Improving STEM's Calculus education using cross-countries best teaching practices

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Abstract – **A recently completed study by the Mathematical Association of America outlined the main points for improving Calculus education for STEM students based on the experience of teaching math in the USA. In this paper, we expand this study with the best calculus teaching practices from multiple countries. Our methodology initially followed the Program for International Student Assessment (PISA) study and then was significantly modified due to the difference between school and University education. We started by comparing the practices of two countries with traditionally different education systems. We described and analyzed factual differences and similarities of content, pedagogy and socio economics, based on an adjusted to higher education international comparison system developed by Organization for Economic Cooperation and Development for high school education. We outlined our work on culturally independent comparison methods with a goal of improving or adjusting Calculus education.**

Index Terms – Calculus education for STEM, PISA

INTRODUCTION

Calculus is an important part of STEM education. On the one hand, calculus is a branch of mathematics that comprises study of change (differential calculus) and accumulation (integral calculus). On the other hand, calculus based on clear geometric ideas shows what can be achieved by using scientific / analytic / logical approach for developing and applying new ideas.

Both aspects demonstrate the importance of the Calculus in STEM education. The essence of the engineering mind is an ability to quickly identify a problem, find practical solutions and then choose the most optimal one. While some real-world problems may be expressed and solved without complex mathematics, most do require systematic scientific approach. Calculus becomes invaluable as a way of studying change and accumulation using application examples based on a scientific/logical approach, and thereby acquiring problem solving skills.

Calculus education has always depended on culture and on the socio economic of engineering [1]. We will skip the history of the Calculus to concentrate on the current study of calculus education by the Mathematical Association of America (MAA) [1]. We focus instead on the developing a framework that lays out how students can succeed in calculus and that leverages improvement in Calculus instruction across the United States. Although our goals coincide with the MAA's, our approach is different. We look at the study of Calculus beyond the boundaries of one country. To rank the best cross-countries practices, we had to identify comparison measures independent of cultural differences.

We use a methodology similar to one developed by Organization for Economic Co-operation and Development (OECD) [2] which conducted a cross countries assessment of high school education outcomes (PISA) [3]. Note that a feasibility study for cross-countries assessment for the Universities/Colleges was discussed in [4].

Our paper deals with the cross-countries best practices of Calculus education. It contains a report on the completion of stage one of the study. We describe differences in Calculus education for two countries and suggest a method for adjusting students' assessment from [3] and surveys from [1] for students, instructors and administrators. The paper proceeds as follows. This section explains why Calculus is important in the STEM education, outlines the main points, and describes the structure of the paper. We outline the studies serving as the main basis for our work. After that, we describe a multitude of Calculus programs and explain why we chose specific Universities for comparison. Then we present the existing evaluation system. Next, we outline the suggested process for studying the cross countries' comparison system. In the next section, we present our findings upon completion of stage one. We proceed by describing the foundation and process of development of culture-independent comparison. The final section provides a summary and outlines future work.

RELATED WORK

Insights and Recommendations (I&R) [1] is a significant effort of the MAA (the final report has 180 pages and summarizes more than 5 years of study) to collect and analyze data about a multitude of Calculus programs in the US Universities and Colleges. It starts by classifying student categories based on school grades, required assessment exams, grading system, grade statistics, background, etc. Data for the study was collected for over 4 years. Surveys were adjusted and tuned up for 2 years during the preliminary stage. Next, MAA moved to the main study. Data was collected from students, instructors and administrator surveys (once at the beginning and once at the end of the semester) and from the institutional research departments. Based on the results of the initial analysis they selected 20 Universities, where the Calculus program was viewed as a success. Universities represent four groups classified by University size, endorsement and maximal granting math degree. The statistical quintessence of the study is the hierarchical linear model [11] constructed of factors that can potentially influence students' attitude towards Calculus. High values of coefficients in the linear model reflect correlation and not causality; nevertheless, they were used to select institutions that produce special results. The model considers over 120 variables of interest. These variables characterize instructor pedagogical methods, behavior and characteristics (over 60 variables) as well as institutional

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characteristics (over 60 variables). Instructors related variables constitute three groups: good teaching, technology and ambition teaching. The latter includes variants of work in groups (most highly rated by students), non-standard questions in homework (medium rating), non-standard questions in class and self-study (low rating) and lectures (negative rating). Institutional characteristics describe student-centered programs: department mentors and guest lecturers (highest rating in category), TA quality: TA preparation seminars, tutoring centers, such as internet resources and computer-aided instructions (highest rating), small group tutoring (medium rating). In addition, these characteristics include questions about supporting technology, such as availability of Math oriented programs like Mathematica, Maple and Matlab in the labs and tutoring centers with appropriate training courses (high rating). The study conclusion recommends using high and medium rated practices to improve the retention of STEM students as well as their attitude and results in Calculus courses. The study also recommends making the placement procedure more efficient. A comparison approach for high school education systems across different countries was suggested in PISA [5]. An overview of the mathematics-related part of PISA is described in [6]. PISA considers three basic categories for a comparison across different cultures. The **content** with an alignment between intended curriculum (i.e. stated standards, syllabi) and implemented curriculum (i.e. the content taught) is evaluated. To compare **teaching practices,** "Teaching and Learning International Survey" (TALIS) was developed based on 13 of the most typical teaching practices, grouped into three dimensions: structure practices, student-oriented practices and activities. To evaluate **teaching quality,** special "culturally independent" tests are developed. To ensure these tests are culturally independent, the double back-translation approach is used.

GENERAL APPROACH TO COMPARISON; CALCULUS EDUCATION FOR STEM MAJORS

Evaluation of cross-country education, main points

Our choice of Calculus for the study was based on the fact, that it is a part of the Core Curriculum for every STEM major. It fits into every class sequence for every major and satisfies every requirement of STEM majors. As a result, every University offers a broad variety of major oriented sequences of Calculus classes. These classes, being mostly similar content-wise, differ by depth of study, types of applications studied, complexity of problems, number of lecture and practice hours, class sizes, etc. While working on comparing different approaches to teaching Calculus, all these aspects should be studied and taken into consideration. We chose USA and RF (Russian Federation) education systems for comparison because they represent almost opposite approaches to education. The USA system implies minimum restrictions, allowing almost complete freedom of choice to students, the only restrictions being required core classes and prerequisites. At the same time the RF system offers a rigid schedule, where each class has its preset place and no changes or modifications of schedules are allowed. Another difference: The RF system on average offers more versions of Calculus classes custom-made for specific majors. Both systems have pros and cons, which we will discuss later. We identify and analyze the pros and cons with the goal of creating a synthetic approach by combining pros and eliminating most if not all, cons of both systems.

Description of the chosen universities, motivation for choice. The choice of the universities for the first stage of comparison was based on similarity of the three factors: *Student body, faculty body, educational environment.* Based on these criteria for our research we chose middle-sized city placed universities that offer either baccalaureate or both baccalaureate and graduate programs. Of these universities, we picked the middle range of top 10% of national universities for both countries. We believe that this choice represents the most typical student and faculty body of STEM majors, provides a similar environment in which students live and learn, and thus allows us to identify the main adjustment for the verifiable comparison of the respective education systems.

EXISTING EVALUATION SYSTEM (PISA) FOR COMPARISON OF CROSS COUNTRIES EDUCATION SYSTEMS

As outlined above, the PISA approach compares three major measures of the education process: content, teaching practices, and teaching quality. PISA used it to study high school systems. With necessary changes, this approach can be used for higher education systems comparison as well.

First, in terms of content, it is important to compare content selection, structure and presentation. Another important comparison measure is the relation between "content learned" and "content taught."

Second, while comparing teaching practices one needs to take in consideration all kinds of practices, including teacher-directed, student-directed activities, and a variety of assessments. Such practices should be identified, matched, and compared across the cultures.

Third, learning environments can be compared by the quality of teaching, i.e. "how teachers deliver content and practices in the classroom." PISA outlines three dimensions of the quality of teaching: classroom organization, emotional support, and instructional support. For cross-countries comparison, the above mentioned double back translation is used. The latter means that first a test is translated from English to a respective language and then back to identify and eliminate any possible discrepancies. The "double" part means that it is done twice by independent translators in order to minimize errors.

PISA also compares the institutions using parameters outlined in Table 1. In this paper we compare colleges using the following parameters:

TABLE I

PARAMETERS USED IN PISA

School size	medium size 15-40 thousand city located
Gender proportion	no selectivity gender bias
School type	both public and private schools are considered
Availability of computers/internet	available both on personal level and via University in both cases - both WiFi and computer labs
Quantity of teaching staff at school	about 10 students per one instructor
Class size	RF – practice classes ≤ 25 / lecture classes ≤ 150, classes are formed based on major, can differ in size
	US - bigger campuses follow same outline, some (usually smaller) campuses use joint classes ≤ 35
Extra-curricular/ mathematical activities at school	RF - various math competitions
	US - preparation and participation in various math competitions/math clubs

Extended curriculum	US - offers Honor Programs
School selectivity	considered are medium selectivity universities in both cases to make proper comparison
School responsibility for curriculum and assessment	curriculum is created on school level in collaboration with majors representatives but should be approved by a State Board

SUGGESTED COMPARISON OF UNIVERSITY CALCULUS EDUCATION SYSTEMS IN THE US AND RF

To compare Calculus educational systems in order to gain an understanding of best practices in each country, we did the following. During Stage 1, we identified a small number of Universities with a sufficiently representative Calculus education, described differences in the educational systems, and suggested a balanced approach to the assessment of the education outcome based on [5]. Then we adjusted surveys from [1] for students, instructors and administrators. Results of Stage 1 constitute the content of the paper. For the next stage, we plan to check the feasibility of the suggested in Stage 1 surveys, assessment system. For the sake of balance, we run our study on a wider range of Universities, verifying that the assessment system produces consistent results and that obtained results actually allow us to find the best practices. The next stage of the study expands the number of Universities to cover all groups generating statistically significant best practices. On the Stage 4, we plan to increase the number of countries participating in the study.

RESULTS OF THE STAGE ONE STUDY

Calculus II and I are required for all technical majors. The percentage of math majors between them is relatively small. That is why we concentrated on these levels of Calculus for technical non-math majors.

I. Comparison of the time allocated for course

Table II shows that in the US the ratio of lectures to practice classes is higher. We believe this is due to a higher level of major specific applications in RF courses.

TABLE II

TIME ALLOCATED IN HOURS (CAN VARY FROM MAJOR TO MAJOR)

	Calculus I	Calculus II
RF	34 lectures/48 practice	34 lectures/48 practice
US	30 lectures/30 practice	30 lectures/30 practice

II. Content comparison

The basic sequence and material covered are mostly the same in both countries, but in RF, due to a more fine-grained engineering majors list, more customized curricula are used. In Tables IIIa, b (see Appendix) we compare Basic Engineering Calculus (US) and Technical Calculus (RF), as similar in both content and assigned hours. The only significant observable difference (see Table IIIb excerpt below) in terms of content is that in US Calculus II includes Series but does not include Double and Triple Integrals, which constitute part of the optional for most majors Calculus III course. At the same time in RF it goes opposite way and Calculus II does include Double and Triple Integrals leaving Series for Calculus III. Both ways have their pros and cons since Series provides fine tools for approximate calculations while Double and Triple Integrals in turn provide tools for calculating areas and volumes of more complex surfaces and bodies. It is also worth noting that more proofs are taught in RF as compared to the US.

TABLE IIIb

EXCERPT CALCULUS II RF VS US CONTENT COMPARISON

RF (in US covered in Calculus III)	US (in RF covered in Calculus III)
Functions of many variables. Gradient and directional derivative. Partial derivatives and differentials of higher orders. Local extrema. Conditional extrema and Lagrange multipliers. Maximum and minimum function values on closed bounded area. Double and Triple integrals.	Series, Alternating Series Integral Test, p-series Comparison Tests Absolute Convergence, Ratio & Root Tests Error Estimate Power Series Reps of Functions as Power Series Taylor & Maclaurin Series

III. comparison of assessment

As seen from Table Iva, Assessments used in the US and RF follow the same outline. However, there are two differences:

- Little Homework is graded in the RF as compared to the US.
- Final Exams differ significantly. See Table IVb.

Specifically, the RF Final Exam tests for both understanding of basic concepts and ability of solving complicated problems involving several different concepts and methods at the same time.

TABLE IVa

ASSESSMENTS SIDE-BY-SIDE COMPARISON

RF	US
Homeworks (each class)	Homeworks (each class) – no grading
Quizzes (weekly/bi-weekly)	Control Homeworks (3-6 times per semester)
Midterm Exams (1-3)	Control Assignments (per subject covered)
Final Exam	Final Exam

TABLE IVb

FINAL EXAMS SIDE-BY-SIDE COMPARISON

RF	US
8-12 problems	25-30 problems
all problems are open ended	multiple choice mostly, can include several open ended problems
can include theory questions	no theory questions
aside of understanding basic concepts checking ability of using them to solve complicated problems	checking understanding of basic concepts mostly

IV. Usage of obtained knowledge for majors

It is typical for RF to have a rigid class sequence, which allows tighter incorporation of Calculus classes into a sequence of the major classes in such a way that necessity of Calculus tools and concepts for acquiring and working with the major specific knowledge becomes more apparent.

At the same time, in the US core courses requirements and the classes' prerequisites are the major tools that determine which classes should be taken and in what order. Such an approach leaves a significant role in deciding which classes will be included in a semester's schedule to the students. As a result, students are able to create individualized schedules more fitting to their needs. Such schedules may have gaps between the courses and their prerequisites, or some classes with co-requisites can have gaps between them, while students could benefit from taking them in parallel. For example, if Physics and Chemistry classes requiring Calculus are taken right after that Calculus they will benefit from both Calculus material being fresh learnt and natural connections and interdependencies in subjects helping better learn each.

V. *Students with STEM majors career perspectives comparison*

In the US, Engineering students are earning on average about 30% above the average starting salary over all programs of study. In terms of post-graduation employment, engineering graduates have a first-year related employment rate of 54% compared to 47% for all programs graduates. In the RF, the situation for engineering majors is even better: Engineering graduates have the highest employment rate of 80% among the country graduates, with average starting salaries being twice the national median [7], [8], [16], [17]. Such a significant difference in post-graduation employment rates may be explained by the fact that the process of US industry outsourcing started much earlier and went much further than in RF, thus shrinking the job market.

VI. *Working conditions for educators*

Social status and job satisfaction: In both countries social status and job satisfaction seems relatively high. This may stem from traditional respect for the teaching profession and from a relatively stable employment status in comparison to employment in industry.

Pay: Average pay for professors at all levels is well above country median salary, being respectively higher in the US, where it reaches double or triple median salary for mathematics professors. However, instructors/lecturers and even more so adjunct professors earn on average median level or below. [9], [10], [18].

Stability of the job: Professor and instructor positions are quite stable compared to the non-academic job market, but instructors and adjuncts are more vulnerable. We can state that stability of tenured positions is balanced by the more volatile positions of non-tenured and part-time faculty.

WORK IN PROGRESS: DEVELOPING ASSESSMENT THAT ALLOWS COMPARISON BETWEEN EDUCATIONAL SYSTEMS

To this point, we have not discussed the objective ways of comparing the teaching quality of both systems. One way is to follow the PISA approach and develop a culturally independent assessment test to be administered to students in both countries. However, in view of visible discrepancies in the content as discussed earlier, such a test should cover only the common part of the curriculum. Fortunately, this decision is not too restrictive. According to Tables III, the content discrepancies between the US and RF calculus curricula are relatively minor. We plan to use the double back translation approach as outlined above to achieve cultural independency. While necessary, this approach does not seem to be sufficient for college level Calculus taught to STEM majors. One reasons as stated earlier, is that the goals of both educational systems is differ in several aspects. These differences can be split into four categories:

1. Depth of theoretical material.
2. Complexity levels of problems offered in the course.
3. Motivation behind notions and overall Calculus approach.
4. Quantity of the assessment problems.

From the data collected and outlined above it seems that in the RF system, theoretical material is offered in greater depth with more results given with complete proofs. In the US, more attention is paid to explaining ideas behind the notions, theorems and methods of Calculus. At the same time in the RF, the complexity of the problems offered both in class and in homework is higher than in US. In the US we observed a significantly higher number of problems, probably due to requirement to have a higher productivity. We interpret these differences as a result of different requirements the calculus students have from both upcoming major classes and their future professional careers. Therefore, we consider them to be cultural differences too, and they also should be considered while developing the comparison questionnaire and assessment test. To weigh out the complexity and quantity aspects (see categories 2) and 4) above), we develop the test following the Content Inventory approach [12] or [14]. This approach tests the understanding of all basic concepts introduced in the course (**comprehension part**). The common principles for developing such tests and usage of technology while teaching the subject are outlined in [15]. We plan on adding a separate section with problems (possibly application problems) of different levels of complexity to research how well both systems prepare students to deal with complicated (real life) problems. This is a very important aspect of most STEM careers (**in-depth application part**). Thus, we clarify the basis for cross countries assessment of Calculus education outcomes. This concludes Stage 1 of our study. In addition to the assessment of the Calculus education outcome, we are adjusting the surveys from [1]. We want to know what students think is essential to success in a course, and we want to ask instructors to evaluate their plans and realization of the course education. We are interested in how administrators provide proper support for students' study.

CONCLUSION AND FUTURE WORK

Our goal is to find ways to improve Calculus education and adjust it to current educational environment using best cross countries practices of Calculus education.

Our work is based on two main sources: study of the best practices of Calculus education by MAA [1] and comparison of education results of high school done by OEAD (PISA) [2].

The novelty of the paper is that we presented cross country evaluation criteria (comp. [1]) that is applicable and significant in University Education (comp. [2]).

The paper reports on the general process and results of Stage 1 of our work, that is the choice of countries and Universities' candidates and analysis of our results. We identified differences in the courses and suggested an assessment that is a mixture of testing "understanding concepts" and deeper questions. In parallel, we outlined socio economic effects and we plan on using them in the adjustment of student, instructor and administration surveys. Our next steps are to run the assessment in chosen countries and with Universities candidates to find appropriate weighting in for comprehension and in-depth application parts. We also plan on running adjusted surveys for students, instructors and administrators.

Finally, we plan on tuning the assessment and surveys and running them on a wide range of participating countries and Universities to identify the best international practices, and thus recommend them for the improvement of Calculus education.

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APPENDIX. TABLE

TABLE IIIa
CALCULUS I RF VS US CONTENT COMPARISON

RF	US
Calculus as a tool for creating math models. Math symbols. Number sets. Functions of one variable.	Mostly introduced in Pre-calculus
Sequences, limits and their properties. Limit of a function, continuity.	Sequences (Calculus II) Limit of a Function
Infinitesimals, properties and types. Special limits. Asymptotic approximations of basic elementary functions and function behavior around special points.	Calculating limits Limits at Infinity; Horizontal Asymptotes
Local and global properties of continuous functions, points of discontinuity.	Continuity
Functions at infinity. Asymptotes. Hyperbolic functions.	Curve Sketching
Derivatives and differentials, their applications. Derivatives and continuity.. Tangent and normal lines	Derivatives & Rate of Change Derivative as a function Linear approximation & differentials
Differentiation rules, tables of derivatives.	Differentiation formulas
Derivatives and differentials of higher order.	Covered in Calculus III
Mean value theorem	Mean Value Theorem
Monotonicity conditions.	Derivatives and graphs
Defining and finding local extrema.	Maximum and minimum values Derivatives and graphs Optimization Problems
Maximum and Minimum Values on an Interval. Applications.	Derivatives and graphs Optimization Problems
L'Hospital's Rule, application to function sketching.	Indeterminate Forms/L'Hospital's Rule (Calculus II)
Taylor and Maclaurin formulas.	(Calculus II)
Taylor formula – finding limits and approximations.	Covered in Calculus III
Concavity and inflection points. Curve sketching.	Curve Sketching
Linear approximation & differentials	Linear approximation & differentials

TABLE IIIb
CALCULUS II RF VS US CONTENT COMPARISON

RF	US
Antiderivatives and Indefinite Integrals. Substitution and Integration by parts. Integration strategies.	Antiderivatives (Calculus I) Indefinite Integrals (Calculus I) Substitution Rule (Calculus I) Integration by Parts Strategies for Integration
Complex numbers/polynomial' zeros. Integrals of rational functions	Complex numbers/Fundamental theorem of Algebra (Precalc) Int of Rat'l Functions/Partial Functions
Integrals of trigonometric and some other irrational functions.	Trigonometric Integrals Trigonometric Substitution Natural Exponential Function Gen Log and Exp Functions Inverse Functions Inverse Trigonometric Functions
Definite integrals, geometric meaning, existence and properties.	The Definite Integral (Calc I) Areas between curves (Calc I)
Mean Value Theorem for Integrals. Derivative of definite integral over variable upper limit.	Mean Value Theorem for Integrals (Calculus I)