Multiple Representations in Physics: Deliberate Practice Does Not Improve Exam Scores

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***Abstract* — Physics problem solving requires the use of verbal, pictorial, diagrammatic, and mathematical representations in order to translate a problem into its underlying mathematical components. This study investigated the effect of explicit deliberate practice on representation skills in introductory physics at the University of Louisville. In a controlled, randomized design, physics students received either deliberate practice or traditional solve-though practice weekly throughout the semester. The deliberate practice instruction focused on translating from verbal to diagrammatic and diagrammatic to mathematical representations. The deliberate practice problem solving intervention had no impact on exam performance or an encoding task developed to measure facility with diagrammatic representations of physics problems. However, performance on the encoding task positively predicted exam performance across conditions. These results suggest that deliberate practice on multiple representations did not demonstrably improve exam scores. However, the ability to encode problem features predicts exam performance more generally.**

Keywords— physics education, multiple representations, problem solving, deliberate practice.

I. Introduction

A first-semester physics course introduces students not only to fundamental physics principles, but also to new problem-solving [1–3], mathematization [4], and representational [5, 6] skills. Each of these skills is refined over time, and research has indicated that physics novices and experts differ significantly in the way they approach and solve novel problems [7–10]. Chi, Feltovich and Glaser [7] found that immediately after reading through a physics problem, expert physicists (professors) categorize it using the fundamental physics concept by which it can be solved, whereas novices (students who had taken only one semester of physics and received an 'A') categorize the problem by its surface features (such as an inclined plane, spring, or pulley). Kohl and Finkelstein [8] monitored the act of solving physics problems, and found that experts move between representations more quickly than novices, and also choose representations strategically at specific points in the problem-solving timeline. Their work indicates the importance of both when and how multiple representations are used in problem solving.

The multiple representations used in solving physics problems include verbal, visual, pictorial, diagrammatic, and mathematical (e.g. [5]), typically in that order, if they are all needed. These representations are useful because they organize information differently [6, 11], facilitating goal formulation and memory load during the problem-solving process [2]. Physics education researcher Van Heuvelen [12] stated that a diagrammatic representation "summarizes the prominent features of a process while removing noisy details that distract from understanding – in short, the diagram contributes to understanding and to physical intuition" (p. 891).

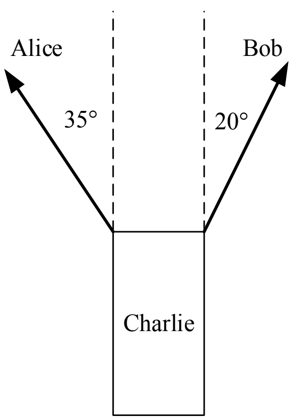
For example, Newton's Second Law (NSL) is one of the fundamental concepts of physics introduced early in a first-semester course. NSL states that the acceleration of an object is proportional to the net force on the object and inversely proportional to the object's mass: $a = \sum F / m$, or $\sum F = ma$. An example problem based on NSL is given in Fig. 1. The example problem is in verbal form, and is highly contextualized (Fig. 1a). Prior to learning physics, students most likely will represent the scenario in three dimensional detail as they would see it visually (Fig. 1b). To solve the problem, students must translate the important information into a pictorial diagram (Fig. 1c) and then a vector diagram (Fig. 1d). A vector diagram is a primary representational structure of physics problems because of its utility. One type of vector diagram is a free-body diagram which represents an object as a single point on the origin. The vector diagram in the example problem is a free-body diagram. A vector, whether drawn with an arrow or written as a set of x and y components, contains both direction and magnitude. From a vector diagram, it is easier to break apart the components for a mathematical representation, as shown in the subscripts of Fig. 1e. Drawing the vector diagram helps students determine correct trigonometric functions to apply in the equations. By constructing the problem using multiple representations, the path to a solution becomes clear.

As students work through the multiple representations of many problems, they can develop schemas for typical problems and procedures. Schemas are long-term memory structures that organize information about an item, situation, or task [13]. By allowing students to readily access related concepts and procedures, schemas reduce the demand on students' working *(a)* *Verbal Representation:*

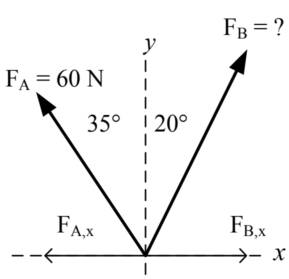
Alice and Bob are pulling their son Charlie on a sled. Alice is pulling the sled with a force of 60N at an angle of 35° to the left of the forward direction. Bob is pulling the sled at an angle of 20° to the right of the forward direction. With what force should Bob pull the sled so that the sled will move straight forward?

(b) *Visual Representation:*

(c) *Pictorial Diagram Representation:*



(d) *Vector (free-body) Diagram Representation:*



(e) *Mathematical Representation:*

$$\sum F_x = -F_{A,x} + F_{B,x} = 0$$

$$F_{A,x} = 60\ [N] * \sin(35°)$$

$$F_{B,x} = F_B * \sin(20°)$$

Fig. 1: An example problem with multiple representations.

memory—the temporary store that enables students to attend to and work with the problem at hand [14]. By creating organized schemas, students that gain expertise in multiple representations should be able to remember more when first reading through a problem or seeing a diagram. Experts may rely more on mental representations than external representations because of their advanced schemas [7]. Entry-level students, however, must first become familiar and flexible with multiple representations in order to begin gaining this expertise.

Van Heuvelen [12] suggested that teaching the use of multiple representations explicitly may be best for student understanding. The ability to extract and re-organize the information may be a skill that would benefit from explicit definition and practice. *Deliberate practice* is a teaching technique that uses a series of activities designed to improve the current level of performance and has been shown to benefit skills required for expertise [10]. In this case, deliberate practice is repeating the action of translating between the multiple representations in physics. No research has investigated the impact of deliberate practice in physics with a controlled experimental design. Kohl, Rosengrant, and Finkelstein [9] investigated two different courses at two universities, one with a strongly-directed approach to multiple representations, and one with a weakly-directed approach. They found that, for success on a challenging problem, it was critical that students draw a correct free-body diagram. Students in the weakly-directed course were less likely to do this even after drawing a correct pictorial diagram. However, students in the strongly-directed approach had more difficulty drawing the correct pictorial representation prior to drawing a free-body diagram. The authors were unable to compare results with statistical analysis because the question was administered differently in each course (multiple choice or free response). In their comparison between courses on the likelihood of multiple representation use, they found that both courses had a very large percentage of students who did use diagrams (approximately 90%). They use this finding to conclude that multiple representation use can be successfully taught in multiple ways. The authors noted several limitations of the study, such as different student populations and different teachers, as well as the difference between assessment question types.

The main objective of this study was to use a controlled experimental methodology to measure the effect of an in-class, multiple representations deliberate practice intervention on student exam performance.

II. Methodology

This study investigated deliberate practice with multiple representations in a randomized, controlled experimental design. Students in a first-semester algebra-based physics course were randomly assigned to one of two groups: *deliberate representations practice*, or *solve-through practice*. Groups met in different classrooms for one class per week. In the deliberate practice classroom, the instructor selected specific representational steps of the solution method and had students practice only one step for several problems in a row. In the solve-through classroom, the instructor went through problems step by step from beginning to end. We expected that students in the deliberate practice condition would develop stronger schemas than students in the solve-through condition. In order to analyze this, outcome measures included performance on unit tests and the final exam. Additionally, we designed encoding tasks that included diagram re-draw questions as well as verbal-to-diagram multiple choice questions. Encoding tasks were administered just prior to each of the exams.

MANOVAs were used to compare the two student groups in performance on the four exams and encoding tasks. We expected that the deliberate practice group would outperform the solve-through group on all outcome measures, perhaps with greater differences over time, or differences only occurring in later assessments. However, due to the exploratory nature of this study, it was also possible that no difference between conditions would be found. Students may need more than 1-hr per week with this intervention to develop enough expertise to differ from their peers on assessments, or the assessments themselves may not have been designed to capture any differences. This would be important to know as well. A third hypothesis was that, for both conditions, performance on the encoding task would predict performance on the exams.

A. Participants

All students who registered for *Fundamentals of Physics I* at a large U. S. Midwestern university were participants in the study ($N = 111$). Student data was only included in the analysis if the student had attendance of at least 5 out of 10 experimental class days and took all four exams ($N = 87$). Additionally, one low-performing outlier was removed from the analysis (final $N = 86$). Other similar cuts showed similar results, and 5/10 was selected to preserve N while still requiring students to have multiple exposures to the intervention.

B. Course Format

The *Fundamentals of Physics I* course is the first of an algebra-based introductory physics sequence. Course grades were based on homework and four in-class exams, spaced throughout the semester. The final exam was cumulative and worth more points than the other three exams. Class met for 50 minutes on Mondays, Wednesdays and Fridays. Pre-lecture materials were assigned to be completed prior to every class. These assignments were intended to cover material that students could learn individually to give them a base set of knowledge to help them understand the material covered in class. Most of the pre-lecture assignments included a preparatory short video and an assigned reading. Additionally, homework was assigned weekly. Class time was spent discussing conceptual examples and actively working problems. Review days were given preceding exam days, when students would be asked “Which type of problem is this?” and were instructed to vote between four options listed on the screen. Multiple choice answers included “Conservation of Energy,” “Projectile Motion,” and other physics problem categories. This review was intended to consolidate and organize students’ understanding of different types of problems. At the beginning of the review day, an encoding task was given, framed as a “warm-up” practice for the students.

C. Procedures

The experimental manipulation occurred on Friday classes throughout the semester. Students were randomly divided into two groups at the beginning of the semester and were instructed to go to one of two classrooms on Fridays. In both classrooms, students were instructed to sit in randomized groups of three (shown on the projector screen at the beginning of class). Groups changed every week. In the *deliberate practice* condition, students were given specific tasks to perform within a series of problems. For example, students would be instructed to “draw a pictorial representation of the problem” for three different but similar problems in a row. After practicing this step three times, the next prompt would be to “draw a vector diagram based on the picture shown” and students would be asked to do this step of the solution procedure again three times in a row with different problems. The representations practiced were *pictorial*, where every item described in the problem was represented (Alice, Bob, Charlie, and sled in Fig. 1c), *diagrammatic*, in a free-body or vector diagram (Fig. 1d), and *mathematical* (Fig. 1e). In the *solve-through* condition, students worked through problems from start to finish. The instructor and a physics graduate student Teaching Assistant alternated each week between classrooms to avoid an instructor bias within the study.

D. Materials

A series of eight problems was prepared for each week of the intervention (10 Fridays in total). One slideshow was made for the *solve-through* group that consisted of a series of problems, followed by full solution procedures. A separate slideshow was

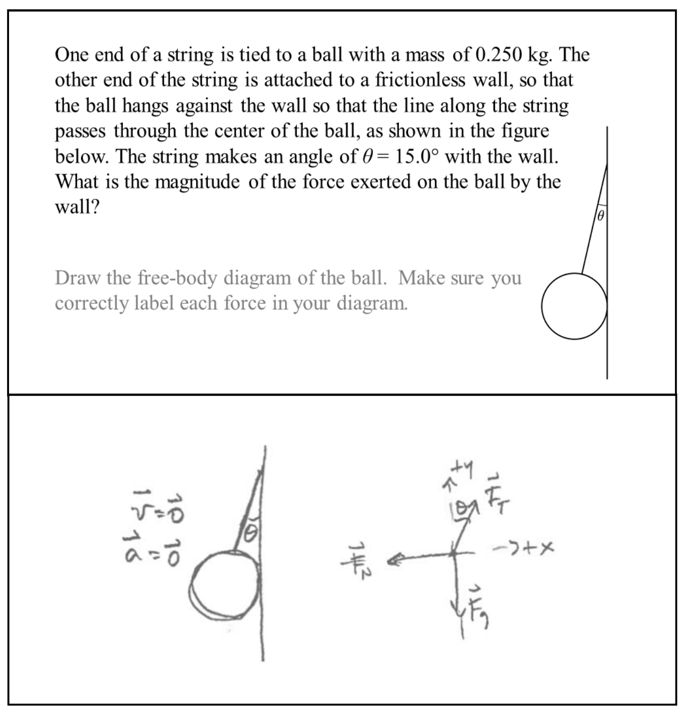


Fig. 2. Example of presentation style in the *deliberate practice* condition. Below the full problem, in red, is the request for a single representational step: drawing a free-body diagram (a type of vector diagram). This step is then reviewed on the following slide.

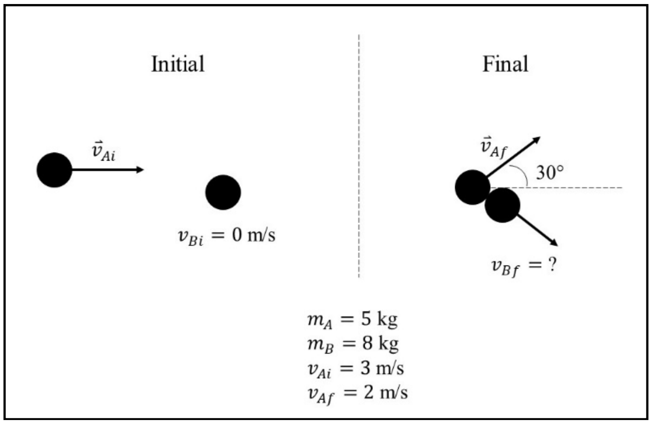


Fig. 3. Example of a diagram encoding problem with critical shape, variable, and question information and distracting values. This picture was presented for 15 seconds, and students were instructed to re-draw as much information as they could remember.

Table 1:
Encoding Shape Rubric Example

Give the student a 1 only if they have:

- An initial drawing and a final drawing;
- The initial drawing has two circles; the circle on the left has a horizontal velocity vector pointing to the right.
- The final drawing shows the two circles colliding; each circle has a velocity vector at some angle from the horizontal, one angled up from the horizontal and the other angled down from the horizontal.

Give the student a 0.5 if they do one of the following:

- In the initial drawing, the nonmoving circle is forgotten;
- In the final drawing, one vector is horizontal;
- One shape (circle or vector) defined above is incorrect.

Give the student a 0 if they:

- Do not include arrows on the moving balls or have incorrect arrows (i.e. pointing straight up);
- More than one element (circle or vector) is incorrect.

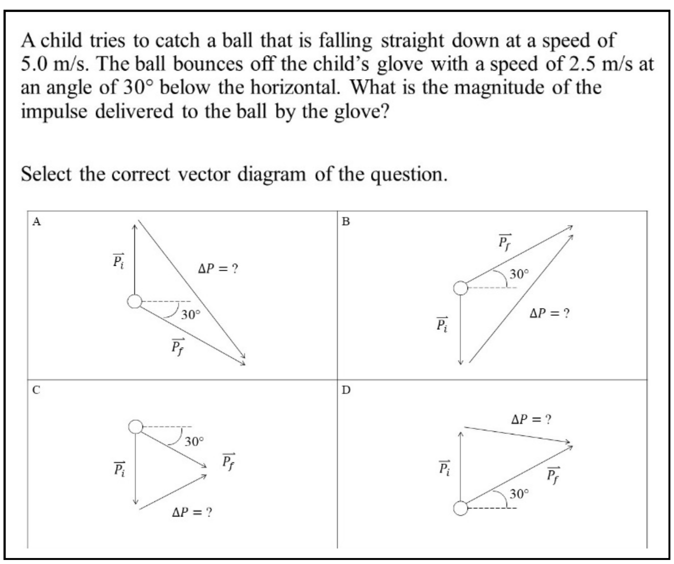


Fig. 4. Example of a multiple choice encoding problem. A verbal problem was presented for 20-30 seconds with a specific representational step of the solution method queried below it. Students then selected the correct representation from four options given on the next slide.

made for the *deliberate practice* group which had a problem with the intended step explicitly stated (see Fig. 2), followed by that specific step worked out. In both conditions, the instructor would show a problem on the screen and students would work on it in groups. After students had generally completed the given problem, the instructor would review the solution step(s), and then move on to the next problem. Typically, the *deliberate practice* group was exposed to at least one step for all 8 problems in the 50-minute period. For example, Step 1 might be presented for problems 1-3. Step 2 would then be presented for problems 3-6, Step 3 would be presented for problems 6-8, and then Step 4 would be presented for problem 8. The *solve-through* group worked through all steps of three-to-four problems. Exposure to more problems in the same amount of time was an additional intended benefit of the *deliberate practice* condition.

The *encoding task* consisted of three diagram problems and three multiple choice problems. In the diagram problems, students were instructed to re-draw a pictorial diagram that was presented on the screen for 10-15 seconds (see Fig. 3). The diagrams were intended to be complex, with more information included than was possible to encode and remember in the amount of time given. Diagrams included critical shape, variable, and question information as well as extraneous values (values that were reasonable to include in the diagram but weren't critical to understanding the problem, and considered to be distractors). Student memory of each of these pieces of information was coded separately for data analysis, and we were most interested in performance on the shape factor, as that would most likely detect schemas.

The encoding task multiple choice questions were verbal questions shown for 20-30 seconds, followed by four possible diagrams, pictorial or vector, depending on the problem (see Fig. 4). Students were instructed to find the matching representation for the word problem.

E. Encoding Task Coding

The diagram portion of the encoding task was coded by two research assistants. Each diagram question was coded for four different elements: shape, critical variables, question, and values. Each element was coded with a 1, 0.5, or 0, using a detailed rubric. This study specifically focused on the shape component, which was expected to demonstrate development of schemas. The shape score was based on whether or not the student drew the correct shapes and vectors with the correct directional movement or forces indicated. There were often many different vectors and shapes in a single problem, and a score of 1 indicated that all shapes and vectors were present and correct. A half point was assigned if only one item was missing or incorrect, or a specific misconception occurred, and 0 was given if there were two or more items incorrect. The shape rubric for the example shown in Fig. 3 is shown in Table 1. There was moderate agreement between the two coders, (Cohen's $\kappa = 0.42$). Results reported here are based on the coder with more physics experience.

F. Analyses

One-way multivariate analyses of variance (MANOVAs) were conducted to assess the impact of deliberate practice on performance on (a) the exams (b) the encoding shape scores, and (c) the encoding multiple choice scores. If deliberate practice enhanced physics schema acquisition, then there would be a significant main effect of condition in any or all of these outcome variables. Specifically, exam performance across all four exams should be greater in the deliberate practice condition than in the solve-through condition. If the deliberate practice was additive over time, differences may only appear on later exams and encoding scores.

In addition to examining condition differences, the relationship between encoding scores and exam scores were examined using regression analyses, collapsing across conditions.

III. Results

Results from the study of deliberate representations practice in the physics classroom are described in three sections based on outcome variables: results of condition, relationship between encoding and exam performance, and instructor observations.

A. *Deliberate Practice MANOVAs*

The main effect of condition was not significant for any outcome variable: exam scores, $F(4, 81) < 1$, Wilks' $\Lambda = 0.99$, $p = 0.958$; encoding shape scores, $F(4, 46) = 1.34$, Wilks' $\Lambda = 0.90$, $p = 0.271$; and encoding multiple choice (M.C.) scores, $F(4, 45) = 1.01$, Wilks' $\Lambda = 0.92$, $p = 0.410$. Means and standard errors (SE) are reported in Table 2. The number of students is lower for encoding scores than exam scores because some students missed one or more review day, but individual ANOVAs with all participants for each encoding task confirmed no statistical differences due to condition.

Table 2:
Descriptive Statistics Based on Condition

	Deliberate Practice		Solve-Through	
Assessment	*N*	Mean (*SE*)	*N*	Mean (*SE*)
Exam 1	42	0.76 (0.17)	44	0.76 (0.16)
Exam 2	42	0.68 (0.20)	44	0.69 (0.21)
Exam 3	42	0.80 (0.18)	44	0.82 (0.18)
Exam 4	42	0.84 (0.13)	44	0.83 (0.12)
Encoding				
Shape 1	27	1.00 (0.14)	24	1.04 (0.15)
Shape 2	27	2.12 (0.15)	24	1.94 (0.16)
Shape 3	27	2.30 (0.11)	24	2.52 (0.11)
Shape 4	27	2.30 (0.13)	24	2.38 (0.14)
M.C. 1	27	2.31 (0.16)	24	2.04 (0.16)
M.C. 2	27	2.20 (0.18)	24	1.96 (0.18)
M.C. 3	27	1.27 (0.18)	24	1.42 (0.18)
M.C. 4	27	2.27 (0.15)	24	2.46 (0.15)

B. *Regression – Encoding Scores and Exam Performance*

Because no differences based on condition were found, subsequent analyses were conducted collapsing across this factor. This analysis examined the general effects of encoding ability on exam performance. A backward regression method was used. In this method, a series of analyses is performed, starting with some potential variables and removing non-significant variables, one at a time. The variables included initially were encoding shape and multiple choice score, as these were expected to reflect schema acquisition.

Regression analyses showed that for three of the four exams, encoding multiple choice score was not a significant indicator of exam performance (Exam 1: $\beta = 0.012$, $p = 0.917$; Exam 2: $\beta = 0.102$, $p = 0.380$; Exam 4: $\beta = 0.11$ $p = 0.369$). In Exam 3, multiple choice score did predict exam scores ($\beta = 0.22$, $t(73) = 2.08$, $p = 0.041$).

Encoding shape score significantly predicted exam score and explained a significant proportion of variance in all exams (Exam 1: $\beta = 0.33$, $t(68) = 2.87$, $R^2 = 0.11$, $p = 0.005$; Exam 2: $\beta = 0.27$, $t(72) = 2.33$, $R^2 = 0.07$, $p = 0.023$; Exam 3: $\beta = 0.36$, $t(73) = 3.28$, $p = 0.002$; overall $R^2 = 0.18$; Exam 4: $\beta = 0.23$, $t(69) = 2.00$, $R^2 = 0.06$, $p = 0.049$).

C. *Instructor Observations*

Although no significant differences were found between mean exam performances, the instructor anecdotally observed differences in students' attitudes during and following the Friday sessions throughout the semester. Because the deliberate practice group had shorter problems, it only took 5 minutes to review the first step of the problem and get everyone engaged. By the third problem, students were generally working steadily and confidently. In the solve-through group, some students would wait until the answer was given until they worked on their own, which was nearly 15 to 20 minutes into the class. Also, immediately following the weekly manipulations, deliberate practice students walked out of the classroom with positive comments, appreciating the exposure as well as a sense of achievement getting through all of the problems quickly, whereas the solve-through students did not express the same positive feedback.

IV. Discussion

A. *Deliberate Practice*

We did not find significant differences in exam scores between the students who received deliberate practice on representations and the students who received traditional solve-through practice. These findings were surprising, given the anecdotal differences observed between students' attitudes in the different groups. Specifically, the deliberate practice group appeared to be more comfortable with the course content.

There are several possible explanations for these results. One possibility is that students in the deliberate practice condition did not work as much outside of class, and the students in the solve-through condition "caught-up" before exams with additional self-practice. Specifically, the deliberate practice may have given students a higher sense of *fluency*. Fluency can lead to overconfidence and poor awareness of one's own understanding, which can result in reduced effort and practice time [15, 16]. We do not have evidence to support this idea, except for the knowledge that many factors affect exam performance beyond classroom instruction (such as achievement goals and study strategies, see [17]).

Another explanation for the lack of observed differences could be that the subject matter assessed on exams does not reflect the types of learning gains obtained by deliberately practicing problem representations. Specifically, the exams were largely procedure-based (i.e., requiring problem solving). Deliberate practice may have benefitted representational flexibility, or even conceptual understanding, which were not critical for exam performance.

It is also possible that any benefits are more long-term in nature. By the end of the course, students were still relative physics novices. Because representational flexibility is assumed for experts, increasing this flexibility early on may help lead to deeper understanding of the physics concepts as students progress through the material. Or, perhaps additional deliberate practice (e.g., in subsequent semesters) is needed in order to begin to see learning gains. Thus, learning benefits may only appear in future learning scenarios or conceptual knowledge assessments [18].

With respect to alternative outcome measures, we did not directly examine the use of multiple representations on the exams. There could be differences in the errors that students make in problem-solving, representational or otherwise. As discussed by Kohl, Rosengrant, and Finkelstein [9], having a correct free-body diagram is necessary for reaching a correct solution for difficult problems. However, there are a few opportunities to make mistakes; some students could misinterpret a problem and draw an incorrect picture, leading to an incorrect vector diagram, whereas other students fail to generate a correct vector diagram based on a correct picture. In addition to those errors discussed by others, incorrect equation selection and wrong mathematical procedures are other ways to get an incorrect answer. Although we have a measure of students' ability to reach the correct solution (exam grade), we do not have information on what kind of errors are made. Future analyses will examine these errors as a function of condition.

It is also possible that the existing course design already teaches multiple representations well, and that our deliberate practice was superfluous. Van Heuvelen [12] recommended several classroom practices to facilitate physics learning: active, cooperative learning in lectures; repeated exposure over an extended time interval in a variety of contexts; and explicit instruction and practice with individual skills needed to represent and solve problems. The original partially-flipped design of this physics course already included some of these best practices. Students were being exposed to course content outside of class, and the instructor was using examples and engaging students in class, making connections between their prior knowledge, intuitions, the outside world, and new content. In addition, on each exam review day, students in both conditions were shown problems and given the opportunity to vote on the type of problem it was and discuss what process they would use to solve it. This, with the rest of the course design, may have been sufficient to give students flexibility with the multiple representations required in novice physics problem-solving.

B. *Encoding Task*

Across both conditions, performance on the diagram shape component of the re-draw encoding task predicted exam performance. This finding indicates that representational flexibility is a critical component of physics problem-solving, validating other physics education research (e.g. [9]). This finding also presents a novel method of assessing schema

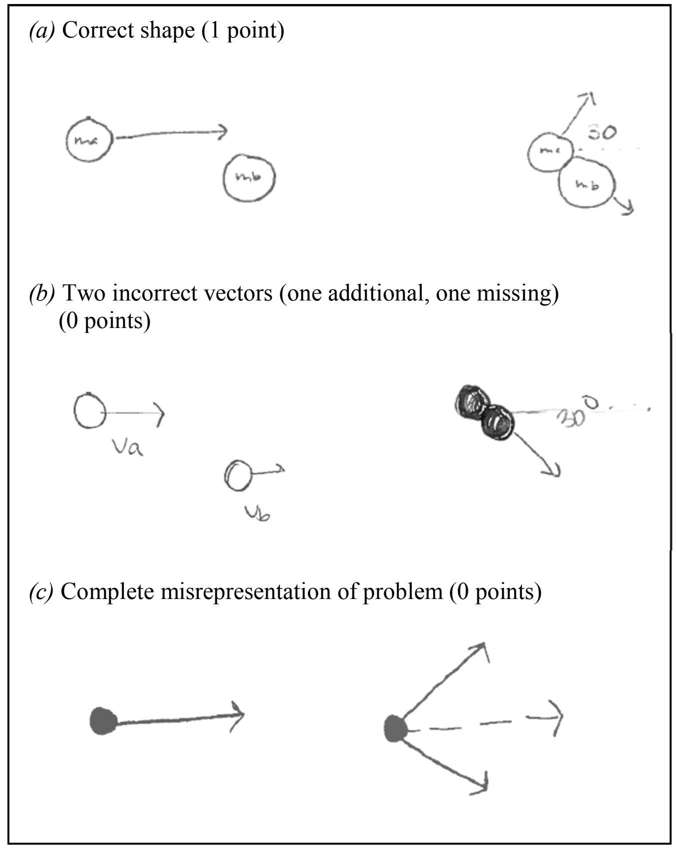


Fig. 5. Student solutions to the diagram re-draw encoding problem in Fig. 3. Students have a wide variety of success capturing the shape of the problem (two circles, one moving, colliding and moving away from each other. The correct shape is shown at the top of the figure.

formation in physics novices (for similar assessments in early mathematics, see [19]). Whereas problem categorization [7] and problem-solving [8] can only be done by students who have already learned the tested physics concepts, this assessment allows insight into the development of diagram schemas in first-semester physics students. Errors made in this re-draw task can indicate flaws in understanding and possibly big misconceptions.

There were many different errors seen in student drawings. For example, three student solutions are shown in Fig. 5. The first figure (Fig. 5a) shows the correct shape (see Fig. 3 for reference). The second picture (Fig. 5b) has an extra velocity vector in the initial state, and a missing velocity vector in the final state. This picture indicates that the student does not have a solid schema for collision; in most collision problems, two objects hit each other and move apart; the velocity vectors define the problem, and these specific elements are not captured by this image. In the last example (Fig. 5c), there are not even two different objects at two points in time. These three images also show the variety of time spent on the surface detail of the circle being filled in.

V. CONCLUSIONS

This paper discusses the results of a controlled experiment of a multiple-representations deliberate practice intervention in a physics classroom. Results showed that the intervention did not affect student exam scores. These findings complement and validate the findings from the between-course analysis done by Kohl, Rosengrant, and Finkelstein [9], demonstrating that multiple representations are well-learned even when instructional techniques do not explicitly target this procedure. Performance on the exams in the current study was predicted by performance on an encoding task, also complementing the finding that flexibility with multiple representations is a critical skill to be developed in a first-year physics course.

Future work should investigate whether deliberate practice effects occur on other measures, such as in a deeper analysis of specific exam errors and other aspects of encoding. Future studies might also investigate motivational measures such as interest and enjoyment and judgment of course difficulty. In addition, future research should isolate the effects of deliberate practice from other beneficial instructional elements, such as the review day practice.

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