ASSESSING STUDENTS' MATHEMATICAL CHALLENGES IN AN INTRODUCTION TO SIGNAL PROCESSING COURSE AND THE EFFECTS OF COOPERATIVE LEARNING

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***Abstract*—Signal processing is a specialization in electrical engineering which is often considered mathematically challenging for students. It is difficult in part due to the abstract nature of the topics, extensive use of mathematical formulations and mathematical limitations and deficiencies from prior courses. Professors have often identified deficiencies related to complex numbers and signal transformations, which are key for fundamental topics such as convolution and frequency analysis. Although previous work has assessed the effects of alternative teaching methods in this course, no prior studies have focus on complex numbers and signal transformations and their related misconceptions and deficiencies. In this work, we aim to identify students' mathematical deficiencies by building upon prior knowledge through a series of tutorials, designed following backward design, and which followed an informal cooperative learning approach. Data was collected through pre- and post- tutorial tests and self-confidence surveys. Results showed an overall increase in performance in post-tutorial assessments as well as increased reported self-confidence. Skills that remain more challenging are those requiring plotting mathematical functions.**

Keywords— *Informal Cooperative Learning, Signals and Systems, Signal Transformations, Complex Numbers*

1. INTRODUCTION

Introduction to Signal Processing, or Signals and Systems, is often considered a challenging course in electrical engineering. This course is crucial as it is the pre-requisite to signal processing and other courses relying on linear systems theory. Prior studies have reported challenges related to the abstract nature of the material presented in this class and its lack of direct connection with real life [1], [2]. Moreover, there is a consensus about the mathematical limitations, deficiencies and misconceptions [1], [2], [3], [4], [5].

Several works have addressed different issues in this course over the last decades. Buck and Wage [6]–[8] investigated conceptual understandings for this course and developed a concept inventory as a measure of conceptual understanding. So [2], presented a refined lecture method in which the instructor combined lectures and step-by-step problem solving, called "chalk-and-talk". This method was compared to power point lectures and it was found that the students' performance increased in the "chalk-and-talk" lectures. In another project, Vaz and Arcolano [9], studied the combination of outcome-based learning and assessments. In their work, they considered a series of projects which led students to reflect about their own learning, in connection with previous knowledge. Students formed a portfolio in which they reflected about their learning experiences. An emphasis on simulation tools was presented. In another work, McClellan and Rosenthal [10] used MATLAB graphical user interfaces to animate and visualize abstract concepts.

Although several works have investigated this course in a larger scope, professors have often identified mathematical deficiencies related to complex numbers and signal transformations. Such skills are relevant for topics such as convolution and frequency analysis. There is no prior literature assessing the study of such topics in the context of signals and systems theory, nor which are the particularities of such topics relevant to signals and systems that are the most challenging. These are topics covered on previous courses, but to what extent students understand and master the skills needed for applying those successfully in this class is not known.

The contributions of this study are two-fold. First, we aim to identify which skills related to complex numbers and signal transformations present major challenges for students and reveal related misconceptions. Since students are familiar with both topics from previous courses, we focus on scaffolding understanding to construct new skills upon what students know. Second, we aim to assess the effects of scaffolding and cooperative learning in students' self-confidence and performance in solving problems on such topics.

We proposed to have a series of tutorial sessions outside the class time to identify student's difficulties and provide the skills needed for the class. The tutorials are designed

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following backward design to ensure alignment of the teaching goals, assessments and the instruction [11]. Instruction followed an informal cooperative learning format, consisting of short lecture periods followed by short periods for individual and peer problem solving. This structure allows students to cognitively organize the material being taught, that is, to reason about the procedures they follow, and it allows both instructors and students to identify misconceptions and gaps in comprehension [12].

This paper is organized as follows. Section 2 presents the methods, in which backward design is described in more detail with respect to complex numbers and signal transformations, and the tutorial's structure is discussed. Section 3 presents results, from both qualitative and quantitative observations. Finally, in Section 4 conclusions are presented.

2. METHODS

In this section, we discuss how the tutorials were designed following backward design, and further details on their structure.

2.1 Backward design

In order to ensure alignment between instruction, assessments and teaching goals, tutorials are designed following backward design [11]. In backward design, the big ideas, knowledge or understandings, and learning goals are identified first. Once the learning goals are established, assessments are determined. Assessments should provide evidence that will help to determine whether the learning goals have been achieved. Finally, teaching methods that allow students to achieve the learning goals are selected.

2.1.1 Learning goals

In complex numbers, we want to ensure that students understand and are comfortable with the fundamentals of complex numbers, such as representations in the plane, conversion between coordinate systems (Cartesian and polar), properties of complex numbers and conjugate. More advanced goals include complex exponentials, Euler's relation, and plots of magnitude and phase. Complex exponentials and Euler's relation are extensively used in frequency analysis through the course.

In signal transformations, we aim to reinforce fundamental operations on the independent (time) variable: time-shift, time-reverse, and time-scaling; and dependent variable (amplitude): amplitude-shift, amplitude-scaling, amplitude-reverse. We want to emphasize on the combination of transformations, with emphasis on time transformations. Such transformations are fundamental signal manipulations and are the core of topics such as convolution.

2.1.2 Assessments

Assessments are the instruments to collect information and provide evidence on whether the goals are accomplished [11]. In this study, assessments included pre- and post- self-confidence surveys and pre- and post- short tests. The pre- and post- tutorial surveys asked students to rate how confident they were about solving problems related to each of the learning goals corresponding to that tutorial (complex numbers or signal transformations). The scale was "None", "Low", "Medium", and "High". Both pre- and post- tutorial surveys asked the same questions. Additional questions presented along with the self-confidence surveys included demographic questions, such as when they studied the topics for first time in the pre- tutorial survey, and feedback about the tutorials in the post- tutorial survey.

In addition to pre- and post- self-confidence tutorials, students completed pre- and post- short tests. For both complex numbers and signal transformations tutorial sessions, the problems presented covered the learning goals presented in Section 2.1.1. The problems fall in three main categories: mathematics' fundamentals (to assess potential mathematical misconceptions), basic problems that students are expected to master from prior courses, and more advanced problems that are emphasized in the class. Both pre- and post-tutorial tests follow the same structure.

In complex numbers, the misconceptions problems were true or false questions, which required students to explain their answer [13]. Such questions included to determine whether a complex number given in rectangular form is greater or less than another, like 2 + j3 < 3 + j4 ($j = \sqrt{-1}$). This problem requires them to know that in order to compare two complex numbers they need to consider either their magnitude, real, or imaginary parts separately. Another related problem was whether $\sqrt{-9} \times \sqrt{-9} = 9$, which aims to check if students recall order of operations and fundamentals of complex numbers.

Basic problems in complex numbers included converting a complex number from one coordinate system to another (polar to Cartesian and vice-versa), representing a complex number in the plane, and expressing complex exponentials such as $e^{j\frac{\pi}{2}}$ in rectangular form. More advanced problems included computing and plotting the magnitude and phase of $X(e^{j\omega}) = e^{j\omega}\cos(\omega)$ and $X(e^{j\omega}) = 1 + e^{j2\omega}$. Such problems require students to make use of exponent rules, Euler's relations, properties of complex numbers for computing the magnitude and phase, and skills representing mathematical expressions in plots.

For signal transformations, we asked students to identify the operation to which each character referred to in (i) $x(\pm at - b)$ and (ii) $\pm ax(t) - b$. The options are time-shift, time-reverse, time-scaling, amplitude scaling, amplitude shift and amplitude reverse. Basic problems required students to plot signals such as $x(t) = u(t-1) - u(t-2)$ and set up the integral to compute the area under the curve. Such problem requires both operations on the independent and dependent variables. More advanced problems included combination of transformations, such as if $x(\tau) = u(\tau - 2)$, plot $x(\tau)$, $g(\tau) = x(-\tau)$, and $g(\tau - t)$.

2.2.3 Instruction Method

The last step in backward design is determining the instruction methods [11]. Instruction methods include lecture, informal cooperative learning, active cooperative learning, flipped classroom, and problem-based learning. For this research, we chose informal cooperative learning (ICL). In ICL, the instruction time is allocated in intervals consisting of short periods of lecture, followed by individual and peer activities [12]. Figure 1 shows a sample structure of ICL. Such structure allows to focus the student's attention on the material to be learned, set up an environment that is conducive to learning, and provides time for problem solving and conceptual questions. By interleaving short periods of lecture and discussions with problem solving, students can cognitively organize the material being taught. Moreover, it is possible to identify and correct misconceptions and gaps in comprehension right when they occur. In addition, this method provides personalized learning experiences to students [12].

Distinct teaching methods are more appropriate for different learning goals. We choose ICL over other teaching methods since this is the instruction method that provides the mechanisms to achieve our objectives. Lectures are appropriate when it is desired to share information that is not available around and when the contents are too complex that the students cannot learn them on their own. Active cooperative learning, on the other hand, is appropriate when peer instruction is a priority, and knowledge is discovered and constructed by students. In flipped-classroom, the place of homework and lecture is replaced, and students are responsible of their own learning. Problem-based learning requires the learning process to begin with a problem, and the students determine the tools and procedures more appropriate to solve such problem. Since in this work students are already familiarized with the material and the goals is to identify difficulties and construct new knowledge upon what students know, we follow ICL.

2.2 Tutorials

2.2.1 Complex numbers

Table 1 shows the allocation of time intervals for complex numbers. The first and last 15 minutes were allocated for consent forms and pre-tutorial tests. The first block focused on stablishing the foundations, including coordinate systems, representations in the plane and the application of properties of complex numbers. Once the fundamentals were stablished and questions were addressed during the discussion of related pre-assessments and the individual problem solving and group discussion time intervals. Sample problems include converting a complex number in polar coordinates $z_1 = 5e^{j\left(\frac{\pi}{3}\right)}$ to rectangular coordinates, which requires that the students identify the complex number in the form $z = re^{j(\theta)}$, and employ the equations to obtain the x and y coordinates. Another sample problem is to compute $\int_0^{T_o} \left|e^{j\omega_o t}\right|^2 dt$. Such problem requires the students to know the properties of complex number as well as combining the integration with a complex exponential.

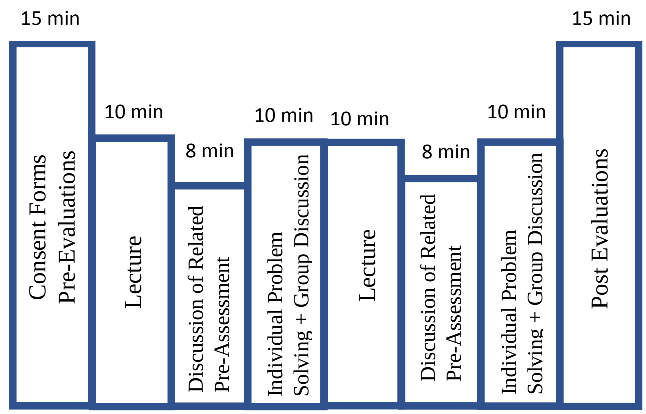


Figure 1. Informal cooperative learning focused discussions. Adapted from [12].

Table 1. Time allocations and activities for complex number tutorials following an informal cooperative learning structure.

Time Duration (min)	Activity
15	Consent forms and pre-tutorial tests
10	Lecture: Basics of Complex Numbers
8	Discussion: Related pre-tutorial test problems
10	Individual problem solving and group discussion
10	Lecture: Complex exponentials and Euler's identity
8	Discussion: Related pre-tutorial test problems
10	Individual problem solving and group discussion
15	Post-tutorial tests

The second block focused on Euler's identity and advanced operations involving complex exponentials. Sample problems include re-expressing $x(t) = 1 + \frac{1}{4}e^{j2\pi t} + \frac{1}{4}e^{-j2\pi t}$ in to a simpler form and computing the magnitude and phase of $x(t) = e^{j2t} + \frac{1}{4}e^{j3t}$. Structuring the instruction in this manner allowed students to immediately apply what they were learning and gradually increase the level of difficulty. In the first problem, Euler's identities can be applied directly. The second problem requires more expertise and imagination using the laws of exponents to factorize the exponentials.

2.2.2 *Signal Transformations*

Table 2 shows the structure of the tutorial sessions on signal transformations. During the first block, the time-shift, time-reverse, and time-scaling operations were revisited within the lecture time. Immediately after that, related problems presented in the pre-assessment were discussed and students were provided with a period of time for individual problem solving followed by group discussions. The problems focused on plotting functions such as $x(t-3)$ (time-shift), $x(3t)$ (time-scaling), and $x(t) = u(t-1) - u(t-2)$ (time-shift). The latter problem requires students to combine both time and amplitude operations, which often pose difficulties to students, as well as working with the step function, which they have already seen in class. During the second block, we focused on combinations of transformations, of the type $x(\pm t \pm b)$ and $x(at \pm b)$. Examples include if $x(t) = u(t+1) - u(t-2)$, plot $x(-t-2)$ and if $x(t) = u(t+1) - u(t-2)$, plot $x(2-t)$.

Table 2. Time allocations and activities for complex number tutorials following an informal cooperative learning structure.

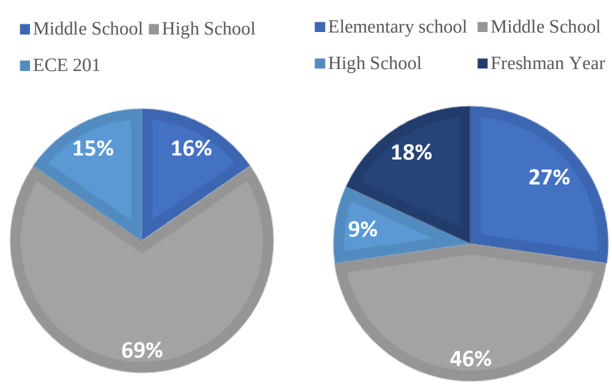
Time Duration (min)	Activity
15	Consent forms and pre-tutorial tests
10	Lecture: Fundamentals of signal transformations
8	Discussion: Related pre-tutorial test problems
10	Individual problem solving and group discussion
10	Lecture: Rules of combinations of transformations in the independent variable.
8	Discussion: Related pre-tutorial test problems
10	Individual problem solving and group discussion
15	Post-tutorial tests

3. RESULTS

This project was approved by the Institutional Review Board (IRB). All participants signed a consent form and all data was de-identified. A total of N = 24 students participated in the study: 12 participated in the complex number tutorials, 12 participated in the signal transformations tutorials, 4 students participated in both tutorials.

3.1 *Demographics*

We collected demographic information in which we asked students when they learned about complex numbers and signal transformations for first time. Figure 2 (a) show that most students learned complex numbers for first time in high school (69%) with a few students reporting basic circuits as the first time learning about complex numbers. Topics related to signal transformations or operations on functions have a wider time span (Figure 2 (b)), with some students reporting they have seen such topics for first time in elementary school and others in their freshman year.



(a) (b)

Figure 2. Demographics of when students learned (a) complex numbers, (b) signal transformations, for first time.

3.2 *Reported self-confidence*

Figure 3 and 4 show the reported self-confidence from the pre- and post- tutorials, respectively. As observed on Figure 3, the skills students reported to be less confident prior the tutorial are complex exponentials and Euler's identities, with no one reporting high confidence. This changed considerably in the post-tutorial surveys, where no student reported to lack of total self-confidence. In order to assess the statistical significance of self-confidence responses, each response was assigned a score as follows: "None" = 0, "Low" = 1, "Medium" = 2, and "High" = 3. Skills that reported a significant increase in self-confidence are representations in the plane ($p < 0.05$), conversions between coordinate systems ($p < 0.05$), complex exponentials ($p < 0.001$) and Euler's identities ($p < 0.001$). Conjugate, properties of complex numbers and arithmetic are all basic skills and did not show a significant increase in reported self-confidence.

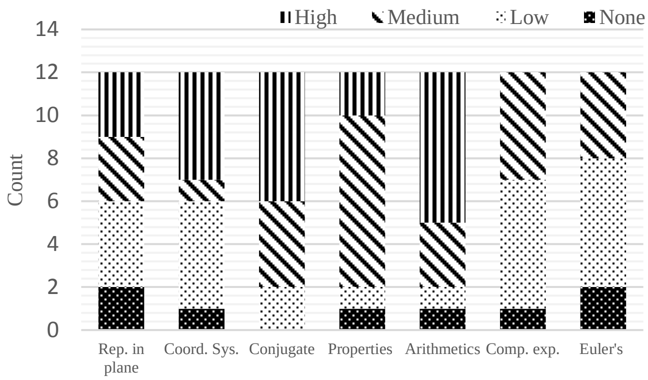


Figure 3. Reported pre-tutorial self-confidence in complex numbers.

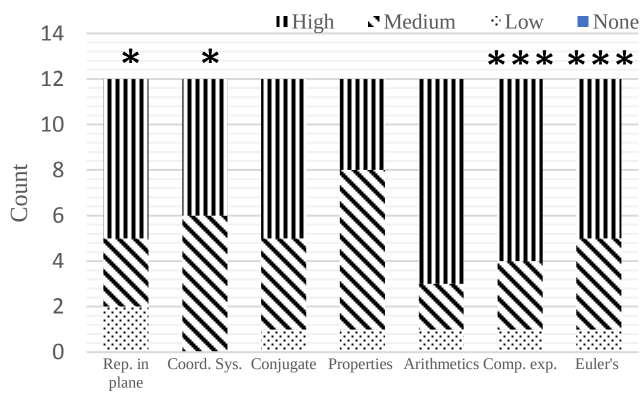


Figure 4. Reported post-tutorial self-confidence in complex numbers. * $p < 0.05$, ** $p < 0.01$, and *** $p < 0.001$.

Similarly, Figure 5 and 6 show the reported self-confidence from signal transformations tutorials. Prior to the tutorial, signal sketching was the skill students reported being the less confident, while operations on the independent and dependent variables resulted mostly on medium self-confidence. This changed drastically in the post-tutorial surveys, where most students reported high self-confidence for all skills ($p < 0.01$).

3.3 *Tutorial assessments*

To qualitatively assess the effect of the tutorials on the student's performance in solving problems, we considered the overall score from pre- and post- tutorial tests. The rubric

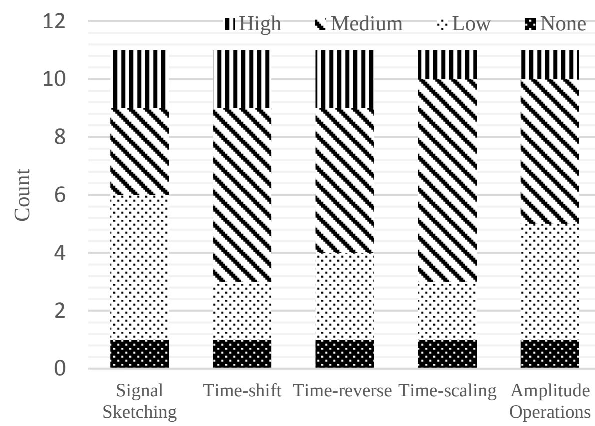


Figure 5. Reported pre-tutorial self-confidence in signal transformations.

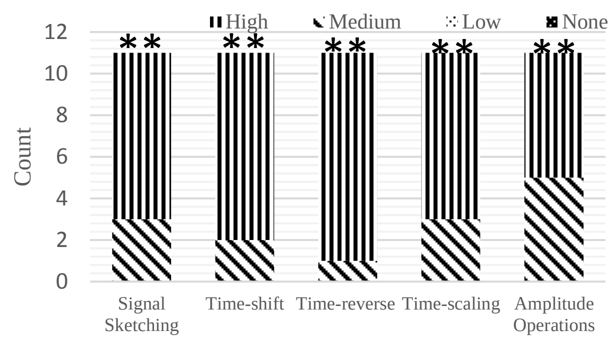


Figure 6. Reported post-tutorial self-confidence in signal transformations. * $p < 0.05$, ** $p < 0.01$, and *** $p < 0.001$.

was such that each problem counted for a maximum of 2 points. Table 3 summarizes the rubric used while grading such problems. The maximum total points per problem was 2 points. A 2 was obtained if the problem was solved as expected, including explanations (if required), steps demonstrating how to arrive at the solution, and correct plots with labels. A 1 was obtained if some steps toward the solution were missing, some labels on plots or explanations (if required). A 0 was obtained if there was no evidence about how the solution was obtained, or plots were missing.

We computed the improvement gain from pre- to post-assessment (Buck and Wage, 2005) which is given by

$$\langle g \rangle = \frac{Post - Pre}{100 - Pre} \tag{1}$$

The improvement gain $< g >$ is between 0 and 1, with an improvement gain equal to 0 when there is no gain from pre- to post- scores and is a maximum of 1 in the case that the pre- equals 0 and the post equals 100, which reflects the maximum improvement gain. In the case of both the pre- and post- being equal to 100 the improvement gain is 0.

Table 3. Rubric for pre- and post- tutorial tests.

Criteria (2 points max.)	Achievement Level 3: Exceptional (2 points)	Achievement Level 2: Acceptable (1 points)	Achievement Level 1: Not Acceptable (0 points)
For any problem	• Correct solution. • Explanation (if required) • Steps demonstrating how to arrive at the solution • Correct plots with labels.	• Some steps toward the solution are missing. • Missing some labels on plots. • Missing explanations (if required).	• No evidence on how the final solution was obtained. • Missing plots.

Table 4 shows the grand average of pre- and post- test scores from the complex numbers tutorials. The post-test scores are significantly greater than the pre-test scores ($p < 0.01$, t-test), and the improvement gain was $\langle g \rangle = 39.70 \pm 35.92$. Similarly, Table 5 shows the grand average of pre- and post- test scores from the signal transformation tutorials. The post-test scores are significantly greater than the pre-test scores ($p < 0.01$, t-test) as well, and the improvement gain was greater than that from complex numbers, $\langle g \rangle = 55.00 \pm 39.90$.

Table 4. Complex numbers pre- and post- tutorial scores.

Pre-test	Post-test	p-value	<g>
37.50 ± 30.67	65.97 ± 13.03	0.004	39.70 ± 35.92

Table 5. Signal transformations pre- and post- tutorial scores

Pre-test	Post-test	p-value	<g>
69.31 ± 15.16	88.63 ± 8.75	0.002	55.00 ± 39.90

Figure 7 shows the performance in grand average counting with different topics from complex numbers. Skills with significant improvement are properties, Euler's identities and complex exponentials. Misconceptions, representations and plots did not show significant improvement. An improvement on mathematical misconceptions would require addressing a broader range of related skills, which is difficult to predict. Plots of magnitude and phase still present a major challenge to students. In addition to difficulties in visualizing mathematical functions, the concepts of magnitude and phase are even more abstract and less intuitive. In signal transformations time-scaling was the only skill that did not show a significant improvement on the tests scores (Figure 8).

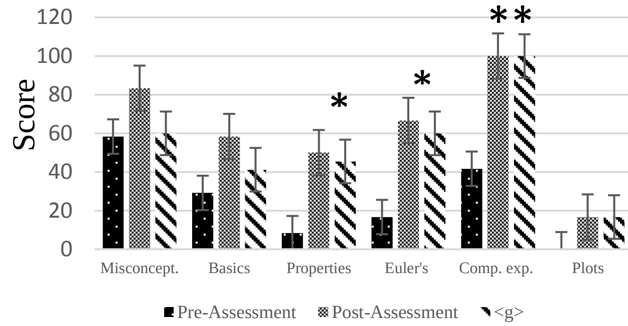


Figure 7. Complex numbers tutorial assessments by skills. * $p < 0.05$, ** $p < 0.01$, and *** $p < 0.001$.

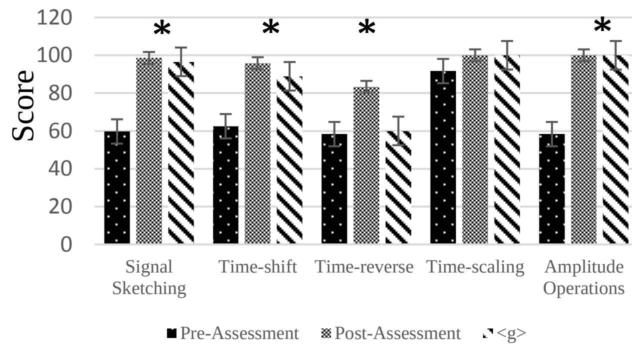


Figure 8. Signal transformations tutorial assessments by skills. * $p < 0.05$, ** $p < 0.01$, and *** $p < 0.001$.

3.4 Students' feedback

In the post-tutorial surveys, we asked students to provide feedback about the tutorials. All comments were positive, including: "Learned properties I did not previously know", "Cleared my confusion", "I liked that there were a lot of examples and that you asked for questions throughout", "covered relevant concepts not covered in class", "It had plenty examples that I had time to solve on my own before we did them as a group", "gave me a good understanding of material I forgot". In addition, students appreciated the opportunity to solve problems by themselves in the classroom, as well as the peer instruction and discussions.

4. CONCLUSIONS

Incorporating cooperative learning activities while teaching complex numbers and signal transformations allows students to reinforce mathematical skills needed for Introduction to Signal Processing courses and increases their self-confidence in solving related problems. Structuring the tutorials to follow an informal cooperative learning instruction method allowed to scaffolding understanding by building new knowledge upon prior knowledge students may have also need to reinforce. Results suggest that these tutorials would be beneficial for students in identifying skills that need reinforcement. Topics reflecting major challenges for students include those involving skills related to representing mathematical expressions visually. More specifically, computing and plotting the magnitude and phase of complex numbers needs to be reinforced. In addition, student's feedback from the tutorials was positive, and they valued the active learning activities, reinforcing topics that are relevant for the class, and the peer instruction.

5. ACKNOWLEDGEMENTS

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