short range exchange interactions:

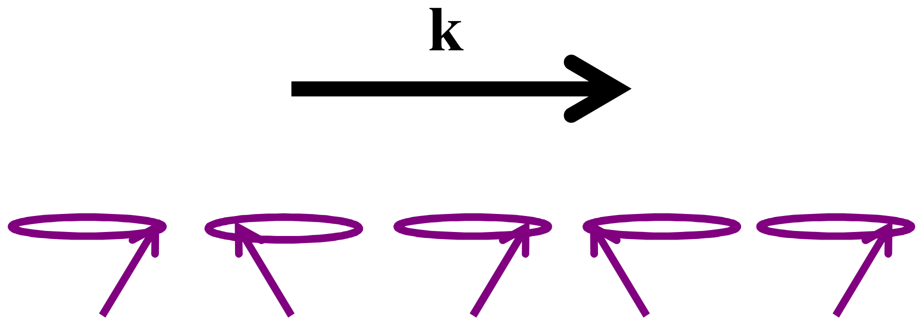


Anisotropic Propagation

Dipolar effect: dependence on propagation direction



small dipolar energy



large dipolar energy

Spin Waves and Micromagnetics

Procedure:

1) **Relax** to steady state
2) Use **broadband** pulse to excite spin waves
3) Record **time** evolution (for spectral analysis)

Example: exciting precession in mumax3 script

```
defregion(1,rect(10e-9,125e-9))
save(regions)
```

} define antenna region

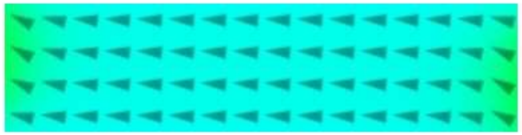
```
driv := 0.001          // amplitude driving field
f    := 1.0e9          // frequency units
fdel := 20.*f*2.*pi    // frequency window
time := 1000./fdel     // evolve time
toff := 3./f           // offset
```

sinc function pulse

```
B_ext = vector(-24.6E-3, 4.3E-3,driv*sin( (t-toff)*fdel )/(2*pi*(t-toff)*fdel))
run(time)
```

Results

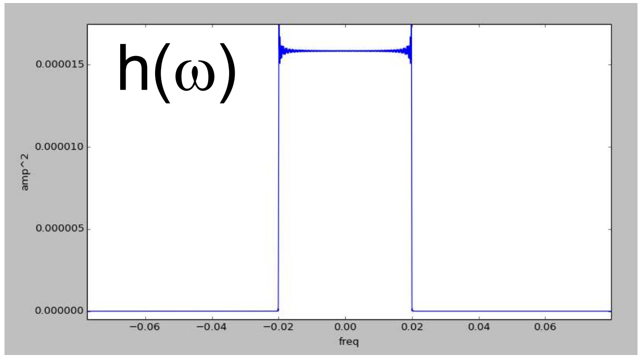
Ground state:

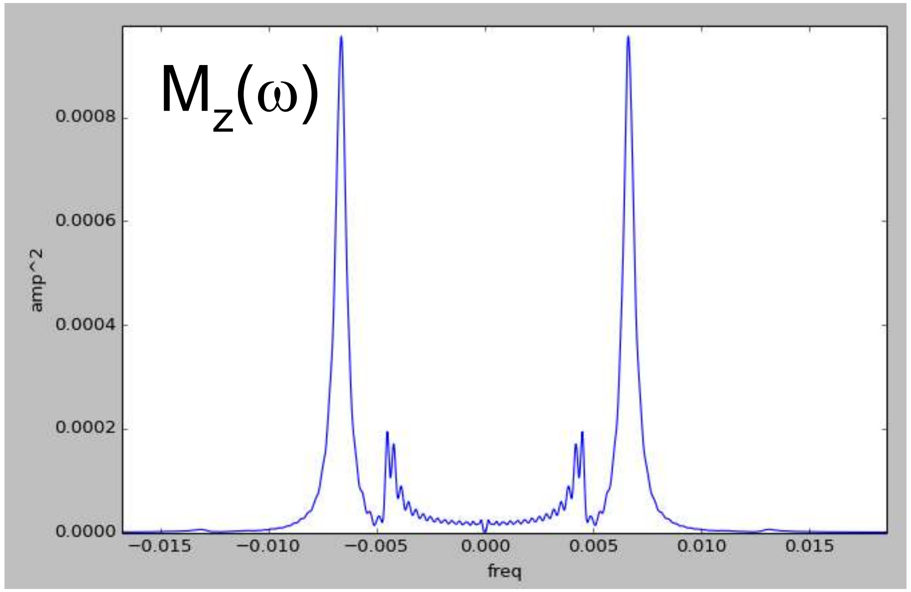


Antenna:

***Note**: Spectral analysis performed separately on mumax generated data.*

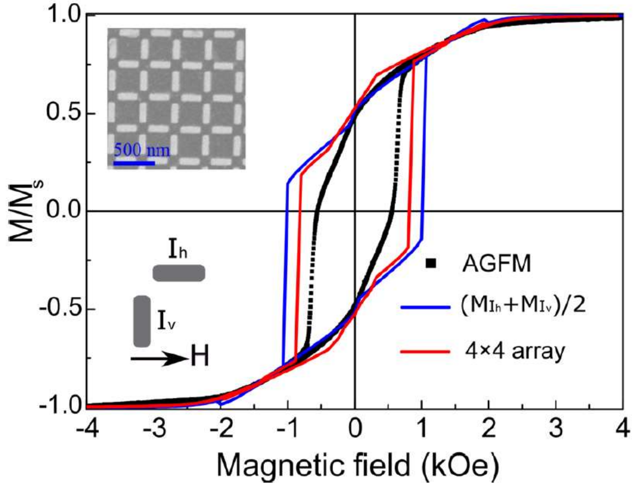
Spectra:

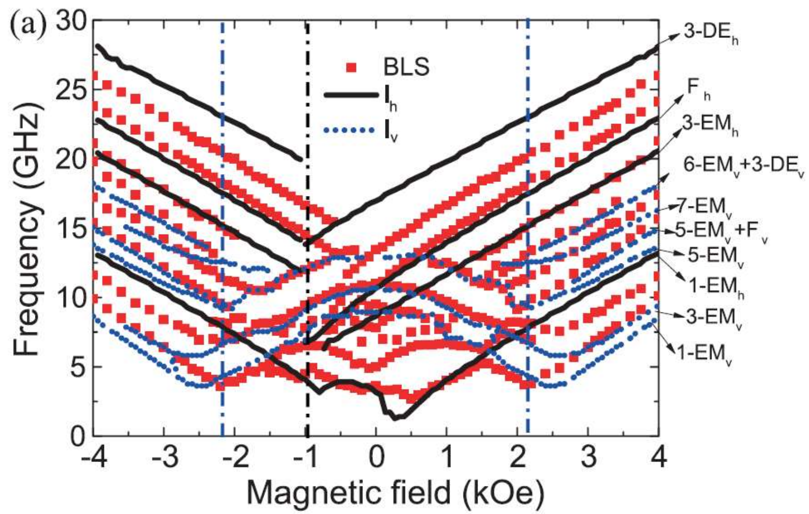


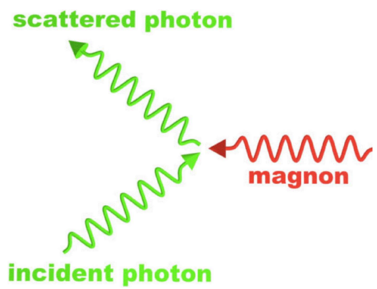


Example:Spin Waves on Spin Ice

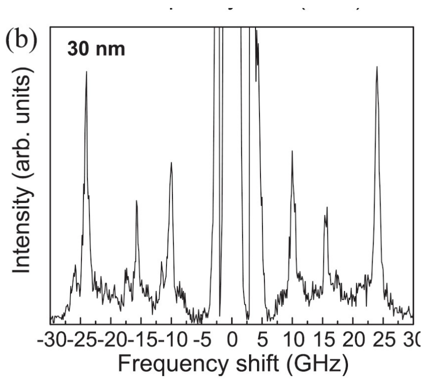
(Yue Li PhD ~2017)

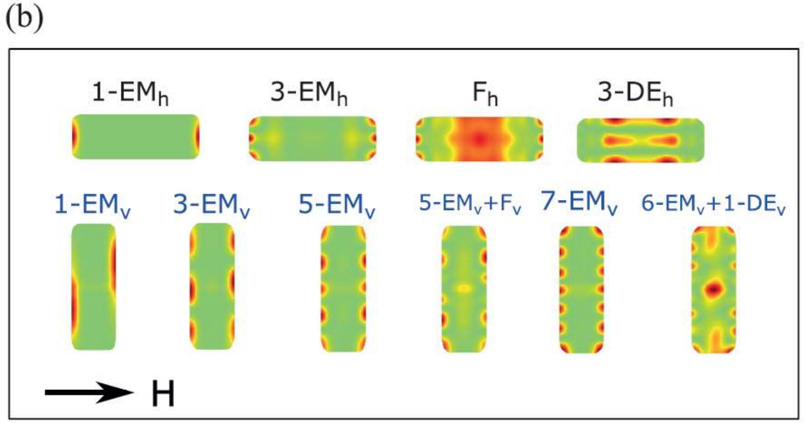






Hillebrands, et al.



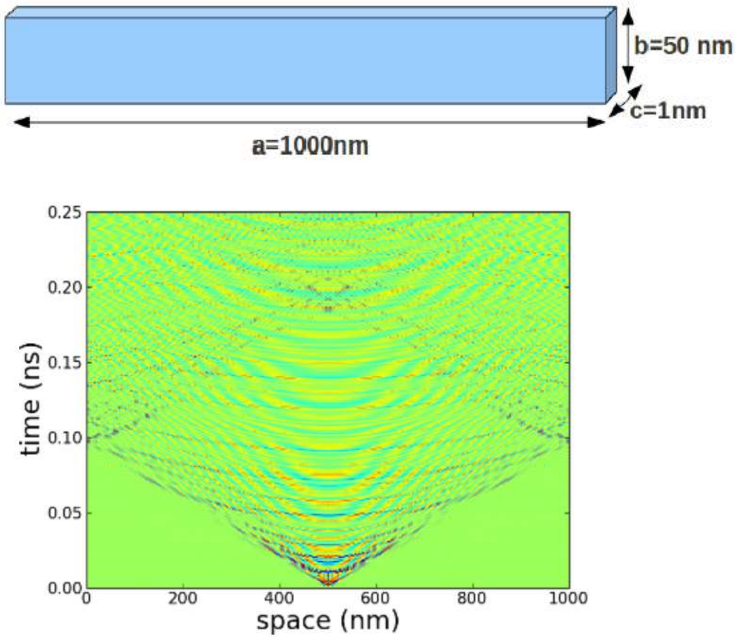


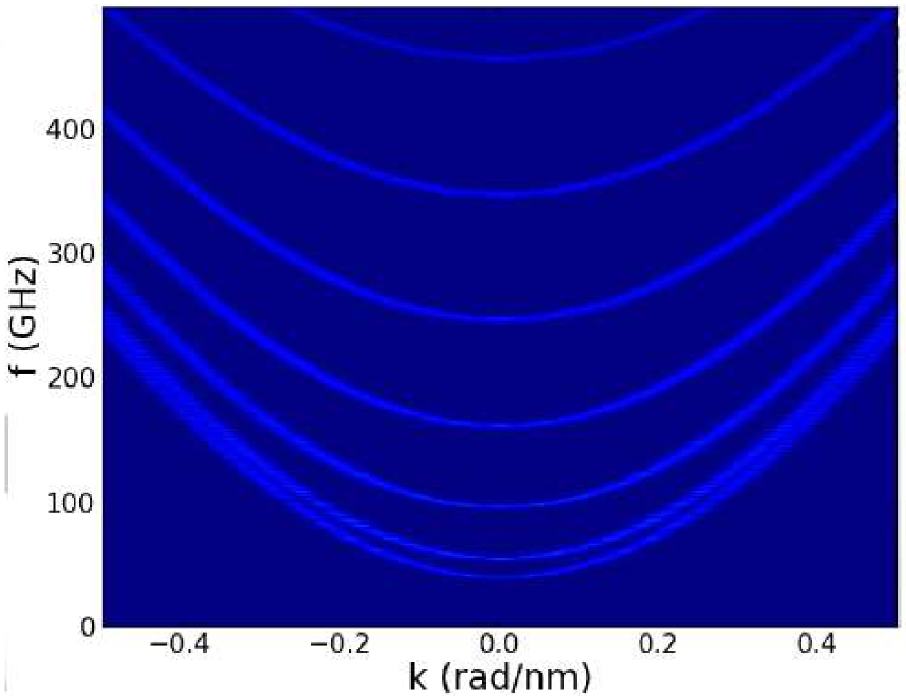
Li, et al. J. Appl. Phys. 2017

Note: Spin Wave Dispersions

Spin wave $\omega(\mathbf{k})$ from micromagnetics: apply 4D pulse

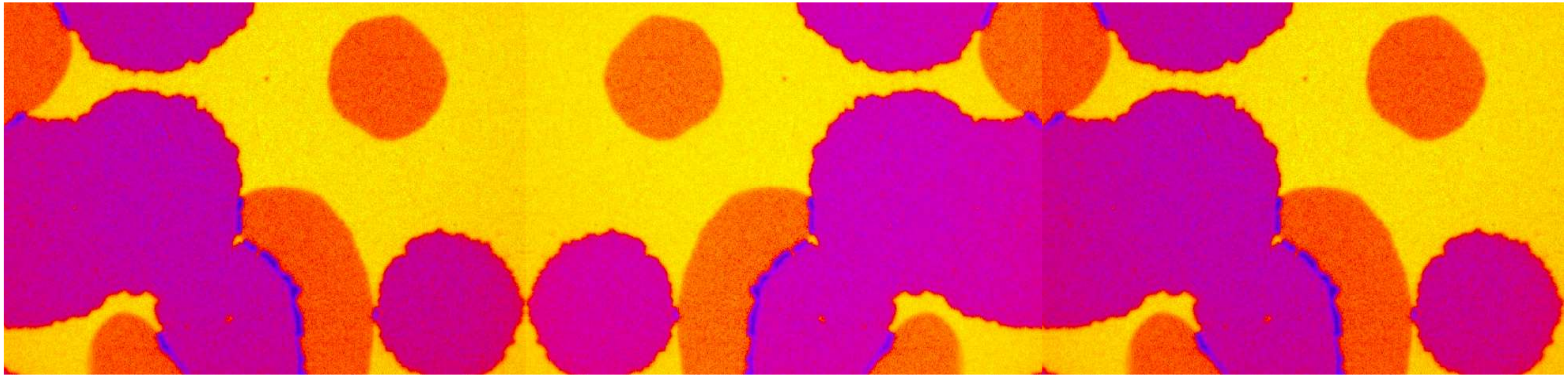
$$h(x,y,z,t) = sinc((x-x')k_x)sinc((y-y')k_y)sinc((t-t')\omega)$$



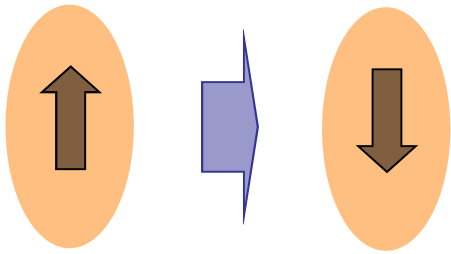


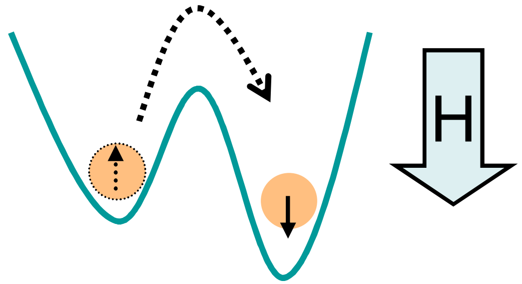
Venkat, Fangohr, et al., IEEE Trans. Magn. 49 (2013)

Reversal Processes, Domains and Domain Walls

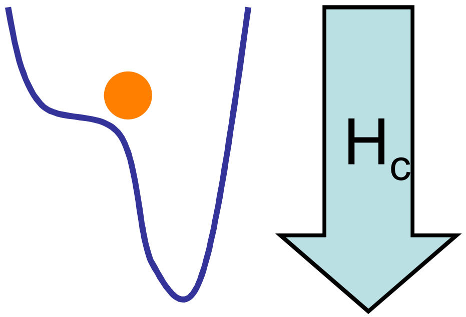


Switching of Single Domain Particles



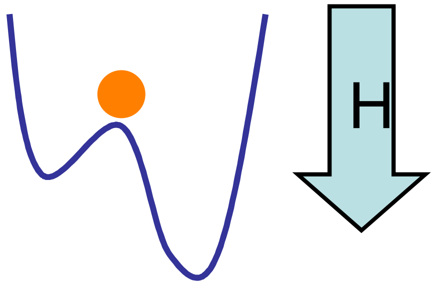


$H \geq H_c$



Dynamics: Precessional reversal

$H < H_c$



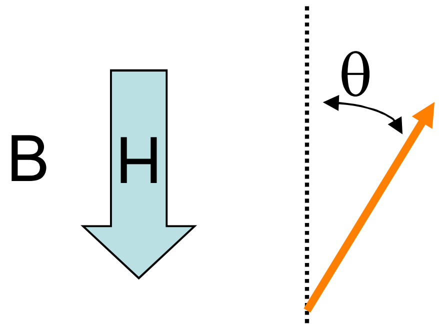
Stability: Thermal activation

Challenge: fluctuations over long time scales

Approach: Stoner-Wohlfarth models

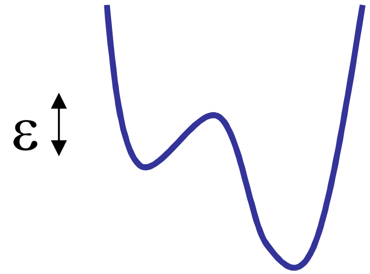
Single Domain Rotation

Approximate reversal as pure **relaxation**:



$$E = V\left(-BM\cos\theta + K\sin^2\theta\right)$$

$$= VK\left[1 + \left(\frac{BM}{2K}\right)^2\right]$$



Stoner-Wohlfarth Model

Rate depends on activation energy and attempt frequency

$$\Gamma = \frac{1}{\tau} = f_o \exp\left(-\varepsilon / k_B T\right)$$

Reversal of a Particle Ensemble

Ensemble of particles: B=0, thermal fluctuations reduce M

Approach to equilibrium: Chemical rate problem

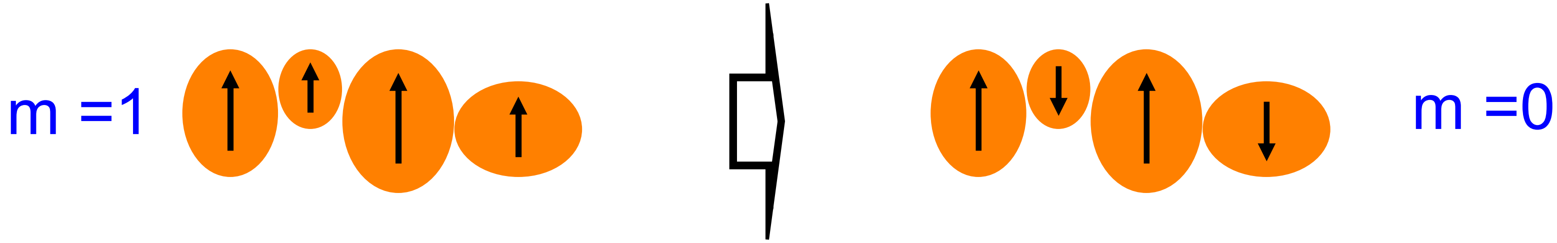
$$\frac{dn_\uparrow}{dt} = W_{\downarrow\uparrow} n_\downarrow - W_{\uparrow\downarrow} n_\uparrow$$

$$\frac{dn_\downarrow}{dt} = W_{\uparrow\downarrow} n_\uparrow - W_{\downarrow\uparrow} n_\downarrow$$

$$m(t) = n_\uparrow - n_\downarrow = A e^{-\Gamma t}$$

Reversal of a Particle Ensemble

Ensemble of particles: H=0, dipolar fields drive m to 0



Approach to equilibrium: Distribution of rates

$$m(t) = A\int P(\Gamma)e^{-\Gamma t}d\Gamma$$

Can one measure the distribution of rates P(Γ)?
(Rebecca Fuller, PhD 2010)

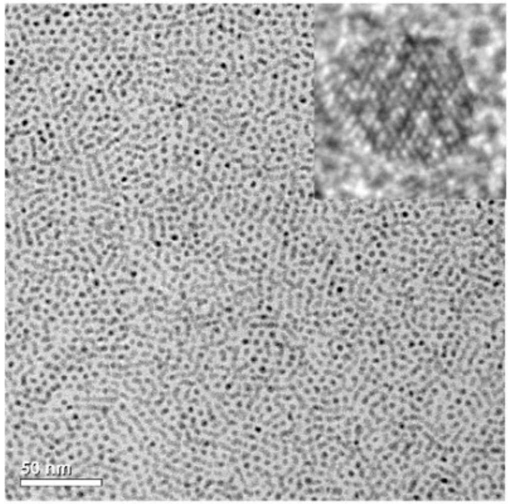
Relaxation: Distributions

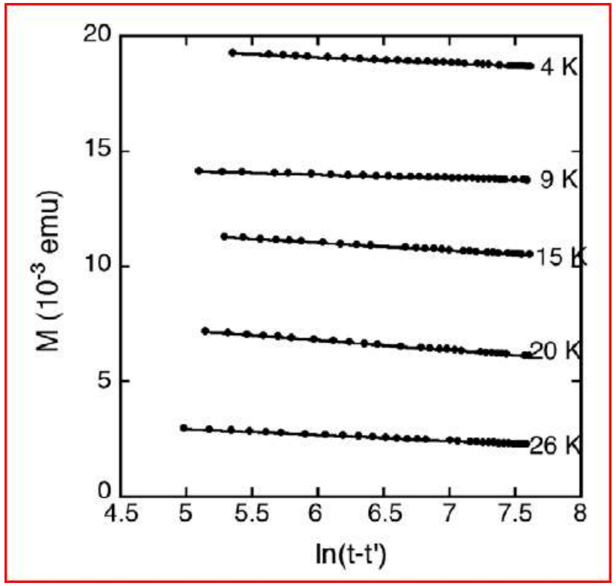
Distribution of energy barriers: $\Gamma = f_o \exp(-\varepsilon / k_B T)$

$$m(t) = m(\infty) + A\int P(\int \Gamma) e^{-t\Gamma} d\Gamma$$

Magnetic viscosity: ln(t) for broad distributions

$$m(t) = C - S(H)\ln(t\Gamma_o)$$

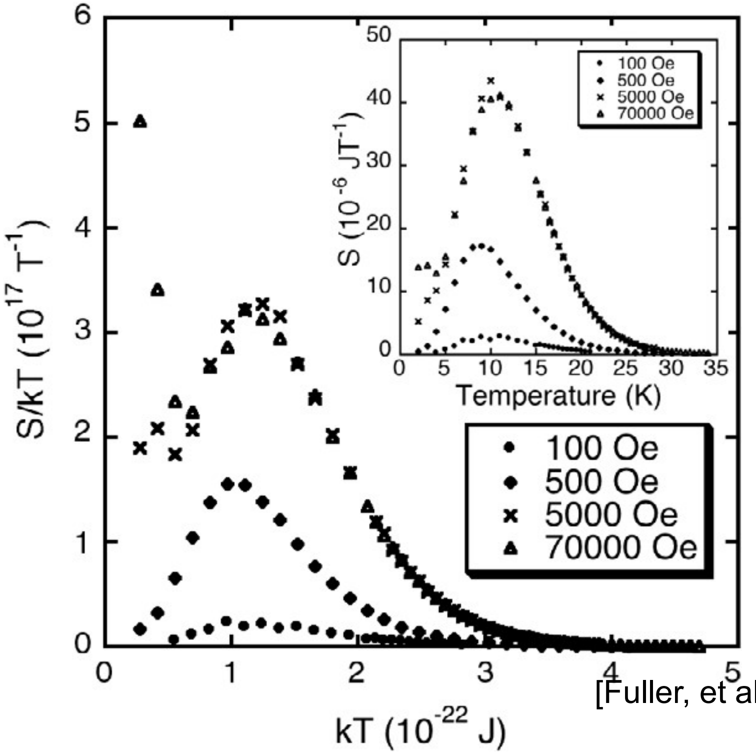




[Fuller, et al., JPCM 2009]

Relaxation: Energy Barriers

Useful measure: study dm/dt at different T



[Fuller, et al., JPCM (2009)]

$$P(\varepsilon) \approx \frac{S}{Ak_B T}$$

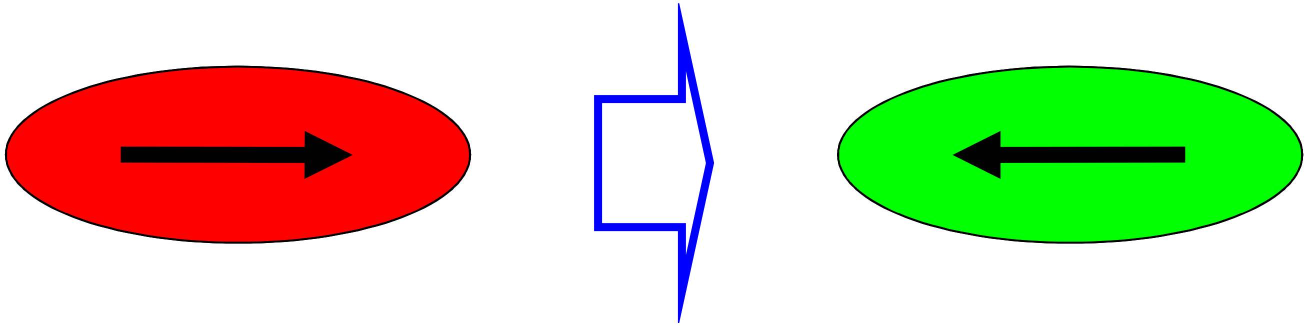
*Viscosity at different **temperatures** and **fields** provides estimates for energy barrier distribution*

Questions?

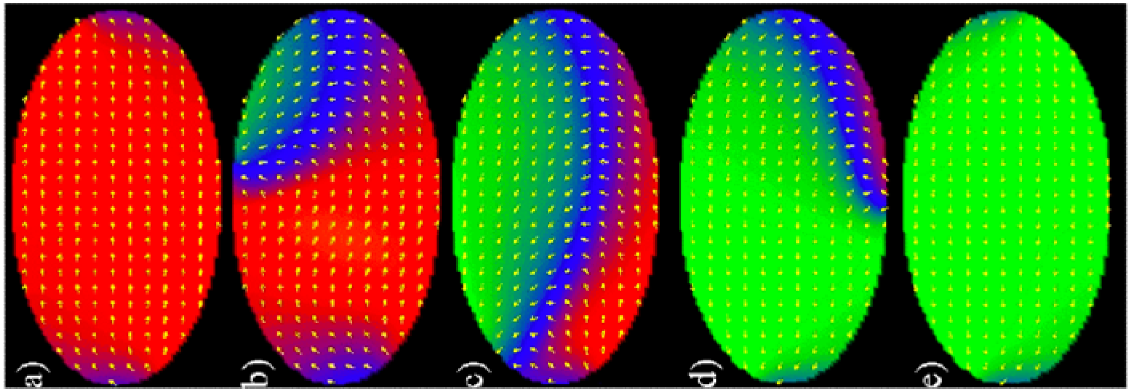

Poker Challenge
CHALLENGE
$ 75,000
$ 50,000
$ 25,000
$ 10,000
$ 5,000
$ 2,500
$ 1,000
$ 0
BANK

Reversal processes, magnetic domains and domain walls

Routes to Reversal

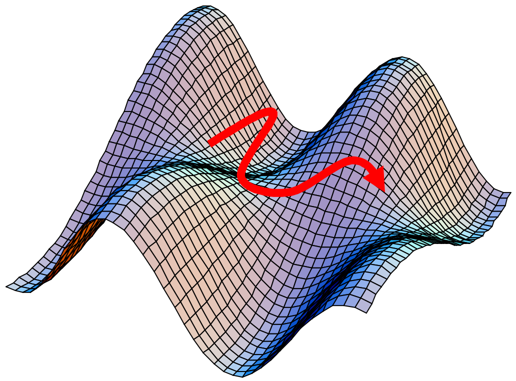


Nucleation of domains and domain walls



[Slaughter, 2000]

Challenge: fluctuations over long times **and** large lengthscales

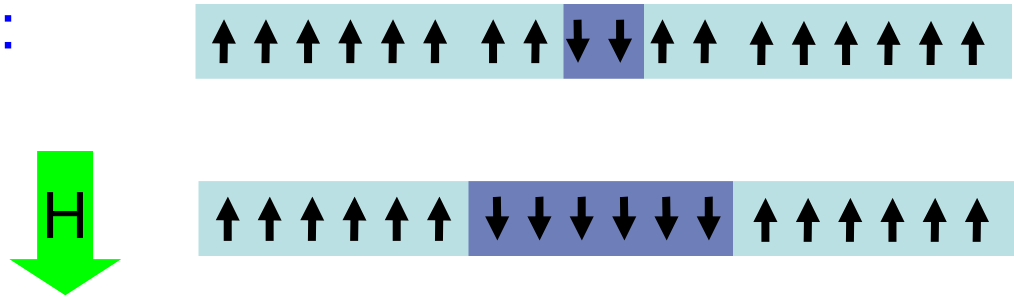


Approach: a Stoner-Wohlfarth model for fluctuating lines

Magnetization Processes & Domains

Nucleation processes:

Growth of a critical domain volume



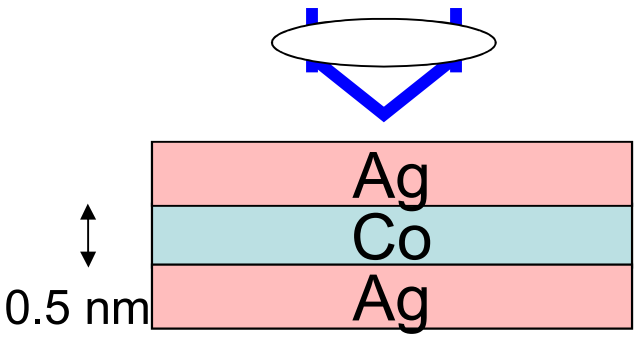
$$E_{Zeeman} = -\mu MVH \qquad E_{DW} = \sigma A$$

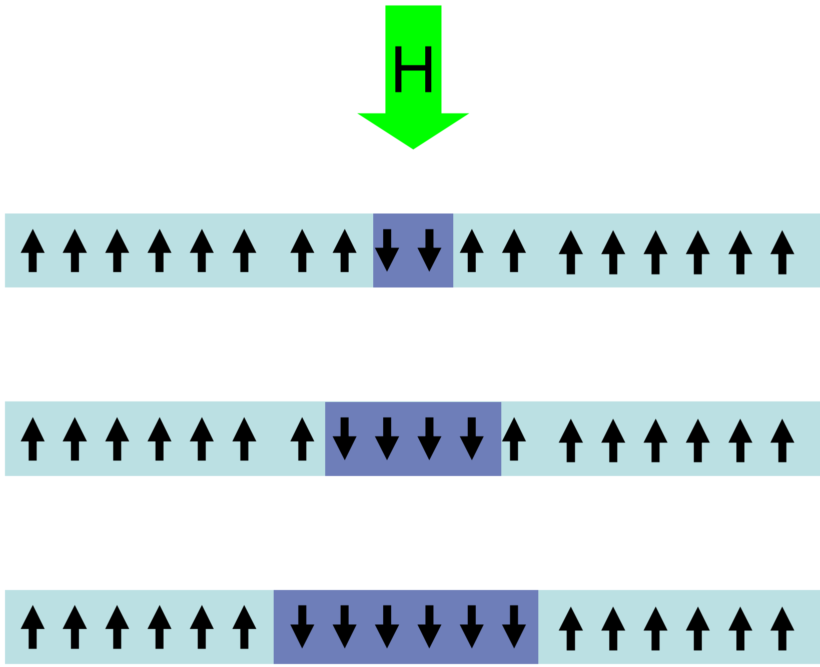
Surface energy

$$\Rightarrow \left(\frac{V}{A}\right)_c = \frac{\sigma}{(\mu MH)}$$

Domain & Wall Dynamics

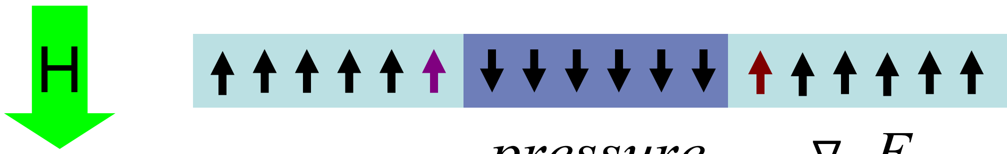
- **Example**: MOKE study
- Perpendicular M in Co
- Method:
  - saturate
  - apply field pulse
  - image & repeat





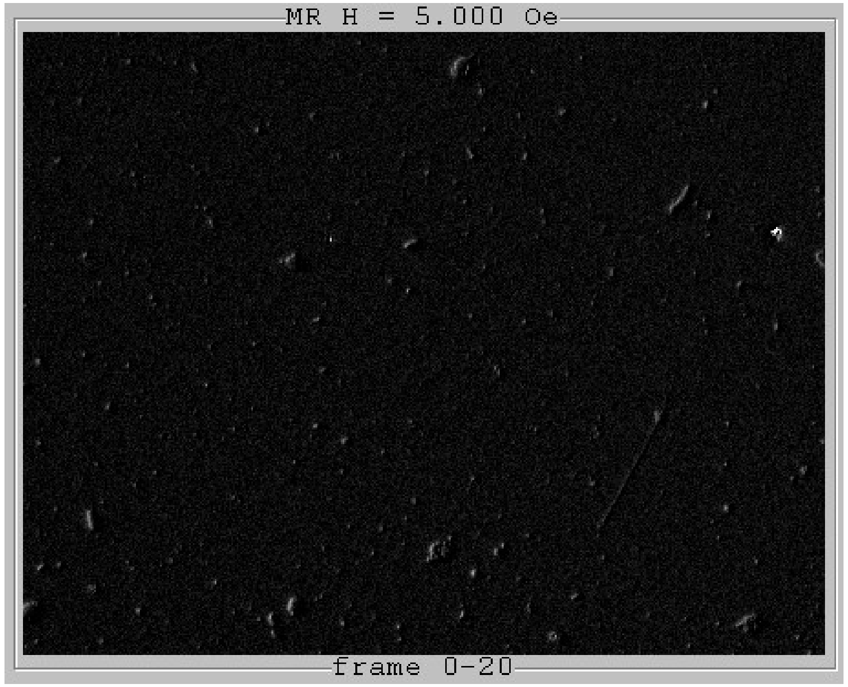
Magnetisation Processes & Domains

Growth stops at local field gradients (pinning 'pressure')



$$pressure \sim -\nabla E_{local}$$

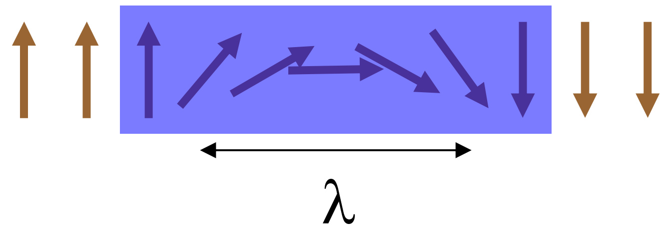
Stroboscopic 'movie' of domain growth



Magnetization Processes & DW's

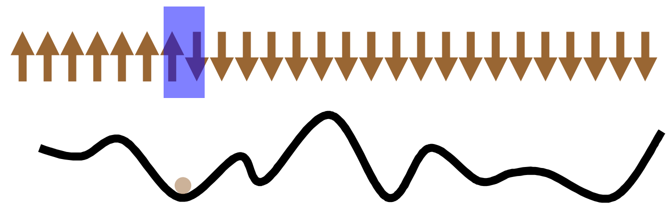
Wall structure:

- Topological excitation
- Surface tension
- Characteristic width



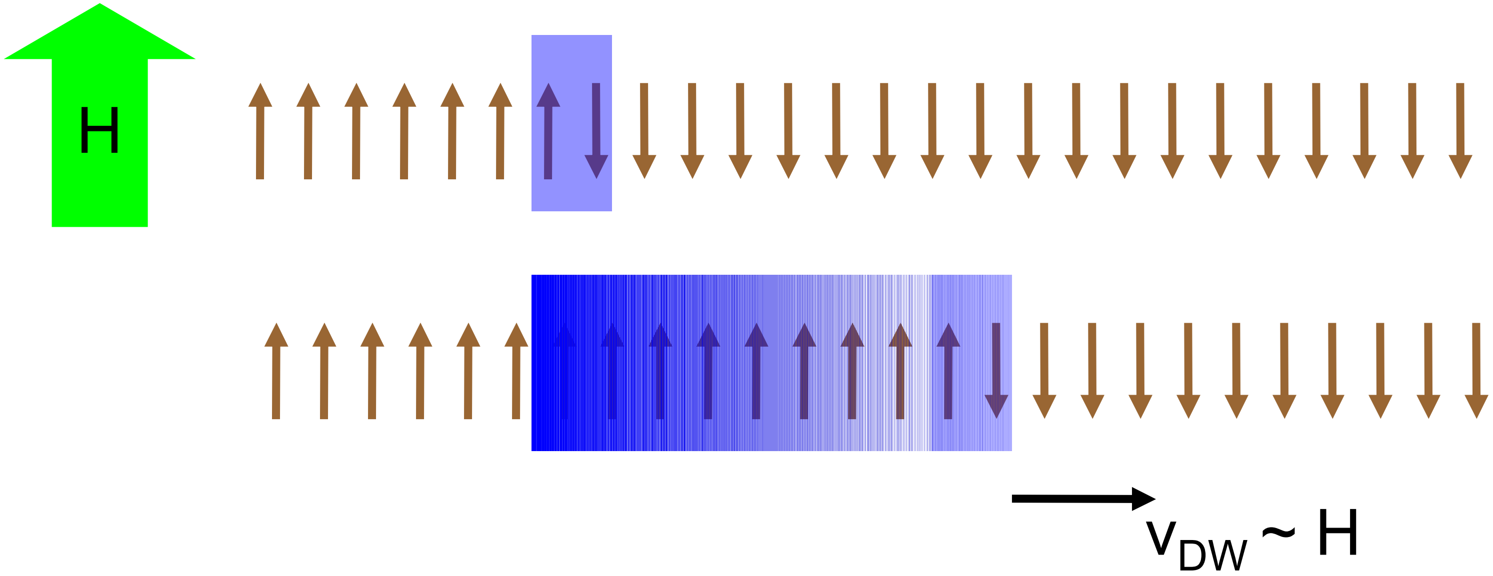
Dynamics:

- **Translation** & fluctuations
- **Pinning & 'creep'**
- Internal modes



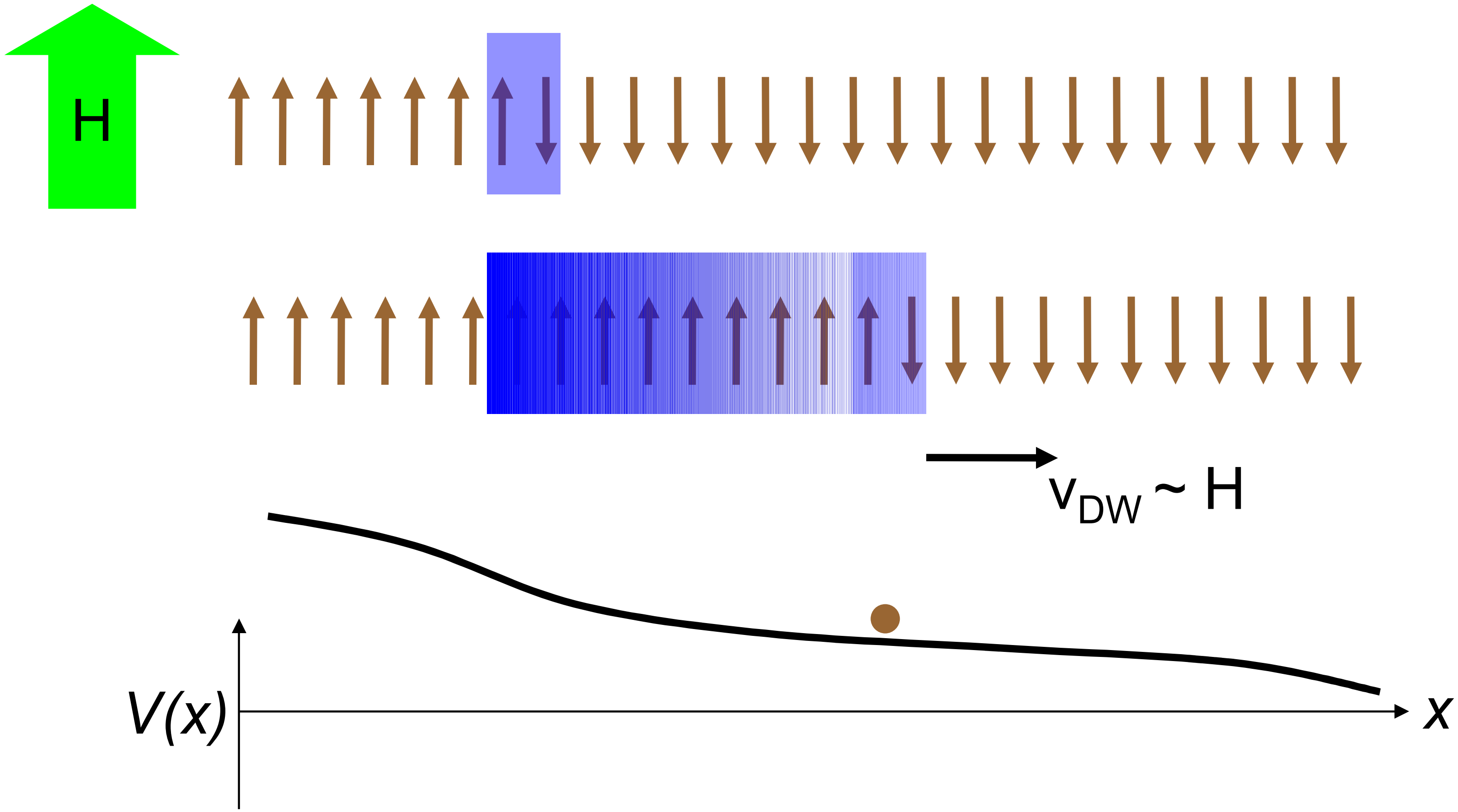
DW Mobility: High Field Flow

Viscous Flow: High field driven dynamics



DW Mobility: High Field Flow

Viscous Flow: High field driven dynamics



DW Mobility Theory: Flow

Torque equations of motion:

$$\frac{\partial m}{\partial t} = \underbrace{-\gamma m \times H}_{\text{precessional torque}} + \frac{\alpha}{M_s} \underbrace{m \times \frac{\partial m}{\partial t}}_{\text{Gilbert damping}}$$

Effective local field: (exchange, anisotropy, dipolar)

$$H = H_{applied} - \frac{\partial E}{\partial \boldsymbol{m}}$$

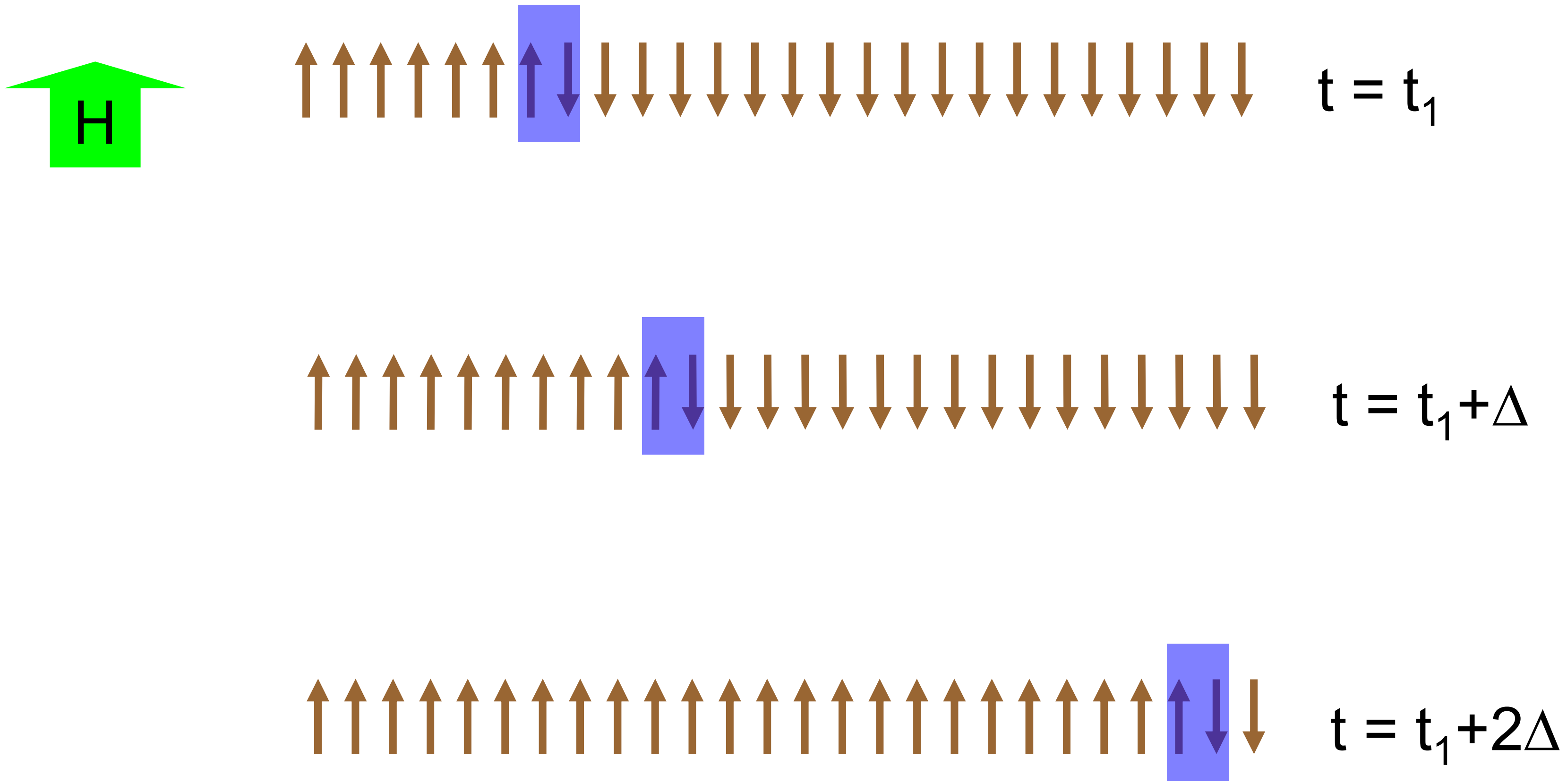
Time averaged velocity (in Flow regime):

$$v \propto \int \overline{|\boldsymbol{m} \times \boldsymbol{H}|^2}\, d^3 x$$

[X Wiang, P Yan, J Lu]

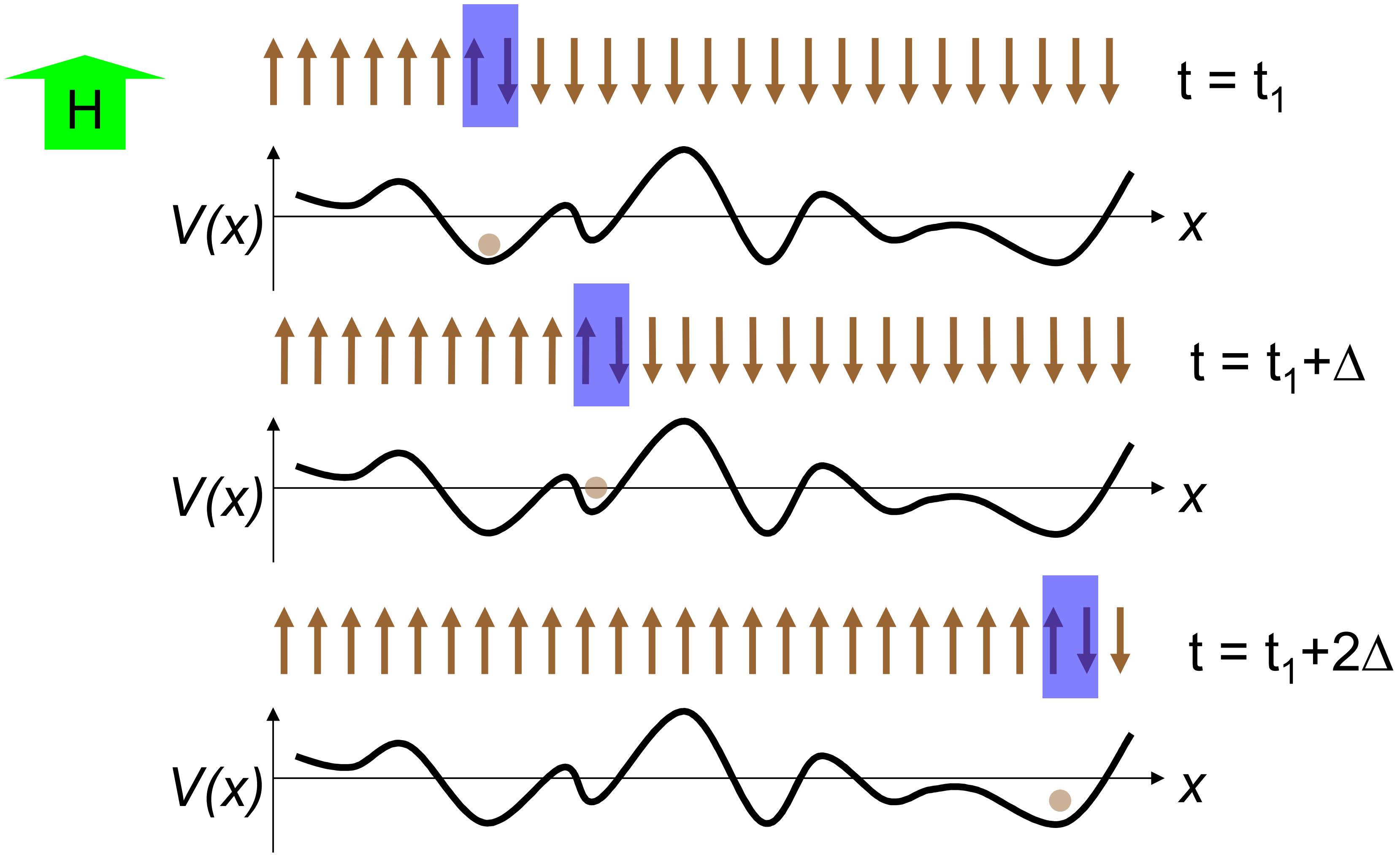
DW Mobility: Low Field Creep

Creep: low field thermally activated dynamics



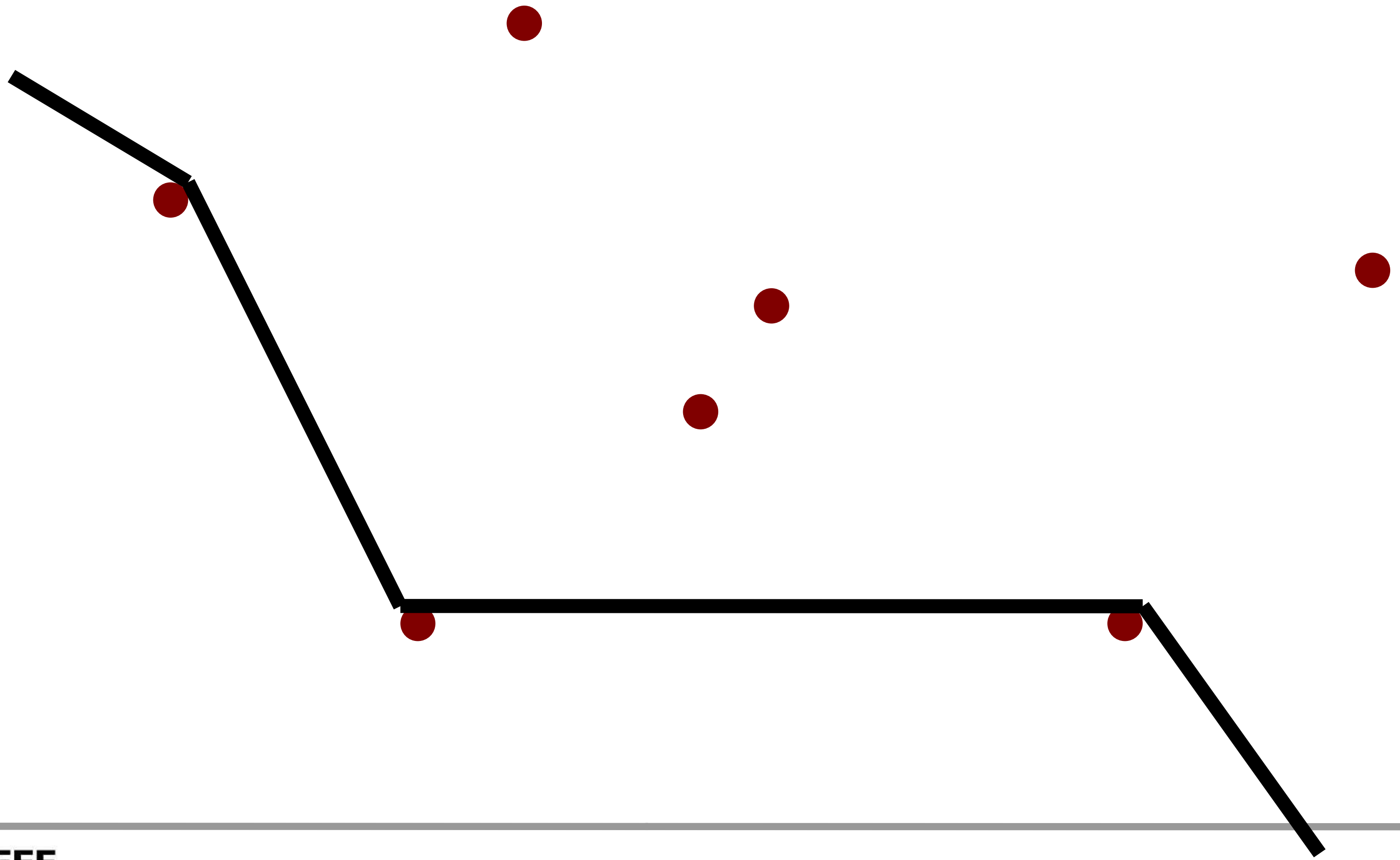
DW Mobility: Low Field Creep

Creep: low field thermally activated dynamics



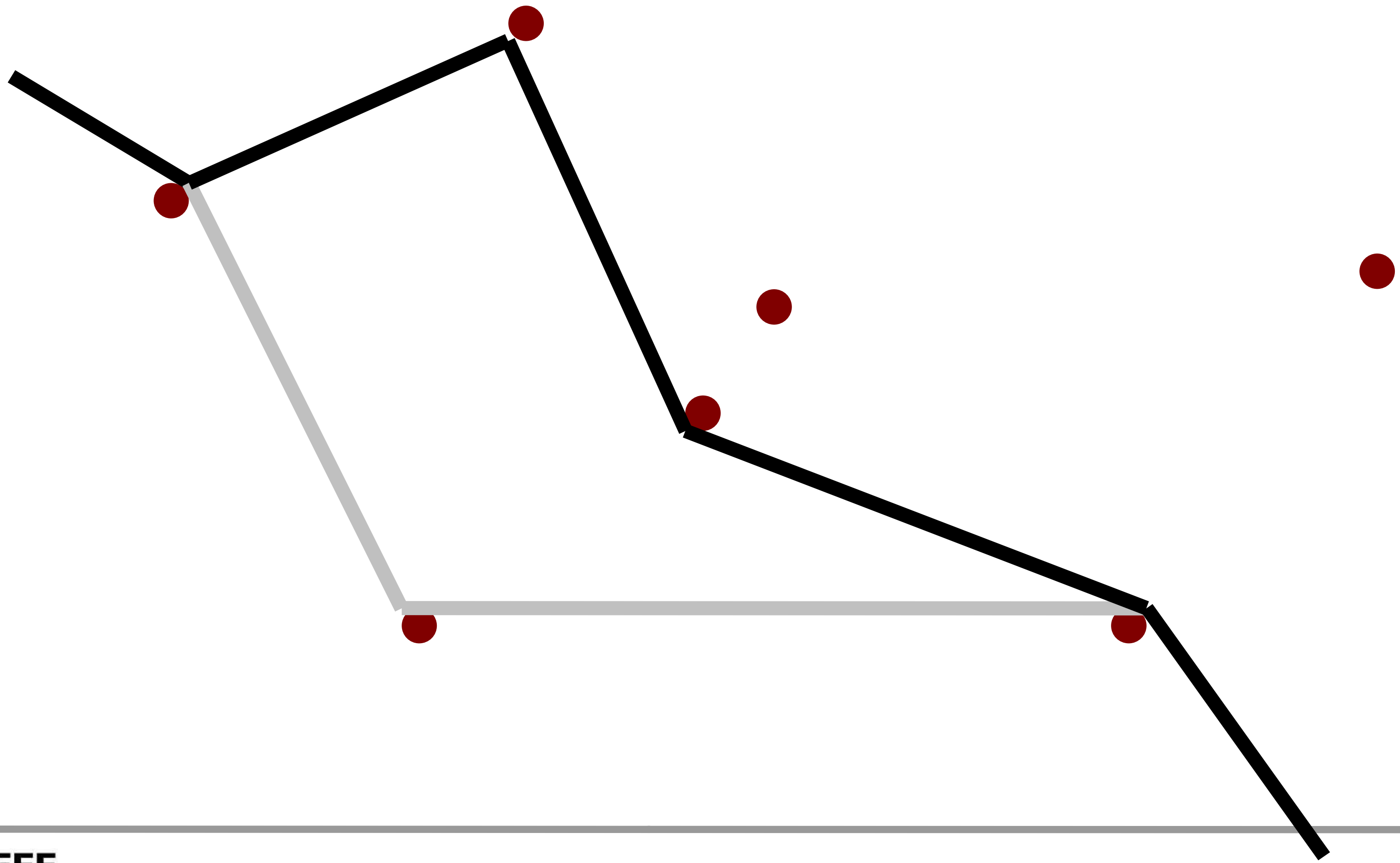
DW Mobility Theory: Creep

Pinning sites oppose wall motion:



DW Mobility Theory: Creep

Pinning sites oppose wall motion:

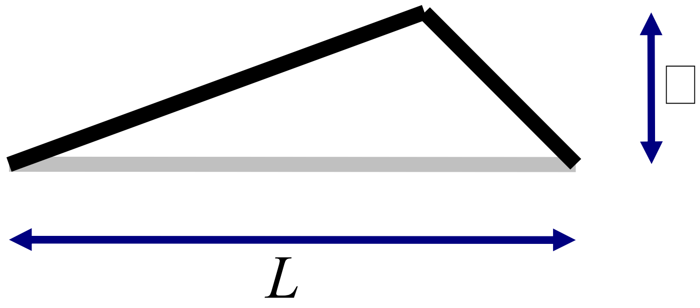


DW Mobility Theory: Creep

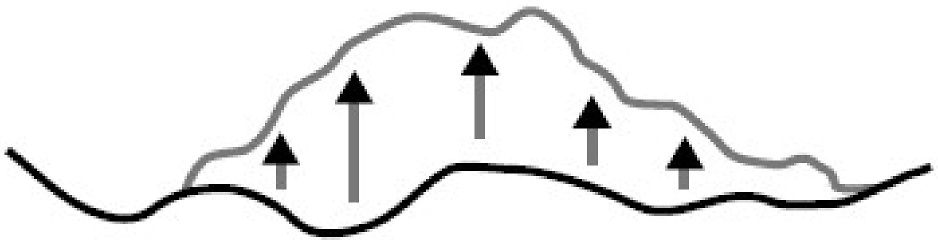
Number of pinning sites:

$$E_p = \sqrt{f_{pin}^2 N_p}\,\xi$$

pin force



Macroscopic wall motion through **avalanche**:



Scaling: **critical field** for avalanche onset

$$[E_{elastic} - E_{Zeeman}] = E_B \approx U_C \left(\frac{H_{dep}}{H_{applied}} \right)^{\frac{2\zeta - 2 + D}{2 - \zeta}}$$

DW Mobility Theory: Creep

Depinning rate:

$$\frac{1}{\tau(L)} = \frac{1}{\tau_0}\exp\left[\frac{-E_B(L)}{k_B T}\right]$$

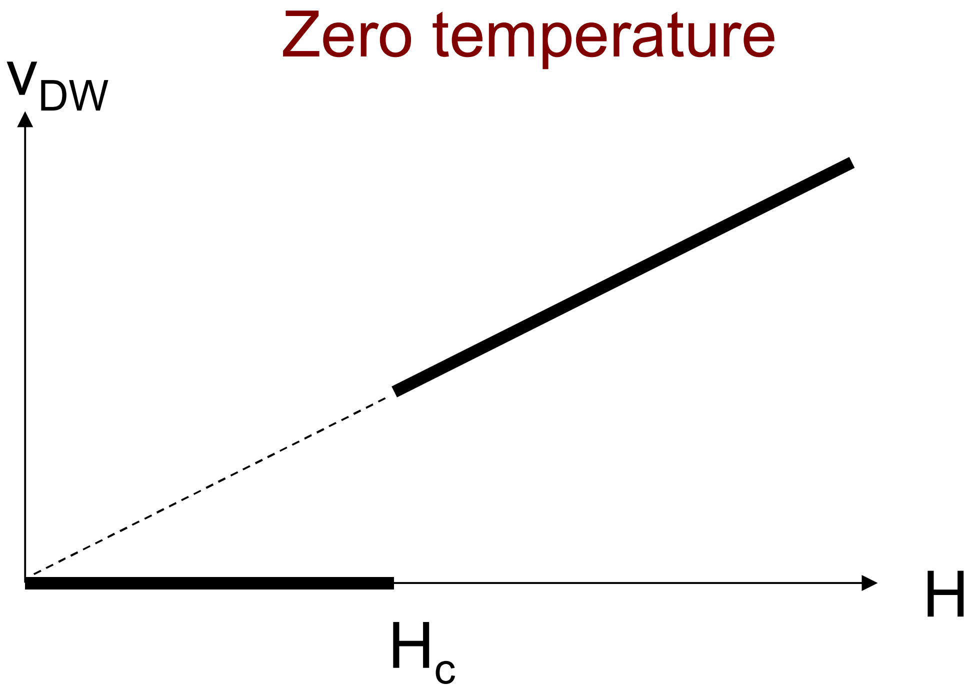
Multiply by distance travelled to give velocity:

$$v = \frac{w(L)}{\tau(L)} \approx \frac{\xi}{\tau_0}\exp\left[\frac{-U_C}{k_B T}\left(\frac{H_{dep}}{H_{applied}}\right)^{\mu}\right]$$

Expect $\mu = 1/4$ for ultra thin films.

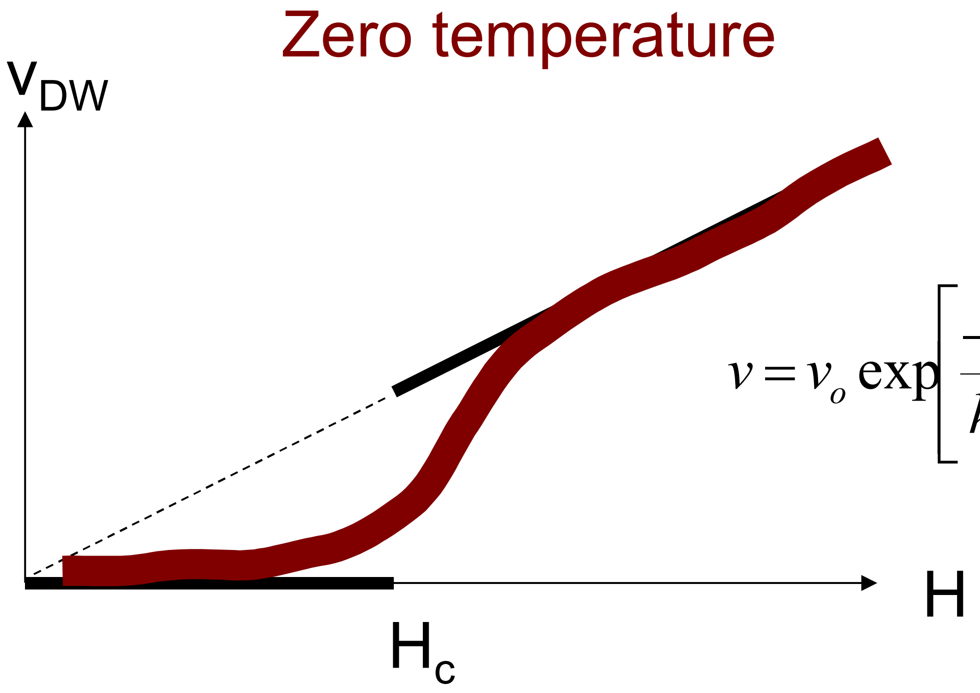
DW Motion: Transition

Threshold: transition from creep to viscous flow



DW Motion: Transition

Threshold: transition from creep to viscous flow

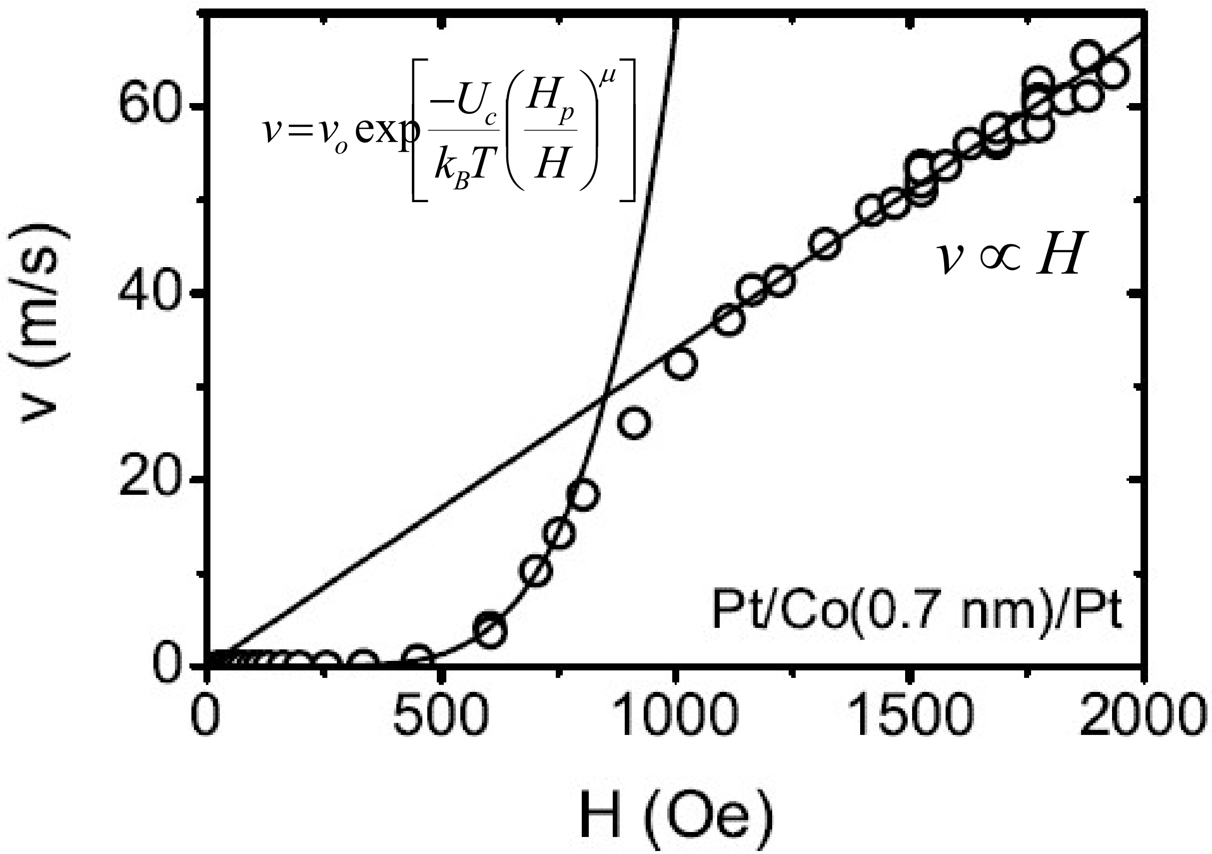


$$v = v_o \exp\left[\frac{-U_c}{k_B T}\left(\frac{H_p}{H}\right)^{\mu}\right]$$

DW Motion: Transition

Observed transition from creep to viscous flow:

(Peter Metaxas, PhD 2009)



[Metaxas, et al., Phys. Rev. Lett., 2007]

Summary

- **Approximations**: Heisenberg exchange, anisotropy, mean field theory

- **Simulations**: Micromagnetic, Monte Carlo

- **Analytic models**: spin waves, domain walls, thermal activation

Thank you!