The thinking line

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Abstract— **A simple problem-solving strategy is suggested, which seems to be universally adaptable to a broad variation of applied mathematics problems. As a first step to use this documentation problem-solving strategy, instructors need to define good (memorization) knowledge, and the students do have to memorize the same. The documentation method is flexible regarding the solution development and allows the instructor to get an insight of students' knowledge. This allows instructors to specifically help students in their individual challenges.**

Keywords—applied mathematics, problem solving, documentation, memorization, regression

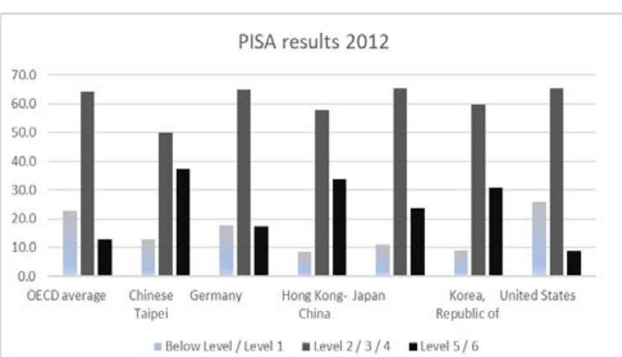
I. Introduction

Working in Electrical and Computer Engineering at UNM, many professors complain that students do not know the basics to solve problems. And solid mathematical abilities are required to solve these problems.

Mathematics education is approached differently in the USA, Germany, and Asia. Asian countries seem to be driven by several factors, including memorization and practice and a difference exists between content (Asia) and process (USA) orientation as memorization (Asia) and meaningful learning (USA) [1]. Memorization and practice are also known in Asia as the two basics of Mathematics teaching [2]. In Germany, every state approaches education differently, but all states still have Gymnasiums. A Gymnasium is a traditional way of getting a school diploma (Abitur) allowing absolvents to study at German Universities.

In fig. 1 the Mathematics literacy is shown according to National Center for Education Statistics [3] based on Program for International Student Assessment [PISA] data from 2012.

Figure 1: PISA results 2012



The USA has 25.9% in rather low competency levels 1 and below 1, Germany 17.7%, the USA has 8.8% in rather high competency levels 5 and 6, Germany 17.5%. Korea, Hong Kong – China, and Japan are in both categories better than the USA and Germany. Germany seems to be in the middle.

In Germany, the Gymnasium prepares around 1/3 of all students for higher education [4]. It cannot be spoken of a highly selective school system. It may even be compared with the USA approach of offering Advanced Placement courses for students at the High School level, who enjoy a higher challenge.

Researchers defined three types of teaching at the Gymnasium [5]. In type I teaching the instructor is interested in learning progress almost in all class periods and expectations are clearly stated. Students work on problems, which have multiple solution ways and most of the hours are disruption-free. Individual support and goal orientation in combination with frequent cognitive activation of learning material can be observed. The students participate actively in problem-solving. Type II and III basically follow type I teaching but with following differences: Type II is characterized by instructors with a less frequent interest in the learning progress of students, type III is characterized by instructors who show little interest in learning progress and more frequent disruptions happen during class. It was stated the type I is predominantly used in the Gymnasium.

A more detailed look at the PISA data reveals a surprise, see fig. 2. The German average is at 514 [5], but the students attending Gymnasiums' achieved 589 points comparable to Asian results in Mathematics. The average of Japan's PISA points was 536 and Korea's 554. The USA achieved an average of 481.

Schoenfeld [6] stated his main research motivator is to find answers to help children to enjoy Mathematics. This study focuses on the relationship between memorization and a German way of problem-solving documentation. An active learning approach is encouraged, and suggestions are made how to teach mathematics in an engineering context.

II. A Theoretical Framework

A. Literature Review

The cognitive information processing theory declares following learning outcomes: declarative knowledge, procedural knowledge, and memory. Anderson [7] stated that the combination of declarative knowledge and procedural knowledge leads to complex cognition. One question arises, what comes first, declarative knowledge or procedural knowledge to achieve complex cognition? According to the empirical evidence, learners need predominantly declarative knowledge first and procedural knowledge during practice [8].

In Asia memorization seems to be more important than procedural knowledge [9]. But both are necessary to acquire knowledge [1]. East Asia is too focused on memorization, and the USA is too process oriented [1]. Might these foci lead to

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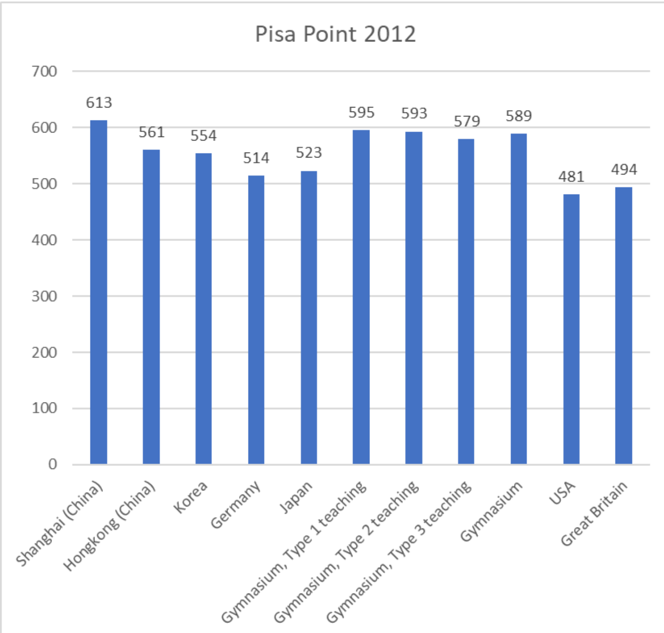
rote memorization of solution ways in Asia or result in a lack of fundamental knowledge in the USA?

FIGURE 2: PISA RESULTS 2012 DETAIL



Some researchers argue that knowledge is constantly changing thus memorization is not important [10]. But the mathematical knowledge taught at a High School or University level to solve engineering problems is rather resistant to change. Boaler [11] stated the common core direction of memorization is questionable even though she presented a method how to memorize content knowledge. Reseachers distinguish between number sense, deep understanding and blind memorization [12], [13]. PISA results support this distinction; low test scores were achieved by students who used a memorization strategy to solve given mathematical problems [13]. Mathematical success is not based on memorization of solution strategies [14].

Memorization has an impact on complex cognition [15], [16]. Fundamental knowledge is important [17]. The lack of simplest algebraic knowledge on community college level leads to superficial conceptual learning [18].

The discussion about memorization exists since over a century. E.g. Atkins [19] stated teachers put too much emphasis on thinking while ignoring the necessity of memorization.

Dewey [20] wrote that general knowledge is required before it may be applied. He also criticized school education, which only amasses factual knowledge or schemata manipulation without recognition of the bigger picture. But he cautioned to condemn all knowledge of definitions. He pointed at the importance of how information chunks are connected and that these connections have to be made by students, ideally in new situations.

Questions come up, what kind of Mathematical knowledge is good memorization, like fundamental facts, and what kind is bad memorization, like solution strategy memorization? How should memorization take place? And does memorization have an impact on the ability to problem solve?

The procedural knowledge of Mathematics is here seen as the ability to problem solve [21], [22]. Problem-solving models can be differentiated in descriptive models, based on empiric data, and normative models, giving interested audience pedagogical help [23]. The following will give an overview of some problem solving normative models and one empirical model.

Dewey [20] described the problem-solving process in five steps, see fig. 3. He wrote that the 1st and 2nd step might occur at the same moment. Betsch, Funke, and Plessner [24] remarked that the first phase is not well researched, but is also not relevant to the daily work inside the classroom. This statement surprises because if students do not feel a difficulty or motivation, no problem solving will take place.

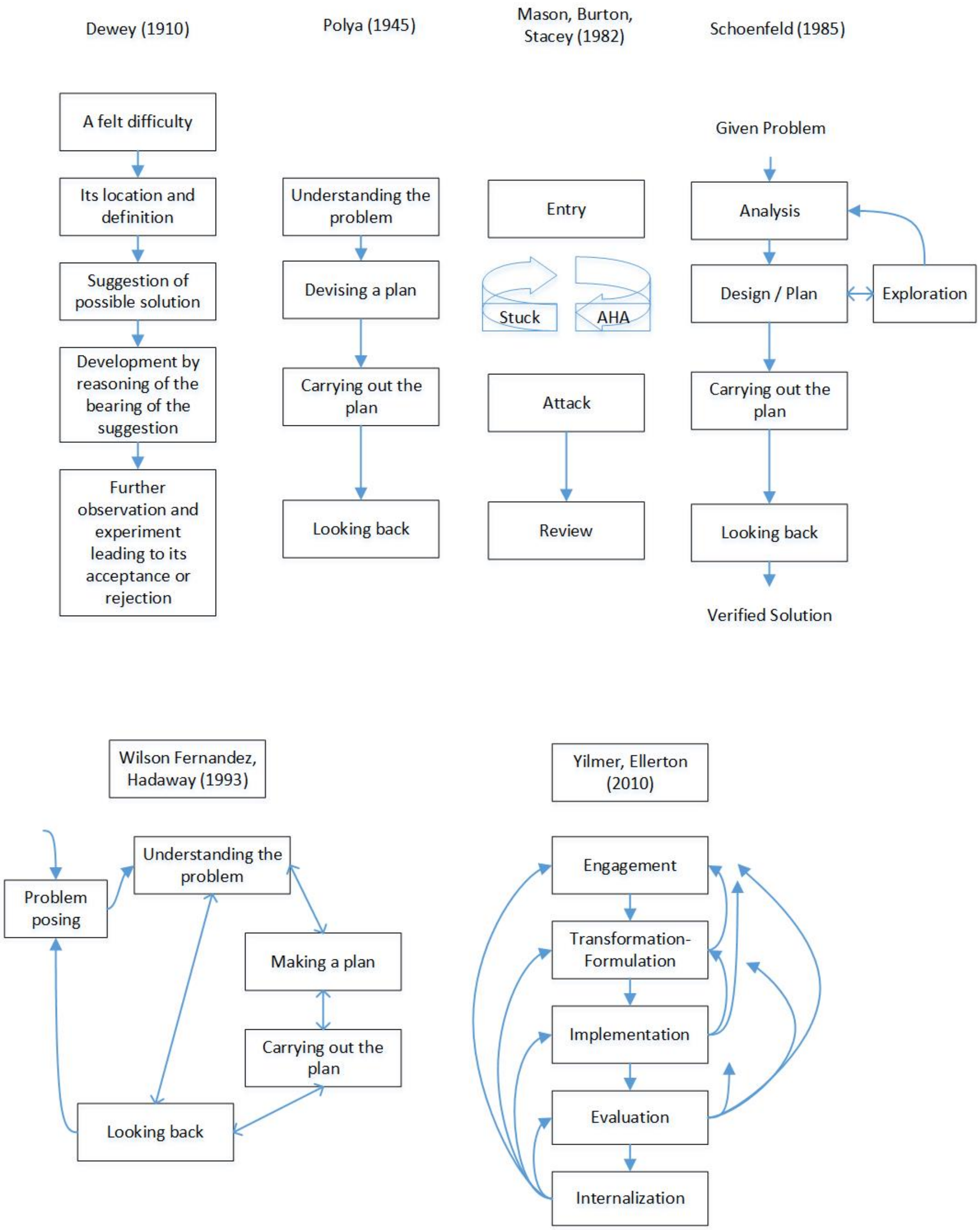
Polya [21] developed a four-phase model, see fig. 3. He synthesized the first two steps of Dewey's model into one step, understanding the problem. The next three phases are similar to Dewey's model. Polya's model is normative as it suggests following predefined steps while solving a problem. He seemed to suggest a linear problem-solving model, which is questionable [25].

Mason, Burton, and Stacey [26] suggested a non-linear model, see fig. 3. The first, third, and fourth phases are similar to Polya's model. But the second phase was based on the thoughts of Wallas [27], emphasizing an intuitive and creative aspect of problem-solving. The second phase is descriptively helpful, but it is very difficult to know what happens during this phase [28]. One improvement may is the introduction of a circular process model.

Schoenfeld [29] suggested a problem-solving model with Polya's phases and the circular process idea but based it on Dewey's logical thinking instead on Wallas's intuitivism. He claimed that students do not need to find a solution plan immediately but can explore the problem. Thus a circular process exists as shown in his model, fig. 3. He also included Dewey's thought of a felt difficult, not expressed as a phase, but mentioned as given problem. He formulated a final product of the problem-solving process, verified solution. This is important from a process point of view. A final product of the problem-solving process is defined, the verified solution, and a starting point, the given problem.

Wilson, Fernandez, & Hadaway [23] suggested an even more circular problem-solving model, see fig. 3. The phases of Polya's model are all present, but feedback between the phases was included. This is basically the visualization of the words "check each step" and "Can you use the result, or the method, for some other problem?" [30]. Additionally, they propose a "Problem-Posing" phase similar to Dewey's and Schoenfeld's first phases. The connection between the looking back phase and the problem-posing phase may indicate a meta-cognitive process.

FIGURE 3: PROBLEM-SOLVING MODELS



Yimer and Ellerton [31] supported Wilson's model, but split the last phase into two phases, "Evaluation" and "Internalization." During the evaluation, the concrete problem is confirmed or rejected. During the internalization, a meta-cognitive process happens, reflection on the solution process. They also confirmed circular processes, but even more extensive as jumps from later phases to earlier phases were observed.

In summary, since the problem-solving description of Dewey and Polya the main steps seem to stay the same – acceptance that a problem exists, analysis, design of plan, attacking the problem, review. The non-linearity of these phases are emphasized and reflection, meta-cognitive learning, also seems to happen.

Additionally, reflection during the problem-solving process [31] and the transition between the phases [29] seem to be important. Reflection might be essential during problem-solving and both may take place, cognitive and meta-cognitive activities [29], [36]. The transition between the phases might be connected with meta-cognitive activities [23]. And an internal monitor might be of importance in the transition of phases [32]. In spite of the importance of problem-solving models surprisingly little research is done to describe these processes [22]. One of these studies was already mentioned [31].

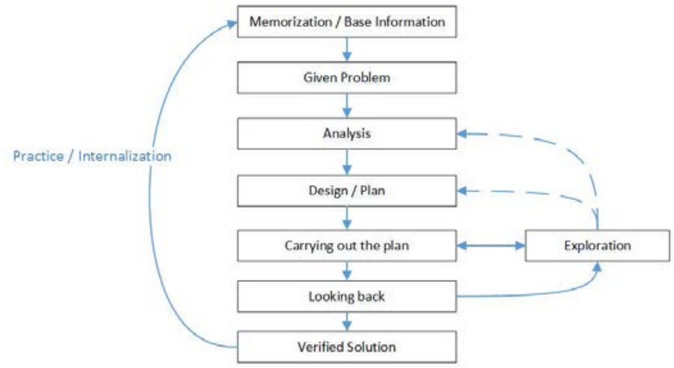
The earlier described models did not address how many persons try to solve a problem. The group approach of problem-sovling might be superior to think aloud strategies in singular work. One study enriched these problem-solving models by a "watch and listen" phase and analyzed group problem-solving behavior [33]. Other problem-solving studies use group and pair work to analyze reflection phases [23]. Teachers were enabled to observe not only the problem-solving steps but also the reflection work during the problem-solving process. The asymmetric communication between students and teacher is well researched, but the symmetric interaction between students is not [34]. Asymmetric communication is not necessarily superior regarding the improvement of problem-solving competency. Yet symmetric interaction requires steering and active collaboration for being productive and to avoid wild goose chases.

Another aspect of problem-solving is how the problem-solving process develops. Successful problem solvers seem to be more systematic in their approach than unsuccessful problem solvers [35]. Self-regulatory behavior is mentioned as an important success factor also by other authors, e.g. [37], [38]. Problem-solving failure is connected to the lack of lock back activities [39].

B. Suggested conceptual framework

All problem-solving models have a coherent challenge for the practitioner, how does an instructor use these to teach students? Dewey [20] believed problem solvers need to have acquired a solid body of knowledge before the problem-solving activities. Polya [21] mentioned a list of typically used mental operations might be useful for the problem solver. Fernandez, Hadaway, and Wilson [23] stated, one goal of instruction is to develop knowledge. As a consequence of this thinking, a memorization/base information phase is suggested as the first step to problem-solving. All other phases and connections are overtaken from known problem-solving models, see fig. 4.

FIGURE 4: SUGGESTED FRAMEWORK



Between the exploration phase and the analysis and design/plan phase, only a dashed line exists. The reason for this is, students may hit a roadblock during the carrying out phase, which requires going back to earlier phases. The same is true for the locking back phase if a student gets confronted with the knowledge of having a wrong solution.

Dewey wrote, "To maintain the state of doubt and to carry on systematic and protracted inquiry – these are the essentials of thinking [20]." The following model, fig. 5, shows how the students were instructed to solve any problem they are confronted with. The blue line was called "Thinking Line". On the left side of the thinking line the concrete steps of problem-solving take place. On the right side of the line the students were expected to write down on what (mathematical) argument they base their concrete step. It is the experience of this researcher that High School students seem to have the tendency to not document their problem-solving steps. One example of how to use the thinking line documentation is given in Appendix A.

Locke [40] stated that thought goes wrong if it depends on others. This is a clear argument against memorization of solution ways. It was one objective of the instruction that students learn to document in a shorthand way to document their problem-solving steps. This is a typical way how German Gymnasium's students get taught. It is important to notice that the suggested solution ways are only one possible way to solve these problems. There are certainly other solution ways.

Students were encouraged to use whatever argument they saw fit. They only had to write their argument down on the right side of the thinking line. And in general, the argument is found within their memorized base information knowledge. Thus, the phase 4 and 5 are constantly repeated creating the phase 6, a documentation of their solution way. Within this process, phase 7 might jump in if students realize they are on the wrong path.

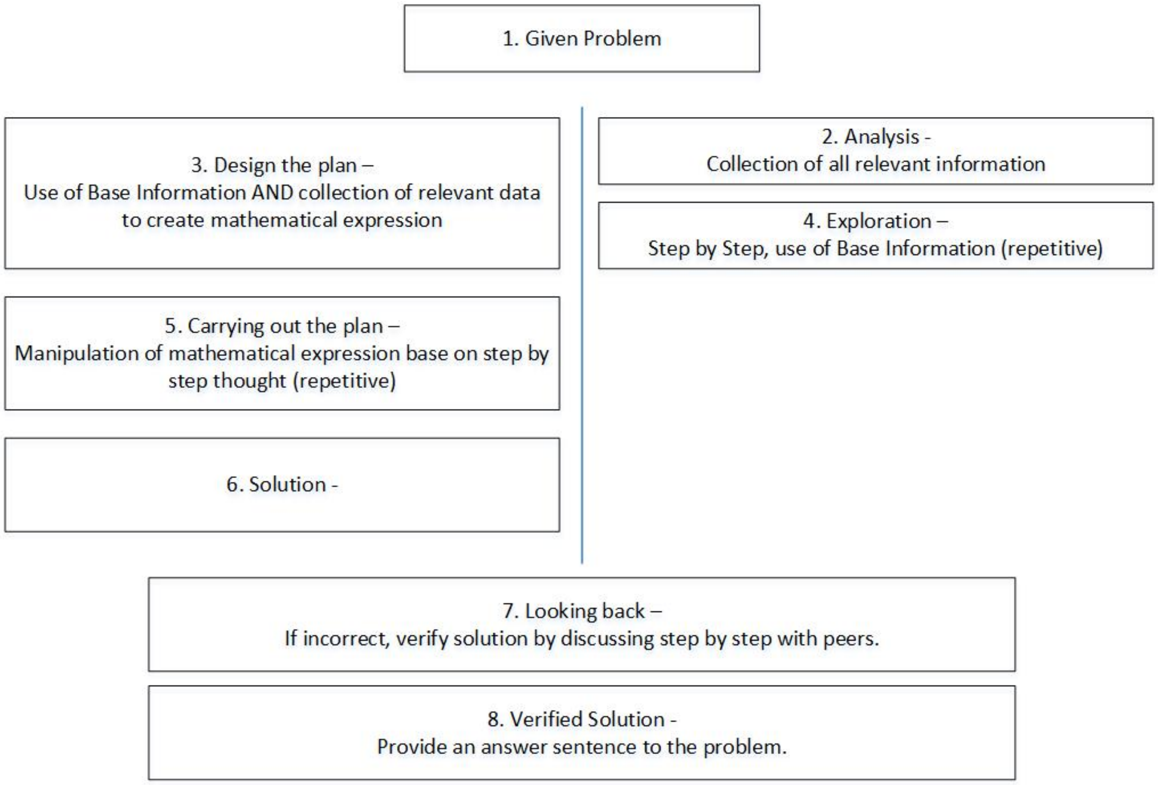


FIGURE 5: PROBLEM SOLVING DOCUMENTATION

This method supports the Type I teaching at the German Gymnasium as explained in the introduction. It also supports the idea of having a systematic approach to a wide variety of Mathematics or applied Mathematics problems.

It is also a graphical organizer, on the right side of the thinking line cognitive processes truly happen. Collection of information and searching / writing down of arguments for immediate action. The only real cognitive task on the left side is the point 3, design the plan. The collected information (in application and more difficult engineering problems) needs to be combined with the memorized base information to create a solvable pure mathematical problem.

Further on the left side, the application of the argument is executed to the mathematical expression, point 5. Only the ability to correctly apply a mathematical rule/law/principal is required.

It is important to allow individual difference [20]. The independent thinking is more import than answering correctly [22]. These thoughts are thoroughly supported by this documentation method. Students can solve problems in any way, certainly sometimes in a rather complicated way, but this does not matter. Also, the problem-solving procedure of looking back is well supported. Even though the students do not think aloud, an instructor can basically see what the thought of the student was by looking on the right side of the thinking line. The instructor can determine if the student used an information chunk inappropriately. The student might not have fundamentally understood the mathematical argument used. Or the student used the correct argument but applied it incorrectly on the left side of the thinking line. In this case may be more practice or reinstruction how to apply this argument is required.

One major advantage of this documentation method is that students can discuss their work. They can compare their solution ways with their peers' solution way. This is especially useful if students are placed in work groups. In this study, students were grouped in parties of three to four. They were allowed to switch groups or also to stand up and go to a different group during class for exchanging thoughts or getting help. This supported not only self-reflective behavior but also group-reflective behavior. The instructor did not lecture more than four to six minutes per class. He supported mainly the groups, discussing their approach, or single steps. By just pointing the student to their true problem in a long solution way, the right answer was not given but the students could focus on the true issue of their solution way. This supported a very effective feedback system, let it be from instructor to student or student to student. This is a pronounced active learning approach.

If a student solves a problem alone, e.g., during an exam, this method has the advantage that partial credit for correct solving steps can be granted. A typical mistake is, e.g., incorrectly multiplying by -1. The solution will be wrong, but all other solution steps might be correct. A grader can follow the student steps and might be able to assign 95% of all possible points for this problem as the mistake is a minor calculation mistake and not a fundamental flaw. A partial grading policy might encourage/obligate students to document their work appropriately. This might avoid cheating problems by simply verifying a solution, which might be right or wrong. Also, students should know that the way how to solve a problem is important and not necessarily the answer.

A difference to Asian teaching is that the students are not encouraged to memorize complete solution ways of problem types. The students are encouraged to play around with the base information memorized by them. They may find out some patterns like one needs to do the opposite from what you see to solve algebraic problems. Or they may observe that some mathematical arguments are quite frequently used in many problem-solving processes, like the distributive property.

Examples of this documentation method are given in appendix A.

III. RESEARCH STUDY

Asia seems to have an overemphasis on memorization by not only lettings students memorize the basic knowledge (like multiplication facts), properties, and laws of Mathematics but also solution ways [1]. In the USA very little emphasis seems to be given to memorization. In this study, the question of what is good memorization vs. bad memorization is not answered. It was simply assumed that the information chunks mentioned in the ACT theory [7], or basic definitions [20], are given by the authors of the workbook. The information chunks to be memorized were taken from the workbook, PreCalculus edition 5 [41]. They seem to be reliable experts to define the good memorization knowledge. This good memorization knowledge seems to be summarized in the blue boxes of the workbook. All blue boxes were enforced to be memorized with the exception of blue boxes which list concrete solution solving steps for given problems.

A. *Participants*

76 students, distributed to three classes, took a PreCalculus class. The students came from a diverse background, 28 Hispanic, 45 White, and 3 black. The school has a poverty level of 52% free or reduced lunch students. 39 students were female, 37 male. In this group of students, 3 were Sophomores, 5 Senior, and 68 Junior.

B. *Research set up*

This study took place during the weeks 1 to 14 of the first semester of PreCalculus. Students were instructed by a teacher, who is very knowledgeable in PreCalculus and the TL method.

Memorization during class seems to be possible through practice, but it was assumed it is a task which could and should be done outside the classroom. Students were expected to memorize the blue boxes at home.

The assessments were based on the suggested assessment questions by the authors of the PreCalculus textbook. However, the assessments always contained the rather “hard” questions and not only pure memorization fact questions.

The TL was promoted by the teacher during the first three assessments through role modeling this documentation method in every single problem-solving explanation during class.

C. *Variables*

From the 2nd week of instruction to the 14th week of instruction the teacher administered through a student response system multi-response memorization quizzes twice a week with the exception of four weeks during which an assessment took place. Each quiz contained between 12 and 32 questions.

At the beginning the questions were only taken form the first unit of the book. If the average of a question was below 60% the same question appeared on the next quiz again. Due to these repeated questions, the amount of questions in a quiz grew up to 32 questions. The main point was to drill students in the factual knowledge and to not let go of knowledge which was not memorized appropriately by the student body. The 60% threshold was taken to emphasize to students that a F grade is not acceptable for this basic knowledge.

I.e. following question was given

$(a+b)^2 = ?$

And the following solutions were offered

a^2+b^2 , a^2+b^2+2ab , $b^2+2ab+a^2$, a^2+ab-b^2 , $b^2+2ba+a^2$, b^2+ba+a^2+ab , $a^2+2ab+b^2$

These quizzes were administered by a clicker response system. The effectiveness of these systems are supported by some authors, e.g. [42], [43], but also questioned, e.g. [44]. Clickers give immediate feedback to students and might provoke active memorization [45]. The quizzes were build up with the new information chunks of the new unit and some information chunks of the former units. The clicker system was a very practical tool to access memorization knowledge and the grading was done automatically by the system. This enabled the teacher to verify students’ memorization knowledge of a huge amount of information chunks effectively.

Only if a student picked all correct answer choices one point was given. By forcing students to think minimal about the information chunks it was tried to exclude guessing strategies. In general these Friday memorization quizzes ended up with an average grade of F. The same quiz was given again on Monday. Even though the same quiz was administered, students were forced to read the answers again and choose carefully by scrambling the questions and answers. It was tried to avoid guessing strategies and short-term memory availability of correct answer options.

The average grade of Monday quizzes improved over time from C to B+. The grade assigned was always the better grade of both days. This grade is the basis for the constructed memorization independent variable of the study.

Each quiz was graded by successfully answered questions divided by all questions. All results of taken quizzes were averaged before an assessment.

The students took an assessment for each instructional unit, two to four. Each assessment contained multiple choice and open answer questions, half and half. Each question was worth one full point. Partial credit was granted for correct process steps and memorization knowledge. The assessment variable contained the percentage of earned points divided by possible points.

For the thinking line variable, the teacher assigned three values to the thinking line ordinal variable of the 4th unit's assessment. 0 if the TL was not used, 0.5 if the TL was partially used, and 1 if the TL was used in more than 90% of the problems. The values of the ordinal variable were chosen to simplify the transition of qualitative data into quantitative data. As the students had never been exposed to a systematic documentation method, no data could be achieved during the first two assessments.

D. Hypothesis

Following the conceptual framework, *good* memorization should have a positive impact on problem-solving assessments.

H1: The higher students score on good memorization knowledge, the better their assessment outcome is.

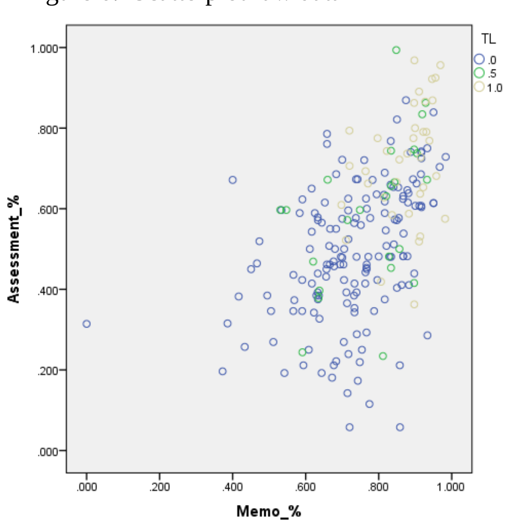
The positive impact of the suggested documentation method should have a positive impact on problem-solving assessments.

H2: Students who use the TL method of documentation achieve better assessment results.

IV. Research Results

A scatterplot provides an overview of the data, see fig. 6. A linear relationship between memorization and assessment might exist and students using the TL method may score higher than students who do not.

Figure 6: Scatterplot raw data



1) Outlier and assumptions

An outlier analysis was conducted based on leverage values, discrepancy, and influence of data [46]. One outlier was identified.

Then all regression assumptions were verified. Normality of dependent variable, independence of independent variables, homogeneity of variance, linearity, normality of unstandardized residuals, fixed X, and multicollinearity [46]. All assumptions were fulfilled.

2) Correlations and regression results

The Pearson correlations are given in Table I.

A regression was conducted to see if Memorization predicted Assessment results of students' achievements in a PreCalculus class. Using the enter method it was found that Memorization explained a significant amount of the variance in the value of Assessment ($F(1, 74) = 24.679$, $p < .01$, $R^2 = .25$, $R^2_{Adjusted} = .24$). The analysis shows that Memorization did significantly predict Assessment ($b = .753$, $t = 4.968$, $p < .01$).

Hypothesis one was supported.

TABLE I. Correlations

	Memorization	Assessment	TL
Memorization	1	.487**	.364**
Assessment	.487**	1	.474**
TL	.364**	.474**	1

**. Correlation is significant at the 0.01 level (2-tailed).

A multiple regression was conducted to see if Memorization and TL predicted Assessment results of students' achievements in a PreCalculus class. Using the enter method it was found that Memorization and TL explained a significant amount of the variance in the value of Assessment ($F(2, 73) = 16.098$, $p < .01$, $R^2 = .306$, $R^2_{Adjusted} = .287$). The analysis shows that Memorization did significantly predicted Assessment ($b = .595$, $t = 3.697$, $p < .01$) and that TL did significantly predicted Assessment ($b = .122$, $t = 2.426$, $p < 0.02$).

Hypothesis two was supported.

V. Discussion

This research undertook an attempt to suggest a documentation method for applied mathematics that students learn how to problem solve and instructors can teach a meta approach to problem-solving.

A model of problem-solving was developed based on the problem-solving theory, the two basics pillars of Asian teaching, and the German Gymnasium teaching approach. The importance (variance explained 23.6%) and existence (significant, $p < 0.01$) of a memorization phase in the problem-solving process was confirmed. It was confirmed that the thinking line documentation method seems to be a promising way to teach students problem-solving in a mathematical context (variance explained 6.8%, significant p < 0.019) while using an active learning approach.

A. Implications for Theory

A new problem-solving model framework was suggested. The new element is the definition of a memorization/base information phase before the actual problem-solving starts. All other aspects of the suggested model are already known, but specifically, the reflection part is formulated differently.

B. Implications for Practice

An issue was raised by actively using the lowest level of Bloom's taxonomy – remembering (facts and basic concepts). What are good vs. bad facts/basic concepts? Following definition is proposed “Good facts are simple blocks of knowledge needed to solve problems in a modular fashion.”

Instructors should reflect on the basic memorization knowledge required in their course. They should point out the *good* knowledge to their students. They should refrain from making students memorize solving steps. When explaining the solution to a given problem, the instructor should role model a step by step solving process with a clear connection to the *good* (memorized) knowledge. Instructors should acknowledge the presentation of students’ knowledge in an assessment by giving partial credit. Partial credit seems to be especially important for problem-solving with the help of Mathematics. A minor mistake makes the whole solution incorrect, even if a student fully understood all taught concepts. Instructors should read the step by step solution documentation of students and identify the true student’s misconception for being able to support the student exactly where the need is.

C. Limitations and Future Research

A shortcoming of this study is the specific student body of rather high achievers taking PreCalculus at a High School. Future research may want to verify the results with a more diverse student body. Another shortcoming is the subject of PreCalculus; future research may want to verify the results for other subjects, e.g. Engineering, and Physics. Another shortcoming is the not addressing of how the students’ memorization took place. Future research may want to define what *good* vs *bad* memorization knowledge is. Future research may want to verify what kind of memorization takes place in Asia, USA, and Germany.

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VI. APPENDIX A

Given Problem

$$\frac{x}{3} - \frac{x}{4} = 12$$

Find the value for x!

$\frac{x}{3} - \frac{x}{4} = 12$	Common denominator
$\frac{4}{4} * \frac{x}{3} - \frac{3}{3} * \frac{x}{4} = 12$	Multiplying and combining like terms
$\frac{x}{12} = 12$	* 12 and golden rule of Algebra
$x = 144$	

The value for x is 144.