Fundamentals of Magnetism – 1

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1. Introduction; basic quantities
2. Magnetic Phenomena.
3. Units

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Lecture 1 covers basic concepts in magnetism; Firstly magnetic moment, magnetization and the two magnetic fields are presented. Internal and external fields are distinguished. Magnetic energy and forces are discussed. Magnetic phenomena exhibited by functional magnetic materials are briefly presented, and ferromagnetic, ferrimagnetic and antiferromagnetic order introduced. SI units are explained, and dimensions are provided for magnetic, electrical and other physical properties.

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An elementary knowledge of vector calculus and electromagnetism is assumed.

Books

Some useful books include:

- J. M. D. Coey; *Magnetism and Magnetic Magnetic Materials*. Cambridge University Press (2010) 614 pp
An up to date, comprehensive general text on magnetism. Indispensable!

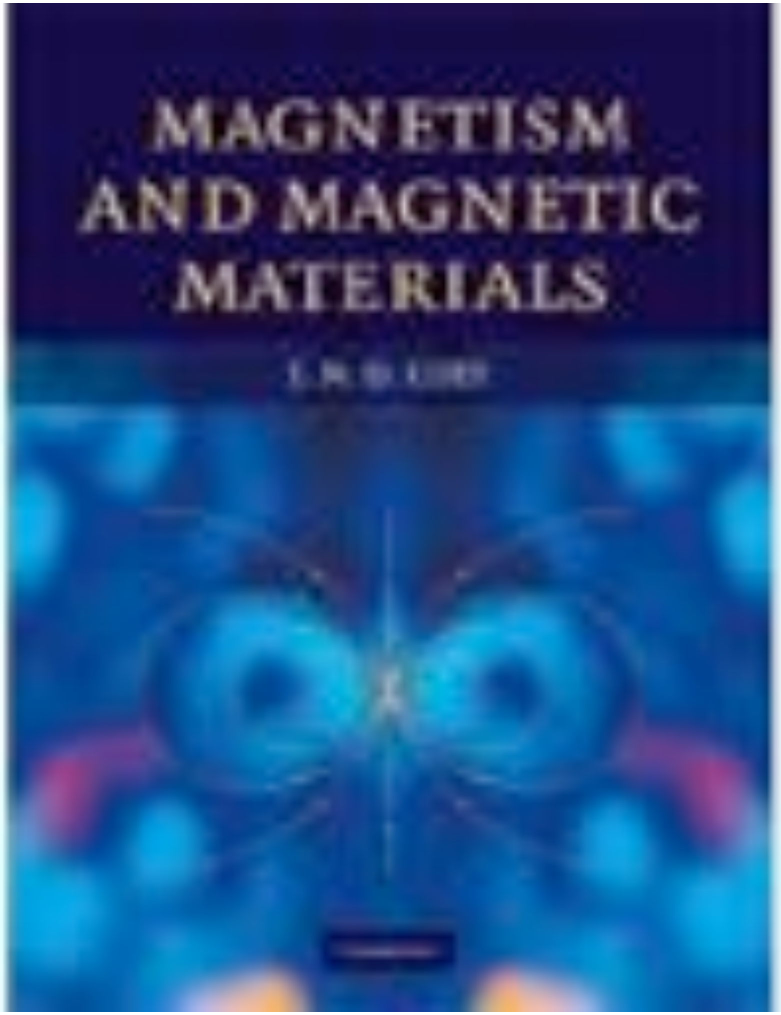
- S. Blundell *Magnetism in Condensed Matter*, Oxford 2001
A good, readable treatment of the basics.

- D. C. Jilles *An Introduction to Magnetism and Magnetic Magnetic Materials*, *Magnetic Sensors and Magnetometers*, 3rd edition CRC Press, 2014 480 pp
Q & A format.

- R. C. O'Handley. *Modern Magnetic Magnetic Materials*, Wiley, 2000, 740 pp
Q & A format.

- J. Stohr and H. C. Siegman *Magnetism: From fundamentals to nanoscale dynamics* Springer 2006, 820 pp.
Good for spin transport and magnetization dynamics. Unconventional definition of ***M***

- K. M. Krishnan *Fundamentals and Applications of Magnetic Material,* Oxford, 2017, 816 pp
Recent general text.. Good for imaging, nanoparticles and medical applications.



614 pages. Published March 2010

Available from Amazon.co.uk ~€50

www.cambridge.org/9780521816144

1 Introduction

- 2 Magnetostatics

3 Magnetism of the electron

4 The many-electron atom

5 Ferromagnetism

6 Antiferromagnetism and other magnetic order

7 Micromagnetism

8 Nanoscale magnetism

9 Magnetic resonance

10 Experimental methods

11 Magnetic materials

12 Soft magnets

13 Hard magnets

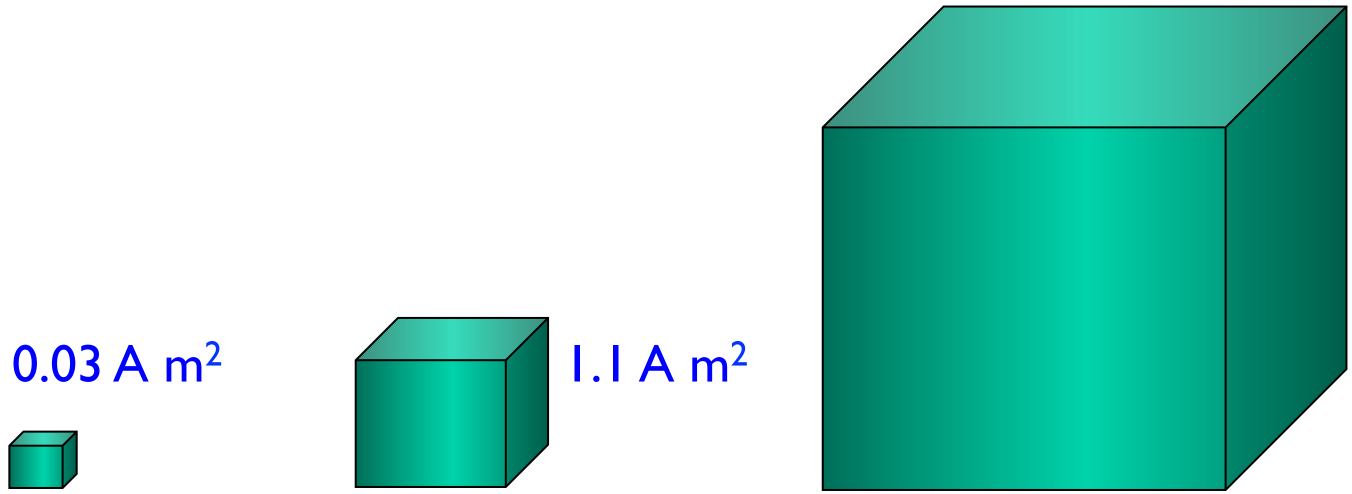
14 Spin electronics and magnetic recording

15 Other topics

Appendices, conversion tables.

1. Introduction; Basic quantities

Magnets and magnetization



$0.03\ \mathrm{A\ m^2}$ — $1.1\ \mathrm{A\ m^2}$ — $17.2\ \mathrm{A\ m^2}$

3 mm — 10 mm — 25 mm

$\boldsymbol{m}$ is the magnetic (dipole) moment of the magnet. It is proportional to volume

$$\boxed{\boldsymbol{m} = \boldsymbol{M}V}$$

magnetic moment

magnetization

volume

Suppose they are made of $Nd_2Fe_{14}B$
($M \approx 1.1\ \mathrm{MA\ m^{-1}}$)

What are the moments?

Magnetization is the intrinsic property of the material;
Magnetic moment is a property of a particular magnet.

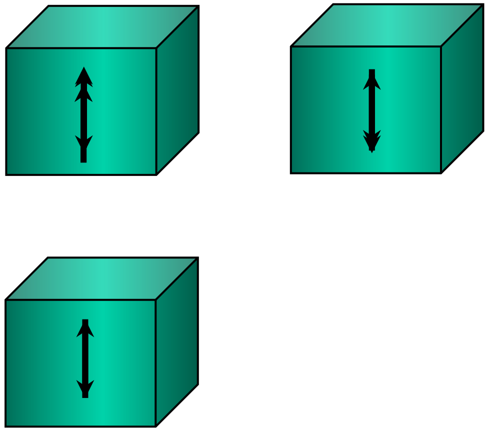
Magnitudes of *M* for some ferromagnets

		M (MAm^{-1})
Permanent magnets	$Nd_2Fe_{14}B$	1.28
	$BaFe_{12}O_{19}$	0.38
Temporary magnets	Fe	1.71
	Co	1.44
	Ni	0.49

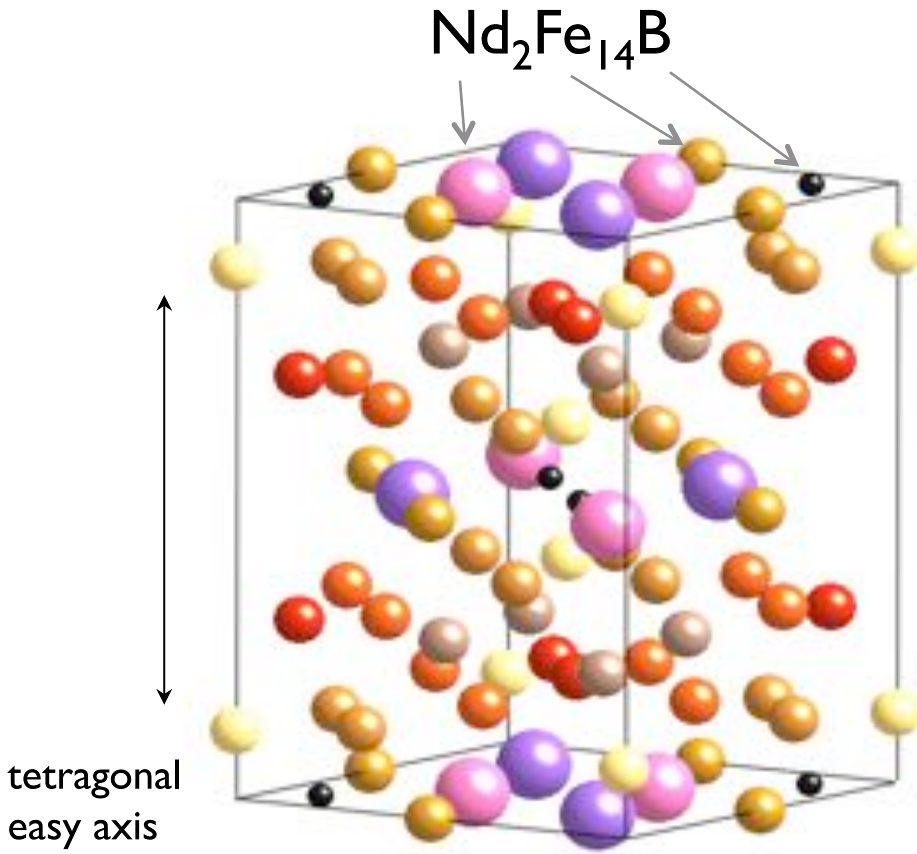
NB: These are the values for the pure phase

Magnetic moment - a vector

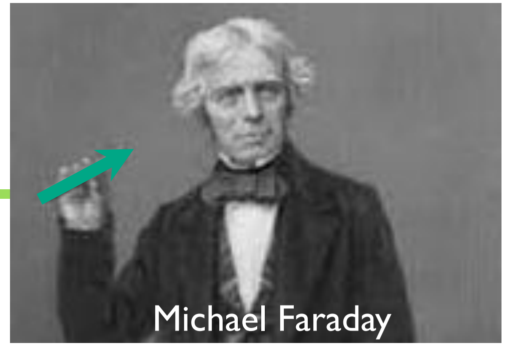
Each magnet creates a field around it. This acts on *any* material in the vicinity but strongly with another magnet. The magnets attract or repel depending on their mutual orientation



↑ ↑	Weak repulsion
↑ ↓	Weak attraction
← ←	Strong attraction
← →	Strong repulsion

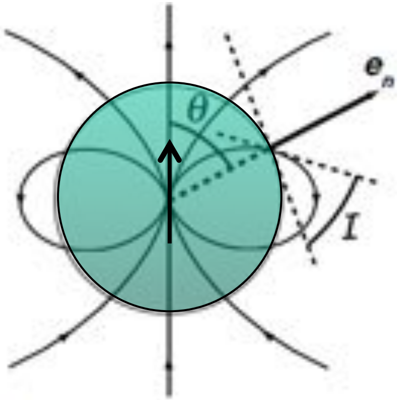


Field due to a magnetic moment *m*

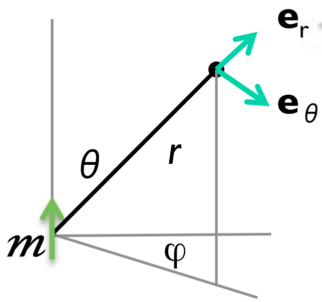


$$\boldsymbol{H}_r = 2\left(\frac{m}{4\pi r^3}\right)\cos\theta; \quad \boldsymbol{H}_\theta = \left(\frac{m}{4\pi r^3}\right)\sin\theta; \quad \boldsymbol{H}_\phi = 0.$$

The Earth's magnetic field is roughly that of a geocentric dipole

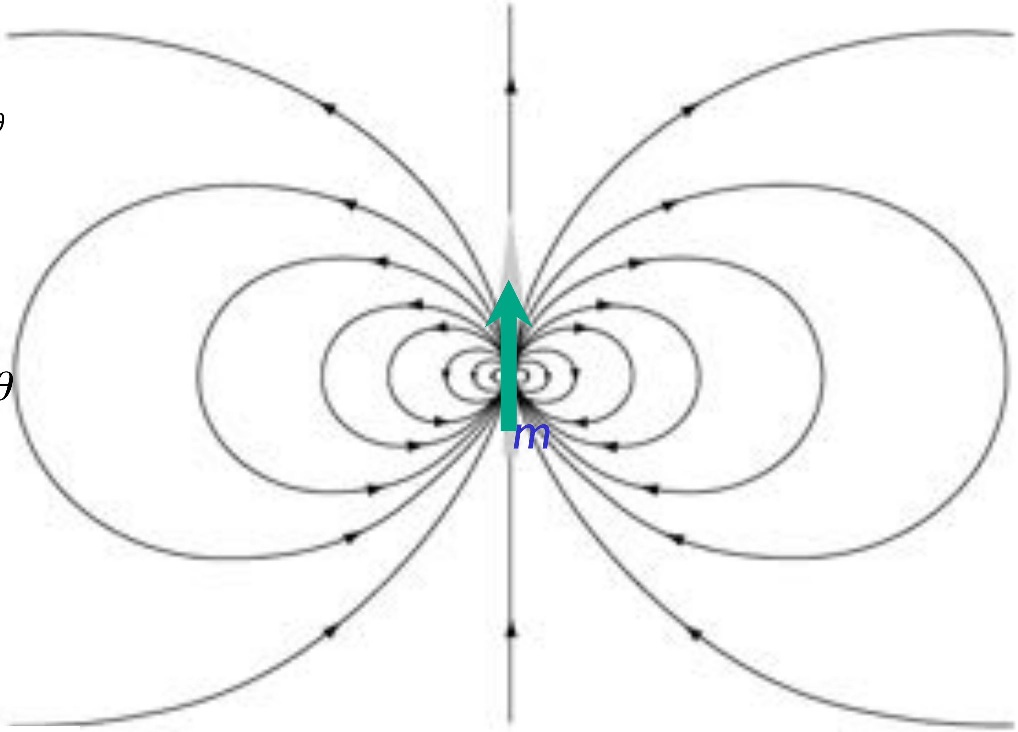


The angle of dip.



$\tan I = B_r/B_\theta = 2\cot\theta$
$dr/rd\theta = 2\cot\theta$

Solutions are
$r = c\sin^2\theta$



Equivalent forms

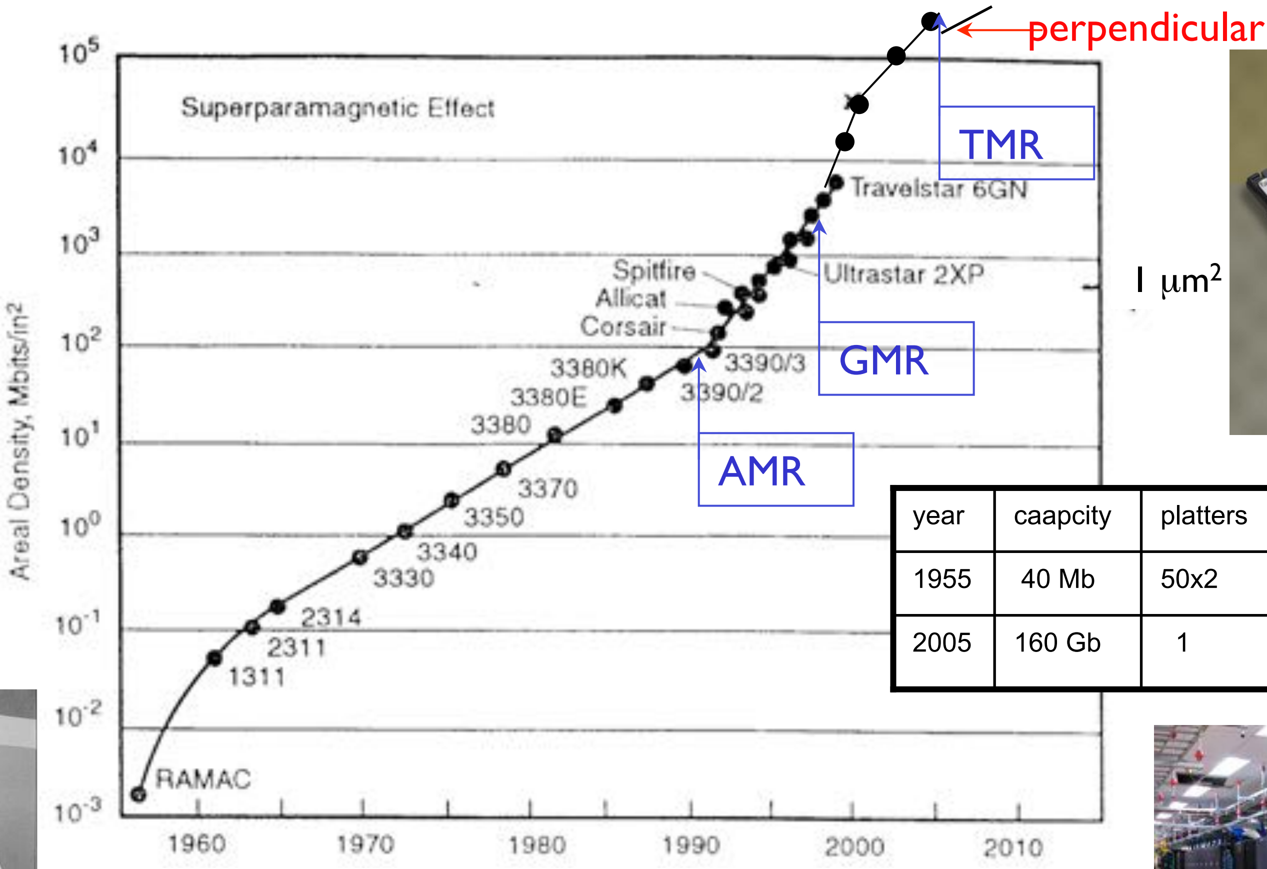
$$\boldsymbol{H} = \frac{m}{4\pi r^5}\left[3xz\mathbf{e}_x + 3yz\mathbf{e}_y + (3z^2 - r^2)\mathbf{e}_z\right]$$

$$\boldsymbol{H} = \frac{1}{4\pi}\left[3\frac{(\mathrm{m}\cdot\boldsymbol{r})\boldsymbol{r}}{r^5} - \frac{\mathrm{m}}{r^3}\right]$$

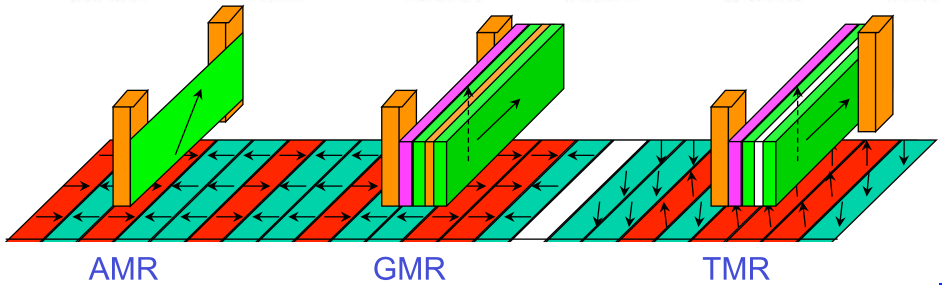
Note. The dipole field is scale-independent. $m \sim d^3$; $H \sim 1/r^3$
Hence $H = f(d/r)$.
Result: 60 years of hard disc recording !

Magnetic recording

A billion-fold increase in recording density



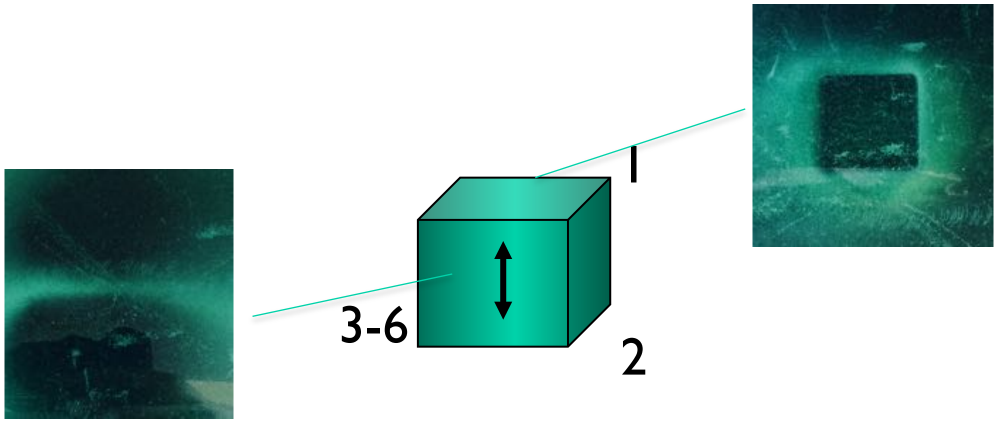
year	caapcity	platters	size	rpm
1955	40 Mb	50x2	24”	1200
2005	160 Gb	1	2.5”	18000



How can you tell which way a magnet is magnetized?

Use the green paper!

It responds (turns dark) to the perpendicular component of ***H***



This tells us the magnetic axis, but not the direction of vector ***m***

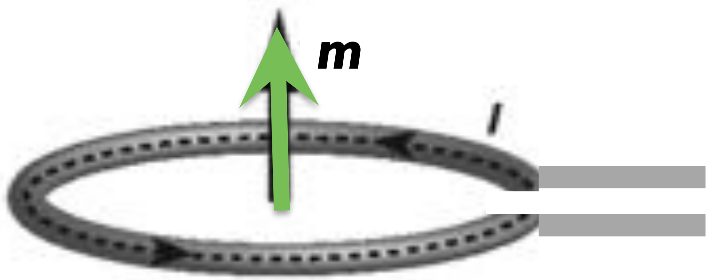
We can decide the relative orientation of two magnets from the forces, but the direction of the arrow is a matter of *convention*.

Equivalence of electricity and magnetism; Units

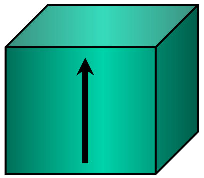
What do the units mean?

m – A m^2

M – A m^{-1}

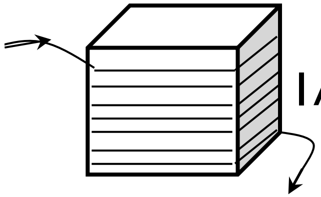


Ampère, 1821. A current loop or coil is equivalent to a magnet



1 A m^2

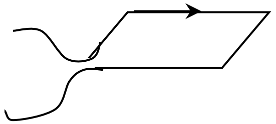
$\equiv$



10,000 turns

1A

$\equiv$



10,000 A

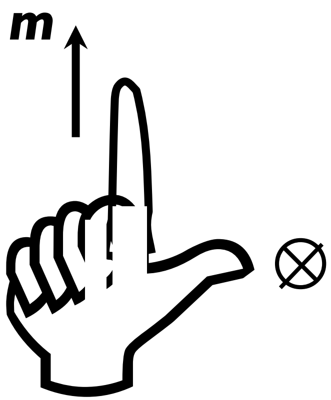
Permanent magnets win over electro-magnets at small sizes

$$\boxed{\boldsymbol{m} = I\mathcal{A}}$$

area of the loop

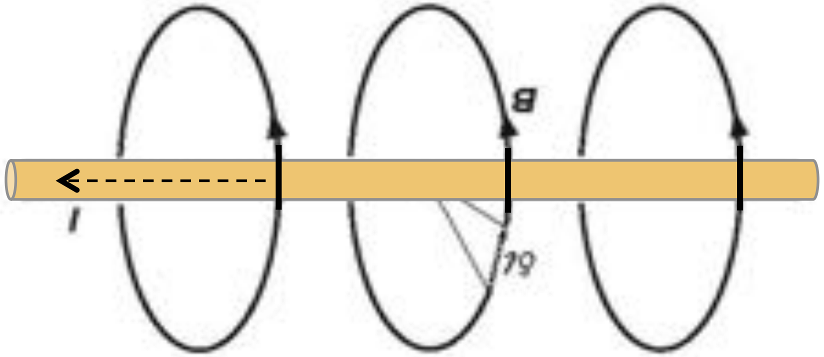
$$\boxed{\boldsymbol{m} = nI\mathcal{A}}$$

number of turns

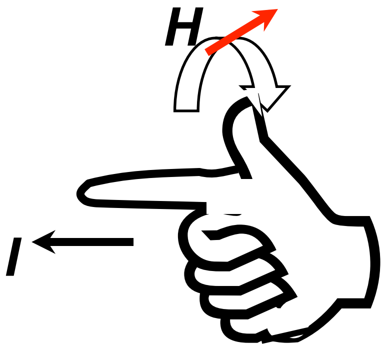


Right-hand corkscrew

Magnetic field *H* – Oersted's discovery



The relation between electric current and magnetic field was discovered by Hans-Christian Øersted, 1820.



Right-hand corkscrew

$$\boxed{\oint H d\ell = I}$$ Ampère's law

Units of H; A m^{-1}

Earth's field ≈ 40 Am^{-1}

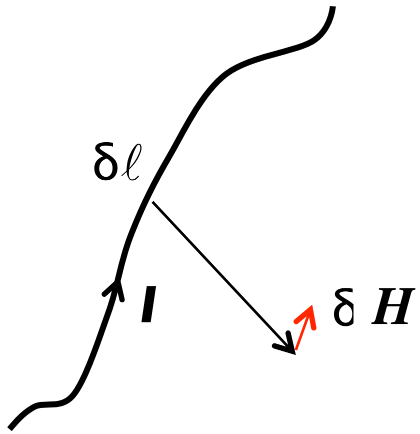
$H = I/2\pi r$

If I = 1 A, r = 1 mm

H = 159 A m^{-1}

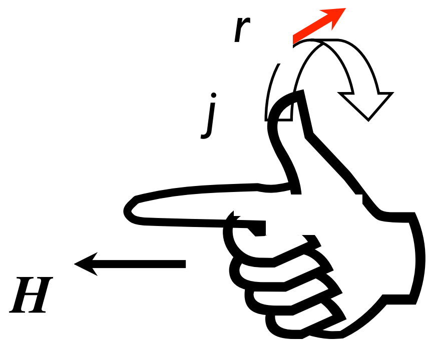
Field due to electric currents

We need a differential form of Ampère's Law; The Biot-Savart Law



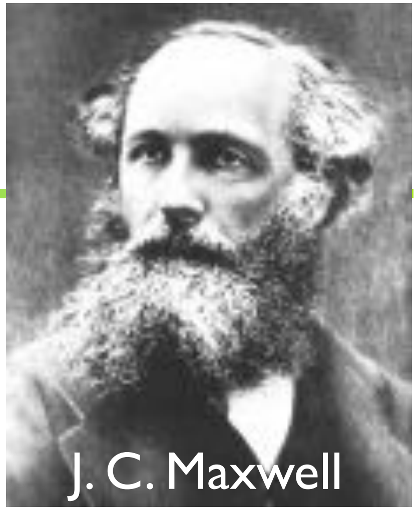
$$\delta \boldsymbol{H} = -\frac{1}{4\pi}\frac{\boldsymbol{r}\times\boldsymbol{j}}{|r^3|}\delta V$$

$$\delta \boldsymbol{H} = -\frac{1}{4\pi} I \frac{\boldsymbol{r}\times\delta\ell}{|r^3|}$$



Right-hand corkscrew (for the vector product)

Magnetostatics



Maxwell's Equations
In a medium;
$\boldsymbol{B} \neq \mu_0 \boldsymbol{H}$

$$\nabla \cdot \boldsymbol{D} = \rho,$$

$$\nabla \cdot \boldsymbol{B} = 0,$$

$$\nabla \times \boldsymbol{E} = \cancel{-\partial \boldsymbol{B}/\partial t},$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{j} + \cancel{\partial \boldsymbol{D}/\partial t}.$$

Electromagnetism with no time-dependence

In magnetostatics, we have only magnetic material and circulating currents in conductors, all in a steady state. The fields are produced by the magnets & the currents

$$\boxed{\nabla \cdot \boldsymbol{j} = 0 \quad \nabla \cdot \boldsymbol{B} = 0 \quad \nabla \times \boldsymbol{H} = \boldsymbol{j}.}$$

B and *H* fields in free space; permeability of free space μ_0

When illustrating Ampère's Law we labelled the magnetic field created by the current, measured in Am^{-1} as ***H***. This is the 'magnetic field strength'

Maxwell's equations have another field, the 'magnetic flux density', labelled ***B***, in the equation $\nabla \cdot \boldsymbol{B} = 0$. It is a different quantity with different dimensions and units. Whenever ***H*** interacts with matter, to generate a force, an energy or an emf, the constant μ_0, the 'permeability of free space' is involved.

In *free space*, the relation between ***B*** and ***H*** is simple. They are numerically proportional to each other

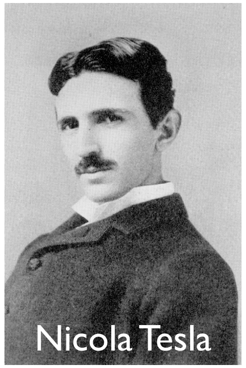
$$\boxed{\boldsymbol{B} = \mu_0 \boldsymbol{H}}$$

B-field
Units Tesla, T

Permeability of free space
Units TmA^{-1}

H-field
Units $A\ m^{-1}$

In practice you can never mix them up. The differ by almost a million! ($795775 \approx 800000$)



μ_0 depends on the definition of the Amp.
It is precisely $4\pi\ 10^{-7}\ T\ m\ A^{-1}$

Magnetic Moment and Magnetization

The atomic magnetic moment $\boldsymbol{m}_a$ is the elementary quantity in atomic-scale magnetism. (L. 2)

Define a local moment density - magnetization – $\boldsymbol{M}(\boldsymbol{r}, t)$ which fluctuates wildly on an atomic, sub-nanometer scale and on a sub-nanosecond time scale.

Define a mesoscopic average magnetization

$$\boxed{\delta \boldsymbol{m} = \boldsymbol{M} \delta V}$$

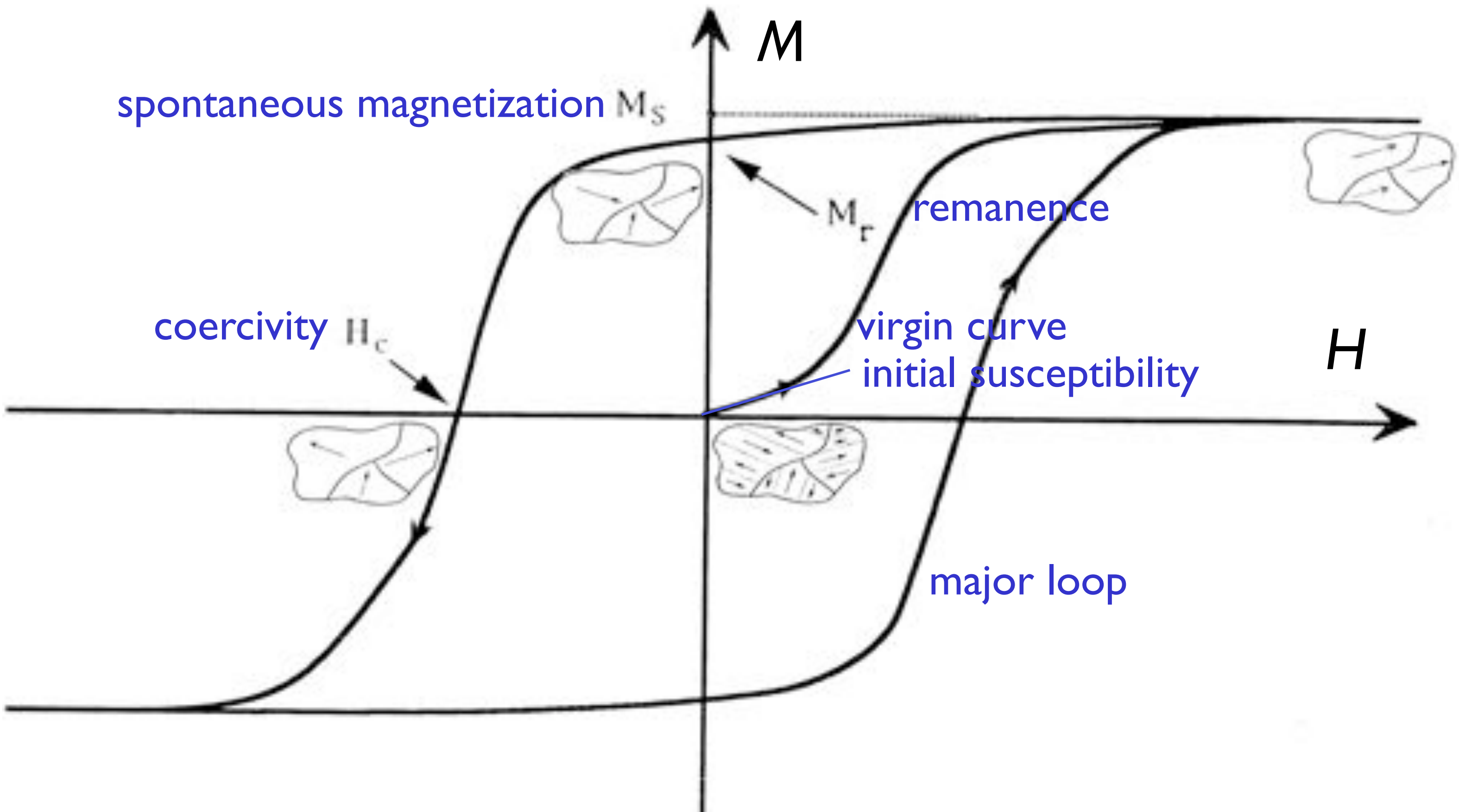
$$\boldsymbol{M} = \delta \boldsymbol{m} / \delta V$$

The continuous medium approximation

$\boldsymbol{M}$ could be the spontaneous magnetization $\boldsymbol{M}_s$ within a ferromagnetic domain or fine particle. Most materials are *not* spontaneously ordered and $\boldsymbol{M} = 0$. The atomic moments $\boldsymbol{m}_a$ then fluctuate rapidly and the time average of any one of them, or the spatial average of an ensemble of atoms is *zero*. $\boldsymbol{M}$ is then the response of the molecular material to an external magnetic field $\boldsymbol{H}$. At very low temperatures the spontaneous fluctuations freeze out.

Initially the response is linear; $\boldsymbol{M} = \chi \boldsymbol{H}$. The dimensionless constant χ is the susceptibility.

Magnetization curves - Hysteresis loop



The hysteresis loop shows the irreversible, nonlinear response of a ferromagnet to a magnetic field . It reflects the arrangement of the magnetization in ferromagnetic *domains*. A broad loop like this is typical of a *hard* or *permanent* magnet. The remanent state is normally *metastable*.

Magnetization and current density

The magnetization of a solid is somehow related to a 'magnetization current density' $\boldsymbol{J}_m$ that produces it.

Since the magnetization is created by bound currents, $\int_s \boldsymbol{J}_m . d\boldsymbol{A} = 0$ over any surface.

Using Stokes theorem $\oint \boldsymbol{M}.d\ell = \int_s (\nabla \times \boldsymbol{M}).d\boldsymbol{A}$ and choosing a path of integration outside the magnetized body, we obtain $\int_s \boldsymbol{M}.d\boldsymbol{A} = 0$, so we can identify $\boldsymbol{J}_m$

$$\boxed{\boldsymbol{J}_m = \nabla \times \boldsymbol{M}}$$

We don't know the details of the magnetization currents, but we can measure the mesoscopic average magnetization and the spontaneous magnetization of a sample.

Magnetic flux density - *B*

Now we discuss the fundamental field in magnetism.

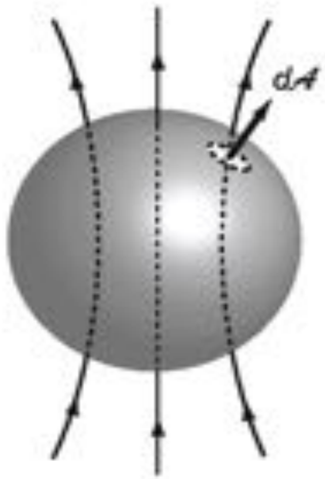
Magnetic poles, analogous to electric charges, *do not exist*. This truth is expressed in Maxwell's equation

$$\boxed{\nabla.\boldsymbol{B} = 0.}$$

This means that the lines of the *B*-field always form complete loops; they never start or finish on magnetic charges, the way the electric *E*-field lines can start and finish on +ve and -ve electric charges.

The same can be written in integral form over *any closed surface S*

$$\boxed{\int_S \boldsymbol{B}.\mathrm{d}\mathcal{A} = 0} \qquad \text{(Gauss's law).}$$



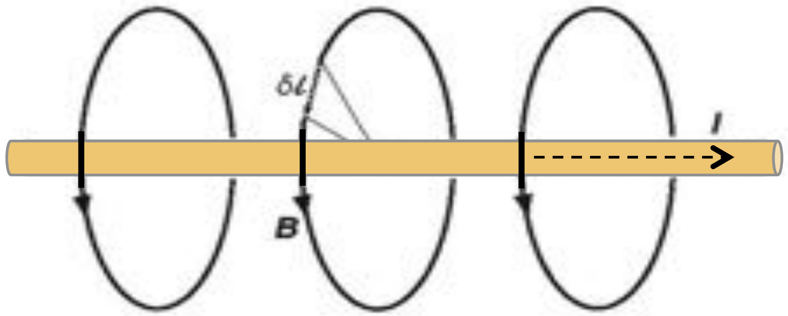
The flux of ***B*** across a surface is $\Phi = \int \boldsymbol{B}.\mathrm{d}\mathcal{A}$. Units are Webers (Wb).
The net flux across any closed surface is zero.
B is known as the flux density; units are Teslas. (T = Wb m^{-2})

Flux quantum Φ_0 = 2.07 10^{15} Wb (Tiny)

The B-field

*Sources of **B***

- electric currents in conductors
- moving charges
- magnetic moments
- time-varying electric fields. (Not in *magnetostatics*)



$B = \mu_0 I/2\pi r$

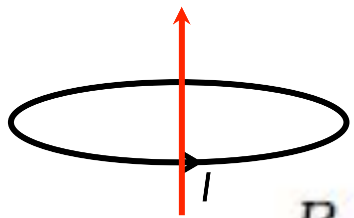
In a steady state: Maxwell's equation

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{j} \qquad \oint \boldsymbol{B} \cdot \mathrm{d}l = \mu_0 I$$

$$\begin{vmatrix} \mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ B_x & B_Y & B_Z \end{vmatrix}$$

(*Stokes theorem*;

$$\int_S (\nabla \times \mathbf{A}) \cdot \mathbf{e}_n dr^2 = \oint \mathbf{A} \cdot d\ell)$$



$$B_0 = \frac{\mu_0 I}{2a}$$

Field at center of current loop

Forces between conductors; Definition of the Amp

$\boxed{\boldsymbol{F} = q(\boldsymbol{E} + \mathbf{v} \times \boldsymbol{B})}$

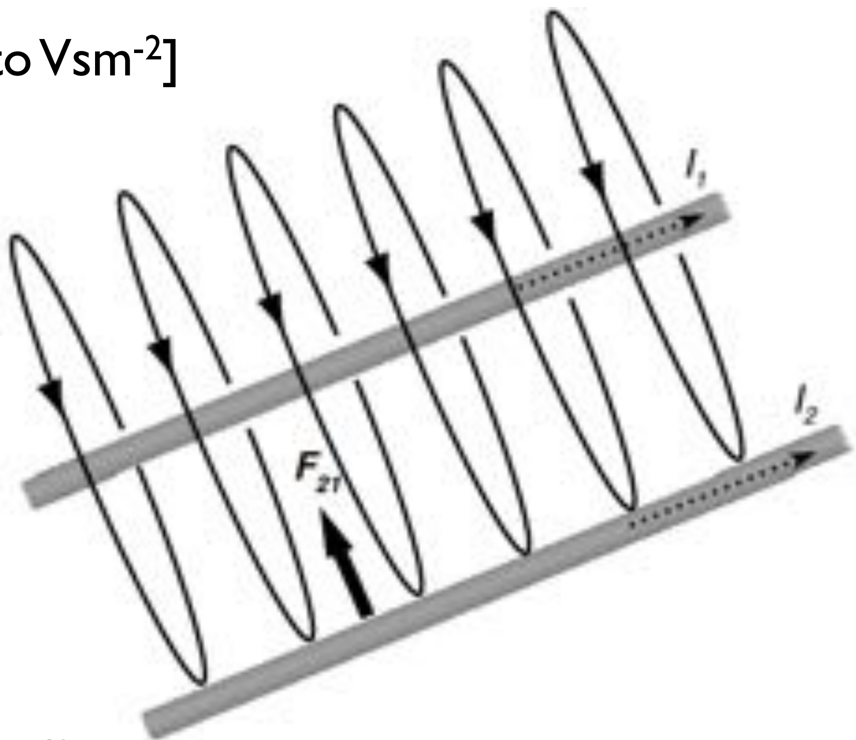
Lorentz expression. Note the Lorentz force does no work on the charge

Gives dimensions of ***B*** and ***E***. [T is equivalent to $\mathrm{Vsm^{-2}}$]

If ***E*** = 0 the force on a straight wire arrying a current ***I*** in a uniform field ***B*** is $F = BI\ell$

The force between two parallel wires each carrying one ampere is precisely $2\ 10^{-7}$ N $\mathrm{m^{-1}}$. (Definition of the Amp)

The field at a distance 1 m from a wire carrying a current of 1 A is 0.2 μT

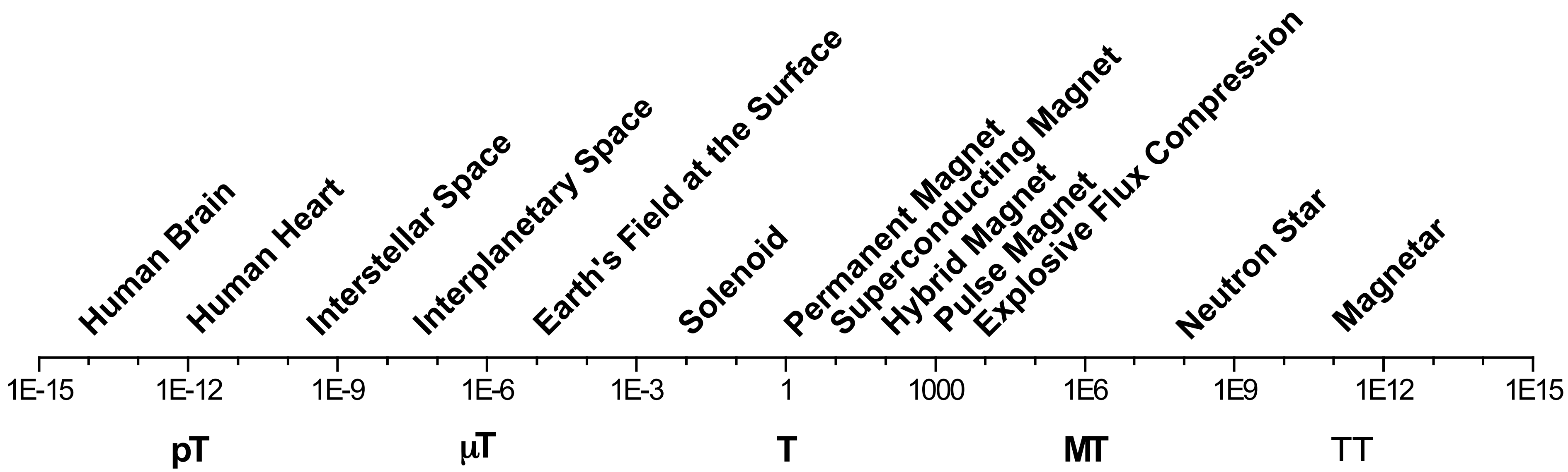


$B = \mu_0 I_1/2\pi r$

Force per meter = $\mu_0 I_1 I_2/2\pi r$

The range of magnitude of *B in Tesla* (*for* H in Am^{-1} multiply by 800.000)

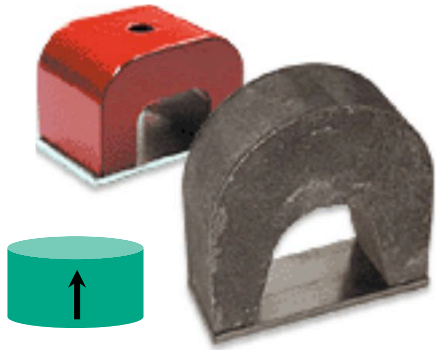
- The tesla is a rather large unit
- Largest continuous laboratory field ever achieved is 45 T



Typical values of *B*

Human brain 1 fT

Earth 50 μT

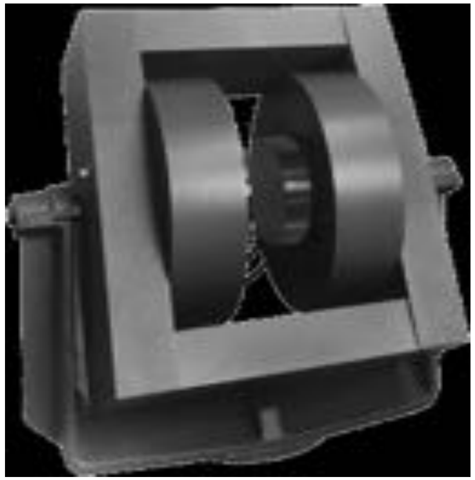


Permanent magnets 0.5 T

Superconducting magnet 10 T

Magnetar 10^{12} T

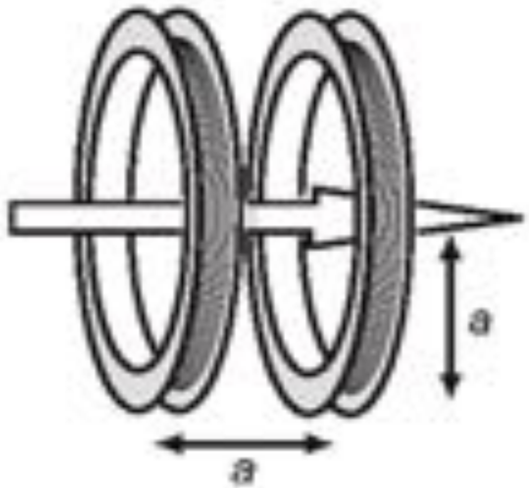
Helmholtz coils 0.01 T



Electromagnet 1 T

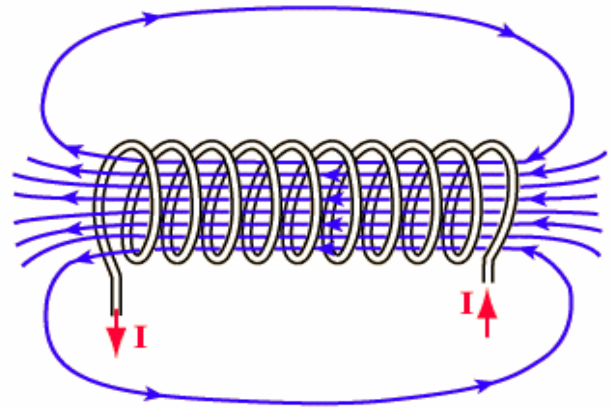
Sources of uniform magnetic fields in free space

Helmholtz coils



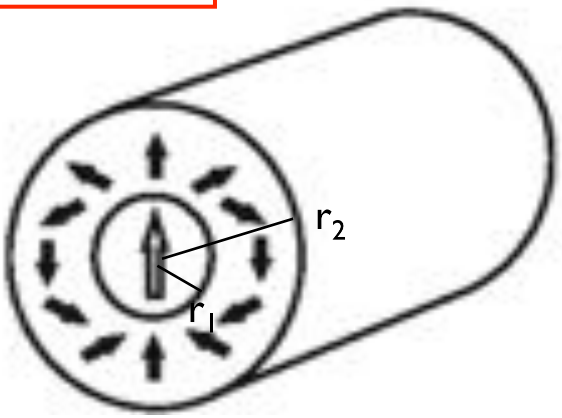
$B = (4/5)^{3/2}\mu_0 N I/a$

Long solenoid



$B = \mu_0 n I$

Halbach cylinder



$$B_r = \left(\frac{\mu_0 \lambda}{2\pi r^2}\right)\cos\theta, \quad B_\theta = \left(\frac{\mu_0 \lambda}{2\pi r^2}\right)\sin\theta, \quad B_z = 0.$$

$B = \mu_0 M_{rem} \ln(r_2/r_1)$

Why the *H*-field ?

Ampère's law for the field is free space is $\nabla \times \boldsymbol{B} = \mu_0(\boldsymbol{j}_c + \boldsymbol{j}_m)$ but $\boldsymbol{j}_m$ cannot be measured !

Only the conduction current $\boldsymbol{j}_c$ is accessible.

We showed that $\boldsymbol{J}_m = \nabla \times \boldsymbol{M}$

Hence $\nabla \times (\boldsymbol{B}/\mu_0 - \boldsymbol{M}) = \mu_0\, \boldsymbol{j}_c$

We can retain Ampère's law is a usable form provided we define $\boldsymbol{H} = \boldsymbol{B}/\mu_0 - \boldsymbol{M}$

Then $\boxed{\nabla \times \boldsymbol{H} = \mu_0\, \boldsymbol{j}_c}$

And $\boxed{\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})}$

The *H*-field.

The *H*-field plays a critical role in condensed matter.
The state of a solid reflects the local value of $\boldsymbol{H}$.
Hysteresis loops are plotted as $M(H)$

Unlike $\boldsymbol{B}$, $\boldsymbol{H}$ is not solenoidal. It has sources and sinks in a magnetic material wherever the magnetization is nonuniform.

$$\nabla.\boldsymbol{H} = -\nabla.\boldsymbol{M}$$

The sources of $\boldsymbol{H}$ (magnetic charge, q_{mag}) are distributed

— in the bulk with charge density $-\nabla.\boldsymbol{M}$
— at the surface with surface charge density $\boldsymbol{M}.\boldsymbol{e}_n$

Coulomb approach to calculate $\boldsymbol{H}$ (in the absence of currents)

Imagine $\boldsymbol{H}$ due to a distribution of magnetic charges q_m (units Am) so that

$$\boldsymbol{H} = q_m\boldsymbol{r}/4\pi r^3 \quad \text{[just like electrostatics]}$$

Potentials for $\boldsymbol{B}$ and $\boldsymbol{H}$

It is convenient to derive a field from a potential, by taking a spatial derivative. For example $\boldsymbol{E} = -\nabla\varphi_e(\boldsymbol{r})$ where $\varphi_e(\boldsymbol{r})$ is the electric potential. Any constant φ_0 can be added.

For $\boldsymbol{B}$, we know from Maxwell's equations that $\nabla.\boldsymbol{B} = 0$. There is a vector identity $\nabla\boldsymbol{\cdot}\,\nabla\times\boldsymbol{X} \equiv 0$. Hence, we can derive $\boldsymbol{B}(\boldsymbol{r})$ from a *vector potential* $\boldsymbol{A}(\boldsymbol{r})$ (units Tm),

$$\boxed{\boldsymbol{B}(\boldsymbol{r}) = \nabla\times\boldsymbol{A}(\boldsymbol{r})}$$

The gradient of any scalar *f* can be added to $\boldsymbol{A}$ (a *gauge* transformation) This is because of another vector identity $\nabla\times\nabla\boldsymbol{\cdot}f \equiv 0$.

Generally, $\boldsymbol{H}(\boldsymbol{r})$ *cannot* be derived from a potential. It satisfies Maxwell's equation $\nabla\times\boldsymbol{H} = \boldsymbol{j}_c + \partial\mathbf{D}/\partial t$. In a static situation, when there are *no conduction currents* present, $\nabla\times\boldsymbol{H} = 0$, and

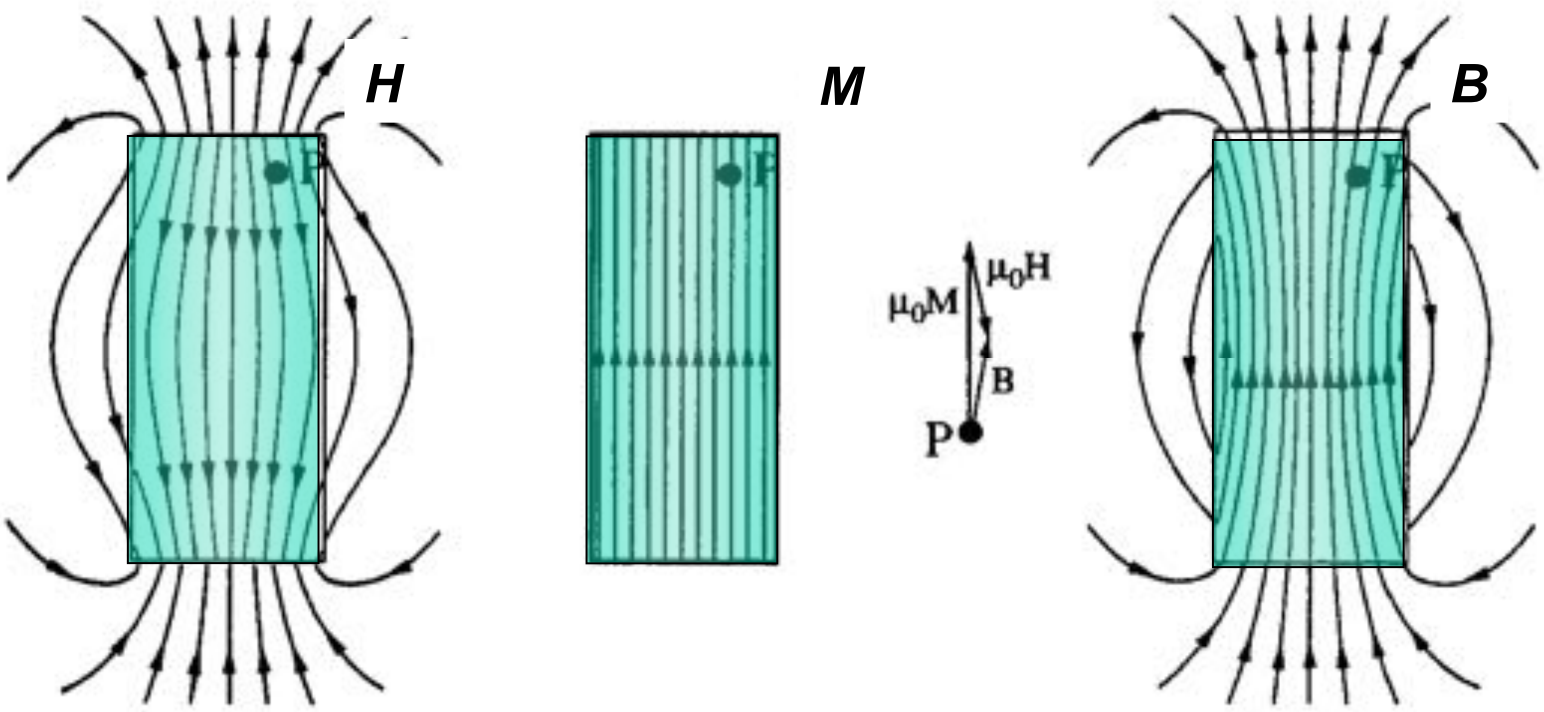
$$\boxed{\boldsymbol{H}(\boldsymbol{r}) = -\nabla\varphi_m(\boldsymbol{r})}$$

In these special conditions, it is possible to derive $\boldsymbol{H}(\boldsymbol{r})$ from a magnetic scalar potential φ_m (units A). We can imagine that $\boldsymbol{H}$ is derived from a distribution of magnetic 'charges' $\pm q_m$.

Relation between ***B*** and ***H*** in a material

The general relation between ***B, H*** and ***M*** is

$$\boxed{\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})} \qquad \text{i.e. } \boldsymbol{H} = \boldsymbol{B}/\mu_0 - \boldsymbol{M}$$

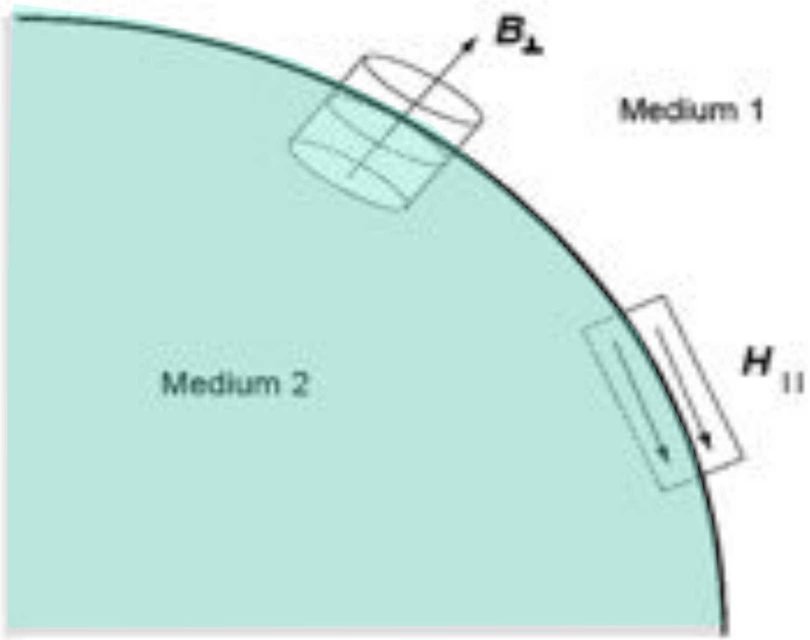


We call the *H*-field due to a magnet; — *stray field* outside the magnet

— *demagnetizing field, H_d,* inside the magnet

Boundary conditions

Conditions on the fields



It follows from Gauss's law

$$\int_S \boldsymbol{B}\cdot d\mathcal{A} = 0$$

that the *perpendicular component* of ***B*** is continuous. $(\boldsymbol{B}_1 - \boldsymbol{B}_2)\cdot\boldsymbol{e}_n = 0$

It follows from from Ampère's law

$$\oint \boldsymbol{H}.d\boldsymbol{l} = I = 0$$

that the *parallel component* of ***H*** is continuous. since there are no conduction currents on the surface. $(\boldsymbol{H}_1 - \boldsymbol{H}_2) \times \mathbf{e}_n = 0$

Conditions on the potentials

Since $\int_S \boldsymbol{B}\cdot d\mathcal{A} = \oint \boldsymbol{A}\cdot d\boldsymbol{l}$ (Stoke's theorem)

$$(\boldsymbol{A}_1 - \boldsymbol{A}_2) \times \mathbf{e}_n = 0$$

The scalar potential is continuous $\varphi_{m1} = \varphi_{m2}$

Boundary conditions in linear, isotropic homogeneous (LIH) media

In LIH media, $\boldsymbol{B} = \mu_0\mu_r\boldsymbol{H}$ $(\mu_r = 1 + \chi)$ since $\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$

Permeability susceptiility

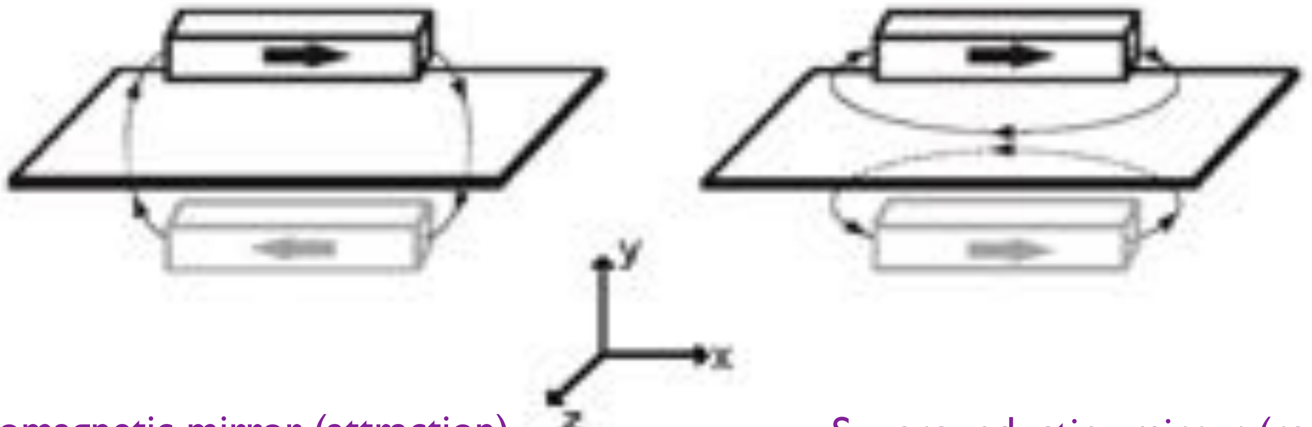
Hence $\boldsymbol{B}_1\boldsymbol{e}_n = \boldsymbol{B}_2\boldsymbol{e}_n$

$$\boldsymbol{H}_1\boldsymbol{e}_n = \mu_{r2}/\mu_{r1}\boldsymbol{H}_2\boldsymbol{e}_n$$

So field lies ~ perpendicular to the surface of soft iron but parallel to the surface of a superconductor.

Diamagnets produce weakly repulsive images.

Paramagnets produce weakly attractive images.



Soft ferromagnetic mirror (attraction) Superconducting mirror (repulsion)

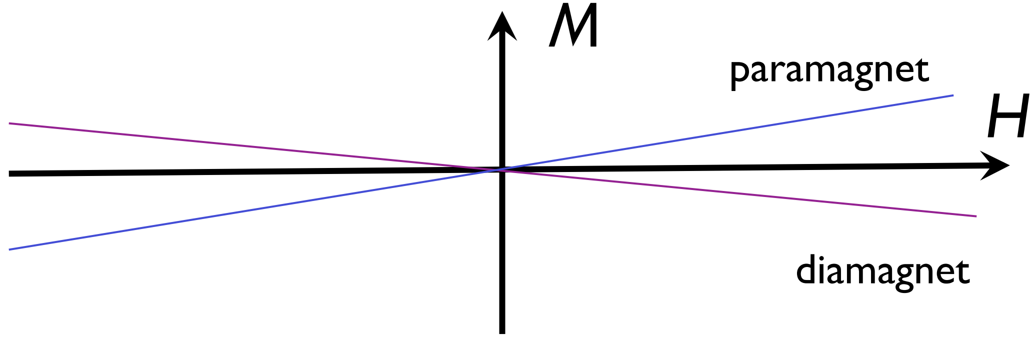
Paramagnets and diamagnets; susceptibility

Only a few elements and alloys are ferromagnetic. (See the magnetic periodic table). The atomic moments in a ferromagnet order spontaneosly parallel to eachother.

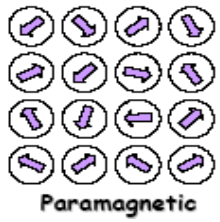
Most have no spontaneous magnetization, and they show only a very weak response to a magnetic field. They are paramgnetic, with a small positive susceptibility when there are disordered atomic moments; otherwise they are diamagnetic.

Here $|\chi| << 1$

χ is 10^{-4} - 10^{-6}

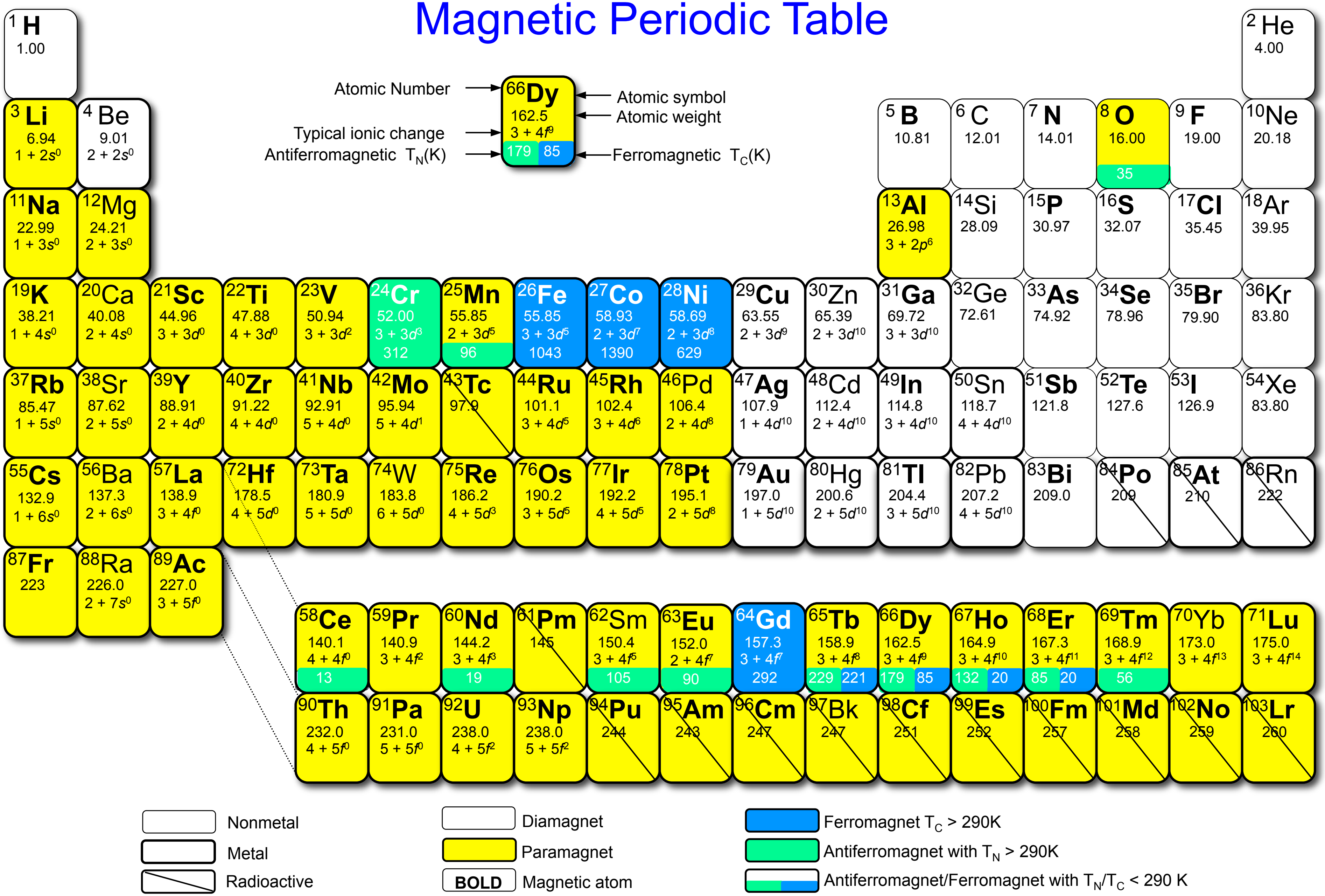


Magnetic moment/volume/field
Magnetic moment/mass/field
Magnetic moment/mole/field



Disordered $T > T_c$

Magnetic Periodic Table



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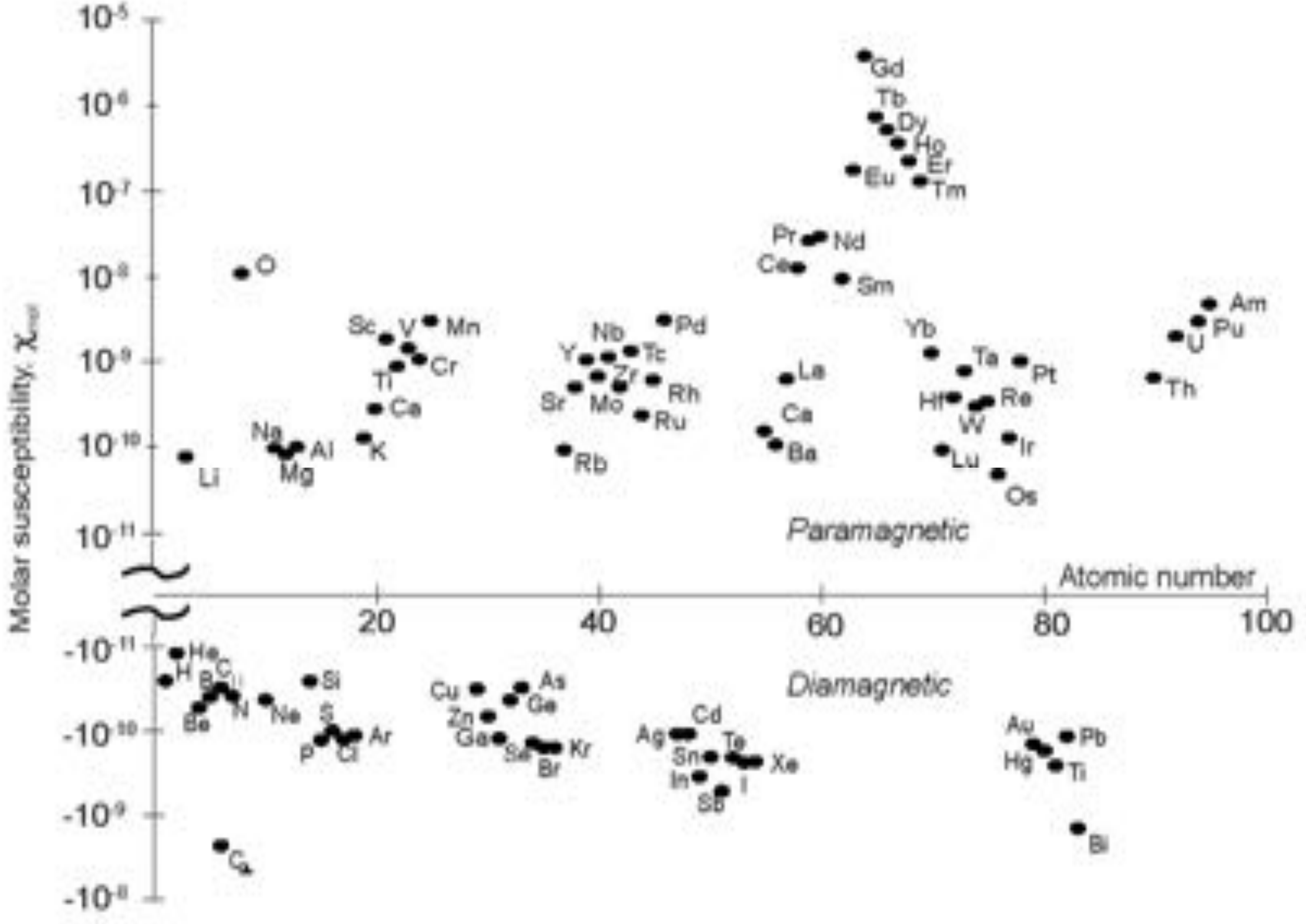
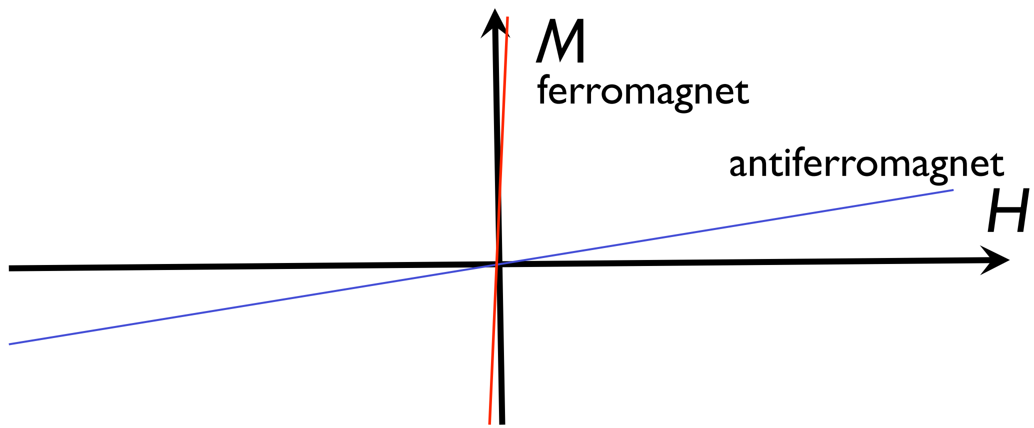
Susceptibility of the elements



Antiferromagnets and ferrimagnets

A few elements and many compounds are antiferromagnetic. The atomic moments order spontaneously antiparallel to eachother in two equivalent sublattices.

Ferrimagnets have two numerically inequivalent sublattices. They respond to a magnetic field like a ferromagnet.

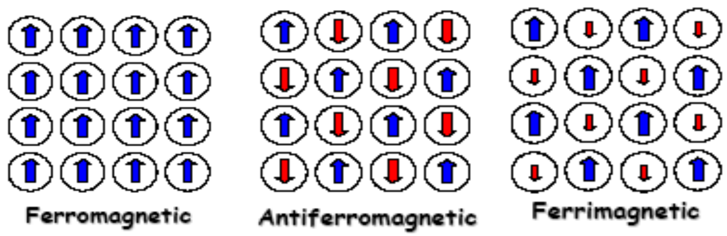


For antiferromagnets

$\chi << 1$

χ is 10^{-4} - 10^{-6}

For ferrimagnets and ferromagnets $\chi \geq 1$



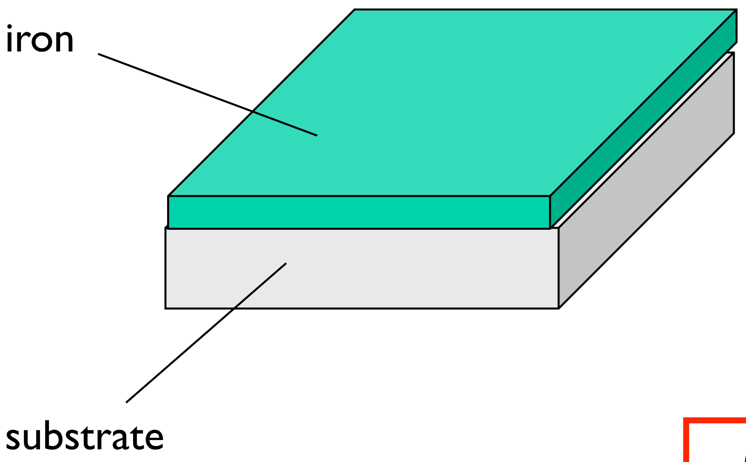
Ordered $T < T_c$

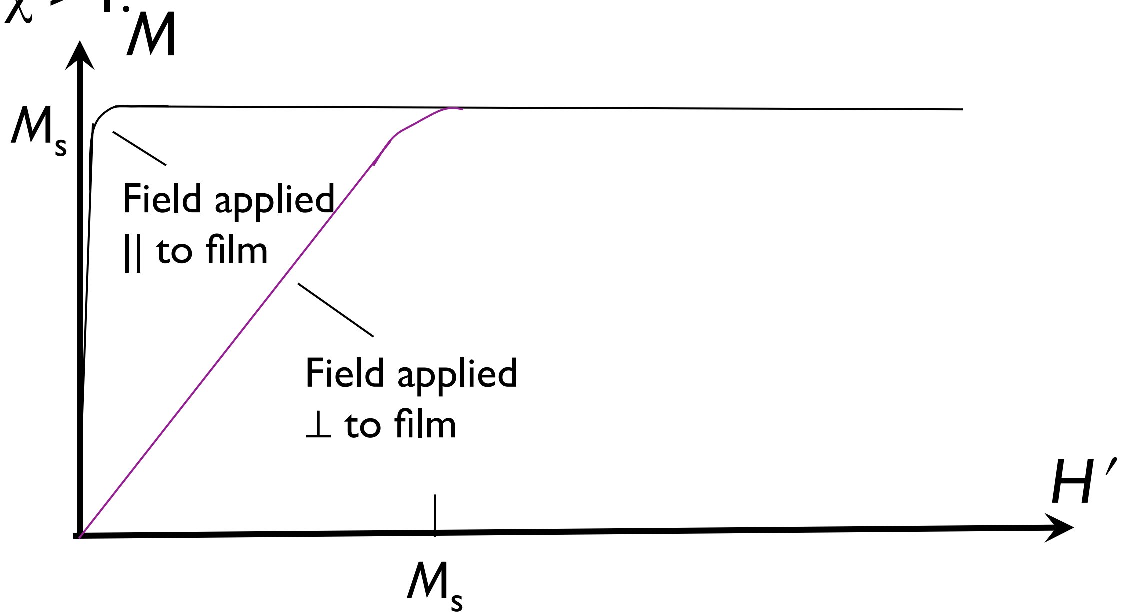
Magnetic fields - Internal and applied fields

In ALL these materials, the *H*-field acting inside the material is not the one you apply.

These are *not* the same. If they were, any applied field would instantly saturate the magnetization of a ferromagnet when $\chi > 1$.

Consider a thin film of iron.





$$\boxed{\boldsymbol{H} = \boldsymbol{H}' + \boldsymbol{H}_d}$$

Internal field — External field — Demagnetizing field

$M_s = 1.72\ \mathrm{MAm^{-1}}$

Demagnetizing field in a material - $\boldsymbol{H}_d$

The demagnetizing field depends on the *shape* of the sample and the direction of magnetization.

For simple uniformly-magnetized shapes (ellipsoids of revolution) the demagnetizing field is related to the magnetization by a proportionality factor $\mathcal{N}$ known as the *demagnetizing factor*. The value of $\mathcal{N}$ can never exceed 1, nor can it be less than 0.

$$\boxed{\boldsymbol{H}_d = -\mathcal{N}\boldsymbol{M}}$$

More generally, this is a tensor relation. $\mathcal{N}$ is then a 3 x 3 matrix, with trace 1. That is

$$\boxed{\mathcal{N}_x + \mathcal{N}_y + \mathcal{N}_z = 1}$$

Note that the internal field H is always less than the applied field H' since

$$\boxed{\boldsymbol{H} = \boldsymbol{H}' - \mathcal{N}\boldsymbol{M}}$$

Demagnetizing factor $\mathcal{N}$ for special shapes.

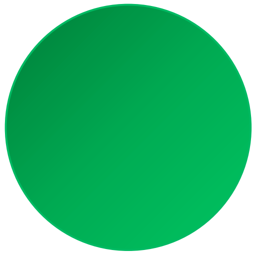
$\mathcal{N} = 0$

Long needle, ***M*** parallel to the long axis

Thin film, **M** parallel to plane

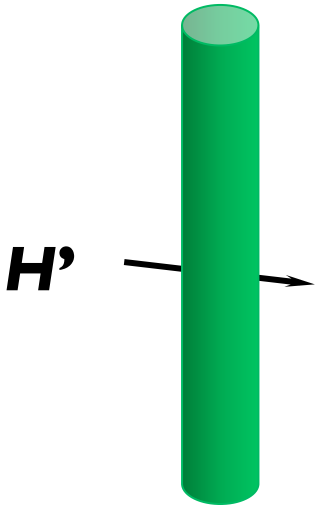
Toroid, **M** perpendicular to **r**

$\mathcal{N} = 1/3$

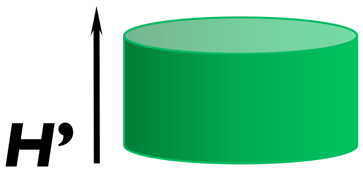


Sphere

$\mathcal{N} = 1/2$



Long needle, ***M*** perpendicular to the long axis

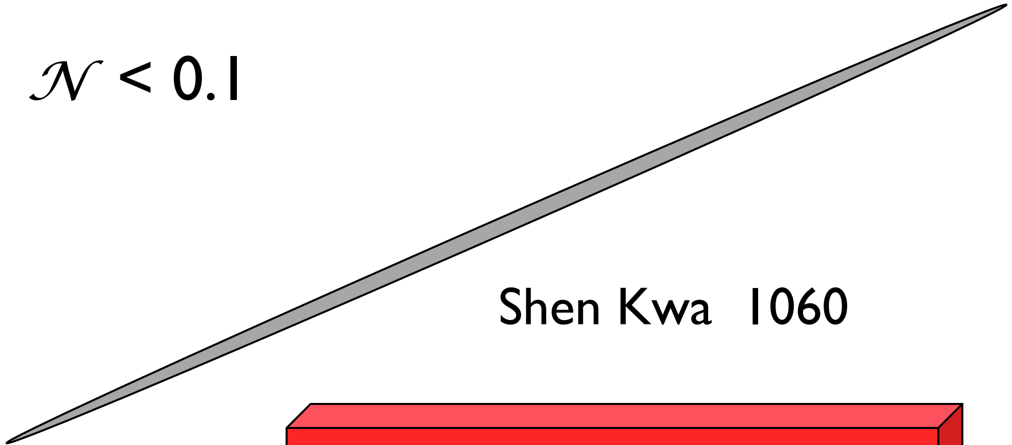


$\mathcal{N} = 1$

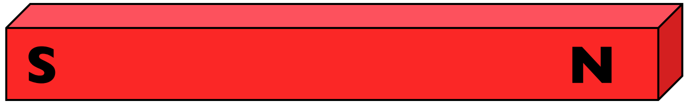
Thin film, **M** perpendicular to plane

The shape barrier

$\mathcal{N} < 0.1$



Shen Kwa 1060



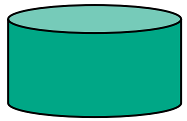
Daniel Bernouilli
1743

Gowind Knight 1760

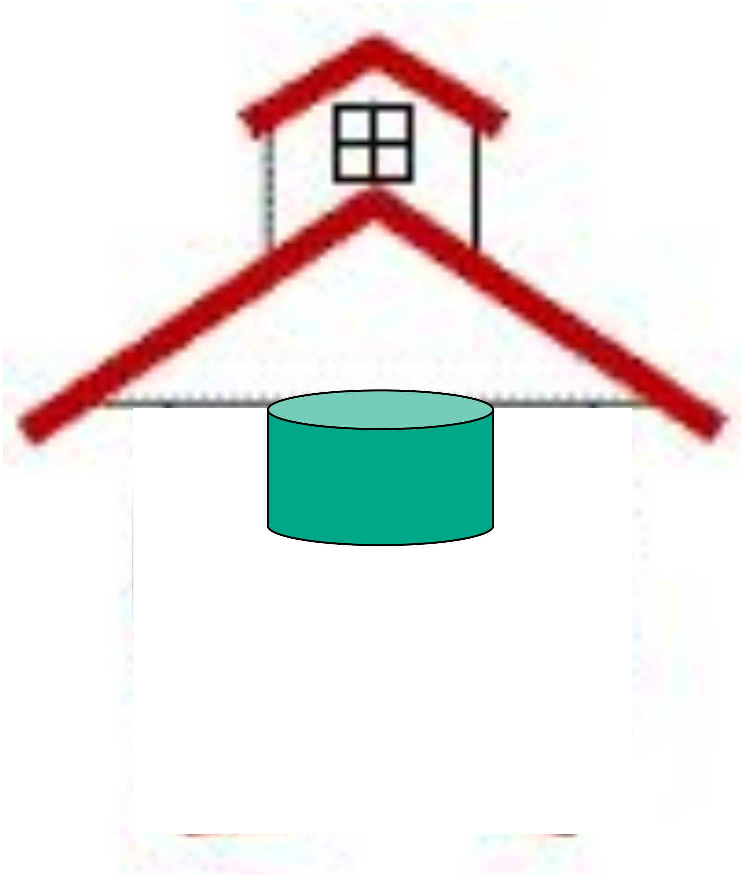
T. Mishima
1931

$\mathcal{N} = 0.5$

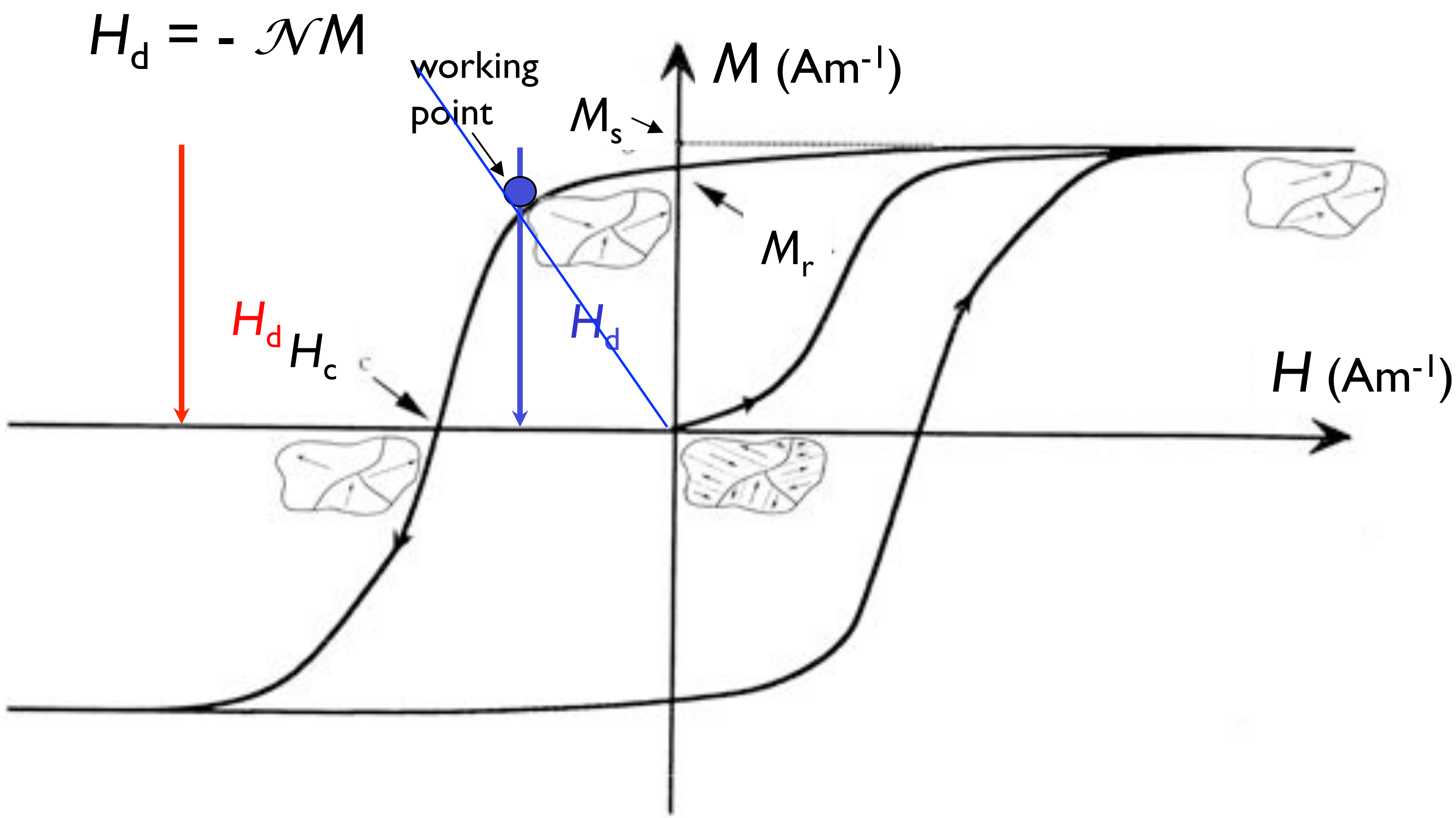
New icon for permanent magnets! $\Rightarrow$



Philips 1952



The shape barrier ovecome !

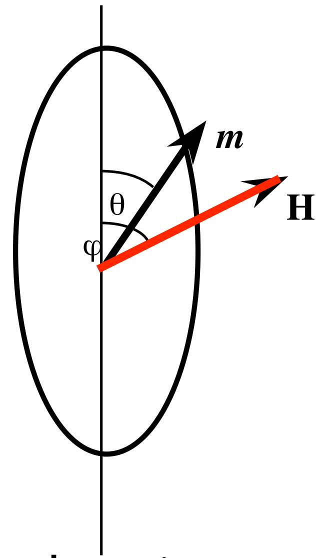


Single-domain particles

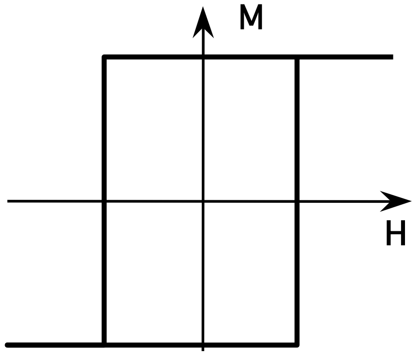
When ferromagnetic particles are no bigger than a few tens of nanometers, it does not pay to form domains. In very small particles, reversal takes place by *coherent rotation* of the magnetic moment ***m***. There must be an easy direction of magnetization.

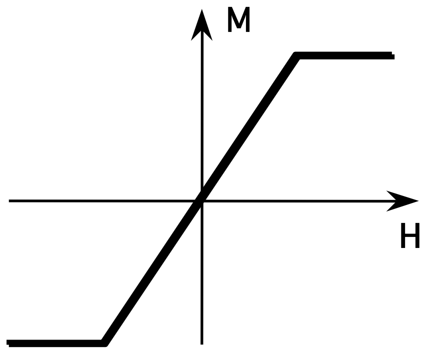
If an external field ***H*** is applied at an angle φ to the easy direction, and the magnetization is at an angle θ to the easy direction, the energy is $E_{tot} = E_a + E_Z$ the sum of anisotropy Zeeman terms. The first $E_a = K_u V \sin^2\theta$, hence there in an energy barrier $K_u V$ to reversal.

$$E_{tot} = K_u V \sin^2\theta - \mu_0 m H \cos(\varphi - \theta)$$



The energy can be minimized, and the hysteresis loop calculated numerically for a general angle φ. This is the Stoner-Wohlfarth model.





A single domain particle where the magnetization rotates coherently.

Paramagnetic and ferromagnetic responses

Susceptibility of linear, isotropic and homogeneous (LIH) materials

$\boldsymbol{M} = \chi' \boldsymbol{H}'$ χ' is external susceptibility (no units)

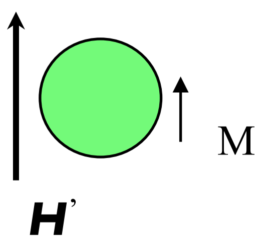
It follows that from $\boldsymbol{H} = \boldsymbol{H}' + \boldsymbol{H}_d$ that

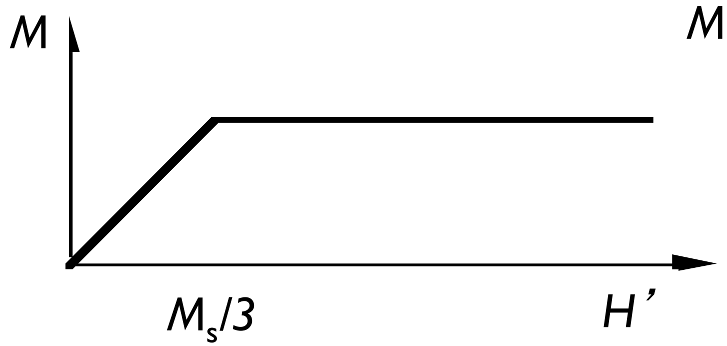
$$\boxed{1/\chi = 1/\chi' - \mathcal{N}}$$

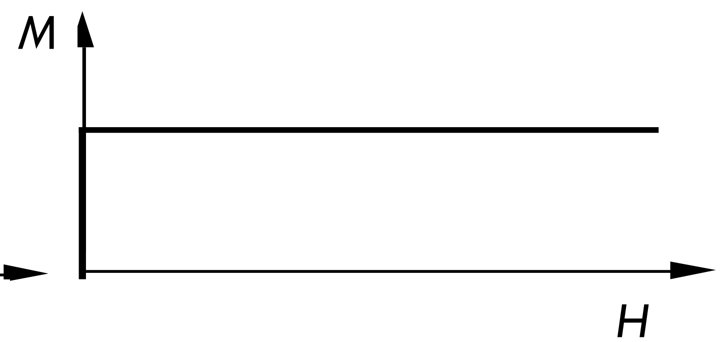
Typical paramagnets and diamagnets: $\chi \approx \chi'$ (10^{-5} to 10^{-3}) Demag field is negligible.

Paramagnets close to the Curie point and ferromagnets:

$\chi >> \chi'$ χ diverges as $T \rightarrow T_C$ but χ' never exceeds $1/\mathcal{N}$.



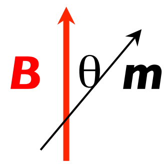




Energy of ferromagnetic bodies

Torque and energy of a magnetic moment m in a uniform field,.

$\boxed{\Gamma = \mathbf{m} \times \mathbf{B}}$ $\boxed{\varepsilon = -\mathbf{m}.\mathbf{B}}$



$\Gamma = mB \sin\theta.$ $\varepsilon = -mB \cos\theta.$

Force

In a non-uniform field, $\boldsymbol{f} = -\nabla\varepsilon$ $\boxed{\boldsymbol{f} = \mathbf{m}.\nabla\mathbf{B}}$

• Magnetostatic (dipole-dipole) forces are long-ranged, but weak. They determine the magnetic microstructure (domains).

• $\frac{1}{2}\mu_0 H^2$ is the energy density associated with a magnetic field $\boldsymbol{H}$

M ≈ 1 MA m^{-1}, $\mu_0 H_d \approx$ 1 T, hence $\mu_0 H_d M \approx 10^6$ J m^{-3}

• Products $\boldsymbol{B}.\boldsymbol{H}$, $\boldsymbol{B}.\boldsymbol{M}$, $\mu_0\boldsymbol{H}^2$, $\mu_0\boldsymbol{M}^2$ are all energies per unit volume.

• Magnetic forces *do no work* on moving charges $\boldsymbol{f} = q(\boldsymbol{v} \times \boldsymbol{B})$ [Lorentz force]

• No potential energy associated with the magnetic force.

Magnetostatic forces

Force density on a magnetized body at constant temperature

$$\boxed{\boldsymbol{F}_m = -\nabla G}$$

$$\boldsymbol{F}_m = \nabla(\mu_0 \boldsymbol{H}'.\boldsymbol{M})$$

$$\nabla(\boldsymbol{H}'.\boldsymbol{M}) = (\boldsymbol{H}'.\nabla)\boldsymbol{M} + (\boldsymbol{M}.\nabla)\boldsymbol{H}'$$

Kelvin force

$$\boxed{\boldsymbol{F}_m = \mu_0(\boldsymbol{M}.\nabla)\boldsymbol{H}'}$$

General expression, when ***M*** is dependent on ***H*** is

$$\boxed{\boldsymbol{F}_m = -\mu_0\nabla\left[\int_0^H \left(\frac{\partial M v}{\partial v}\right)_{H,T} dH\right] + \mu_0(\boldsymbol{M}.\nabla)\boldsymbol{H}.}$$

$v = 1/d$ d is the density

Some expressions involving *B*

$\boldsymbol{F} = q(\boldsymbol{E} + \mathbf{v} \times \boldsymbol{B})$ Force on a charged particle *q*

$\boldsymbol{F} = \boldsymbol{B}\, I\ell$ Force on current-carrying wire

$\mathcal{E} = - d\Phi/dt$ Faraday's law of electromagnetic induction

$E = -\boldsymbol{m}.\boldsymbol{B}$ Energy of a magnetic moment

$\boldsymbol{F} = \nabla\, \boldsymbol{m}.\boldsymbol{B}$ Force on a magnetic moment

$\boldsymbol{\Gamma} = \boldsymbol{m} \times \boldsymbol{B}$ Torque on a magnetic moment

2. Magnetic Phenomena

Functional Magnetic Materials

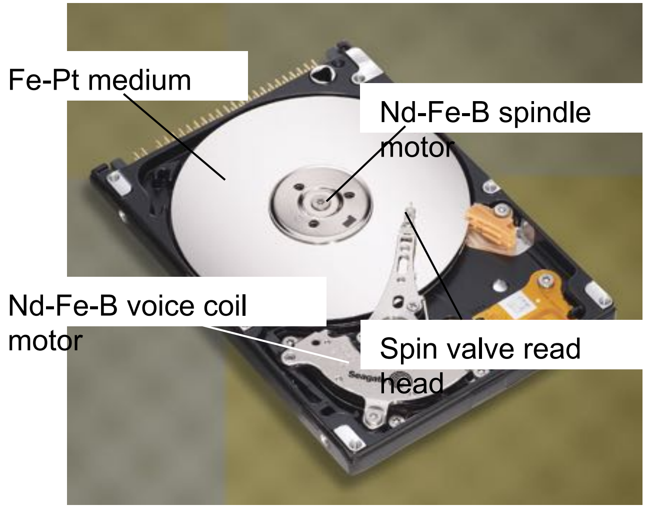
Magnetism is an experimental science that has spawned generations of useful technology, thanks to the range of useful phenomena associated with magnetically-ordered materials.

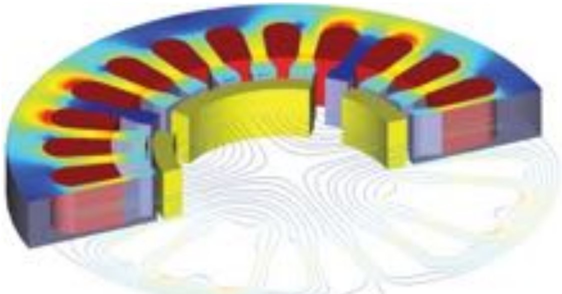
Property	**Effect**	**Magnitude**	**Applications**
Hard ($H_c < M$)	Creates stray field	$H \leq 2M_s$	Motors, actuators, flux sources
Soft ($H_c \approx 0$)	Amplifies flux	$1 < \chi < 10^3$	Electromagnetic drives
Magnetostrictive	Changes length	$0 < \delta\ell/\ell < 10^{-3}$	Actuators Sensors
Magnetoresistive	Changes resistivity	$\delta\rho/\rho \sim 1$ % or 100% in heterostructures	Spin electronics Sensors
Magnetocaloric	Changes temperature	$\delta T \sim 2$ K	Refrigeration

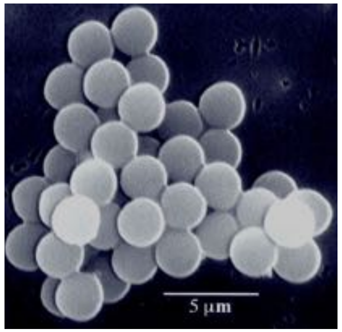
$$\ell = \ell(H)$$

$$\rho = \rho(H)$$

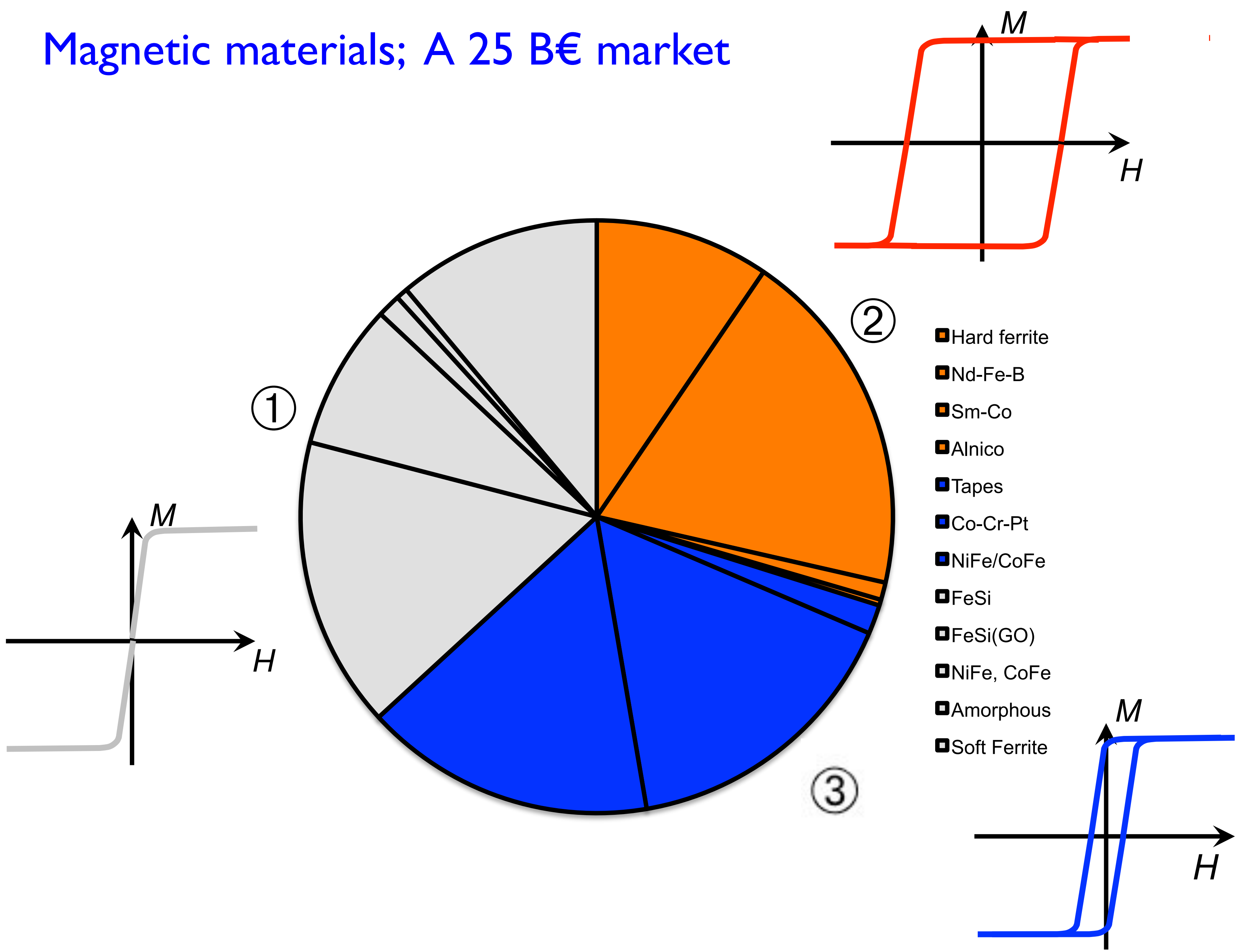
$$T = T(H)$$


Fe-Pt medium
Nd-Fe-B spindle motor
Nd-Fe-B voice coil motor
Spin valve read head




5 μm

Magnetic materials; A 25 B€ market



3. Units

A note on units:

Magnetism is an experimental science, closely linked to electricity, and it is important to be able to calculate numerical values of the physical quantities involved. There is a strong case to use SI consistently

- SI units relate to the practical units of electricity measured on the multimeter and the oscilloscope
- It is possible to check the dimensions of any expression by inspection.
- They are almost universally used in teaching of science and engineering
- Units of ***H***, ***B***, Φ or $d\Phi/dt$ have been introduced.

BUT

Most literature still uses cgs units. You need to understand them too.

SI / cgs conversions:

	SI units		cgs units	
	$\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$		$\boldsymbol{B} = \boldsymbol{H} + 4\pi\boldsymbol{M}$	
m	A m^2		emu	
$\boldsymbol{M}$	A m^{-1}	(10^{-3} emu cc^{-1})	emu cc^{-1}	(1 kA m^{-1})
σ	A m^2 kg^{-1}	(1 emu g^{-1})	emu g^{-1}	(1 A m^2 kg^{-1})
$\boldsymbol{H}$	A m^{-1}	($4\pi/1000 \approx 0.0125$ Oe)	Oersted	($1000/4\pi \approx 80$ A m^{-1})
$\boldsymbol{B}$	Tesla	(10000 G)	Gauss	(10^{-4} T)
Φ	Weber (Tm^2)	(10^8 Mw)	Maxwell (G cm^2)	(10^{-8} Wb)
$d\Phi/dt$	V	(10^8 Mw s^{-1})	Mw s^{-1}	(10 nV)
χ	-	(4π cgs)	-	($1/4\pi$ SI)

Mechanical

Quantity	Symbol	Unit	m	l	t	i	θ
Area	$\mathcal{A}$	m^2	0	2	0	0	0
Volume	V	m^3	0	3	0	0	0
Velocity	v	$\mathrm{m\ s}^{-1}$	0	1	−1	0	0
Acceleration	a	$\mathrm{m\ s}^{-2}$	0	1	−2	0	0
Density	d	$\mathrm{kg\ m}^{-3}$	1	−3	0	0	0
Energy	ε	J	1	2	−2	0	0
Momentum	p	$\mathrm{kg\ m\ s}^{-1}$	1	1	−1	0	0
Angular momentum	L	$\mathrm{kg\ m}^2\ \mathrm{s}^{-1}$	1	2	−1	0	0
Moment of inertia	I	$\mathrm{kg\ m}^2$	1	2	0	0	0
Force	f	N	1	1	−2	0	0
Force density	F	$\mathrm{N\ m}^{-3}$	1	−2	−2	0	0
Power	P	W	1	2	−3	0	0
Pressure	P	Pa	1	−1	−2	0	0
Stress	σ	$\mathrm{N\ m}^{-2}$	1	−1	−2	0	0
Elastic modulus	K	$\mathrm{N\ m}^{-2}$	1	−1	−2	0	0
Frequency	f	s^{-1}	0	0	−1	0	0
Diffusion coefficient	D	$\mathrm{m}^2\ \mathrm{s}^{-1}$	0	2	−1	0	0
Viscosity (dynamic)	η	$\mathrm{N\ s\ m}^{-2}$	1	−1	−1	0	0
Viscosity	ν	$\mathrm{m}^2\ \mathrm{s}^{-1}$	0	2	−1	0	0
Planck's constant	$\hbar$	J s	1	2	−1	0	0

Electrical

Quantity	Symbol	Unit	m	l	t	i	θ
Current	I	A	0	0	0	1	0
Current density	j	$\mathrm{A\,m^{-2}}$	0	−2	0	1	0
Charge	q	C	0	0	1	1	0
Potential	V	V	1	2	−3	−1	0
Electromotive force	$\mathcal{E}$	V	1	2	−3	−1	0
Capacitance	C	F	−1	−2	4	2	0
Resistance	R	Ω	1	2	−3	−2	0
Resistivity	ϱ	$\Omega\,\mathrm{m}$	1	3	−3	−2	0
Conductivity	σ	$\mathrm{S\,m^{-1}}$	−1	−3	3	2	0
Dipole moment	p	C m	0	1	1	1	0
Electric polarization	P	$\mathrm{C\,m^{-2}}$	0	−2	1	1	0
Electric field	E	$\mathrm{V\,m^{-1}}$	1	1	−3	−1	0
Electric displacement	D	$\mathrm{C\,m^{-2}}$	0	−2	1	1	0
Electric flux	Ψ	C	0	0	1	1	0
Permittivity	ε	$\mathrm{F\,m^{-1}}$	−1	−3	4	2	0
Thermopower	S	$\mathrm{V\,K^{-1}}$	1	2	−3	−1	−1
Mobility	μ	$\mathrm{m^2\,V^{-1}\,s^{-1}}$	−1	0	2	1	0

Magnetic							
Quantity	Symbol	Unit	m	l	t	i	θ
Magnetic moment	$\mathfrak{m}$	A m^2	0	2	0	1	0
Magnetization	M	A m^{-1}	0	−1	0	1	0
Specific moment	σ	A m^2 kg^{-1}	−1	2	0	1	0
Magnetic field strength	H	A m^{-1}	0	−1	0	1	0
Magnetic flux	Φ	Wb	1	2	−2	−1	0
Magnetic flux density	B	T	1	0	−2	−1	0
Inductance	L	H	1	2	−2	−2	0
Susceptibility (M/H)	χ		0	0	0	0	0
Permeability (B/H)	μ	H m^{-1}	1	1	−2	−2	0
Magnetic polarization	J	T	1	0	−2	−1	0
Magnetomotive force	$\mathcal{F}$	A	0	0	0	1	0
Magnetic 'charge'	q_m	A m	0	1	0	1	0
Energy product	(BH)	J m^{-3}	1	−1	−2	0	0
Anisotropy energy	K	J m^{-3}	1	−1	−2	0	0
Exchange stiffness	A	J m^{-1}	1	1	−2	0	0
Hall coefficient	R_H	m^3 C^{-1}	0	3	−1	−1	0
Scalar potential	φ	A	0	0	0	1	0
Vector potential	A	T m	1	1	−2	−1	0
Permeance	P_m	T m^2 A^{-1}	1	2	−2	−2	0
Reluctance	R_m	A T^{-1} m^{-2}	−1	−2	2	2	0

Thermal

Quantity	Symbol	Unit	m	l	t	i	θ
Enthalpy	H	J	1	2	−2	0	0
Entropy	S	$\mathrm{J\,K^{-1}}$	1	2	−2	0	−1
Specific heat	C	$\mathrm{J\,K^{-1}\,kg^{-1}}$	0	2	−2	0	−1
Heat capacity	c	$\mathrm{J\,K^{-1}}$	1	2	−2	0	−1
Thermal conductivity	κ	$\mathrm{W\,m^{-1}\,K^{-1}}$	1	1	−3	0	−1
Sommerfeld coefficient	γ	$\mathrm{J\,mol^{-1}\,K^{-1}}$	1	2	−2	0	−1
Boltzmann's constant	k_B	$\mathrm{J\,K^{-1}}$	1	2	−2	0	−1

(1) Kinetic energy of a body: $\varepsilon = \frac{1}{2}mv^2$

$[\varepsilon] = [1, 2, -2, 0, 0]$

$[m] = [1, 0, 0, 0, 0]$

$[v^2] = \dfrac{2[0, -1, -1, 0, 0]}{[1, -2, -2, 0, 0]}$

(2) Lorentz force on a moving charge; $\boldsymbol{f} = q\boldsymbol{v} \times \boldsymbol{B}$

$[f] = [1, 1, -2, 0, 0]$

$[q] = [0, 0, 1, 1, 0]$

$[v] = [0, 1, -1, 0, 0]$

$[B] = \dfrac{[1, 0, -2, -1, 0]}{[1, 1, -2, 0, 0]}$

(3) Domain wall energy $\gamma_w = \sqrt{AK}$ (γ_w is an energy per unit area)

$[\gamma_w] = [\varepsilon A^{-1}]$

$= [1, 2, -2, 0, 0]$

$- [0, 2, 0, 0, 0]$

$= [1, 0, -2, 0, 0]$

$[\sqrt{AK}] = 1/2[AK]$

$[\sqrt{A}] = \frac{1}{2}[1, 1, -2, 0, 0]$

$[\sqrt{K}] = \frac{1}{2}\dfrac{[1, -1, -2, 0, 0]}{[1, 0, -2, 0, 0]}$

(4) Magnetohydrodynamic force on a moving conductor $\boldsymbol{F} = \sigma \boldsymbol{v} \times \boldsymbol{B} \times \boldsymbol{B}$ ($\boldsymbol{F}$ is a force per unit volume)

$$[F] = [FV^{-1}] = [1, 1, -2, 0, 0] - \frac{[0, 3, 0, 0, 0]}{[1, -2, -2, 0, 0]}$$

$$[\sigma] = [-1, -3, 3, 2, 0]$$
$$[v] = [0, 1, -1, 0, 0]$$
$$[B^2] = \frac{2[1, 0, -2, -1, 0]}{[1, -2, -2, 0, 0]}$$

(5) Flux density in a solid $\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$ (note that quantities added or subtracted in a bracket must have the same dimensions)

$$[B] = [1, 0, -2, -1, 0]$$

$$[\mu_0] = [1, 1, -2, -2, 0]$$
$$[M], [H] = \frac{[0, -1, 0, 1, 0]}{[1, 0, -2, -1, 0]}$$

(6) Maxwell's equation $\nabla \times \boldsymbol{H} = \boldsymbol{j} + \mathrm{d}\boldsymbol{D}/\mathrm{d}t$.

$$[\nabla \times \boldsymbol{H}] = [Hr^{-1}] = [0, -1, 0, 1, 0] - [0, 1, 0, 0, 0] = [0, -2, 0, 1, 0]$$

$$[j] = [0, -2, 0, 1, 0]$$

$$[\mathrm{d}\boldsymbol{D}/\mathrm{d}t] = [Dt^{-1}] = [0, -2, 1, 1, 0] - [0, 0, 1, 0, 0] = [0, -2, 0, 1, 0]$$

(7) Ohm's Law $V = IR$

$$= [1, 2, -3, -1, 0]$$

$$[0, 0, 0, 1, 0] + [1, 2, -3, -2, 0] = [1, 2, -3, -1, 0]$$

(8) Faraday's Law $\mathcal{E} = -\partial\Phi/\partial t$

$$= [1, 2, -3, -1, 0]$$

$$[1, 2, -2, -1, 0] - [0, 0, 1, 0, 0] = [1, 2, -3, -1, 0]$$

Table B SI–cgs conversion Locate the quantity you wish to convert in column A (its units are in the same row); to convert to a quantity in row B (its units are in the same column), *multiply* it by the factor in the table. Examples are given in appendix F.

Field conversions

A ↓	B	units →	SI H A m^{-1}	SI B T	cgs H Oe	cgs B G
SI	H	A m^{-1}	1	μ_0	$4\pi \times 10^{-3}$	$4\pi \times 10^{-3}$
SI	B	T	$1/\mu_0$	1	10^4	10^4
cgs	H	Oe	$10^3/4\pi$	10^{-4}	1	1
cgs	B	G	$10^3/4\pi$	10^{-4}	1	1

B–H conversions are valid in free space only

Susceptibility conversions

A ↓		B units →	SI χ	SI χ_m m^3 kg^{-1}	SI χ_{mol} m^3 mol^{-1}	SI χ_0 J T^{-2} kg^{-1}	cgs κ	cgs χ_m emu g^{-1}	cgs χ_{mol} emu mol^{-1}
SI	χ		1	1/d	$10^{-3}\mathcal{M}$/d	$1/\mu_0$d	$1/4\pi$	$10^3/4\pi$d	$10^3\mathcal{M}/4\pi$d
SI	χ_m	m^3 kg^{-1}	d	1	$10^{-3}\mathcal{M}$	$1/\mu_0$	d$/4\pi$	$10^3/4\pi$	$10^3\mathcal{M}/4\pi$
SI	χ_{mol}	m^3 mol^{-1}	10^3d$/\mathcal{M}$	$10^3/\mathcal{M}$	1	$10^3/\mu_0\mathcal{M}$	10^3d$/4\pi\mathcal{M}$	$10^6/4\pi\mathcal{M}$	$10^6/4\pi$
SI	χ_0	J T^{-2} kg^{-1}	μ_0d	μ_0	$10^{-3}\mu_0\mathcal{M}$	1	10^{-7}d	10^{-4}	$10^{-4}\mathcal{M}$
cgs	κ		4π	$4\pi 10^{-3}$/d	$4\pi 10^{-6}\mathcal{M}$/d	10^4/d	1	1/d	$\mathcal{M}$/d
cgs	χ_m	emu g^{-1}	4πd	$4\pi 10^{-3}$	$4\pi 10^{-6}\mathcal{M}$	10^4	d	1	$\mathcal{M}$
cgs	χ_{mol}	emu mol^{-1}	4πd$/\mathcal{M}$	$4\pi 10^{-3}/\mathcal{M}$	$4\pi 10^{-6}$	$10^4/\mathcal{M}$	d$/\mathcal{M}$	$1/\mathcal{M}$	1

$\mathcal{M}$ is molecular weight (in g mol^{-1}), d is density (use SI units in rows 1–4, cgs units in rows 5–7)

Magnetic moment and magnetization conversions

	A ↓	B units →	SI $\mathfrak{m}$ A m^2	SI M A m^{-1}	SI σ A m^2 kg^{-1}	SI σ_{mol} A m^2 mol^{-1}	cgs $\mathfrak{m}$ emu	cgs M emu cm^{-3}	cgs σ emu g^{-1}	cgs σ_{mol} emu mol^{-1}
	$\mathfrak{m}$	μ_B/formula	9.274×10^{-24}	5585d$/\mathcal{M}$	$5585/\mathcal{M}$	5.585	9.274×10^{-21}	5.585d$/\mathcal{M}$	$5585/\mathcal{M}$	5585
SI	$\mathfrak{m}$	A m^2	1	$1/V$	1/dV	$10^{-3}\mathcal{M}$/dV	10^3	$10^{-3}/V$	1/dV	$\mathcal{M}$/dV
SI	M	A m^{-1}	V	1	1/d	$10^{-3}\mathcal{M}$/d	$10^3 V$	10^{-3}	1/d	$\mathcal{M}$/d
SI	σ	A m^2 kg^{-1}	dV	d	1	$10^{-3}\mathcal{M}$	10^3dV	10^{-3}d	1	$\mathcal{M}$
SI	σ_{mol}	A m^2 mol^{-1}	10^2d$V/\mathcal{M}$	10^3d$/\mathcal{M}$	$10^3/\mathcal{M}$	1	10^6d$V/\mathcal{M}$	d$/\mathcal{M}$	$10^3/\mathcal{M}$	10^3
cgs	$\mathfrak{m}$	emu	10^{-3}	$10^3/V$	1/dV	$10^{-3}\mathcal{M}$/dV	1	$1/V$	1/dV	$\mathcal{M}$/dV
cgs	M	emu cm^{-3}	$10^{-3}V$	10^3	1/d	$10^{-3}\mathcal{M}$/d	V	1	1/d	$\mathcal{M}$/d
cgs	σ	emu g^{-1}	10^{-3}dV	10^3d	1	$10^{-3}\mathcal{M}$	dV	d	1	$\mathcal{M}$
cgs	σ_{mol}	emu mol^{-1}	10^{-3}d$V/\mathcal{M}$	10^3d$/\mathcal{M}$	$1/\mathcal{M}$	10^{-3}	d$V/\mathcal{M}$	d$/\mathcal{M}$	$1/\mathcal{M}$	1

$\mathcal{M}$ is molecular weight (g mol^{-1}), d is density, V is sample volume (use SI in rows 1–5, cgc in rows 6–9 for density and volume). Note that the quantity $4\pi M$ is frequently quoted in the cgs system, in units of gauss (G)

To deduce the effective Bohr magneton number $p_{eff} = m_{eff}/\mu_B$ from the molar Curie constant, the relation is $C_{mol} = 1.571\ 10^{-6}\ p_{eff}^2$ in SI, and $C_{mol} = 0.125\ p_{eff}^2$ in cgs.

Review — Lecture 1

You should now know:

- Some good books
- How magnets behave; Magnetic moment ***m*** and magnetization ***M***
- How magnetism is related to electric current
- The two fields; ***B*** and ***H***
- Demagnetizing effects in solids
- Energy and force of magnetic moments
- What magnetic materials are used for
- The units used in magnetism, and understand what they mean
- How to check the dimensions of any quantity or equation.