Incorporating Real World Examples in a Random Signal Analysis Course

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***Abstract*— This Research to Practice Full Paper presents the design and initial evaluation of engaging students in a random signal analysis course through a set of application-oriented mini-projects involving real-world examples. Many students struggle learning the concepts of probability and random processes due to its abstract nature. The goal of these mini-projects is to help students better understand abstract course concepts, let them practice applying course concepts to solve real world problems, and at the same time, improve their critical thinking and problem solving skills. This paper will describe the design and implementation of these mini-projects, as well as their preliminary evaluation.**

***Keywords*— Undergraduate, Junior, Electrical Engineering, Simulation, Critical Thinking, Problem Solving, Student Perception**

I. Introduction

The study of probability and random processes is fundamental to many engineering disciplines. Many undergraduate engineering students often take a course in this subject. In Electrical Engineering, random signal analysis is a required course in the junior year. Students often have trouble grasping concepts in this subject.

Many interventions have been proposed such as active learning exercises, computer simulation and MATLAB-based computer projects to help students understand probability and statistics concepts and to help them see the relevance of these concepts through applications in the electrical engineering discipline.

This paper takes a different approach. Six mini-projects involving applications in the daily life are incorporated into the curriculum, with one real world example in every major topic area of the course spreading throughout the semester. Students either conduct theoretical analysis, do computer simulation or perform data analysis for these mini-projects. The goal is to help students understand abstract course concepts, and to let students practice applying these theoretical concepts to real world situations. With most of these mini-projects being open-ended, they also improve student's critical thinking and problem solving skills.

This paper will discuss how these application-oriented exercises are incorporated into the existing curriculum, specifically how six mini-projects are introduced to use real world examples to teach abstract probability and statistics concepts. The impact of this addition will be evaluated through student surveys. The rest of the paper is organized as follows. First, prior work on probability and random processes education is briefly reviewed in the background section along with an introduction of the random signal analysis course curriculum. The mini-projects design and implementation is described next, followed by the assessment and results. Then future work is presented, followed by conclusion.

II. Background

A. Existing Work

The concepts in probability and random processes presents challenges for students and teachers alike [1-3, 5, 7, 8]. Students struggle due to the abstract nature of the subject. Many interventions have been proposed in the literature to enhance student learning experience.

Active learning exercises are used in [2], where students work on completing worksheet problems in-pairs after a short lecture at the beginning of class. Card games are used in [3] to teach conditional probability. Virtual reality learning modules are developed in [4] to teach the concepts of mean, median and mode in a manufacturing system setting.

MATLAB is used extensively to incorporate hands-on learning experience into a rather theoretical probability course. In [5], lab experiments such as tossing dice are used to teach Central Limit Theorem along with MATLAB computer simulations. MATLAB-based computer projects are introduced in [1, 6, 7] using applications from circuit analysis, digital communication, computer networks, digital image processing and digital speech processing. In [8], a special purpose Graphical User Interface (GUI) is built using MATLAB to teach probabilistic methods through a series of transformations applied to a data set to generate white noise.

In contrast to prior work, mini-projects focusing on applications from daily life are adopted in the random signal analysis course. Students perform theoretical analysis, computer simulation or data analysis to apply abstract course concepts to solve real world problems in these mini-projects.

B. Course Content

Random Signal Analysis in the School of Electrical, Computer and Energy Engineering at Arizona State University is a 3-credit hour 15-week course required by all electrical engineering students during their third year. It is intended to introduce the concepts of probability and random signals, and to discuss their applications to engineering problems. The course is taught in a traditional lecture format. Concepts are explained through examples. Additional worksheet problems are given to students to practice in-class if time permits. Students are assessed through homework and three exams.

The course schedule (not including exams) is shown in Table I including topics covered and the approximate time used. Six mini-projects are developed to help students learn the abstract course concepts through real world examples. Table I also shows when each mini-project is assigned in the curriculum. The details of the mini-projects will be discussed next.

TABLE I. COURSE CONTENT

Course Topics	Mini-Project
Set theory, Probability Axioms, Conditional Probability, The Law of Total Probability, Bayes Theorem, Independence (2 weeks)	1
Tree Diagrams, Counting Methods, Independent Trials (2 weeks)	2
Discrete Random Variables (PMF, CDF, Expected Value, Families of Discrete Random Variables) (2 weeks)	3
Continuous Random Variables (PDF, CDF, Expected Value, Families of Continuous Random Variables) (2 weeks)	4
Multiple Random Variables (Joint/Marginal PMF, PDF, CDF, Covariance, Correlation, Independence) (2 weeks)	5
Derived Random Variables, Central Limit Theorem, Laws of Large Number, Statistical Inference (Sample Mean, Confidence Interval, Binary Hypothesis Testing, P-Value) (2 weeks)	6

III. DESIGN AND IMPLEMENTATION

When designing the mini-projects, one option is to use applications from electrical engineering like many previous efforts from existing work. However, in order to perform in-depth probability and statistical analysis, domain knowledge needs to be introduced, which takes away valuable class time. Since the textbook [9] adopted in the course is written for electrical and computer engineering students, many examples and exercises in the textbook already draw from applications in the electrical and computer engineering domain. Therefore, it is decided to use daily life applications students are familiar with in the mini-projects instead.

Six mini-projects are added to the curriculum to align with the course topics and schedule, and to enhance student's understanding of abstract course concepts. As shown in Table I, the design goal is to have one real world example in every major topic area of the course evenly spread out throughout the semester.

As shown in Table II, the mini-projects fall into three categories: theoretical analysis, computer simulation, and data analysis. Theoretical analysis mini-projects are like expanded version of homework problems. Students apply knowledge learned in class to perform theoretical calculations. Computer simulation mini-project requires student to simulate random phenomena and compare results with theoretical calculations. In data analysis mini-projects, real world data sets are provided, and students perform statistical analysis on the data sets to make sense of data.

TABLE II. MINI-PROJECT TASKS

Mini-Project	Applications and Tasks
1	Finished Product Testing (Theoretical Analysis)
2	Powerball Lottery Game (Theoretical Analysis)
3	Servicing Customers (Theoretical Analysis and Computer Simulation)
4	Who Was Typing? (Data Analysis)
5	California Housing Prices (Data Analysis)
6	A/B Testing (Data Analysis)

Since the random signal analysis course is a junior level course, it is assumed that students already have some programming background. Therefore, for computer simulation and data analysis mini-projects, student can choose to use any programming language they are comfortable with. Some MATLAB programming examples are provided to students since they are already included in the textbook. In the end, most students choose to use MATLAB, Python and Excel for their mini-projects.

The content of the six mini-projects are described in detail next.

A. *Mini-Project 1: Finished Product Testing*

This mini-project familiarizes students with the Law of Total Probability and Bayes' Theorem through a real-world example.

Bayes' Theorem is used to update the predicted probabilities of an event by incorporating new information. It has widespread applications from physics to cancer research, ecology to psychology. In this mini-project, students need to help a manufacturing company choose between two testing methods [10]: one is less expensive but less accurate, and the other is more accurate but more expensive. Students must take into account the different costs and revenue involving selling a product: manufacturing cost, testing cost, sell price, return cost. Students need to calculate the average profit per product by creating a tree diagram and calculating probability. Bayes Theorem also needs to be applied to make recommendation. At the time when this mini-project is assigned, the concept of expectation has not been introduced. The instructor uses the average profit in this mini-project to briefly introduce the concept of expected value.

B. *Mini-Project 2: Powerball Lottery Game*

The goal of this mini-project is to let students practice counting methods in a real-world setting.

In all applications of probability theory, it is important to understand the sample space of an experiment. Students often have trouble determining the number of outcomes in the sample space of a sequential experiment. This mini-project outlines rules of the Powerball lottery game [11]. Students are asked to calculate the odds of winning different prizes and the probability of not winning any prize. To do this, students need to understand sequential experiments and sampling without replacement. And more importantly, they need to figure out how many different combinations, i.e., outcomes, are possible when drawing multiple white balls and a red ball with specific numbers on them.

C. Mini-Project 3: Servicing Customers

This mini-project intends to help students understand the Poisson process and its relationship to Bernoulli trial.

An important problem in many disciplines is the allocation of resources to service customers. In this mini-project, students are asked to help the manager of a supermarket to decide how many express check-out lanes to open in order to satisfy a pre-defined customer wait time requirement [12].

Customer arrivals are often modeled as Poisson processes. In this mini-project, students practice calculating probabilities using Poisson distribution. Instead of given the distribution parameter directly, students need to estimate customer arrival rate based on a week's customer arrival data. In addition, students apply the theory of splitting one Poisson process into multiple Poisson processes in a real-world situation where multiple express check-out lanes are added to a single lane. Moreover, students simulate the Poisson arrival process using Bernoulli trials which helps them understand the relationship between these two stochastic processes. To complete this mini-project, students propose the number of express lanes to open using theoretical analysis then verify through simulation if their recommendation would indeed satisfy the service requirement.

D. Mini-Project 4: Who Was Typing?

This mini-project gives students the opportunity to practice estimating the mean and variance of a normally distributed population. It also exposes students to binary hypothesis testing and likelihood ratio test.

Gaussian distribution is used widely in real-world applications. In this mini-project, students are given keystroke timing information from two different users typing the same paragraph. The keystroke timing information from an unknown user typing a different paragraph is also given. Assuming that the time taken by each user to type various keys is normally distributed, students need to estimate the parameters of these normal distributions from different users. A brief introduction to likelihood function, binary hypothesis testing and likelihood ratio test is then given to students. Students need to decide which user types the new paragraph by performing a likelihood ratio test, after performing parameter estimation from the real world data set.

E. Mini-Project 5: California Housing Prices

This mini-project lets students practice obtaining descriptive statistics from a dataset, such as minimum, maximum, mean, standard deviation and histogram. It also lets students explore the relationship between two random variables by calculating their correlation coefficient.

Analyzing a real-world dataset is a great way of connecting abstract probability concepts to concrete applications. Obtaining descriptive statistics from a dataset is often the first step to understand the data at hand and the necessary step to prepare data for potential machine learning tasks in the future. In this mini-project, students are given a dataset containing California Census data including location, population, income, house value, etc [13]. They are asked to compute various statistics of all attributes such as minimum, maximum, mean, standard deviation and histogram. They also need to analyze the dataset to find which attributes are the best predictors of house value by calculating their correlation coefficients.

F. Mini-Project 6: A/B Testing

This mini-project introduces students to the concepts of confidence interval and p-value.

Some of the most important applications of probability theory involves reasoning in the presence of uncertainty, i.e., analyzing the observations of an experiment to make a decision. Students are introduced to the concept of hypothesis testing in Mini-Project 4. Before this mini-project, students are given a formal lecture on binary hypothesis testing, confidence interval and p-value.

In this mini-project, students practice to make a decision using hypothesis testing and estimate how good their decision it by calculating confidence interval and p-value. Specifically, they are given the student assessment scores from two learning activities. They must determine which activity is more effective in improving learning outcomes by performing a binary hypothesis test. They are asked to calculate the confidence interval of the difference in the mean scores of the two activities as well as its p-value to support their decision making.

IV. Assessment and Results

The six mini-projects are incorporated in the course and piloted in the spring semester of 2020 with 62 students enrolled.

Two surveys are given to students to collect feedback to the mini-projects: one at the one third of the semester and the other at the end of the semester. There are 62 responses in the first survey and 55 responses in the second survey.

The survey questions are shown in Table III. The survey uses a 5-option symmetric disagree-agree scale. Students choose from "strongly disagree", "disagree", "neither agree nor disagree", "agree" and "strongly agree".

TABLE III. Survey Questions

Number	Questions
1	"Finished Product Testing" helps me understand the Law of Total Probability and Bayes Theorem.
2	"Powerball Lottery Game" helps me understand counting methods.
3	"Servicing Customers" helps me understand the relationship between Poisson arrival and Bernoulli trials, and how they can be used to model real-world customer arrival problem to satisfy service requirement.

Number	Questions
4	"Who was typing?" helps me understand how to model real-world data using a Gaussian distribution. I got to practice using likelihood ratio test to do hypothesis testing.
5	"California Housing Prices" gets me to practice obtaining descriptive statistics from a real-world data set, such as minimum, maximum, mean, standard deviation, histogram and correlation coefficient, and see how these statistics can help start to make sense of the data.
6	"A/B Testing" helps me see a real-world application of Central Limit Theorem and gets me to practice calculating confidence interval and p-Value, which are essential to statistical inference.

Besides the survey questions in Table III, there is a question in both surveys asking students the difficulty level of the mini-projects. Students choose from five options: too easy, easy, difficulty level is about right, hard, and too hard.

One free response question is included in the End of the Semester Survey asking students for their overall experience with the mini-projects, their favorites and least favorites, and suggestions for improvement.

A. Survey Results

Student's responses to the questions in Table III are shown in Fig. 1.

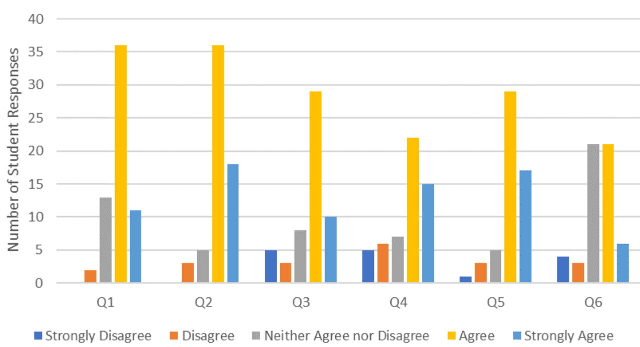

Fig. 1. Student responses to survey questions 1 through 6 in Table III

The figure shows that students overall have positive experience with the mini-projects. Out of all student responses, 76%, 87%, 71%, 67%, 84% and 50%, choose "agree" or "strongly agree" that the six respective mini-projects help them learn the corresponding course concepts. "Powerball Lottery Game" and "California Housing Prices" seem to do a better job while "A/B Testing" seems to be the least effective.

Fig. 2 shows student responses to the difficulty level of the mini-projects. Mini-project 1 and 2 are assigned before the One Third Semester Survey, and mini-project 3 through 6 are assigned during the rest of the semester. Survey results show that students find mini-project 3 through 6 more difficult. This is no surprise since mini-projects 3 through 6 require computer simulation and data analysis, while mini-projects 1 and 2 only involve theoretical calculations.

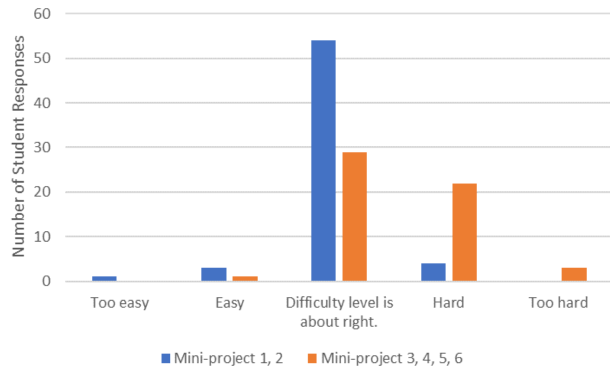

Fig. 2. Student responses to difficulty level of the mini-projects

Another point to consider when interpreting the survey results is that the responses regarding mini-project 1 and 2 were collected in the One Third Semester Survey, which was given at the end of February 2020, right before the pandemic started. The survey questions regarding mini-project 3 through 6 were given in the End of the Semester Survey. Students attending class remotely for two thirds of the semester could impact their ability completing mini-project 3 through 6.

As shown in Fig. 1 and Fig. 2, overall the survey results are encouraging, but more work is needed to improve student experience as will be discussed in the next section.

B. Student Comments

Student's comments given to the free response question in the End of Semester Survey provide additional insight into the overall impact of the mini-projects.

Overall students feel that the mini-projects are interesting and give the class more value. They think the mini-projects help them better understand the material that is presented in lecture. For example, they help them visually understand course materials, and allow them to see different ways to apply concepts learned in class. More importantly, the mini-projects help them "increase their confidence in probability."

Students particularly like the real-world applications because they allow them to know exactly why they are learning what they are learning and learn how to apply what they have been learning to real life scenarios. The mini-projects are useful because they employ theory to solve actual problems. "They all seem like things that one may actually do in industry."

Students feel that the mini-projects are both more valuable and more interesting than the tedious textbook problems. They think they are "a different type of assignment that stimulates thinking more and in a unique way". The required "bit of extra thinking" helps them think critically and their open-endedness force them to problem solve.

Some students struggle with the programming aspect of the mini-projects. They feel like the mini-projects are useful except they are expected to know how to program. It takes them a lot of time to try to remember how to code specific things. They think more refreshers and hints as to how to complete the assignments would be helpful. Despite the difficulty, many students think struggling to complete the mini-projects helps them understand the concepts better, and they end up enjoying them because they give practical examples to the course.

"A/B testing" receives the worst score from Fig. 1. One student shed some light on the reason behind it: "since we just went over the material and there are not many examples, it was difficult to know how to do it correctly." This is an accurate description, since the course topic related to this mini-project is covered during the last week of class before the final exam review. Students have only one week to complete this project before the final exam.

There are quite a few students complaining about the wording of the mini-projects. They think some instructions are vague and it is difficult for them to figure out what some of the mini-projects are actually asking for. They would take them less time if the instructions were more clear.

Out of all survey respondents, only one student complains that there could have been more of a focus on engineering related issues instead of general problems.

As for student's favorite mini-project, "California Housing Prices" tops the list because students feel that it is the most useful since it applies almost all calculations that they learn in class. "Who was typing" comes in second because students think the application seems the most interesting.

V. FUTURE WORK

Student's suggestions for improvement include making the mini-project instructions less vague and easily understandable. They also want more hint and help on the programming aspect of the mini-projects. In addition, more time and examples could be allocated and given to the last mini-project. These will be the focus areas of improvement in the next round of course offering. Another experiment to try out is to spend more time in lectures working on these mini-projects, i.e., to shift most of the problem-solving part of these mini-projects to in-class discussion utilizing teamwork and collaborative learning.

VI. CONCLUSION

Six mini-projects involving real-life examples are implemented in a random signal analysis course. Preliminary survey results show that most students enjoy the mini-projects. They acknowledge that the mini-projects help them understand abstract course concepts and apply them in real-world situations. They also help to improve their critical thinking and problem-solving skills. Although the initial student feedback is encouraging, more work needs to be done to improve the mini-projects in future course offerings.

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