Evaluation of acoustical position determination in a classroom scenario

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***Abstract*—This study evaluates whether ultrasound tones can be used to determine position of mobile devices in classrooms. The aim is to use this position to find the seating scheme of students for another study in the area of e-learning. We have conducted several experiments to test mobile devices for their suitability. We have also performed experiments in real scenarios and implemented a software system that for position determination. We measure distances and apply the least-square algorithm to calculate the position of mobile devices.**

***Index Terms*—Acoustical communication, position determination in classrooms, student-centered learning**

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I. INTRODUCTION.

A. Research questions in the area of learning science.

We started this research for a concrete question in another application domain. In our scenario students are sitting in a classroom. Students will annotate lecture material online according to their understanding. Every student in our scenario uses a mobile device to follow electronic lecture material (mainly slides) which are handed out in parallel to a live presentation [1].

In this work, we focus on the position of students within a lecture room. Therefore, we need to know where everybody sits during an entire semester. We developed a system to measure the sitting positions of students. From positions we can derive density (if students tend to sit together), and proximity to the lecturer. As we also measure positions in subsequent lectures we can also derive whether students tend to sit in the same groups regularly. We later determine those attributes through a survey and through the evaluation of comments on lecture material.

In order to classify students we need to find parameters that describe students behavior in regard to interaction with lecture material. Comparable parameters are equality of annotations and annotated content among groups of students, amount of annotations, time between slide shown and annotation (latency), complexity of annotations (either full text or simple marking by clicking a pre-defined button), and post-processing (later changes to annotations).

In our theory, students in a peer group have similar attributes. They will most likely prefer to sit in the vicinity of their respective peers in a classroom. In this paper we present the entire setup to later test the following assumptions:

- Students who tend to regularly sit together with the same other students in close vicinity author equal annotations and work with similar lecture content.
- Students acting in groups (those who repeatably sit together) perform annotations a bit later during the lecture than those students who prefer to sit alone most of the time. We assume that they talk to each other and negotiate the matter before annotating.
- Students who sit in front of the class room produce more content than "back benchers".
- In contradiction to the last item, we can also assume that students sitting in the back write more and more indistinct annotations since they are less focused while the lecture content is presented [2] [3].
- Students regularly sitting in groups spend more time in repetition and post-processing of annotations. We assume that the fact that they sit in groups also means that they reflect lecture content together afterwards.

B. Technical idea for position determination.

Acoustical communication in general is known from several research activities, prototypes and even popular products. The basic idea is to use high frequency transmissions above the upper hearing frequency of around 19 kHz to reach neighboring devices. Typical applications are beacons that indicate their presence via ultrasonic sound and low throughput data transmissions. Even viruses use ultrasonic sound for controlling other nodes nowadays. Ultrasonic communication is very cheap. All known applications use built-in speakers and microphones for this purpose. Typical throughput rates are below 100 bit per second.

Ultrasonic beacons can be used to determine whether a device is within reception range of such a beacon. This is more or less a pre-stage of arbitrary position determination in space. If the beacon signal can be modulated more precise positioning is possible in space when the position of beacons is known.

An obvious idea is to determine the position through measuring distances to devices with known positions. Early examples range back to the 1980s [4]. Signal strengths and round-trip-times are common physical values that depend on distance to a signal source. So, it is obvious again, to utilize either one or both of these values to determine the relative positions of nodes in an acoustical network. We want to renounce beacons, because in our concrete scenario beacons are hard to install. Instead, we determine the relative position of all mobile devices to each other.

C. Outline of this paper.

As this technical paper was highly influenced by a real application scenario we start with an explanation of that scenario in section III. We also present the state of the art of acoustic communication and general positioning methods in the same section. The state of the art is presented for both, the application domain of learning theory in section II and positioning methods in section III. The following section IV starts with a more detailed description of the application (section IV-A). We derive requirements and desired parameters from this scenario and present the idea of how to determine the position of mobile devices in the same section in subsections IV-B and IV-C. After explaining the general idea we need to set up some experiments to show the feasibility and chances for success of our approach. We describe experiments about hardware capabilities and to gather information about real scenarios in section V. We briefly explain the implementation of our idea and evaluate it in section VI.

II. State of the art in learning theory.

For our research, we briefly highlight two aspects. First, the observation of active knowledge processing from the perspective of Generative-Learning [5]. Second, the changes in knowledge processing by students' cooperative work.

First, to observe the way, students actively process information. In previous research, we have built a system to connect students and lecture presentations. Students receive lecture slides as list on their client devices and build individual scripts by assigning badges, bookmarks, notes and hashtags, during the lecture. We then analyze students' interactions in terms of Coding [6], defined as active intervention to received stimuli to generate meaning.

Second, cooperative work is one important design principle in student's self-regulated learning. Especially in heterogeneous small groups, students benefit from different view-angles on concepts, interpretation and solutions. One example is the identification of misconceptions [7]. Furthermore, we assume groups in lectures negotiate about issues on lecture content and tend to align to common remaining problems. This process could emerge to problem based grouping. These groups could enable dedicated problem based teacher activities in lecture scenarios as promoted in [8].

III. State of the art in positioning and acoustic communication on mobile devices.

A. Acoustic communication.

The highest audible frequency for humans is dependent on particular person and age around 20 kHz. Normal frequency ranges reach up to 18 kHz. We've chosen to start with 19 kHz [9], hence, we have a remaining bandwidth of about 3 kHz available for communication.

For first tests we developed a web based prototype using javascript and modern browsers, described in [10]. First tests on ultrasound communication with mobile devices produced the following insights.

The first prototype uses a straight forward frequency hopping of four different signal frequencies, separated by gap frequencies. The prototype achieved a data rate of 185 bit/s. Some reasons behind this are: a slow propagation of acoustic waves with only 340 m/s, resulting in a higher sensitivity to the Doppler-Effect on senders or receivers movement (we assumed a maximum of 0.5 m/s), and higher interference with reflections compared to electromagnetic signals.

Nevertheless, we only send timed identification information. The amount of data send is not very important, so we did not do much effort in optimization of bandwidth yet.

In first test, we did not implement any forward error correction yet. Due to the broadcasting nature of acoustic signal propagation and therefore medium access, a request of correction information from sender is not practicable for our scenario. Nevertheless, in first tests we reached an accuracy of 1 m and better in ranges up to 18 m.

B. Positioning methods.

There are many position determination schemes available for mobile devices. We would like to give a brief overview here. Generally positions can be determined from inside the mobile node or from an outside viewer onto the mobile node. The first is usually referred to as positioning, the second as tracking. In our scenario we cannot clearly distinguish between positioning and tracking since all devices are set up to determine the distance to neighbor nodes.

There are many publications focusing on positioning in sensor networks which in general reflect our scenario [11][12][13] [14].

Positioning means to either determine that a fixed beacon is in the vicinity of a mobile device (yes-no) or to more precisely determine the relative position to at least one fixpoint (distance 1 meters, 4 meters and so on). For the latter we can again use two methods, tri-angulation or tri-lateration [15]. Tri-angulation means to determine the angle between two fixpoints seen from the mobile device. Alternatively one fixpoint can be substituted by a compass direction instead and the angle between compass direction (such as north) and the other fixpoint is measured. This is usually called bearing. Tri-angulation does not work in our scenario and does not work well in general acoustic scenarios because it is very hard to measure the direction of arrival for incoming sound signals with built-in microphones. For Tri-lateration the mobile device measures its distance to at least three fixpoints (in 2-dimensional scenarios) in order to determine its relative position. This can be achieved be measuring any physical value that depends on distance.

Signal strength would be a first choice since the mobile device can easily measure it. But, with general sensitivity and directional characteristic of different microphones and speakers and attenuation of obstacles between devices in mind, we consider this approach unsuitable for our scenario. See section V for a more concrete consideration of this approach.

Measuring signal traveling times between devices is more precise since it is not influenced by the potential measuring errors mentioned above. Furthermore, acoustical signals have a slow traveling speed in air. Hence, errors on measure traveling times have small influence on calculating the distance.

Traditional methods use fixpoints to which all other mobile devices measure their respective distance in order to calculate their positions in space. In most cases distances to more than one fixpoint will be calculated. If the number of fixpoints available is larger than the number of dimensions plus one the calculation is over-determined. Naturally occurring measurement errors will lead to erroneous positions [16]. In an over-determined situation the nature of error can be described more precisely. Error ellipses are used to describe this effect. There are well researched methods to limit the error and to calculate the most accurate position [17].

Sources of error are the replacement of devices between two measurements, general clock errors causing imprecise measuring, for instance through low clock frequencies, and possible physical effects such as reflections and interference. Because we have a beaconless scenario we obviously need to determine the distance between many devices to form a grid structure. The precision of relative positions between devices depends much on the degree of over-determination. Higher degrees are possible if the grid structure is very dense, which is usually the case in our scenario. Section VI illustrates expected density, number of devices, range of transmission etc. more precisely. Concrete implementations are described in [18] and [19].

In order to measure the distance between two devices by measuring signal traveling times either round-trip-time or single leg latency can be used. The latter has the disadvantage that it needs two precisely synchronized clocks (eg. real time clocks with precise synchronization). Measuring the round-trip-time only relies on one clock in the sending device which also receives the answer. The channel between device A and B has to be symmetric in terms of latency, which is the case in our scenario. Letting very special physical effects aside, the sound travels with constant speed. The node that reflects the ping signal has to answer with a constant in-device latency.

The position can be described according to different reference systems. In our scenario it is sufficient to describe the position of devices relative to each other. The orientation of devices is unknown. This means, there is no chance to determine the orientation of the entire grid of devices within the classroom, so, we never know who sits left and who sits right. Furthermore different coordinate systems can be used to describe the position. A 2D Cartesian coordinate system with the teacher's position defined as (0, 0) is adequate for our purposes.

IV. Concept.

In this section we answer the following questions in regard to position determination: What is the application scenario? What are the restrictions? Which assumptions can be made? Which algorithm is used to determine the positions out of the distance measurements? What is the complexity of this algorithm? How can we limit the complexity in case the degree of over-determined calculations is very high?

A. Position determination in a class room.

In our scenario we have a number of students in a lecture hall or classroom, carrying mobile devices connected via IP based networks, preferably WiFi. We use round-trip-time measurements to determine the distance. For this a device B reflects a signal sent by device A. Device A uses half of the time between sending and receiving the reflected signal, to measure signal propagation without synchronizing clocks. Nevertheless, constraining time between receiving and reflecting a signal needs to be constrained to real time.

The signal is modulated and contains a watermark which guarantees that external ultrasonic noise does not influence the measurements. We consider a sequence of 4 bits as sufficient for this watermark. This value can be changed if needed. The longer the watermark is, the more time the measurement takes.

All devices are connected to a traditional computer network for the main purpose of distributing lecture materials to student devices (described in [1]). A central controller steers devices and defines which device sends latency check signals to which other device that reflects the signal via a control unit. The controller collects measurement reports from all devices which have performed the ping-pong-procedure. Figure 1 shows the general architecture.

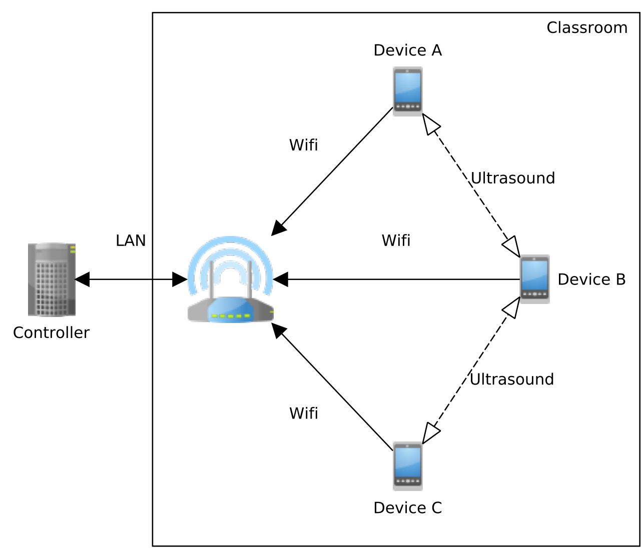

Fig. 1. General architecture with three mobile devices and the controller.

The controller calculates the relative positions of all devices according to those measurement reports. In most cases the calculation will be over-determined (multi-lateration instead of tri-lateration). The controller limits errors in a way that the overall error inside the grid is minimized. We present the algorithm in more details in section IV-C.

From our scenario we can make assumptions on a variety of facts, such as the density of devices in the room (resulting in number of pairs of devices to determine the distance), the characteristics of devices (clocks, hardware outfit), and distribution of devices in the room (relatively evenly spread vs. mainly clustered).

Parallel measurements such as in TDOA approaches are unfeasible since we have no fixpoints with known positions. Hence, all measurements are performed on a device-to-device basis. The density of devices and transmission range of signals have direct influence on the number of possible device-to-device distance measurements. Since the bandwidth of ultrasonic acoustical signals is limited to 3 kHz, the data rate is limited. The more devices are in range of each other the more time would it take to determine the distance to all neighbors. We needed to conduct a study to model density and distribution of devices in space. The results are presented in section V.

The signal range depends on the quality of microphones and speakers. Both have to reach the designated frequencies above 19 kHz. We conducted a study to learn about the capabilities of mobile devices. We also wanted to know if space multiplex can be applied in large classrooms, meaning that two independent pairs of devices measure their distance at the same time in different corners of the room.

Alternatively instead of measuring the round-trip-time between two mobile devices we can also measure the time between controller, device A, device B and back to the controller, whereas the line segments from and to the controller are formed by IP based computer networks. The latency of line segments controller to A and B to controller can be determined by traditional network protocols (ICMP echo request aka. ping) and will be deducted from the overall round trip time, leaving the latency between A and B.

B. Requirements and constraints.

We have the following requirements for the application:

- Time to fix is less than 20 seconds. The purpose of this constraint is to limit the movement of students between measurements. We assume students to be sitting down during the measurements. Subsequent measurements to update positions should not take longer.
- Up to 100 students in classroom. This has influence on the maximum density of devices in the room as well as on the number of pairs for measurements.
- Classroom size up to 20 by 20 m. The longest distance between devices could be around 30 m. We do not need to cover this distance with ultrasonic test tones, since we can stitch measurements together to reach the entire room.

C. Positioning algorithm for our approach.

The controller starts with two devices A and B and determines the distance between them. It maintains a list of all devices in the classroom and arbitrary chooses two devices to measure the distance. In case these two devices are not in range with each other, the controller chooses another device. Optionally the round-trip-time can be measured again with flipped roles to increase precision.

Alternatively the controller can initialize a pre step to find neighbor devices. For this the controller iterates over all mobile devices and requests them to emit a beacon sound. All devices that receive this beacon sound send a report via the IP based computer network to the controller. This pre step is especially helpful in larger classrooms where not all devices are within range of each other.

In the second step the controller chooses a third device C. Distances from A to C and from B to C are determined. Up to this point there is no possibility for an over-determined calculation. The appearance of a fourth node will change that. Instead of tri-lateration with two neighbors for each device (actually number of dimensions minus one) we can calculate the relative positions with multi-lateration.

With four nodes in range of each other there are four triangles in which tri-lateration can be performed. This reflects the questions how many elements a powerset has which contains all subsets with three devices.

This has an exponential growth. The controller can decide to exclude certain devices from measurements and calculation. The controller will ignore those devices in measurements and calculation that have the least potential influence on precision of the position of other devices. The controller excludes devices whose position errors are already estimated very low through other measurements and, hence, do not need to be considered much further when the controller chooses additional pairs of devices for distance measurement. The estimated positioning error depends on the accuracy of distance measurements and the geometry of devices to each other.

Since early astronomic and geodetic research during the end of the 18th century the method of least squares is a standard approach in regression analysis to the approximate solution of over-determined systems [20]. Following this approach, the controller estimates the real distances between devices by finding the minimum of squared residuals.

V. Technical studies and experiments.

In this section we define the experimental setups for a variety of devices. We need to answer the following questions:

- What is the signal range under several conditions and with several devices?
- What is the available and needed clock precision to measure the round-trip-time for several devices?
- How long does it take to perform one measurement and all needed measurements?
- What is the device density in typical classroom situation?

For all experiments we use a set of non moving mobile devices. We collected a list of typical devices owned by students. The following devices where used for the experiments:

- Smart phones (Samsung Galaxy Note 4, Apple iPhone 4 and 6, Samsung Galaxy S3 and S6)
- Tablet computers (Apple iPad 2 and 3, Samsung Galaxy Tab 2)
- Notebook computers (Apple MacBook Pro Mid 2014, Lenovo T61, Apple MacBook Pro 2010, Dell Latitude E5430)

We conducted several experiment in three different lecture rooms, one with 84 seats, one with 20 seats and one with 60 seats. All three rooms are equipped with typical classroom furniture.

A. *Determination of signal range under several conditions and with several devices*

Signal range depends mainly on three factors: volume (power) of transmission, sensitivity of microphones and environmental noise, all together resulting in a signal-to-noise-ratio. Typical speakers in mobile devices have a nonlinear frequency response. In higher-grade devices a digital sound processor compensates some of the non linearity by amplifying certain frequency ranges. The maximum transmission power (volume at the speaker) can be measured under controlled environment in a sound laboratory as well as the microphone sensitivity. In practical tests we evaluate signal ranges under real environments in different situations. We measure noise levels in classrooms with the real equipment and without high precision instruments.

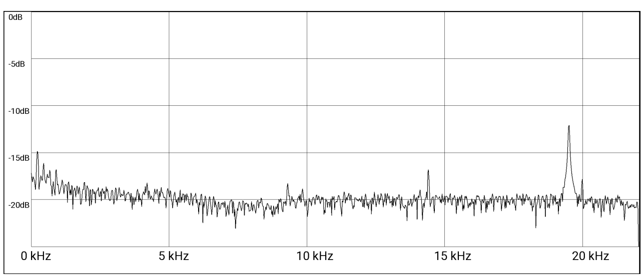

Fig. 2. Sound spectrum with test tone. The test tone is approx. 12 dB louder than the general noise floor in the example (measured with a Samsung Galaxy Note 4 as receiver and a Apple MacBook Pro as sender at a distance of 6 m).

All tested devices are capable of generating ultrasonic tones above 19 kHz. We believe that relatively small built-in speakers are beneficial for this. We also noticed that all tested microphones can handle those frequencies well. Even under noisy conditions ranges of up to 20 m are easily possible.

B. *Determination of clock precision to measure the round-trip-time for several devices*

The following criteria must be met when we implement the software:

- Is there a precise timer to measure round-trip-times (RTT)? Sound travels at 340 m/s. In order to reach an accuracy of 1 m when measuring RTT the timer must have a resolution of less then 7 ms.
- The function that starts and stops the timer must be called often enough to reach the necessary precision.
- The process that listens to incoming ultrasound tones must also be agile enough to avoid jitter and to reach the needed precision. There are two of those listeners. One listens on the mobile device that reflects the incoming signal. The other one listens to the returning signal at the mobile node that measures the distance.

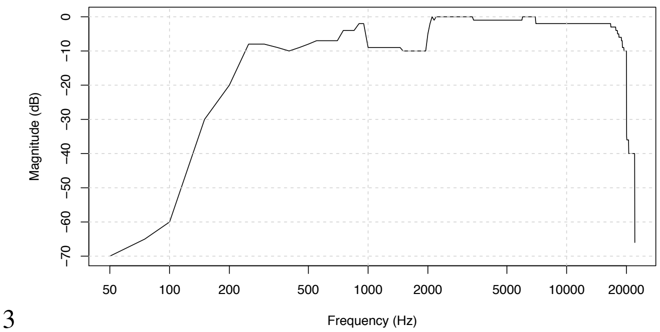

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Fig. 3. Smoothed frequency response of a smart phone (Samsung Galaxy Note 4). There is a steep drop around 20 kHz, but the phone is still sensitive enough for our purposes.

There are several clocks inside a computer. A quartz powers the main clock. On higher software levels such as JavaScript inside a browser there are much less precise clocks. Depending on the software environment there are more or less precise clocks available. A prototype of the lecture content delivery software runs on JavaScript in a web browser. Javascript's "Date.now()" returns the current time in milliseconds.

The precision of callback functions, such as "window.setTimeout()", vary in several milliseconds due to garbage collection or graphical tasks. However, the "Web Audio API" exposes a more accurate timing than the default clock. Events scheduled using the Audio API are performed in a separate thread.

Response times of the application that reflects the incoming signal have to be limited or at least constant and known. For lower systems, one approach is to measure the response time on the client and report it to the controller. The controller then refits the round trip time accordingly.

C. *Determination of the time needed for one measurement.*

We need to transfer a couple of bytes between sender and receiver in order to guarantee that noise is not considered as a valid signal. Since the data rate of our acoustical channel is very low, only 2 measurements per second can be performed on average. This limits the number of possible measurements significantly.

D. *Determination of device density in typical classroom situations.*

Knowing the signal range we can estimate the number of neighbors and, hence, the density of the grid. Since students use seats in lecture rooms we can easily deliver a seating map in many situations be visiting lectures and noting occupied seats. We visited a classroom 20 times in a week and marked the occupied seats. A typical scenario is depicted in Figure 4. There seem to be certain patterns how students prefer to sit with each other. This is not subject of our research. We are interested in distances and density only at this time. For simplicity we assume that every student carries a mobile device and we virtually placed it in the middle of the seat. In order to estimate the density of mobile devices in the classroom we calculated the distances for all pairs of mobile devices. Figure 5 shows the distribution of distances in our example classroom. This distribution seems to be typical according to our observations.

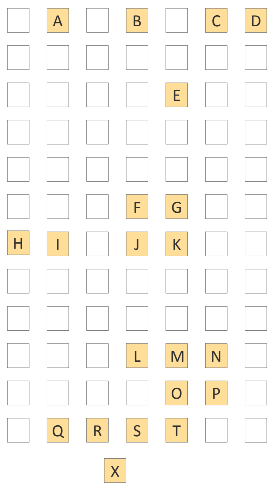

Fig. 4. Example seating map of students with mobile devices in a classroom.

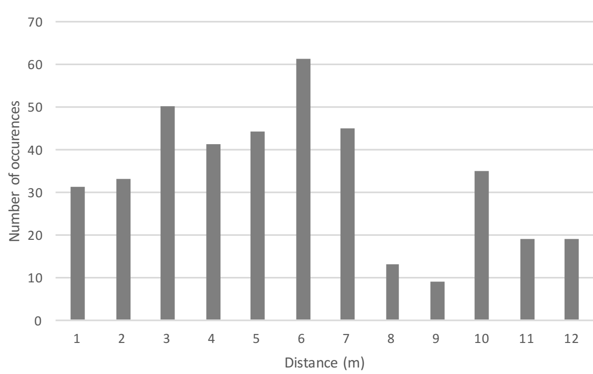

Fig. 5. Distribution of distances between all possible pairs of mobile devices.

Neither maximum distances between devices nor density of mobile devices impose any restrictions for our implementation. The controller is almost free to choose pairs of nodes. Only in some rare cases not all devices are in range of each other.

VI. Implementation and testing.

We have implemented the controller on PC hardware. All test rooms are equipped with WiFi networks. We use the WiFi network to control mobile devices. Mobile devices run a JavaScript application. This application is downloaded with the presentation program for lecture materials.

The controller is aware of the number of mobile devices in the room. It decides if all possible pairs of devices are used for distance measurements based on the constraint of time to first fix, the density and estimated distribution of devices in the room and the degree of over-determination. The least-square algorithm is able to determine the expected positioning error for each mobile devices. Hence, the controller can request other devices with better position estimations to measure the distance to those devices with higher errors.

A. *Simulation.*

In order to test the algorithm we set up a simulation based on a real scenario. We changed the distances by up to 10 % according to a simple error model. This model reflects typical sensor errors and measurement discrepancies. The controller determined the position of devices with the least-square algorithm. The average error was well below one meter. We could determine the correct seats for all students.

B. *Evaluation in real environment.*

We were able to test the entire system in a small scenario with 10 students in a classroom with 20 seats. We asked the students to take seats in 10 different configurations. We calculated their positions. In all cases we were able to locate all students according to their seats. We did not add much contribution to the research area of positioning as we already found a large variety of publications. We did not find any implementation of acoustic positioning in over-determined scenarios.

We believe our approach can deliver position information that is very helpful for our application scenario. In this other research we use the seating map with students to evaluate the thesis that students that share interests and behavior prefer to sit next to each other. This seems to be obvious but no research has been performed to proof this thesis before. We enabled further research to answer this question in a less intuitive and more scientific way.

VII. First answers to research questions.

As described in section I-A, we have a teacher who transmits sequences of information to the audience and an audience consisting of students using a system to observe students' Coding in terms of Generative Learning described in section II.

While our first approach focused on changes in coding through student's differences such as previous knowledge and processing strategies, we now take social grouping and cooperation between students into account. The system described in this work enables grouping of a lecture's audience by proximity. Furthermore, we assume social dependencies by reoccurring groupings of students. Connected with the observation system described in section II we are able to correlate students' active processing, student's position within classroom as well as the grouping of students by density.

First insights on the questions raised in section I-A:

- We found students who tend to sit together regularly do not significantly correlate their notes with each other. We assume students do not negotiate processing of information as expected but benefit from later sharing annotations from different perspectives.
- Due to a less intense negotiation of content processing we did not identify significant delays on annotating lecture information in student groups. However, we noticed groups of students who sit in front of the class room perform more annotations than "back benchers". These annotations are more often revised during lecture related to discussions with the lecturer.
- In contradiction to the last item students sitting in the back write more indistinct annotations since their discussion and revision process is less intense.
- Finally we noticed that groups of students more intensively share annotation afterwards. This leads to an improvement of notes with different perspectives, and to different demands. Furthermore, groups taking part in intense discussions benefit from a higher quality in revised annotations.

More detailed analyses in the application domain will be published as the become available after larger experiments.

Open science repository.

Measurement data, seating maps, controller and client applications can be downloaded at our open science repository at https://opsci.informatik.uni-rostock.de/.

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