Dynamics Modelling & Simulation

Robert Stamps received BS and MS degrees from the University of Colorado, and a PhD in Physics from Colorado State University. He was with the University of Western Australia until 2010, and is currently Professor of Solid State Physics at the University of Glasgow in Scotland. He was an IEEE Magnetics Society Distinguished Lecturer in 2008 (including visits to CBPF and elsewhere in Brazil), and he was the IEEE/IOP Wohlfarth Lecturer in 2004. He is chair of the IRUK IEEE Magnetics Society Chapter, was chair of the 2007 MML Symposium, and will co-chair the Joint European Magnetics Symposia in 2016. This is the fourth time he has lectured at an IEEE Magnetics School.
Dynamics: Modelling and Simulation

Robert Stamps
IEEE Magnetics Society School 2017
Aim of lectures:

To provide an introduction to the philosophy and art of modelling of the essential physics at play in dynamic magnetic systems.

Examples will be given of how simple models can be constructed and applied to understand and interpret observable phenomena, ranging from magnetisation processes to high frequency spin wave dynamics.

Along the way, an introduction to some general tools will be provided, including Monte Carlo models and micromagnetics.
Outline

• Modelling Dynamics: where to start?
  – Starting points
  – Phenomenology

• Some generic tools:
  – Micromagnetics
  – Mean field theory & Monte Carlo

• Spin dynamics
  – Torque equations
  – Spinwaves & resonances

• Domains and domain walls
  – Stoner-Wohlfarth models
  – Magnetic domains and domain walls

With examples from PhD works!
Modelling: where to start?
Models for research & development: magnetic ordering, dynamics, transport ...

Some starting points for model makers
Tools

1) Simulations do not by themselves provide interpretations or insights

2) Analytic/conceptual models often go where simulations cannot
The dark arts of simplification:

*Energies, symmetry and phenomenology*
Energies

Relevant energy scales (P. W. Anderson, 1953):

\begin{itemize}
\item 1 – 10 eV  
  Atomic Coulomb integrals
  Hund's rule exchange energy
  Electronic band widths
  Energy/state at $\varepsilon_f$

\item 0.1 – 1.0 eV  
  Crystal field splitting

\item $10^{-2} – 10^{-1}$ eV  
  Spin-orbit coupling
  $k_B T_C$ or $k_B T_N$

\item 10^{-4} eV  
  Magnetic spin-spin coupling
  Interaction of a spin with 10 kG field

\item 10^{-6} – 10^{-5}$ eV  
  Hyperfine electron-neuclear coupling
\end{itemize}
Concept: Exchange Energy

Pauli exclusion **separates** like spins:

Can be **energetically favourable**: suppose alignment determines average separation. Then *if*:

\[ \langle r_a \rangle \sim 0.3 \text{ nm} \quad \Rightarrow \quad \frac{e^2}{r_a} \sim 4.8 \text{ eV} \]

\[ \langle r_p \rangle \sim 0.31 \text{ nm} \quad \Rightarrow \quad \frac{e^2}{r_p} \sim 4.75 \text{ eV} \]

\[ E_{\uparrow \uparrow} - E_{\uparrow \downarrow} = 0.05 \text{ eV} (580 K) \]

\[ \frac{E_{\uparrow \uparrow} - E_{\uparrow \downarrow}}{\mu_B} = 870 T \]

... equivalent field: \[ \frac{E_{\uparrow \uparrow} - E_{\uparrow \downarrow}}{\mu_B} = 870 T \]
Exchange Interactions

**Exchange**: electrostatic repulsion + quantum mechanics.

Hamiltonian as **spin functions**: (Dirac & Heisenberg)

\[
H = -J_{1,2} \mathbf{\sigma}_1 \cdot \mathbf{\sigma}_2
\]

Pauli spin matrices

Generalised for multi-electron orbitals (**van Vleck**):

\[
H_{ex} = -\sum J(r_a - r_b) S(r_a) \cdot S(r_b)
\]

total spin at sites \( r \)
Using Symmetry: Exchange

Measurable moment **density** (not an operator):

\[ m(r) = \text{Tr}(\rho \hat{M}(r)) \]

**Exchange** still in Heisenberg form:

\[ E_{\text{ex}} = \sum J m(r_j) \cdot m(r_{j+\delta}) \]

Atomic to continuum: Expand \( m \) field about \( r_j \)

\[ m(r_{j+\delta}) = m(r_j) + [(\delta \cdot \nabla) m(r_{j+\delta})]_{j=\delta} + \frac{1}{2} [(\delta \cdot \nabla)^2 m(r_{j+\delta})]_{j=\delta} + \ldots \]
Using Symmetry: Exchange

When lattice symmetry allows:

$$\delta_{x} \frac{\partial m}{\partial x} + (- \delta_{x}) \frac{\partial m}{\partial x} = 0$$

Example: isotropic medium

$$E_{ex} = m_{x} (\nabla^{2} m_{x}) + m_{y} (\nabla^{2} m_{y}) + m_{z} (\nabla^{2} m_{z})$$

Exchange energy must be compatible with symmetry of the crystal

$$E_{ex} = \sum C_{kl} \frac{\partial m_{\alpha}(r)}{\partial r_{k}} \frac{\partial m_{\alpha}(r)}{\partial r_{l}}$$
Dzyaloshinski-Moriya Interaction

**Asymmetric** interaction possible when **inversion symmetry** is absent:

\[
H = \sum [JS_i \cdot S_j - D \cdot (S_i \times S_j)]
\]

Describes weak ferromagnetism of canted antiferromagnets:

\[
D = 0 \quad D \neq 0
\]
Using Symmetry: Anisotropy

Local **atomic environment** affects spin orientation:

Anisotropies & **symmetries**: \((u = \frac{m}{M_s})\)

- **Uniaxial**: 
  \[
  E_{ani}(u_z) = E_{ani}(-u_z)
  \]
  \[
  E_{ani} = -K_u^{(1)}u_z^2 - K_u^{(2)}u_z^4 + \ldots
  \]

- **Cubic**: 
  \[
  E_{ani}(u_x, u_y, u_z) = E_{ani}(-u_x, u_y, u_z), \text{ etc.}
  \]
  \[
  E_{ani} = K_4(u_x^2u_y^2 + u_x^2u_z^2 + u_y^2u_z^2) + \ldots
  \]
Using Symmetry: Dipolar Fields

All moments interact throughout a sample via dipolar fields. Sample shape creates an effective anisotropy:

\[ E_{ani} = \frac{M^2 V}{2 \mu_0} \left( N_x \sin^2 \theta \cos^2 \phi + N_y \sin^2 \theta \sin^2 \phi + N_z \cos^2 \theta \right) \]

Easy direction  Hard direction
It’s only Angular Momentum

Everything (nearly) important for magnetic dynamics can be understood from a toy...

- Precession
- Dissipation
- Instabilities
It's Only Angular Momentum

Bohr and Pauli Study Angular Momentum

Interesting video at: https://www.youtube.com/watch?v=58sryfWQOa0
Some generic tools
1) **Simulations** do not by themselves provide interpretations or insights

2) **Analytic/conceptual models** often go where simulations cannot
Domain Patterns

Pattern detail depends on competition between dipolar interactions, exchange and anisotropy.
The Problem of Dipolar Interactions

Magnetic fields decrease slowly with distance—sample shape matters

Magnetisation is generally not uniform:

wikipedia.org

Nmag.soton.ac.uk
Tools: Micromagnetics
Minimising the Energy

**Goal**: find stable (and metastable) configurations that define minima of the total energy $E$

$$E(\hat{u}) = \int \left[ A(\nabla \hat{u})^2 - K_n (\hat{n} \cdot \hat{u})^2 - \mu_0 M_s (\hat{u} \cdot \vec{H}_a + \hat{u} \cdot \vec{h}_d) \right] dV$$

- exchange
- anisotropy
- applied field
- magnetostatic

Minimisation = vanishing torques:

$$\delta E = 0 \quad \Rightarrow \quad \hat{u} \times \left(-\frac{\partial E}{\partial \hat{u}}\right) = 0$$
A Numerical Method: Finite Differences

Convert differential equations to difference equations:

\[
\begin{align*}
    u_\beta(x + \Delta x) &= u_\beta(x) + \Delta x \frac{\partial}{\partial x} u_\beta(x) + \frac{1}{2} (\Delta)^2 \frac{\partial^2}{\partial x^2} u_\beta(x) \\
    u_\beta(x - \Delta x) &= u_\beta(x) - \Delta x \frac{\partial}{\partial x} u_\beta(x) + \frac{1}{2} (\Delta)^2 \frac{\partial^2}{\partial x^2} u_\beta(x)
\end{align*}
\]

Divide \textit{magnetisation} into blocks, replace differentials, construct torque equations for \textbf{each} block
Magnetostatic Terms

Maxwell equations can define a magnetostatic potential (if we are not worried about an electric field during dynamics)

\[
\begin{align*}
\vec{B} &= \mu_0 \left( \vec{H} + \vec{M} \right) \\
\nabla \cdot \vec{B} &= 0 \\
\nabla \times \vec{H} &= \frac{\partial \vec{D}}{\partial t} \approx 0
\end{align*}
\]

\[
\begin{align*}
\vec{H} &= -\nabla \Phi \\
\nabla^2 \Phi &= -\nabla \vec{M}
\end{align*}
\]

Blocks are sources of \( H \) field

The magnetostatic terms link all blocks throughout the sample
Note: Micromagnetics and GPU's

The magnetostatic calculation involves convolution over all blocks:

\[ \vec{H}(i) = \hat{K}(i, j) \ast \vec{M}(j) \]

Accelerate calculations using Graphical Processing Units

Example: Mumax3

// Standard Problem #4

SetGridsize(256, 64, 1)
SetCellsize(500e-9/256, 125e-9/64, 3e-9)

Msat = 800e3
Aex = 13e-12
alpha = 0.02

m = uniform(1, .1, 0)
relax()
save(m) // relaxed state

autosave(m, 200e-12)
tableautosave(10e-12)

B_ext = vector(-24.6E-3, 4.3E-3, 0)
run(1e-9)

Information & Download:
http://mumax.github.io/index.html
Run Standard Problem 4

https://www.youtube.com/watch?v=DPQFppEbqf4
Approaches (with example codes)

**Finite difference**: mumax3, OOMMF

**Finite element**: useful for complex geometries

- **Nmag**
  - [http://nmag.soton.ac.uk/nmag/](http://nmag.soton.ac.uk/nmag/)
  - [http://magnet.atp.tuwien.ac.at/](http://magnet.atp.tuwien.ac.at/)

**Atomistic**: model atomic lattice scale variations

- **VAMPIRE**
  - [http://www-users.york.ac.uk/~rfle500/research/vampire/](http://www-users.york.ac.uk/~rfle500/research/vampire/)

... and many more!
Limitations!

Lengthscales are limited

Shapes are approximate

Timescales are limited

Classical limits: dynamics & thermodynamics
Questions?
Tools:
Mean field approximation
Thermal Fluctuations

Reduction in magnetisation:

\[ M(T) \sim S \]

Replace local site field with averaged effective field:

\[
\begin{align*}
    \text{Dynamic correlations are replaced by a static field:} \\
    H &= -2 \sum J_{ex} S_i \cdot S_j \approx -2 \sum S_i \cdot B_{ex} \\
    B_{ex} &= \frac{2ZJ_{ex}}{Ng\mu_B} S
\end{align*}
\]
Heisenberg Model and Mean Field

Heisenberg exchange energy:

\[ H = -\sum J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Thermal averaged magnetisation (N moments):

\[ \vec{M} = Ng\mu_B \vec{S} \]

Fluctuations:

\[ \vec{s}_i = \vec{S}_i - \vec{S} \]

\[ H = -\sum J_{ij} (\vec{s}_i + \vec{S}) \cdot (\vec{s}_j + \vec{S}) \]
Heisenberg Model and Mean Field

Z near neighbours:

\[ H = -J \sum \vec{s}_i \cdot \vec{s}_j - 2ZJ \sum \vec{S}_i \cdot \vec{S} + ZN|\vec{S}|^2 \]

Second term is the mean field:

\[ \vec{B}_{ex} = -2ZJ \vec{S} \]

Mean field approximation: neglect first term (correlations)

\[ H_{fluctuations} = -J \sum \vec{s}_i \cdot \vec{s}_j \]
Reminder: Paramagnetism

Probabilities to be antiparallel (down) and parallel (up):

\[
\frac{n_{\downarrow}}{N} \propto \exp\left(\frac{-\mu_B B}{k_B T}\right) \quad \frac{n_{\uparrow}}{N} \propto \exp\left(\frac{\mu_B B}{k_B T}\right)
\]

Magnetisation = difference:

\[
S = \left(\frac{N_{\uparrow} - N_{\downarrow}}{N}\right) = \tanh\left(\frac{\mu_B B}{k_B T}\right)
\]
Generalised Paramagnetism

Angular momentum states
(\( J = 1/2, 3/2, 5/2, \ldots \)): 

Brillouin function for any \( J \):

\[
B_J(x) = \frac{2J+1}{2J} \coth\left(\frac{2J+1}{2J} x\right) - \frac{1}{2J} \coth\left(\frac{1}{2J} x\right)
\]

\[
x = \frac{gJ\mu_B B}{k_B T}
\]

Average magnetisation from:

\[
M \propto S = B_j(x)
\]
Exchange: Replace $B$ by $B_{\text{ex}}$

Average $M$ with mean field $B_{\text{ex}}$: \[
\bar{S} = B_J \left( \frac{g\mu_B Z J_{\text{ex}} \bar{S}}{k_B T} \right)
\]

Plot left and right hand sides to see graphical solution:
Note: Landau Ginzburg Theory

A **general form** for mean field theory, created by Landau and Ginzburg, begins with an energy that is a function of an **order parameter** $\psi$:

$$F = \int \left[ F_0 + \frac{1}{2} a \mid \psi \mid^2 + \frac{1}{4} b \mid \psi \mid^4 + \ldots + \frac{1}{2} \lambda \mid \nabla \psi \mid^2 \ldots \right] d^3r$$

Allowed terms must be consistent with the **symmetries** of the problem that the order parameter $\psi$ must obey. **The equilibrium value of the order parameter** minimises $F$. The coefficients represent various contributions to the system's energy. **Temperature** is introduced in the first coefficient:

$$a = \alpha(T - T_c)$$
L-G and the Ferromagnet

Let the order parameter be the ferromagnetic $M$ that is uniformly magnetised over a volume $V$:

$$\psi = M \quad \Rightarrow \quad \frac{F}{V} = F_0 + \frac{1}{2} \alpha (T - T_c) M^2 + \frac{1}{4} b M^4$$

This energy is easily minimised with respect to $M$:

$$\frac{d}{dM} \left( \frac{F}{V} \right) = 0 \quad \Rightarrow \quad \alpha (T - T_c) M + b M^3 = 0$$

$$M = \pm \sqrt{\alpha (T_c - T) / b}$$
Energy Landscapes

This can be pictured using a plot of the energy landscape for $F(M)$:

For $T > T_c$:

$$M = \pm \sqrt{\frac{\alpha (T_c - T)}{b}}$$

For $T < T_c$:

The diagram shows the transition from a single minimum at $T > T_c$ to a double minimum at $T < T_c$.
Example: Multiferroics

**Coupled** order parameters: \( M \) & \( P \)

\[(M = \text{sum of canted antiferromagnetic sublattices})\]

**Challenges:**

- correlations between spin and charge distributions
- how to describe dynamics?
- how to describe effects of thermal fluctuations?
Example Application: Multiferroics

**Coupled** order parameters: $\mathbf{M} \& \mathbf{P}$

Approach: *(Vincinsius Gunawan PhD 2012)*

Mean field approximation for **free energy**:

$$F = F_{FE}(\mathbf{P}) - \mathbf{P} \cdot \mathbf{E} - \lambda \mathbf{m}_a \cdot \mathbf{m}_b - K \left( m_{az}^2 + m_{bz}^2 \right) - \mathbf{m} \cdot \mathbf{H} + F_{ME}$$

- **polarization part**
- **magnetization part**
Example Application: Multiferroics

Brillouin function for components of \( \mathbf{m} \):

\[
m_{s,\alpha} = g\mu_B J B_J (\bar{m}_s \cdot \bar{B}_s)
\]

Landau-Ginzburg mean field theory for \( P \):

Minimise free energy for \( P \) and \( \theta \):

\[
\frac{d}{d\theta} F = 0
\]

\[
\frac{d}{dP} F = 0
\]

\[
F_{FE} (P) = \alpha_o (T - T_c) P^2 + \beta P^4
\]

[Gunawan et al., JPCM (2011)]
Break!
Tools:
Monte Carlo methods
Ising model and Monte Carlo

Suppose two possible states: 'up' and 'down'

Suppose near neighbour interactions. Probability to flip depends on 4 neighbours:

\[ P(-S_i) \sim \exp \left( \frac{-J(\sum S_i)}{k_B T} \right) \]
Sampling Random Fluctuations

Thermal fluctuations and 2 dimensional Ising model:

Low T  
Near $T_c$  
Above $T_c$
Constructing Averages

Fluctuations drive the system towards thermal equilibrium.

Sample a distribution for averages:

\[ P_{\uparrow \downarrow} \sim \exp\left( -\frac{\Delta E(\uparrow \Rightarrow \downarrow)}{k_B T} \right) \]

\[ P_{\downarrow \uparrow} \sim \exp\left( -\frac{\Delta E(\downarrow \Rightarrow \uparrow)}{k_B T} \right) \]

\[ A = \sum A(\sigma) \rho(\sigma) \quad \rho(\sigma) = \frac{1}{Z} \exp\left( -\frac{E(\sigma)}{k_B T} \right) \]

**Key idea:** \( \sigma \) is a configuration from the ensemble of equilibrium spin configurations.
The Metropolis Algorithm

Sample from \{\sigma\}: Start with some \xi, generate a \sigma' with a single spin flip.

Rules: Calculate \Delta E = E(\xi) - E(\sigma')
1) If \Delta E < 0, accept \sigma' as an equilibrium fluctuation
2) If \Delta E > 0, accept \sigma' if \( P(\Delta E) < 1 \)

For equilibrium fluctuations, \( P(\Delta E) \) must satisfy detailed balance:

\[
P(\sigma')W(\uparrow \Rightarrow \downarrow) = P(\xi)W(\downarrow \Rightarrow \uparrow)
\]

\[
\frac{W(\uparrow \Rightarrow \downarrow)}{W(\downarrow \Rightarrow \uparrow)} = \frac{P(\xi)}{P(\sigma')} = P(\Delta E) = \exp\left(-\frac{E(\xi) - E(\sigma')}{k_BT}\right)
\]
Monte Carlo for the Ising Model
Note on phase transitions: Scaling near critical points
Schematic of the Transition (2\textsuperscript{nd} order)

[Weiss & Forrer]

Linear spinwaves

Large amplitude fluctuations

[Weiss & Forrer]
Scaling

Mean field theory: \( M(T) \sim (T - T_C)^{1/2} \)

Reality includes correlations: \( M(T) \sim (T - T_C)^{\beta} \) \( \beta \approx 0.34 \)

**Note** on dimensionality:
- Ultra thin films \( \sim \) two dimensional systems
- fluctuations destroy long range order
- nano-thermodynamics for small elements (\( \sim \) 0 D!)

*Remember this for later when we talk about domain wall creep*
Example: Interacting magnetic particles

Challenges:
- large arrays of submicron elements
- super-paramagnetic
- long range interactions

Approach: (Zoe Budrikis, PhD 2012)
Combine Mean Field & Monte Carlo
An Artificial Antiferromagnet (artificial square spin ice)

Shape anisotropy: Ising spins
Dipolar interactions
6 interactions but can only minimise 4

Configurations

Local spin configurations:

Type I (ground)
Type II (wall)
Type III (defect)

Nisoli et al., Nature Physics (2010)
Growth of Domains and Wall Motion

Type I domains separated by Type II walls:

Type III 'charge' production during wall motion
Thermal evolution of domains

Thermal fluctuations on 2 timescales:
- small volumes (reversal)
- thermal reduction of element M

\[ \frac{KV}{k_B T} \sim 1 \]

**enhancement**

**suppression**

*Configuration dependent local M*
Mean field model: thermal dynamics

**Mean field** model for *element* magnetisations:

\[ m_j = B_{1/2} \left[ \beta m_j \cdot (h_c + \sum J_{j,k} m_k) \right] \quad h_c = K m_j \]

**Algorithm:**
- self consistent iteration for \(<m_j>\)
- stochastic reversal (*Monte Carlo*)

**Disorder:** uniform distribution for \(K\) centred on \(K_o\)

\[ K = K_o \left( 1 + \frac{r}{2} \right) \quad r \in [-\Delta, \Delta] \]
Thermal fluctuations at walls

(Karen Livesey PhD 2010)

- $M = 1$
- $M = 0.1$

Thermal fluctuations largest on domain walls
Challenge: modelling kinetics in real time with Monte Carlo
Continuous Time Monte Carlo

Probability for acceptance of a single flip (out of N spins):

\[ Q = \frac{1}{N} \sum n(\Delta E)P(\Delta E) \]

number of spins with \( \Delta E \)

Probability that a spin will flip in time \( \Delta t \):

\[ P_{\text{flip}}(\Delta t) = \exp\left(-\frac{\Delta t}{\tau}Q\right) \]

Rejection free algorithm:
1) track all possible transitions
2) accept one according to random \( R \)
3) time update determined by \( R \)

\[ \Delta t = -\frac{\tau}{Q} \ln R \]
Example: Exchange Bias

Thermal setting of bias:

Time dependent coercivity:
Field sweep rates

M Kirschner http://magnet.atp.tuwien.ac.at

Stamps, PRB 2000
Spin Wave Dynamics
Low Temperature Fluctuations

Energy to reverse one spin: $2 J$

$$H = \sum_{ij} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Superposition of ways to flip one spin:

$$| n = 1 \rangle = |\uparrow\downarrow\uparrow\uparrow\uparrow\rangle + |\uparrow\uparrow\downarrow\uparrow\uparrow\rangle + |\uparrow\uparrow\uparrow\downarrow\uparrow\rangle + \ldots$$

Spinwave excitation
Model: Torque equations
Excitations: Spin Waves

Ground state magnetic orderings:

Excitations: Precessional dynamics

Note: The excitations are bosons!
Classical Precession

Transverse oscillations define wavelength
Equations of Motion

Torque equations:

$$\frac{\partial}{\partial t} \mathbf{m}(\mathbf{r}) = -\gamma \mathbf{m}(\mathbf{r}) \times \mathbf{H}_{eff}$$

$$\mathbf{H}_{eff} = \mathbf{H}_a + A\nabla^2 \mathbf{m}(\mathbf{r}) + K(\mathbf{m}(\mathbf{r})) - h_{dip}$$

Note: Dissipation adds additional torques

$$\frac{\partial}{\partial t} \mathbf{m}(\mathbf{r}) = -\gamma \mathbf{m}(\mathbf{r}) \times \mathbf{H}_{eff} + \lambda \mathbf{m}(\mathbf{r}) \times \frac{\partial \mathbf{m}(\mathbf{r})}{\partial t}$$
Equations of Motion: FMR

No exchange contribution for uniform precession and dipole field modelled as shape anisotropy ($K \sim M_s$).

\[ \frac{d}{dt} m_x = -\gamma [H_a m_y - 2K M_z m_y] \]
\[ \frac{d}{dt} m_y = \gamma [H_a m_x] \]
\[ \frac{d}{dt} M_z = \gamma 2K m_x m_y \]

Linearisation:
\[ \frac{d}{dt} M_z \approx 0 \]
\[ (m_x, m_y) \sim e^{-i\omega t} \]
\[ \frac{\omega^2}{\gamma^2} = H_a (H_a - 2K M_s) \]

Anisotropy shifts frequency
With Exchange: Dispersion

Effect of interactions: exchange

\[(m_x, m_y) \sim \exp[-i(\omega t - \mathbf{k} \cdot \mathbf{r})]\]

\[H_{ex} \sim A\nabla^2 \mathbf{m}(\mathbf{r}) \quad \text{vs.} \quad H_{ex} \sim -Ak^2 \mathbf{m}(\mathbf{r})\]

\[
\frac{\omega^2}{\gamma^2} = (H_a + Ak^2)(H_a - 2KM_s + Ak^2)
\]

\[\omega \quad \text{field + anisotropy} \quad k\]
Dzyaloshinskii-Moriya Interactions

*Interface-driven* DMI interaction in ultrathin ferromagnets

\[
\mathcal{H}_{\text{DM}} = -\vec{D}_{12} \cdot (\vec{S}_1 \times \vec{S}_2)
\]

**Effects of DMI on spin waves:**

\[
\omega(k) = \Omega (k^2) \pm Dk_{||}
\]

Moon et al. PRB 2013
Iguchi et al., ArXiv 2015

A. Fert, Mat. Sci. Forum (1990)
Dispersion: Dipole Exchange Modes

Dipolar: long range interaction terms compete with short range exchange interactions:
Anisotropic Propagation

Dipolar effect: dependence on propagation direction

small dipolar energy

large dipolar energy
Spin Waves and Micromagnetics

Procedure:
1) Relax to steady state
2) Use broadband pulse to excite spin waves
3) Record time evolution (for spectral analysis)

Example: exciting precession in mumax3 script

```python
defregion(1, rect(10e-9, 125e-9))
save(regions)

driv := 0.001  // amplitude driving field
f := 1.0e9     // frequency units
fdel := 20.*f*2.*pi  // frequency window
time := 1000./fdel  // evolve time
toff := 3./f     // offset

B_ext = vector(-24.6E-3, 4.3E-3, driv*sin((t-toff)*fdel)/(2*pi*(t-toff)*fdel))
run(time)
```

define antenna region
sinc function pulse
Results

Ground state:

Antenna:

Note: Spectral analysis performed separately on mumax generated data.
Example: Spin Waves on Spin Ice

(Yue Li PhD ~2017)

Note: Spin Wave Dispersions

Spin wave $\omega(k)$ from micromagnetics: apply 4D pulse

$$h(x, y, z, t) = \text{sinc}((x - x')k_x)\text{sinc}((y - y')k_y)\text{sinc}((t - t')\omega)$$

Reversal Processes, Domains and Domain Walls
Switching of Single Domain Particles

Dynamics:
- Precessional reversal

Stability:
- Thermal activation

\[ H \geq H_c \]

\[ H < H_c \]
Challenge: fluctuations over long time scales

Approach: Stoner-Wohlfarth models
Single Domain Rotation

Approximate reversal as pure relaxation:

\[ E = V\left(- BM\cos\theta + K\sin^2\theta\right) \]

\[ = VK\left[1 + \left(\frac{BM}{2K}\right)^2\right] \]

Stoner-Wohlfarth Model

Rate depends on activation energy and attempt frequency

\[ \Gamma = \frac{1}{\tau} = f_o \exp\left(-\frac{\varepsilon}{k_B T}\right) \]
Reversal of a Particle Ensemble

**Ensemble of particles:** $B=0$, thermal fluctuations reduce $M$

**Approach to equilibrium:** Chemical rate problem

\[
\begin{align*}
\frac{dn_{\uparrow}}{dt} &= W_{\downarrow\uparrow} n_{\downarrow} - W_{\uparrow\downarrow} n_{\uparrow} \\
\frac{dn_{\downarrow}}{dt} &= W_{\uparrow\downarrow} n_{\uparrow} - W_{\downarrow\uparrow} n_{\downarrow}
\end{align*}
\]

\[m(t) = n_{\uparrow} - n_{\downarrow} = Ae^{-\Gamma t}\]
Reversal of a Particle Ensemble

Ensemble of particles: $H=0$, dipolar fields drive $m$ to 0

Approach to equilibrium: Distribution of rates

$$m(t) = A \int P(\Gamma) e^{-\Gamma t} d\Gamma$$

Can one measure the distribution of rates $P(\Gamma)$? (Rebecca Fuller, PhD 2010)
Relaxation: Distributions

Distribution of energy barriers:

\[ \Gamma = f_0 \exp(-\varepsilon / k_B T) \]

\[ m(t) = m(\infty) + A \int P(\Gamma) e^{-t\Gamma} d\Gamma \]

**Magnetic viscosity:** ln(t) for broad distributions

\[ m(t) = C - S(H) \ln(t\Gamma_o) \]

[Fuller, et al., JPCM 2009]
Relaxation: Energy Barriers

Useful measure: study $d\text{m}/d\text{t}$ at different $T$

$$P(\epsilon) \approx \frac{S}{A k_B T}$$

Viscosity at different temperatures and fields provides estimates for energy barrier distribution

[Fuller, et al., JPCM (2009)]
Questions?
Reversal processes, magnetic domains and domain walls
Routes to Reversal

Nucleation of domains and domain walls

[Slaughter, 2000]
Challenge: fluctuations over long times and large lengthscales

Approach: a Stoner-Wohlfarth model for fluctuating lines
**Magnetization Processes & Domains**

**Nucleation processes:**

Growth of a critical domain volume

\[
E_{Zeeman} = -\mu MVH \\
E_{DW} = \sigma A
\]

\[
\left( \frac{V}{A} \right)_c = \frac{\sigma}{(\mu MH)}
\]

**Surface energy**
Domain & Wall Dynamics

- **Example**: MOKE study
- Perpendicular \( M \) in Co
- Method:
  - saturate
  - apply field pulse
  - image & repeat

\[ \text{Ag} \quad \text{Co} \quad \text{Ag} \]

0.5 nm

\[ H \]

\[ \begin{align*}
\uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow & \quad \downarrow \downarrow \downarrow \downarrow \downarrow \uparrow \uparrow \uparrow \uparrow \\
\uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow & \quad \downarrow \downarrow \downarrow \downarrow \downarrow \uparrow \uparrow \uparrow \uparrow \\
\uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow & \quad \downarrow \downarrow \downarrow \downarrow \downarrow \uparrow \uparrow \uparrow \uparrow
\end{align*} \]
Magnetisation Processes & Domains

Growth stops at local field gradients (pinning 'pressure')

$\text{pressure} \sim - \nabla E_{\text{local}}$

Stroboscopic 'movie' of domain growth
Magnetization Processes & DW's

Wall structure:
- Topological excitation
- Surface tension
- Characteristic width

Dynamics:
- Translation & fluctuations
- Pinning & 'creep'
- Internal modes
DW Mobility: High Field Flow

**Viscous Flow**: High field driven dynamics

$v_{DW} \sim H$
DW Mobility: High Field Flow

Viscous Flow: High field driven dynamics

\[ v_{DW} \sim H \]
DW Mobility Theory: Flow

Torque equations of motion:
\[
\frac{\partial m}{\partial t} = -\gamma m \times H + \frac{\alpha}{M_s} m \times \frac{\partial m}{\partial t}
\]

precessional torque
Gilbert damping

Effective local field: (exchange, anisotropy, dipolar)
\[
H = H_{\text{applied}} - \frac{\partial E}{\partial m}
\]

Time averaged velocity (in Flow regime):
\[
v \propto \int |m \times H|^2 \, d^3 x
\]

[X Wiang, P Yan, J Lu]
**DW Mobility: Low Field Creep**

*Creep*: low field thermally activated dynamics

\[ t = t_1 \]

\[ t = t_1 + \Delta \]

\[ t = t_1 + 2\Delta \]
DW Mobility: Low Field Creep

**Creep**: low field thermally activated dynamics

\[ V(x) \]

\[ t = t_1 \]

\[ V(x) \]

\[ t = t_1 + \Delta \]

\[ V(x) \]

\[ t = t_1 + 2\Delta \]
DW Mobility Theory: Creep

Pinning sites oppose wall motion:
DW Mobility Theory: Creep

Pinning sites oppose wall motion:
DW Mobility Theory: Creep

Number of pinning sites:

\[ E_p = \sqrt{f_{\text{pin}}^2 N_p \xi} \]

Macroscopic wall motion through avalanche:

Scaling: critical field for avalanche onset

\[
\left[ E_{\text{elastic}} - E_{\text{Zeeman}} \right] = E_B \approx U C \left( \frac{H_{\text{dep}}}{H_{\text{applied}}} \right)^{2\xi - 2 + D} \left( 2 - \zeta \right)
\]
DW Mobility Theory: Creep

Depinning rate:

\[
\frac{1}{\tau(L)} = \frac{1}{\tau_0} \exp \left[ - \frac{E_B(L)}{k_B T} \right]
\]

Multiply by distance travelled to give velocity:

\[
v = \frac{w(L)}{\tau(L)} \approx \frac{\xi}{\tau_0} \exp \left[ - \frac{U_C}{k_B T} \left( \frac{H_{dep}}{H_{applied}} \right)^\mu \right]
\]

Expect \( \mu = \frac{1}{4} \) for ultra thin films.
**DW Motion: Transition**

**Threshold:** transition from creep to viscous flow

Zero temperature

![Diagram](attachment:image.png)
DW Motion: Transition

**Threshold:** transition from creep to viscous flow

\[ v = v_o \exp \left[ \frac{-U_c}{k_B T} \left( \frac{H_p}{H} \right)^\mu \right] \]

Zero temperature

- **$v_{DW}$**
- **$H_c$**
- **$H$**
DW Motion: Transition

**Observed** transition from creep to viscous flow:

*(Peter Metaxas, PhD 2009)*

\[ v = v_o \exp \left[ \frac{-U_c}{k_B T} \left( \frac{H_p}{H} \right) \right] \]

\[ v \propto H \]

Summary

• **Approximations**: Heisenberg exchange, anisotropy, mean field theory

• **Simulations**: Micromagnetic, Monte Carlo

• **Analytic models**: spin waves, domain walls, thermal activation
Thank you!